Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.7-d-trig-^m-a+b-c-sec-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [470]. This is test number [126].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.57 (468)	% 0.43 (2)
Mathematica	% 90.21 (424)	% 9.79 (46)
Maple	% 91.7 (431)	% 8.3 (39)
Maxima	% 29.79 (140)	% 70.21 (330)
Fricas	% 85.32 (401)	% 14.68 (69)
Sympy	% 4.04 (19)	% 95.96 (451)
Giac	% 51.49 (242)	% 48.51 (228)

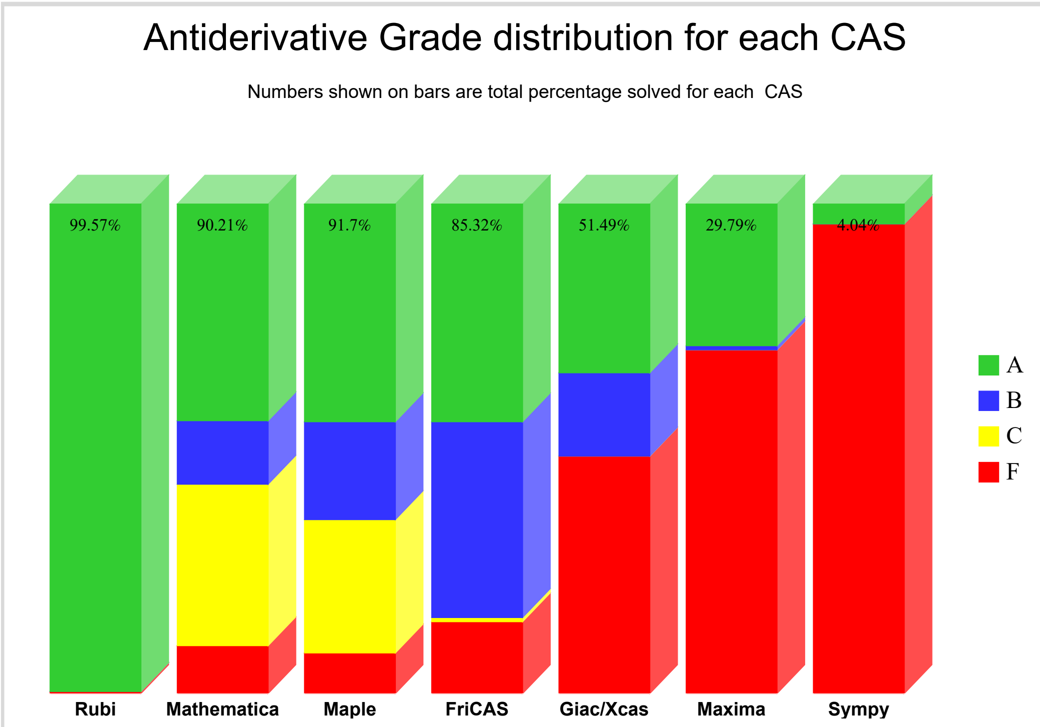
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

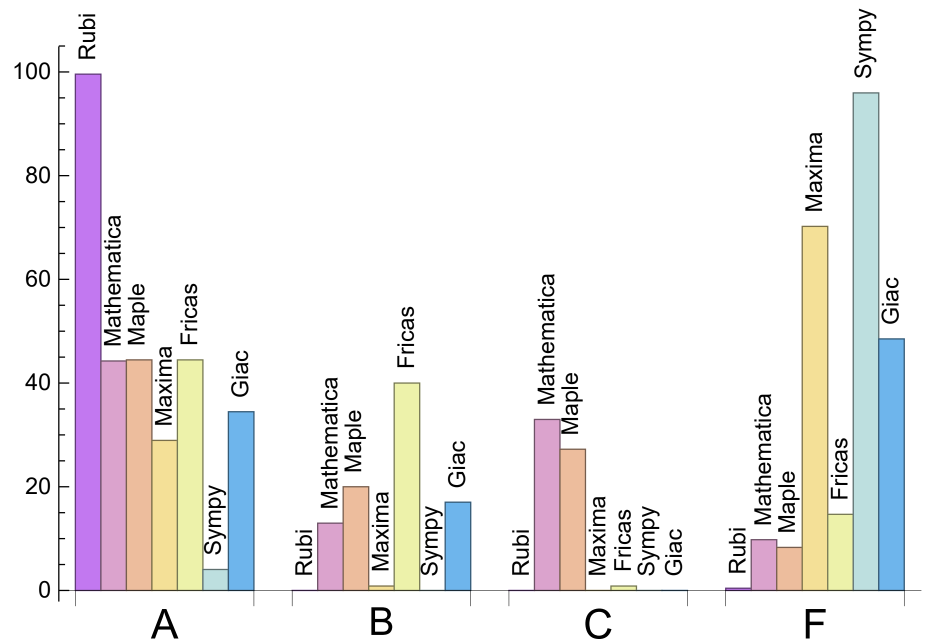
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.57	0.	0.	0.43
Mathematica	44.26	12.98	32.98	9.79
Maple	44.47	20.	27.23	8.3
Maxima	28.94	0.85	0.	70.21
Fricas	44.47	40.	0.85	14.68
Sympy	4.04	0.	0.	95.96
Giac	34.47	17.02	0.	48.51

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	127.59	1.	105.	1.
Mathematica	4.07	456.34	3.89	172.	1.45
Maple	0.35	2747.22	15.29	260.	1.89
Maxima	1.1	110.93	1.73	84.	1.49
Fricas	3.07	1404.58	10.63	941.	8.55
Sympy	14.5	79.05	1.76	66.	1.34
Giac	1.47	291.31	3.23	200.5	1.91

1.4 list of integrals that has no closed form antiderivative

{462, 466, 467, 468, 469, 470}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {34, 47, 48, 49, 53, 55, 60, 61, 62, 63, 64, 65, 66, 78, 79, 125, 126, 128, 132, 133, 136, 137, 138, 139, 140, 180, 181, 193, 196, 199, 205, 208, 209, 211, 215, 217, 218, 219, 240, 254, 264, 268, 269, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 358, 360, 361, 362, 370, 371, 372, 373, 374, 375, 380, 392, 393, 394, 416, 428, 430, 437, 441, 447, 448, 449, 450, 451, 463, 464}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

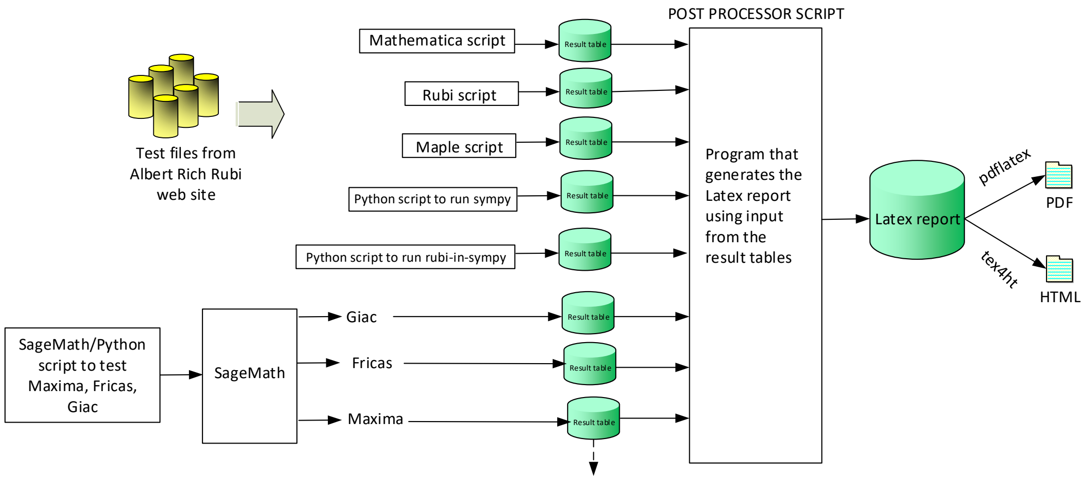
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470 }

B grade: { }

C grade: { }

F grade: { 132, 298}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 23, 26, 67, 68, 69, 70, 71, 72, 80, 81, 83, 84, 85, 87, 93, 94, 95, 96, 97, 99, 100, 101, 103, 104, 105, 107, 108, 109, 112, 113, 114, 116, 117, 118, 119, 120, 121, 129, 130, 131, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 182, 183, 184, 185, 188, 190, 191, 192, 194, 195, 197, 198, 200, 203, 204, 206, 207, 210, 212, 216, 220, 221, 222, 223, 224, 225, 226, 227, 231, 238, 239, 245, 252, 253, 259, 267, 271, 278, 283, 290, 304, 305, 306, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 382, 383, 386, 387, 388, 395, 396, 408, 409, 412, 413, 414, 421, 422, 425, 426, 427, 438, 439, 440, 442, 443, 444, 445, 446, 452, 453, 454, 455, 456, 459, 462, 463, 464, 465, 466, 467, 468, 469, 470 }

B grade: { 5, 6, 7, 18, 19, 21, 24, 25, 27, 102, 106, 115, 125, 126, 127, 132, 136, 137, 138, 139, 140, 174, 178, 179, 236, 256, 265, 266, 279, 291, 297, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 330, 331, 332, 333, 334, 335, 336, 378, 411, 423, 424, 434, 435, 436, 441, 447, 448, 449, 450, 451 }

C grade: { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 77, 78, 79, 82, 88, 89, 90, 91, 92, 98, 110, 111, 122, 123, 124, 128, 148, 149, 150, 151, 180, 181, 186, 187, 189, 193, 196, 199, 201, 202, 205, 208, 209, 211, 213, 214, 215, 217, 218, 219, 232, 234, 235, 240, 246, 247, 248, 249, 250, 251, 254, 255, 260, 263, 264, 268, 269, 272, 276, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 321, 322, 323, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 391, 392, 393, 394, 397, 398, 399, 400, 401, 416, 417, 428, 429, 430, 437, 457, 458, 460, 461 }

F grade: { 73, 74, 76, 86, 228, 229, 230, 233, 237, 241, 242, 243, 244, 257, 258, 261, 262, 270, 273, 274, 275, 285, 286, 287, 288, 376, 377, 379, 381, 385, 389, 390, 402, 403, 404, 405, 406, 407, 410, 415, 418, 419, 420, 431, 432, 433}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 64, 66, 69, 82, 93, 94, 95, 103, 104, 105, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 214, 216, 217, 218, 220, 221, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 358, 359, 360, 361, 362, 363, 364, 365, 374, 375, 378, 391, 404, 417, 430, 454, 455, 456, 457, 458, 459, 460, 462, 466, 467, 468, 469, 470 }

B grade: { 5, 6, 18, 19, 20, 28, 32, 33, 34, 35, 46, 47, 57, 58, 59, 60, 63, 65, 67, 68, 70, 71, 72, 80, 81, 83, 84, 85, 96, 97, 98, 106, 107, 109, 110, 111, 122, 123, 124, 192, 212, 213, 215, 219, 222, 223, 224, 225, 226, 227, 341, 342, 343, 344, 354, 355, 356, 357, 366, 367, 368, 369, 370, 371, 372, 373, 376, 377, 379, 380, 381, 389, 390, 392, 393, 394, 402, 403, 405, 406, 407, 415, 416, 418, 419, 420, 428, 429, 431, 432, 433, 452, 453, 461 }

C grade: { 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 99, 100, 101, 102, 112, 113, 114, 115, 125, 126, 127, 128, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 382, 383, 384, 385, 386, 387, 388, 395, 396, 397, 398, 399, 400, 401, 408, 409, 410, 411, 412, 413, 414, 421, 422, 423, 424, 425, 426, 427, 434, 435, 436, 437, 438, 439, 440 }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 93, 94, 95, 106, 107, 108, 119, 120, 121, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 220, 221, 222, 223, 224, 225, 226, 227, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 363, 364, 365, 366, 452, 453, 454, 455, 456, 459, 462, 466, 467, 468, 469, 470

B grade: { 297, 355, 367, 368 }

C grade: { }

F grade: { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 457, 458, 460, 461, 463, 464, 465 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 41, 42, 43, 47, 48, 49, 54, 55, 56, 60, 61, 67, 68, 69, 70, 71, 73, 74, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 112, 113, 116, 117, 118, 119, 120, 121, 125, 126, 129, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 198, 203, 204, 205, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 239, 240, 247, 248, 249, 253, 263, 268, 269, 276, 281, 282, 290, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 345, 346, 350, 351, 352, 353, 363, 364, 365, 382, 395, 396, 452, 453, 454, 455, 456, 459, 462, 466, 467, 468, 469, 470 }

B grade: { 5, 6, 7, 11, 19, 20, 33, 39, 40, 44, 45, 46, 50, 51, 52, 53, 57, 58, 59, 62, 63, 64, 65, 66, 72, 75, 76, 77, 78, 79, 88, 89, 101, 102, 109, 110, 111, 114, 115, 122, 123, 124, 127, 128, 130, 131, 149, 150, 151, 161, 186, 187, 188, 197, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 236, 237, 238, 250, 251, 252, 254, 255, 256, 264, 265, 266, 267, 277, 278, 279, 280, 289, 291, 292, 293, 294, 295, 296, 297, 320, 322, 323, 335, 336, 342, 343, 344, 347, 348, 349, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade: { 457, 458, 460, 461 }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 228, 229, 230, 231, 232, 233, 241, 242, 243, 244, 245, 246, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

2.1.6 Sympy

A grade: { 162, 311, 312, 313, 317, 318, 319, 324, 325, 326, 339, 417, 430, 452, 453, 454, 459, 468, 469 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 314, 315, 316, 320, 321, 322, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 455, 456, 457, 458, 460, 461, 462, 463, 464, 465, 466, 467, 470 }

2.1.7 Giac

A grade: { 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 42, 43, 47, 48, 49, 50, 51, 52, 53, 55, 56, 60, 61, 62, 63, 64, 65, 66, 68, 69, 81, 82, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 238, 239, 317, 318, 319, 320, 330, 331, 332, 333, 334, 343, 344, 345, 346, 347, 348, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 455, 462, 466, 467, 468, 469, 470 }

B grade: { 1, 2, 5, 6, 7, 15, 18, 19, 20, 28, 32, 33, 41, 44, 45, 46, 54, 57, 58, 59, 95, 148, 149, 150, 151, 222, 223, 225, 226, 227, 278, 290, 291, 311, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 335, 336, 337, 338, 339, 340, 341, 342, 349, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 378, 403, 404, 417, 430, 452, 453, 454, 456, 458, 459 }

C grade: { }

F grade: { 67, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 457, 460, 461, 463, 464, 465 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	120	102	99	186	0	389
normalized size	1	1.	1.45	1.23	1.19	2.24	0.	4.69
time (sec)	N/A	0.061	0.084	0.047	1.003	0.928	0.	1.199

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	88	82	78	150	0	288
normalized size	1	1.	1.33	1.24	1.18	2.27	0.	4.36
time (sec)	N/A	0.051	0.045	0.043	1.013	0.839	0.	1.221

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	62	54	100	0	82
normalized size	1	1.	1.2	1.41	1.23	2.27	0.	1.86
time (sec)	N/A	0.039	0.032	0.041	0.976	1.019	0.	1.274

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	25	34	57	0	38
normalized size	1	1.	1.46	1.04	1.42	2.38	0.	1.58
time (sec)	N/A	0.02	0.018	0.019	1.007	0.901	0.	1.298

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	84	57	59	178	0	82
normalized size	1	1.	3.11	2.11	2.19	6.59	0.	3.04
time (sec)	N/A	0.031	0.04	0.04	1.026	0.837	0.	1.343

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	236	100	103	325	0	278
normalized size	1	1.	4.45	1.89	1.94	6.13	0.	5.25
time (sec)	N/A	0.052	0.388	0.05	1.013	0.815	0.	1.266

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	198	142	136	481	0	379
normalized size	1	1.	2.44	1.75	1.68	5.94	0.	4.68
time (sec)	N/A	0.075	1.843	0.049	1.017	0.772	0.	1.238

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	78	112	150	220	0	153
normalized size	1	1.	0.8	1.14	1.53	2.24	0.	1.56
time (sec)	N/A	0.104	0.308	0.046	1.481	0.502	0.	1.243

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	54	92	111	167	0	120
normalized size	1	1.	0.77	1.31	1.59	2.39	0.	1.71
time (sec)	N/A	0.064	0.303	0.046	1.505	0.49	0.	1.152

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	46	63	123	0	69
normalized size	1	1.	1.29	1.1	1.5	2.93	0.	1.64
time (sec)	N/A	0.044	0.096	0.038	1.487	0.478	0.	1.159

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	76	0	22
normalized size	1	1.	1.	1.07	1.33	5.07	0.	1.47
time (sec)	N/A	0.012	0.003	0.015	0.977	0.463	0.	1.27

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	36	43	35	85	0	38
normalized size	1	1.	1.38	1.65	1.35	3.27	0.	1.46
time (sec)	N/A	0.034	0.064	0.039	0.967	0.454	0.	1.312

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	73	58	163	0	73
normalized size	1	1.	1.83	1.59	1.26	3.54	0.	1.59
time (sec)	N/A	0.046	0.046	0.048	1.003	0.457	0.	1.299

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	128	101	86	236	0	111
normalized size	1	1.	1.88	1.49	1.26	3.47	0.	1.63
time (sec)	N/A	0.056	0.046	0.053	1.013	0.47	0.	1.339

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	118	155	120	217	0	602
normalized size	1	1.	1.22	1.6	1.24	2.24	0.	6.21
time (sec)	N/A	0.09	0.633	0.053	1.005	0.512	0.	1.303

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	83	125	90	158	0	131
normalized size	1	1.	1.15	1.74	1.25	2.19	0.	1.82
time (sec)	N/A	0.072	0.465	0.053	1.013	0.496	0.	1.337

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	75	42	57	104	0	63
normalized size	1	1.	1.63	0.91	1.24	2.26	0.	1.37
time (sec)	N/A	0.035	0.114	0.028	0.976	0.485	0.	1.274

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	108	117	111	271	0	244
normalized size	1	1.	2.08	2.25	2.13	5.21	0.	4.69
time (sec)	N/A	0.066	0.532	0.05	1.021	0.517	0.	1.293

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	1021	195	170	471	0	491
normalized size	1	1.	9.82	1.88	1.63	4.53	0.	4.72
time (sec)	N/A	0.11	6.575	0.063	1.047	0.541	0.	1.292

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	218	264	223	713	0	713
normalized size	1	1.	1.55	1.87	1.58	5.06	0.	5.06
time (sec)	N/A	0.138	1.897	0.069	1.036	0.537	0.	1.313

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	499	199	221	320	0	266
normalized size	1	1.	3.37	1.34	1.49	2.16	0.	1.8
time (sec)	N/A	0.177	1.505	0.056	1.559	0.55	0.	1.266

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	153	123	162	257	0	178
normalized size	1	1.	1.34	1.08	1.42	2.25	0.	1.56
time (sec)	N/A	0.121	1.721	0.054	1.53	0.522	0.	1.299

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	126	71	90	189	0	97
normalized size	1	1.	1.73	0.97	1.23	2.59	0.	1.33
time (sec)	N/A	0.099	0.989	0.049	1.485	0.504	0.	1.219

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	59	142	0	72
normalized size	1	1.	2.65	1.2	1.48	3.55	0.	1.8
time (sec)	N/A	0.029	0.358	0.032	1.013	0.487	0.	1.155

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	109	96	73	163	0	86
normalized size	1	1.	2.18	1.92	1.46	3.26	0.	1.72
time (sec)	N/A	0.058	1.019	0.053	1.014	0.483	0.	1.264

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	151	144	108	240	0	142
normalized size	1	1.	1.99	1.89	1.42	3.16	0.	1.87
time (sec)	N/A	0.076	1.307	0.061	0.988	0.478	0.	1.277

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	353	190	144	346	0	204
normalized size	1	1.	3.43	1.84	1.4	3.36	0.	1.98
time (sec)	N/A	0.097	1.772	0.061	1.	0.497	0.	1.169

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	425	183	0	555	0	504
normalized size	1	1.	4.34	1.87	0.	5.66	0.	5.14
time (sec)	N/A	0.105	3.518	0.066	0.	0.57	0.	1.155

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	376	103	0	381	0	120
normalized size	1	1.	5.3	1.45	0.	5.37	0.	1.69
time (sec)	N/A	0.084	1.626	0.064	0.	0.552	0.	1.187

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	329	46	0	265	0	59
normalized size	1	1.	7.	0.98	0.	5.64	0.	1.26
time (sec)	N/A	0.041	0.957	0.033	0.	0.534	0.	1.214

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	239	76	0	410	0	116
normalized size	1	1.	4.35	1.38	0.	7.45	0.	2.11
time (sec)	N/A	0.071	0.624	0.066	0.	0.565	0.	1.148

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	371	158	0	848	0	294
normalized size	1	1.	4.31	1.84	0.	9.86	0.	3.42
time (sec)	N/A	0.1	1.866	0.081	0.	0.675	0.	1.222

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	549	296	0	1643	0	551
normalized size	1	1.	4.26	2.29	0.	12.74	0.	4.27
time (sec)	N/A	0.154	5.598	0.084	0.	0.8	0.	1.282

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	357	460	0	1019	0	338
normalized size	1	1.	2.15	2.77	0.	6.14	0.	2.04
time (sec)	N/A	0.335	4.349	0.102	0.	0.625	0.	1.279

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	303	260	0	792	0	216
normalized size	1	1.	2.59	2.22	0.	6.77	0.	1.85
time (sec)	N/A	0.169	2.138	0.097	0.	0.592	0.	1.289

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	245	124	0	625	0	131
normalized size	1	1.	3.22	1.63	0.	8.22	0.	1.72
time (sec)	N/A	0.098	0.896	0.085	0.	0.563	0.	1.237

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	48	0	544	0	92
normalized size	1	1.	4.04	1.07	0.	12.09	0.	2.04
time (sec)	N/A	0.044	0.293	0.07	0.	0.557	0.	1.257

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	189	54	0	655	0	100
normalized size	1	1.	3.5	1.	0.	12.13	0.	1.85
time (sec)	N/A	0.073	0.67	0.084	0.	0.556	0.	1.274

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	226	74	0	954	0	144
normalized size	1	1.	2.97	0.97	0.	12.55	0.	1.89
time (sec)	N/A	0.094	2.137	0.097	0.	0.573	0.	1.304

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	318	116	0	1397	0	243
normalized size	1	1.	3.03	1.1	0.	13.3	0.	2.31
time (sec)	N/A	0.139	1.829	0.115	0.	0.606	0.	1.323

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	454	276	0	941	0	736
normalized size	1	1.	2.82	1.71	0.	5.84	0.	4.57
time (sec)	N/A	0.178	6.823	0.097	0.	0.65	0.	1.169

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	403	165	0	684	0	193
normalized size	1	1.	3.54	1.45	0.	6.	0.	1.69
time (sec)	N/A	0.112	4.214	0.082	0.	0.589	0.	1.172

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	393	75	0	478	0	103
normalized size	1	1.	4.68	0.89	0.	5.69	0.	1.23
time (sec)	N/A	0.051	3.876	0.042	0.	0.553	0.	1.209

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	384	172	0	932	0	385
normalized size	1	1.	3.88	1.74	0.	9.41	0.	3.89
time (sec)	N/A	0.107	1.138	0.085	0.	0.727	0.	1.165

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	468	250	0	1613	0	787
normalized size	1	1.	3.18	1.7	0.	10.97	0.	5.35
time (sec)	N/A	0.173	1.903	0.11	0.	0.822	0.	1.241

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	450	390	0	2747	0	950
normalized size	1	1.	2.28	1.98	0.	13.94	0.	4.82
time (sec)	N/A	0.246	2.325	0.119	0.	0.974	0.	1.327

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	2738	555	0	1577	0	420
normalized size	1	1.	10.25	2.08	0.	5.91	0.	1.57
time (sec)	N/A	0.426	24.021	0.112	0.	0.74	0.	1.262

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	1105	323	0	1211	0	275
normalized size	1	1.	5.79	1.69	0.	6.34	0.	1.44
time (sec)	N/A	0.255	14.277	0.102	0.	0.664	0.	1.25

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	825	155	0	1046	0	213
normalized size	1	1.	6.35	1.19	0.	8.05	0.	1.64
time (sec)	N/A	0.169	11.834	0.098	0.	0.627	0.	1.268

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	240	127	0	1027	0	161
normalized size	1	1.	2.61	1.38	0.	11.16	0.	1.75
time (sec)	N/A	0.086	2.018	0.083	0.	0.608	0.	1.123

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	242	86	0	975	0	180
normalized size	1	1.	2.66	0.95	0.	10.71	0.	1.98
time (sec)	N/A	0.085	2.326	0.11	0.	0.6	0.	1.27

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	637	160	0	1534	0	259
normalized size	1	1.	5.18	1.3	0.	12.47	0.	2.11
time (sec)	N/A	0.175	6.768	0.106	0.	0.688	0.	1.289

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	777	189	0	2260	0	355
normalized size	1	1.	4.13	1.01	0.	12.02	0.	1.89
time (sec)	N/A	0.264	3.538	0.118	0.	0.746	0.	1.311

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	1641	374	0	1361	0	1130
normalized size	1	1.	7.67	1.75	0.	6.36	0.	5.28
time (sec)	N/A	0.255	12.255	0.098	0.	0.741	0.	1.244

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	1392	231	0	1007	0	247
normalized size	1	1.	9.04	1.5	0.	6.54	0.	1.6
time (sec)	N/A	0.188	11.69	0.093	0.	0.679	0.	1.21

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	656	108	0	713	0	131
normalized size	1	1.	5.66	0.93	0.	6.15	0.	1.13
time (sec)	N/A	0.068	7.149	0.046	0.	0.603	0.	1.331

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	447	352	0	1773	0	844
normalized size	1	1.	2.9	2.29	0.	11.51	0.	5.48
time (sec)	N/A	0.196	2.619	0.099	0.	1.01	0.	1.277

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	532	430	0	2946	0	1068
normalized size	1	1.	2.5	2.02	0.	13.83	0.	5.01
time (sec)	N/A	0.312	3.651	0.114	0.	1.165	0.	1.492

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	549	567	0	4163	0	1889
normalized size	1	1.	2.14	2.21	0.	16.2	0.	7.35
time (sec)	N/A	0.366	5.231	0.123	0.	1.365	0.	1.392

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	314	314	1639	689	0	2167	0	502
normalized size	1	1.	5.22	2.19	0.	6.9	0.	1.6
time (sec)	N/A	0.505	19.802	0.126	0.	0.889	0.	1.325

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	238	238	2469	423	0	1859	0	439
normalized size	1	1.	10.37	1.78	0.	7.81	0.	1.84
time (sec)	N/A	0.34	25.858	0.117	0.	0.796	0.	1.369

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	1915	314	0	1872	0	296
normalized size	1	1.	10.41	1.71	0.	10.17	0.	1.61
time (sec)	N/A	0.278	19.24	0.101	0.	0.754	0.	1.296

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	332	321	0	1854	0	277
normalized size	1	1.	2.31	2.23	0.	12.88	0.	1.92
time (sec)	N/A	0.188	5.876	0.083	0.	0.713	0.	1.133

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	987	157	0	1430	0	248
normalized size	1	1.	7.96	1.27	0.	11.53	0.	2.
time (sec)	N/A	0.111	6.794	0.104	0.	0.702	0.	1.288

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	994	306	0	2288	0	371
normalized size	1	1.	6.06	1.87	0.	13.95	0.	2.26
time (sec)	N/A	0.25	4.429	0.122	0.	0.788	0.	1.314

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	479	411	0	3312	0	516
normalized size	1	1.	1.98	1.7	0.	13.69	0.	2.13
time (sec)	N/A	0.37	6.067	0.136	0.	0.898	0.	1.292

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	152	1840	0	734	0	0
normalized size	1	1.	1.09	13.24	0.	5.28	0.	0.
time (sec)	N/A	0.147	0.889	0.745	0.	1.189	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	120	1525	0	578	0	120
normalized size	1	1.	1.2	15.25	0.	5.78	0.	1.2
time (sec)	N/A	0.094	0.379	0.384	0.	1.121	0.	1.319

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	98	93	0	468	0	78
normalized size	1	1.	1.48	1.41	0.	7.09	0.	1.18
time (sec)	N/A	0.053	0.15	0.077	0.	1.079	0.	1.259

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	119	688	0	1281	0	0
normalized size	1	1.	1.45	8.39	0.	15.62	0.	0.
time (sec)	N/A	0.09	0.131	0.401	0.	0.778	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	163	3015	0	2234	0	0
normalized size	1	1.	1.31	24.31	0.	18.02	0.	0.
time (sec)	N/A	0.136	0.44	0.447	0.	1.051	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	198	9758	0	3656	0	0
normalized size	1	1.	1.08	53.32	0.	19.98	0.	0.
time (sec)	N/A	0.22	1.407	0.472	0.	2.166	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	0	2665	0	4146	0	0
normalized size	1	1.	0.	11.1	0.	17.27	0.	0.
time (sec)	N/A	0.384	9.038	0.752	0.	9.966	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	0	1940	0	3776	0	0
normalized size	1	1.	0.	10.72	0.	20.86	0.	0.
time (sec)	N/A	0.221	5.425	0.411	0.	3.474	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	432	1290	0	3478	0	0
normalized size	1	1.	3.51	10.49	0.	28.28	0.	0.
time (sec)	N/A	0.131	5.752	0.321	0.	1.671	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-2)	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	589	0	2984	0	0
normalized size	1	1.	0.	7.46	0.	37.77	0.	0.
time (sec)	N/A	0.051	1.81	0.511	0.	1.189	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	1003	0	780	0	0
normalized size	1	1.	0.9	14.75	0.	11.47	0.	0.
time (sec)	N/A	0.081	0.227	0.499	0.	0.701	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	285	3847	0	1092	0	0
normalized size	1	1.	2.71	36.64	0.	10.4	0.	0.
time (sec)	N/A	0.097	3.876	0.415	0.	1.275	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	941	8587	0	1639	0	0
normalized size	1	1.	6.32	57.63	0.	11.	0.	0.
time (sec)	N/A	0.14	9.56	0.58	0.	4.453	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	188	2537	0	876	0	0
normalized size	1	1.	0.96	12.94	0.	4.47	0.	0.
time (sec)	N/A	0.18	1.478	0.624	0.	1.724	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	164	1913	0	710	0	219
normalized size	1	1.	1.01	11.81	0.	4.38	0.	1.35
time (sec)	N/A	0.142	0.791	0.42	0.	1.582	0.	1.414

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	121	0	591	0	124
normalized size	1	1.	0.73	1.21	0.	5.91	0.	1.24
time (sec)	N/A	0.07	0.665	0.059	0.	1.184	0.	1.305

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	171	2563	0	1867	0	0
normalized size	1	1.	1.4	21.01	0.	15.3	0.	0.
time (sec)	N/A	0.136	0.59	0.329	0.	1.096	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	202	5178	0	2477	0	0
normalized size	1	1.	1.25	32.16	0.	15.39	0.	0.
time (sec)	N/A	0.203	1.499	0.378	0.	1.125	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	262	10199	0	3794	0	0
normalized size	1	1.	1.2	46.78	0.	17.4	0.	0.
time (sec)	N/A	0.336	3.573	0.428	0.	1.323	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	0	3067	0	4521	0	0
normalized size	1	1.	0.	10.29	0.	15.17	0.	0.
time (sec)	N/A	0.474	10.573	0.834	0.	34.262	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	211	2309	0	4078	0	0
normalized size	1	1.	0.97	10.64	0.	18.79	0.	0.
time (sec)	N/A	0.341	5.642	0.541	0.	11.429	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	493	1583	0	3771	0	0
normalized size	1	1.	3.06	9.83	0.	23.42	0.	0.
time (sec)	N/A	0.198	5.889	0.362	0.	3.922	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1556	0	3602	0	0
normalized size	1	1.	4.47	13.19	0.	30.53	0.	0.
time (sec)	N/A	0.096	5.542	0.309	0.	1.767	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	64	2032	0	932	0	0
normalized size	1	1.	0.61	19.35	0.	8.88	0.	0.
time (sec)	N/A	0.105	0.221	0.345	0.	1.308	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	369	3925	0	1180	0	0
normalized size	1	1.	2.15	22.82	0.	6.86	0.	0.
time (sec)	N/A	0.153	8.626	0.396	0.	4.958	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	512	8726	0	1732	0	0
normalized size	1	1.	2.45	41.75	0.	8.29	0.	0.
time (sec)	N/A	0.205	10.446	0.951	0.	20.366	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	93	105	219	213	0	0
normalized size	1	1.	0.76	0.85	1.78	1.73	0.	0.
time (sec)	N/A	0.139	0.98	0.427	1.211	0.573	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	69	112	139	0	0
normalized size	1	1.	0.86	0.93	1.51	1.88	0.	0.
time (sec)	N/A	0.091	0.295	0.344	1.024	0.551	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	48	31	38	88	0	81
normalized size	1	1.	1.6	1.03	1.27	2.93	0.	2.7
time (sec)	N/A	0.042	0.114	0.076	0.995	0.511	0.	1.663

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	86	280	0	360	0	0
normalized size	1	1.	2.	6.51	0.	8.37	0.	0.
time (sec)	N/A	0.068	0.106	0.379	0.	0.698	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	140	2199	0	765	0	0
normalized size	1	1.	1.61	25.28	0.	8.79	0.	0.
time (sec)	N/A	0.11	1.033	0.414	0.	0.794	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	78	4983	0	1185	0	0
normalized size	1	1.	0.57	36.11	0.	8.59	0.	0.
time (sec)	N/A	0.165	0.182	0.412	0.	0.919	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	163	2425	0	1543	0	0
normalized size	1	1.	0.84	12.56	0.	7.99	0.	0.
time (sec)	N/A	0.278	1.647	0.62	0.	4.389	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	145	1701	0	1362	0	0
normalized size	1	1.	1.07	12.6	0.	10.09	0.	0.
time (sec)	N/A	0.152	0.606	0.425	0.	1.617	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	125	1055	0	1218	0	0
normalized size	1	1.	1.47	12.41	0.	14.33	0.	0.
time (sec)	N/A	0.109	0.275	0.355	0.	0.931	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	0	976	0	0
normalized size	1	1.	2.23	9.74	0.	25.03	0.	0.
time (sec)	N/A	0.029	0.067	0.409	0.	0.762	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	55	48	0	113	0	0
normalized size	1	1.	1.67	1.45	0.	3.42	0.	0.
time (sec)	N/A	0.071	0.113	0.322	0.	0.529	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	74	66	0	235	0	0
normalized size	1	1.	0.95	0.85	0.	3.01	0.	0.
time (sec)	N/A	0.097	0.209	0.343	0.	0.729	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	100	101	0	409	0	0
normalized size	1	1.	0.76	0.77	0.	3.1	0.	0.
time (sec)	N/A	0.14	0.351	0.383	0.	1.82	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	432	35190	338	324	0	0
normalized size	1	1.	2.53	205.79	1.98	1.89	0.	0.
time (sec)	N/A	0.187	7.611	2.064	1.045	0.864	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	93	12782	190	228	0	0
normalized size	1	1.	0.82	112.12	1.67	2.	0.	0.
time (sec)	N/A	0.117	3.687	0.89	1.02	0.723	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	64	59	77	158	0	0
normalized size	1	1.	1.03	0.95	1.24	2.55	0.	0.
time (sec)	N/A	0.056	1.421	0.052	1.003	0.607	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	113	1094	0	821	0	0
normalized size	1	1.	1.41	13.68	0.	10.26	0.	0.
time (sec)	N/A	0.092	0.838	0.355	0.	0.847	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	97	3289	0	1277	0	0
normalized size	1	1.	0.77	26.1	0.	10.13	0.	0.
time (sec)	N/A	0.153	0.353	0.391	0.	1.038	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	100	8268	0	1989	0	0
normalized size	1	1.	0.56	46.71	0.	11.24	0.	0.
time (sec)	N/A	0.216	0.505	0.446	0.	1.241	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	256	2437	0	1932	0	0
normalized size	1	1.	1.06	10.07	0.	7.98	0.	0.
time (sec)	N/A	0.374	8.645	0.695	0.	20.115	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	229	1714	0	1667	0	0
normalized size	1	1.	1.31	9.79	0.	9.53	0.	0.
time (sec)	N/A	0.217	3.717	0.424	0.	6.326	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	190	1069	0	1458	0	0
normalized size	1	1.	1.57	8.83	0.	12.05	0.	0.
time (sec)	N/A	0.15	1.33	0.314	0.	2.101	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	0	1438	0	0
normalized size	1	1.	2.18	13.08	0.	18.68	0.	0.
time (sec)	N/A	0.05	1.388	0.408	0.	1.04	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	76	89	0	238	0	0
normalized size	1	1.	1.12	1.31	0.	3.5	0.	0.
time (sec)	N/A	0.091	1.77	0.282	0.	0.853	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	102	137	0	428	0	0
normalized size	1	1.	0.83	1.11	0.	3.48	0.	0.
time (sec)	N/A	0.128	0.669	0.316	0.	2.216	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	126	204	0	706	0	0
normalized size	1	1.	0.69	1.11	0.	3.86	0.	0.
time (sec)	N/A	0.182	0.986	0.413	0.	7.921	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	182	229	451	441	0	0
normalized size	1	1.	0.83	1.05	2.06	2.01	0.	0.
time (sec)	N/A	0.215	3.42	2.01	1.017	1.438	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	129	159	263	321	0	0
normalized size	1	1.	0.88	1.09	1.8	2.2	0.	0.
time (sec)	N/A	0.138	2.522	1.063	0.99	1.102	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	90	116	243	0	0
normalized size	1	1.	0.91	0.93	1.2	2.51	0.	0.
time (sec)	N/A	0.068	1.455	0.052	1.	0.813	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	108	5056	0	1341	0	0
normalized size	1	1.	0.85	39.81	0.	10.56	0.	0.
time (sec)	N/A	0.142	5.338	0.68	0.	0.988	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	151	11110	0	2086	0	0
normalized size	1	1.	0.88	64.97	0.	12.2	0.	0.
time (sec)	N/A	0.206	1.548	1.021	0.	1.313	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	129	15551	0	2969	0	0
normalized size	1	1.	0.55	66.46	0.	12.69	0.	0.
time (sec)	N/A	0.324	1.923	1.572	0.	1.807	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	288	288	1705	4477	0	2375	0	0
normalized size	1	1.	5.92	15.55	0.	8.25	0.	0.
time (sec)	N/A	0.442	19.577	1.371	0.	72.545	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	1315	3223	0	2060	0	0
normalized size	1	1.	5.79	14.2	0.	9.07	0.	0.
time (sec)	N/A	0.302	14.36	0.744	0.	24.578	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	983	3158	0	2037	0	0
normalized size	1	1.	5.89	18.91	0.	12.2	0.	0.
time (sec)	N/A	0.202	10.746	0.582	0.	7.396	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3024	0	2021	0	0
normalized size	1	1.	15.42	24.19	0.	16.17	0.	0.
time (sec)	N/A	0.099	17.282	0.423	0.	2.311	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	108	146	0	429	0	0
normalized size	1	1.	1.02	1.38	0.	4.05	0.	0.
time (sec)	N/A	0.108	2.097	0.403	0.	2.401	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	138	225	0	701	0	0
normalized size	1	1.	0.87	1.42	0.	4.44	0.	0.
time (sec)	N/A	0.161	4.64	0.411	0.	8.207	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	173	324	0	1033	0	0
normalized size	1	1.	0.77	1.43	0.	4.57	0.	0.
time (sec)	N/A	0.24	7.31	0.409	0.	29.085	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	123	0	286	0	0	0	0	0
normalized size	1	0.	2.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	4.055	1.088	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	253	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	7.79	1.09	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	178	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	3.849	0.972	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	1.665	0.497	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	1532	0	0	0	0	0
normalized size	1	1.	19.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	17.116	0.383	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	266	0	0	0	0	0
normalized size	1	1.	3.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	4.47	0.385	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	5878	0	0	0	0	0
normalized size	1	1.	66.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	25.793	1.049	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	3781	0	0	0	0	0
normalized size	1	1.	42.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	21.91	0.872	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0
normalized size	1	1.	25.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	15.313	0.275	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	72	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	1.095	0.411	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	132	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	2.281	0.398	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	149	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	1.987	0.454	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	72	125	174	207	0	89
normalized size	1	1.	0.97	1.69	2.35	2.8	0.	1.2
time (sec)	N/A	0.047	0.039	0.03	1.015	0.499	0.	1.301

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	58	81	109	167	0	72
normalized size	1	1.	1.04	1.45	1.95	2.98	0.	1.29
time (sec)	N/A	0.039	0.031	0.026	1.045	0.485	0.	1.175

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	49	61	128	0	53
normalized size	1	1.	1.11	1.29	1.61	3.37	0.	1.39
time (sec)	N/A	0.03	0.02	0.024	1.002	0.476	0.	1.276

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	22	76	0	22
normalized size	1	1.	1.62	1.06	1.38	4.75	0.	1.38
time (sec)	N/A	0.013	0.006	0.014	1.051	0.466	0.	1.315

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	31	31	35	73	0	61
normalized size	1	1.	1.63	1.63	1.84	3.84	0.	3.21
time (sec)	N/A	0.021	0.028	0.047	1.526	0.464	0.	1.295

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	47	54	177	0	108
normalized size	1	1.	0.97	1.27	1.46	4.78	0.	2.92
time (sec)	N/A	0.03	0.027	0.05	1.572	0.473	0.	1.2

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	63	68	274	0	150
normalized size	1	1.	0.65	1.15	1.24	4.98	0.	2.73
time (sec)	N/A	0.039	0.047	0.049	1.519	0.484	0.	1.31

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	36	79	81	374	0	188
normalized size	1	1.	0.49	1.08	1.11	5.12	0.	2.58
time (sec)	N/A	0.045	0.018	0.05	1.513	0.494	0.	1.15

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	75	138	170	293	0	176
normalized size	1	1.	0.77	1.41	1.73	2.99	0.	1.8
time (sec)	N/A	0.059	0.314	0.031	0.998	0.518	0.	1.326

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	54	98	131	243	0	142
normalized size	1	1.	0.77	1.4	1.87	3.47	0.	2.03
time (sec)	N/A	0.046	0.124	0.028	0.994	0.508	0.	1.292

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	59	78	192	0	86
normalized size	1	1.	1.2	1.48	1.95	4.8	0.	2.15
time (sec)	N/A	0.025	0.021	0.027	0.987	0.495	0.	1.294

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	32	51	107	0	58
normalized size	1	1.	1.46	1.33	2.12	4.46	0.	2.42
time (sec)	N/A	0.028	0.017	0.046	0.987	0.501	0.	1.286

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	50	33	36	69	0	50
normalized size	1	1.	1.67	1.1	1.2	2.3	0.	1.67
time (sec)	N/A	0.045	0.023	0.05	0.992	0.472	0.	1.308

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	54	58	113	0	84
normalized size	1	1.	1.42	1.08	1.16	2.26	0.	1.68
time (sec)	N/A	0.066	0.024	0.055	1.001	0.478	0.	1.222

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	81	188	0	116
normalized size	1	1.	0.93	0.9	0.93	2.16	0.	1.33
time (sec)	N/A	0.049	0.311	0.03	0.983	0.475	0.	1.322

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	58	58	140	0	84
normalized size	1	1.	0.94	0.89	0.89	2.15	0.	1.29
time (sec)	N/A	0.043	0.195	0.032	1.012	0.468	0.	1.291

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	46	95	0	50
normalized size	1	1.	0.84	0.81	1.07	2.21	0.	1.16
time (sec)	N/A	0.038	0.088	0.028	1.023	0.458	0.	1.266

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	76	0	22
normalized size	1	1.	1.	1.07	1.33	5.07	0.	1.47
time (sec)	N/A	0.013	0.003	0.013	0.964	0.467	0.	1.296

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	37	50	72	51	54
normalized size	1	1.	1.06	1.19	1.61	2.32	1.65	1.74
time (sec)	N/A	0.027	0.031	0.054	1.5	0.475	21.281	1.242

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	65	99	119	0	107
normalized size	1	1.	0.74	1.07	1.62	1.95	0.	1.75
time (sec)	N/A	0.041	0.088	0.058	1.482	0.484	0.	1.345

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	86	139	167	0	140
normalized size	1	1.	0.76	0.97	1.56	1.88	0.	1.57
time (sec)	N/A	0.053	0.103	0.053	1.486	0.496	0.	1.342

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	119	256	270	428	0	319
normalized size	1	1.	0.72	1.55	1.64	2.59	0.	1.93
time (sec)	N/A	0.141	0.535	0.041	1.026	0.548	0.	1.343

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	94	191	224	355	0	263
normalized size	1	1.	0.73	1.48	1.74	2.75	0.	2.04
time (sec)	N/A	0.134	0.398	0.037	1.007	0.528	0.	1.275

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	63	125	161	284	0	171
normalized size	1	1.	0.69	1.37	1.77	3.12	0.	1.88
time (sec)	N/A	0.074	0.136	0.037	1.015	0.511	0.	1.218

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	80	78	117	239	0	113
normalized size	1	1.	1.43	1.39	2.09	4.27	0.	2.02
time (sec)	N/A	0.067	0.033	0.055	1.01	0.515	0.	1.191

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	72	72	85	165	0	101
normalized size	1	1.	1.47	1.47	1.73	3.37	0.	2.06
time (sec)	N/A	0.061	0.025	0.059	0.987	0.513	0.	1.275

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	106	67	74	142	0	111
normalized size	1	1.	2.	1.26	1.4	2.68	0.	2.09
time (sec)	N/A	0.066	0.026	0.065	0.973	0.487	0.	1.292

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	96	134	139	300	0	221
normalized size	1	1.	0.91	1.26	1.31	2.83	0.	2.08
time (sec)	N/A	0.089	0.416	0.039	1.01	0.508	0.	1.283

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	75	104	109	234	0	166
normalized size	1	1.	0.94	1.3	1.36	2.92	0.	2.08
time (sec)	N/A	0.076	0.385	0.038	1.	0.49	0.	1.247

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	71	96	167	0	111
normalized size	1	1.	0.91	1.34	1.81	3.15	0.	2.09
time (sec)	N/A	0.063	0.265	0.032	0.987	0.477	0.	1.354

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	59	142	0	72
normalized size	1	1.	2.65	1.2	1.48	3.55	0.	1.8
time (sec)	N/A	0.029	0.397	0.038	1.019	0.482	0.	1.323

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	51	72	134	0	77
normalized size	1	1.	1.11	1.09	1.53	2.85	0.	1.64
time (sec)	N/A	0.072	0.152	0.05	1.506	0.488	0.	1.426

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	58	78	117	143	0	126
normalized size	1	1.	0.72	0.96	1.44	1.77	0.	1.56
time (sec)	N/A	0.086	0.126	0.057	1.471	0.49	0.	1.339

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	116	182	212	0	217
normalized size	1	1.	0.83	0.97	1.53	1.78	0.	1.82
time (sec)	N/A	0.146	0.196	0.059	1.503	0.504	0.	1.311

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	268	84	112	215	0	123
normalized size	1	1.	3.67	1.15	1.53	2.95	0.	1.68
time (sec)	N/A	0.044	1.029	0.036	0.997	0.498	0.	1.241

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	455	130	181	317	0	200
normalized size	1	1.	4.1	1.17	1.63	2.86	0.	1.8
time (sec)	N/A	0.065	1.615	0.041	1.002	0.528	0.	1.234

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	1195	141	0	687	0	159
normalized size	1	1.	13.9	1.64	0.	7.99	0.	1.85
time (sec)	N/A	0.121	6.457	0.075	0.	0.602	0.	1.255

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	55	55	1022	68	0	405	0	104
normalized size	1	1.	18.58	1.24	0.	7.36	0.	1.89
time (sec)	N/A	0.072	2.027	0.059	0.	0.558	0.	1.308

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	0	271	0	53
normalized size	1	1.	1.	0.78	0.	7.53	0.	1.47
time (sec)	N/A	0.042	0.08	0.065	0.	0.511	0.	1.244

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	0	378	0	74
normalized size	1	1.	1.	0.87	0.	7.27	0.	1.42
time (sec)	N/A	0.063	0.113	0.091	0.	0.536	0.	1.181

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	105	70	0	517	0	120
normalized size	1	1.	1.38	0.92	0.	6.8	0.	1.58
time (sec)	N/A	0.089	0.3	0.097	0.	0.55	0.	1.265

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	136	110	0	689	0	184
normalized size	1	1.	1.26	1.02	0.	6.38	0.	1.7
time (sec)	N/A	0.102	0.725	0.089	0.	0.574	0.	1.237

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	224	79	0	824	0	136
normalized size	1	1.	2.91	1.03	0.	10.7	0.	1.77
time (sec)	N/A	0.088	2.389	0.059	0.	0.565	0.	1.282

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	192	47	0	678	0	93
normalized size	1	1.	3.69	0.9	0.	13.04	0.	1.79
time (sec)	N/A	0.068	0.804	0.059	0.	0.542	0.	1.184

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	0	498	0	68
normalized size	1	1.	1.	0.78	0.	13.83	0.	1.89
time (sec)	N/A	0.057	0.09	0.054	0.	0.547	0.	1.272

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	48	0	544	0	92
normalized size	1	1.	4.04	1.07	0.	12.09	0.	2.04
time (sec)	N/A	0.042	0.319	0.069	0.	0.554	0.	1.344

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	92	0	663	0	134
normalized size	1	1.	0.89	1.23	0.	8.84	0.	1.79
time (sec)	N/A	0.103	0.253	0.101	0.	0.567	0.	1.359

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	95	194	0	811	0	201
normalized size	1	1.	0.81	1.66	0.	6.93	0.	1.72
time (sec)	N/A	0.157	0.464	0.097	0.	0.595	0.	1.297

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	133	359	0	996	0	309
normalized size	1	1.	0.82	2.2	0.	6.11	0.	1.9
time (sec)	N/A	0.245	0.937	0.098	0.	0.612	0.	1.277

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	980	151	0	950	0	177
normalized size	1	1.	9.61	1.48	0.	9.31	0.	1.74
time (sec)	N/A	0.14	4.422	0.087	0.	0.708	0.	1.272

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	88	68	0	601	0	107
normalized size	1	1.	1.19	0.92	0.	8.12	0.	1.45
time (sec)	N/A	0.07	0.304	0.068	0.	0.547	0.	1.33

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	82	80	0	676	0	127
normalized size	1	1.	0.99	0.96	0.	8.14	0.	1.53
time (sec)	N/A	0.068	0.413	0.083	0.	0.559	0.	1.321

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	945	92	0	867	0	158
normalized size	1	1.	9.36	0.91	0.	8.58	0.	1.56
time (sec)	N/A	0.132	3.474	0.095	0.	0.589	0.	1.194

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	139	120	0	1077	0	205
normalized size	1	1.	1.1	0.95	0.	8.55	0.	1.63
time (sec)	N/A	0.155	1.156	0.101	0.	0.651	0.	1.157

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	171	158	0	1304	0	266
normalized size	1	1.	1.09	1.01	0.	8.31	0.	1.69
time (sec)	N/A	0.168	2.141	0.105	0.	0.689	0.	1.186

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	248	128	0	1172	0	169
normalized size	1	1.	2.48	1.28	0.	11.72	0.	1.69
time (sec)	N/A	0.135	2.584	0.069	0.	0.601	0.	1.197

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	84	106	0	936	0	126
normalized size	1	1.	1.02	1.29	0.	11.41	0.	1.54
time (sec)	N/A	0.082	0.31	0.071	0.	0.572	0.	1.389

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	211	66	0	857	0	117
normalized size	1	1.	2.89	0.9	0.	11.74	0.	1.6
time (sec)	N/A	0.074	0.965	0.088	0.	0.565	0.	1.172

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	240	127	0	1027	0	161
normalized size	1	1.	2.61	1.38	0.	11.16	0.	1.75
time (sec)	N/A	0.082	1.944	0.078	0.	0.605	0.	1.222

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	103	174	0	1246	0	274
normalized size	1	1.	0.73	1.23	0.	8.77	0.	1.93
time (sec)	N/A	0.194	1.259	0.109	0.	0.66	0.	1.321

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	138	276	0	1492	0	277
normalized size	1	1.	0.68	1.36	0.	7.35	0.	1.36
time (sec)	N/A	0.29	1.589	0.106	0.	0.711	0.	1.257

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	499	442	0	1801	0	385
normalized size	1	1.	1.79	1.59	0.	6.48	0.	1.38
time (sec)	N/A	0.347	4.495	0.109	0.	0.773	0.	1.287

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	128	108	0	1053	0	165
normalized size	1	1.	1.19	1.	0.	9.75	0.	1.53
time (sec)	N/A	0.099	0.534	0.074	0.	0.608	0.	1.356

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	163	124	0	1183	0	219
normalized size	1	1.	1.3	0.99	0.	9.46	0.	1.75
time (sec)	N/A	0.106	0.62	0.078	0.	0.651	0.	1.301

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	2256	142	0	1331	0	250
normalized size	1	1.	15.67	0.99	0.	9.24	0.	1.74
time (sec)	N/A	0.131	7.362	0.08	0.	0.66	0.	1.316

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	2382	149	0	1594	0	277
normalized size	1	1.	15.27	0.96	0.	10.22	0.	1.78
time (sec)	N/A	0.194	7.524	0.107	0.	0.738	0.	1.239

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	194	177	0	1908	0	323
normalized size	1	1.	1.07	0.98	0.	10.54	0.	1.78
time (sec)	N/A	0.243	4.455	0.102	0.	0.779	0.	1.297

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	2670	214	0	2271	0	383
normalized size	1	1.	12.48	1.	0.	10.61	0.	1.79
time (sec)	N/A	0.258	7.618	0.118	0.	0.846	0.	1.321

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	125	294	0	1597	0	261
normalized size	1	1.	0.88	2.07	0.	11.25	0.	1.84
time (sec)	N/A	0.157	0.995	0.073	0.	0.697	0.	1.403

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	283	238	0	1449	0	231
normalized size	1	1.	2.3	1.93	0.	11.78	0.	1.88
time (sec)	N/A	0.102	3.831	0.071	0.	0.668	0.	1.39

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	265	97	0	1319	0	176
normalized size	1	1.	2.5	0.92	0.	12.44	0.	1.66
time (sec)	N/A	0.082	2.749	0.067	0.	0.645	0.	1.334

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	332	321	0	1854	0	277
normalized size	1	1.	2.31	2.23	0.	12.88	0.	1.92
time (sec)	N/A	0.175	5.762	0.086	0.	0.717	0.	1.273

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	156	366	0	2188	0	323
normalized size	1	1.	0.78	1.82	0.	10.89	0.	1.61
time (sec)	N/A	0.336	3.765	0.113	0.	0.785	0.	1.359

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	1430	470	0	2531	0	663
normalized size	1	1.	5.32	1.75	0.	9.41	0.	2.46
time (sec)	N/A	0.377	6.573	0.126	0.	0.881	0.	1.378

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	1770	636	0	2954	0	498
normalized size	1	1.	5.03	1.81	0.	8.39	0.	1.41
time (sec)	N/A	0.475	6.659	0.114	0.	1.	0.	1.36

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	1411	649	0	2967	0	437
normalized size	1	1.	6.92	3.18	0.	14.54	0.	2.14
time (sec)	N/A	0.335	6.853	0.092	0.	0.883	0.	1.328

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	70	167	109	244	0	293
normalized size	1	1.	0.52	1.25	0.81	1.82	0.	2.19
time (sec)	N/A	0.062	2.206	0.337	1.463	0.528	0.	1.708

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	60	157	84	207	0	246
normalized size	1	1.	0.59	1.55	0.83	2.05	0.	2.44
time (sec)	N/A	0.051	0.554	0.304	1.516	0.511	0.	1.638

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	145	54	167	0	185
normalized size	1	1.	0.75	2.27	0.84	2.61	0.	2.89
time (sec)	N/A	0.041	0.107	0.264	1.488	0.529	0.	1.405

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	108	28	128	0	190
normalized size	1	1.	1.	3.27	0.85	3.88	0.	5.76
time (sec)	N/A	0.03	0.037	0.347	1.465	0.505	0.	1.491

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	75	50	135	0	0
normalized size	1	1.	1.25	2.34	1.56	4.22	0.	0.
time (sec)	N/A	0.035	0.045	0.33	1.515	0.495	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	57	141	81	228	0	285
normalized size	1	1.	0.85	2.1	1.21	3.4	0.	4.25
time (sec)	N/A	0.042	0.136	0.24	1.483	0.504	0.	1.818

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	69	203	107	315	0	369
normalized size	1	1.	0.69	2.03	1.07	3.15	0.	3.69
time (sec)	N/A	0.051	0.259	0.247	1.475	0.512	0.	1.879

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	79	265	127	406	0	383
normalized size	1	1.	0.59	1.99	0.95	3.05	0.	2.88
time (sec)	N/A	0.061	0.338	0.261	1.5	0.525	0.	1.965

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	471	0	6562	0	0	0	0
normalized size	1	1.27	0.	17.64	0.	0.	0.	0.
time (sec)	N/A	0.688	26.686	0.766	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	364	0	4737	0	0	0	0
normalized size	1	1.26	0.	16.45	0.	0.	0.	0.
time (sec)	N/A	0.51	11.498	0.462	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	271	0	3454	0	0	0	0
normalized size	1	1.24	0.	15.84	0.	0.	0.	0.
time (sec)	N/A	0.398	11.568	0.421	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	103	69	3408	0	0	0	0
normalized size	1	1.29	0.86	42.6	0.	0.	0.	0.
time (sec)	N/A	0.15	0.29	0.447	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	299	539	4623	0	0	0	0
normalized size	1	1.22	2.19	18.79	0.	0.	0.	0.
time (sec)	N/A	0.389	8.52	0.721	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	338	400	0	6392	0	0	0	0
normalized size	1	1.18	0.	18.91	0.	0.	0.	0.
time (sec)	N/A	0.571	11.436	0.747	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	968	2518	0	1138	0	0
normalized size	1	1.	5.2	13.54	0.	6.12	0.	0.
time (sec)	N/A	0.17	11.544	0.607	0.	3.429	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	390	1770	0	972	0	0
normalized size	1	1.	3.2	14.51	0.	7.97	0.	0.
time (sec)	N/A	0.106	8.257	0.387	0.	1.121	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	210	1098	0	813	0	0
normalized size	1	1.	2.76	14.45	0.	10.7	0.	0.
time (sec)	N/A	0.081	1.666	0.304	0.	0.704	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-2)	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	589	0	2984	0	0
normalized size	1	1.	0.	7.46	0.	37.77	0.	0.
time (sec)	N/A	0.049	0.089	0.453	0.	1.193	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	136	1066	0	1210	0	112
normalized size	1	1.	1.66	13.	0.	14.76	0.	1.37
time (sec)	N/A	0.092	0.713	0.315	0.	0.879	0.	1.429

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	152	1713	0	1355	0	166
normalized size	1	1.	1.09	12.24	0.	9.68	0.	1.19
time (sec)	N/A	0.123	1.281	0.39	0.	1.514	0.	1.395

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	1902	2436	0	1527	0	0
normalized size	1	1.	9.7	12.43	0.	7.79	0.	0.
time (sec)	N/A	0.206	16.624	0.563	0.	3.946	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	450	572	0	8000	0	0	0	0
normalized size	1	1.27	0.	17.78	0.	0.	0.	0.
time (sec)	N/A	0.894	10.349	1.169	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	470	0	6562	0	0	0	0
normalized size	1	1.27	0.	17.69	0.	0.	0.	0.
time (sec)	N/A	0.7	16.413	0.654	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	290	366	0	5185	0	0	0	0
normalized size	1	1.26	0.	17.88	0.	0.	0.	0.
time (sec)	N/A	0.529	11.594	0.406	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	277	0	3632	0	0	0	0
normalized size	1	1.24	0.	16.21	0.	0.	0.	0.
time (sec)	N/A	0.3	12.611	0.348	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	294	179	5069	0	0	0	0
normalized size	1	1.22	0.74	21.03	0.	0.	0.	0.
time (sec)	N/A	0.421	1.962	0.439	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	395	350	6396	0	0	0	0
normalized size	1	1.24	1.1	20.05	0.	0.	0.	0.
time (sec)	N/A	0.641	9.316	0.623	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	512	3343	0	1381	0	0
normalized size	1	1.	2.11	13.76	0.	5.68	0.	0.
time (sec)	N/A	0.223	11.049	0.836	0.	13.317	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	400	2519	0	1154	0	0
normalized size	1	1.	2.42	15.27	0.	6.99	0.	0.
time (sec)	N/A	0.131	9.385	0.502	0.	3.453	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	84	1768	0	969	0	0
normalized size	1	1.	0.76	15.93	0.	8.73	0.	0.
time (sec)	N/A	0.102	0.326	0.344	0.	1.122	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1557	0	3602	0	0
normalized size	1	1.	4.47	13.19	0.	30.53	0.	0.
time (sec)	N/A	0.092	2.054	0.293	0.	1.778	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	466	1511	0	3456	0	0
normalized size	1	1.	3.76	12.19	0.	27.87	0.	0.
time (sec)	N/A	0.136	7.032	0.306	0.	1.74	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	191	1713	0	1357	0	0
normalized size	1	1.	1.53	13.7	0.	10.86	0.	0.
time (sec)	N/A	0.12	1.03	0.347	0.	1.524	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	165	2439	0	1539	0	0
normalized size	1	1.	0.85	12.64	0.	7.97	0.	0.
time (sec)	N/A	0.162	1.845	0.49	0.	4.041	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	706	2231	0	3947	0	0
normalized size	1	1.	4.25	13.44	0.	23.78	0.	0.
time (sec)	N/A	0.172	9.732	0.622	0.	4.28	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	109	429	0	536	0	0
normalized size	1	1.	2.6	10.21	0.	12.76	0.	0.
time (sec)	N/A	0.037	0.173	0.296	0.	0.542	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	57	190	0	444	0	0
normalized size	1	1.	2.38	7.92	0.	18.5	0.	0.
time (sec)	N/A	0.019	0.05	0.299	0.	0.527	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	380	0	4753	0	0	0	0
normalized size	1	1.15	0.	14.4	0.	0.	0.	0.
time (sec)	N/A	0.572	11.905	0.564	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	202	0	3033	0	0	0	0
normalized size	1	1.19	0.	17.84	0.	0.	0.	0.
time (sec)	N/A	0.371	10.679	0.405	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	103	69	269	0	0	0	0
normalized size	1	1.29	0.86	3.36	0.	0.	0.	0.
time (sec)	N/A	0.272	0.224	0.399	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	128	279	2985	0	0	0	0
normalized size	1	1.22	2.66	28.43	0.	0.	0.	0.
time (sec)	N/A	0.166	5.276	0.424	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	296	0	4640	0	0	0	0
normalized size	1	1.16	0.	18.2	0.	0.	0.	0.
time (sec)	N/A	0.407	7.365	0.534	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	345	395	0	6382	0	0	0	0
normalized size	1	1.14	0.	18.5	0.	0.	0.	0.
time (sec)	N/A	0.574	12.874	0.744	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	326	1756	0	975	0	0
normalized size	1	1.	2.38	12.82	0.	7.12	0.	0.
time (sec)	N/A	0.135	9.035	0.474	0.	1.273	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	326	1086	0	821	0	0
normalized size	1	1.	4.02	13.41	0.	10.14	0.	0.
time (sec)	N/A	0.091	10.167	0.382	0.	0.774	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	379	0	543	0	0
normalized size	1	1.	2.23	9.72	0.	13.92	0.	0.
time (sec)	N/A	0.071	0.147	0.411	0.	0.655	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	0	976	0	0
normalized size	1	1.	2.23	9.74	0.	25.03	0.	0.
time (sec)	N/A	0.03	0.083	0.407	0.	0.792	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	126	1056	0	1215	0	0
normalized size	1	1.	1.45	12.14	0.	13.97	0.	0.
time (sec)	N/A	0.099	0.257	0.379	0.	0.895	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	1840	1701	0	1362	0	0
normalized size	1	1.	12.87	11.9	0.	9.52	0.	0.
time (sec)	N/A	0.142	16.349	0.455	0.	1.508	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	1739	2425	0	1539	0	0
normalized size	1	1.	8.52	11.89	0.	7.54	0.	0.
time (sec)	N/A	0.207	16.618	0.587	0.	3.864	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	367	0	12514	0	0	0	0
normalized size	1	1.27	0.	43.3	0.	0.	0.	0.
time (sec)	N/A	0.59	20.178	0.65	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	182	113	6593	0	0	0	0
normalized size	1	1.21	0.75	43.95	0.	0.	0.	0.
time (sec)	N/A	0.384	2.416	0.49	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	284	822	6593	0	0	0	0
normalized size	1	1.24	3.59	28.79	0.	0.	0.	0.
time (sec)	N/A	0.448	9.489	0.5	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	295	0	8684	0	0	0	0
normalized size	1	1.23	0.	36.18	0.	0.	0.	0.
time (sec)	N/A	0.337	14.268	0.508	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	399	0	11939	0	0	0	0
normalized size	1	1.19	0.	35.64	0.	0.	0.	0.
time (sec)	N/A	0.561	16.006	0.542	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	436	509	0	15199	0	0	0	0
normalized size	1	1.17	0.	34.86	0.	0.	0.	0.
time (sec)	N/A	0.751	16.267	0.92	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	336	4338	0	1245	0	0
normalized size	1	1.	2.43	31.43	0.	9.02	0.	0.
time (sec)	N/A	0.146	15.07	0.435	0.	1.435	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	405	3068	0	1007	0	0
normalized size	1	1.	5.26	39.84	0.	13.08	0.	0.
time (sec)	N/A	0.097	7.108	0.472	0.	0.806	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	57	59	41	159	0	157
normalized size	1	1.	1.78	1.84	1.28	4.97	0.	4.91
time (sec)	N/A	0.075	0.656	0.23	1.078	0.56	0.	1.949

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	0	1438	0	0
normalized size	1	1.	2.18	13.08	0.	18.68	0.	0.
time (sec)	N/A	0.049	1.463	0.401	0.	1.055	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	2059	1646	0	1640	0	0
normalized size	1	1.	15.72	12.56	0.	12.52	0.	0.
time (sec)	N/A	0.155	15.463	0.342	0.	1.939	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	2046	2372	0	1875	0	0
normalized size	1	1.	10.55	12.23	0.	9.66	0.	0.
time (sec)	N/A	0.226	16.921	0.461	0.	5.064	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	2068	3171	0	2190	0	0
normalized size	1	1.	7.63	11.7	0.	8.08	0.	0.
time (sec)	N/A	0.315	20.251	0.666	0.	14.906	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	383	167	14357	0	0	0	0
normalized size	1	1.19	0.52	44.73	0.	0.	0.	0.
time (sec)	N/A	0.637	2.82	0.614	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	381	1156	10271	0	0	0	0
normalized size	1	1.19	3.62	32.2	0.	0.	0.	0.
time (sec)	N/A	0.565	10.283	0.604	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	327	389	0	14353	0	0	0	0
normalized size	1	1.19	0.	43.89	0.	0.	0.	0.
time (sec)	N/A	0.579	12.991	0.612	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	411	0	17494	0	0	0	0
normalized size	1	1.18	0.	50.13	0.	0.	0.	0.
time (sec)	N/A	0.488	16.156	0.899	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	441	512	0	20922	0	0	0	0
normalized size	1	1.16	0.	47.44	0.	0.	0.	0.
time (sec)	N/A	0.731	13.261	1.43	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	559	639	0	26983	0	0	0	0
normalized size	1	1.14	0.	48.27	0.	0.	0.	0.
time (sec)	N/A	0.906	23.448	2.248	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	607	3010	0	1586	0	0
normalized size	1	1.	4.56	22.63	0.	11.92	0.	0.
time (sec)	N/A	0.139	10.07	0.469	0.	1.46	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	74	76	158	313	0	560
normalized size	1	1.	0.94	0.96	2.	3.96	0.	7.09
time (sec)	N/A	0.093	4.55	0.304	1.183	0.843	0.	2.405

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	215	85	82	313	0	562
normalized size	1	1.	3.03	1.2	1.15	4.41	0.	7.92
time (sec)	N/A	0.09	6.134	0.266	1.206	0.912	0.	2.441

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3016	0	2021	0	0
normalized size	1	1.	15.42	24.13	0.	16.17	0.	0.
time (sec)	N/A	0.102	6.517	0.4	0.	2.326	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	1775	4270	0	2330	0	0
normalized size	1	1.	9.49	22.83	0.	12.46	0.	0.
time (sec)	N/A	0.243	17.54	0.646	0.	6.162	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	1777	5600	0	2708	0	0
normalized size	1	1.	6.81	21.46	0.	10.38	0.	0.
time (sec)	N/A	0.338	21.007	0.93	0.	18.104	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	332	332	1776	6934	0	3035	0	0
normalized size	1	1.	5.35	20.89	0.	9.14	0.	0.
time (sec)	N/A	0.429	24.717	1.471	0.	49.999	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	1777	6116	0	2795	0	0
normalized size	1	1.	9.93	34.17	0.	15.61	0.	0.
time (sec)	N/A	0.194	18.758	0.862	0.	7.403	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	37	142	524	177	0	0
normalized size	1	1.	2.64	10.14	37.43	12.64	0.	0.
time (sec)	N/A	0.02	0.025	0.227	1.785	0.509	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	111	0	2195	0	0	0	0	0
normalized size	1	0.	19.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	18.431	1.051	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	124	1989	0	0	0	0	0
normalized size	1	1.2	19.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.223	17.397	0.374	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	124	1995	0	0	0	0	0
normalized size	1	1.2	19.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	16.826	0.395	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	122	1983	0	0	0	0	0
normalized size	1	1.21	19.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	16.778	0.512	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	124	1987	0	0	0	0	0
normalized size	1	1.2	19.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	17.139	1.056	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	124	1997	0	0	0	0	0
normalized size	1	1.2	19.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	17.587	0.939	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	2.054	0.503	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	126	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	2.262	0.426	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.039	0.356	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0
normalized size	1	1.	25.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	6.266	0.007	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0
normalized size	1	1.	23.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	15.868	0.663	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	1912	0	0	0	0	0
normalized size	1	1.	23.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	16.402	0.73	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0
normalized size	1	1.	23.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	17.258	0.758	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	55	65	128	177	116	408
normalized size	1	1.	0.76	0.9	1.78	2.46	1.61	5.67
time (sec)	N/A	0.062	0.177	0.051	1.	0.541	8.289	3.222

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	50	86	128	80	343
normalized size	1	1.	0.88	1.02	1.76	2.61	1.63	7.
time (sec)	N/A	0.049	0.08	0.048	1.033	0.526	2.352	1.833

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	28	45	93	42	275
normalized size	1	1.	1.	0.93	1.5	3.1	1.4	9.17
time (sec)	N/A	0.024	0.019	0.018	0.996	0.533	0.621	1.423

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	44	26	45	99	0	140
normalized size	1	1.	1.57	0.93	1.61	3.54	0.	5.
time (sec)	N/A	0.047	0.031	0.046	1.02	0.525	0.	1.456

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	52	43	39	116	0	204
normalized size	1	1.	1.62	1.34	1.22	3.62	0.	6.38
time (sec)	N/A	0.051	0.165	0.054	0.992	0.529	0.	1.418

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	64	64	66	215	0	339
normalized size	1	1.	1.25	1.25	1.29	4.22	0.	6.65
time (sec)	N/A	0.071	0.175	0.051	0.988	0.552	0.	1.493

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	73	61	76	231	66	82
normalized size	1	1.	1.14	0.95	1.19	3.61	1.03	1.28
time (sec)	N/A	0.062	0.028	0.052	1.492	0.518	7.26	4.795

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	57	50	61	177	54	66
normalized size	1	1.	1.19	1.04	1.27	3.69	1.12	1.38
time (sec)	N/A	0.057	0.021	0.048	1.475	0.496	2.997	2.26

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	41	45	130	42	49
normalized size	1	1.	1.28	1.28	1.41	4.06	1.31	1.53
time (sec)	N/A	0.052	0.015	0.04	1.475	0.486	1.663	1.604

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	76	0	22
normalized size	1	1.	1.	1.07	1.33	5.07	0.	1.47
time (sec)	N/A	0.013	0.003	0.015	0.999	0.474	0.	1.267

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	43	33	34	85	0	77
normalized size	1	1.	2.26	1.74	1.79	4.47	0.	4.05
time (sec)	N/A	0.054	0.03	0.043	1.475	0.482	0.	1.289

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	51	48	55	185	0	161
normalized size	1	1.	1.55	1.45	1.67	5.61	0.	4.88
time (sec)	N/A	0.059	0.022	0.048	1.514	0.478	0.	1.264

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	63	70	286	0	246
normalized size	1	1.	1.	1.24	1.37	5.61	0.	4.82
time (sec)	N/A	0.063	0.03	0.051	1.567	0.492	0.	1.466

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	126	120	198	242	190	628
normalized size	1	1.	1.26	1.2	1.98	2.42	1.9	6.28
time (sec)	N/A	0.101	0.481	0.061	1.056	0.561	21.833	3.196

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	107	103	154	192	128	560
normalized size	1	1.	1.39	1.34	2.	2.49	1.66	7.27
time (sec)	N/A	0.084	0.277	0.056	1.009	0.542	7.923	1.856

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	82	46	90	130	61	486
normalized size	1	1.	1.71	0.96	1.88	2.71	1.27	10.12
time (sec)	N/A	0.042	0.122	0.022	0.981	0.531	2.198	1.37

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	84	60	86	203	0	358
normalized size	1	1.	1.58	1.13	1.62	3.83	0.	6.75
time (sec)	N/A	0.074	0.227	0.057	0.984	0.536	0.	1.395

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	81	78	81	231	0	339
normalized size	1	1.	1.42	1.37	1.42	4.05	0.	5.95
time (sec)	N/A	0.081	0.181	0.063	0.983	0.528	0.	1.402

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	77	87	82	242	0	474
normalized size	1	1.	1.51	1.71	1.61	4.75	0.	9.29
time (sec)	N/A	0.087	0.261	0.06	1.066	0.521	0.	1.399

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	275	105	113	342	0	132
normalized size	1	1.	2.89	1.11	1.19	3.6	0.	1.39
time (sec)	N/A	0.108	2.123	0.07	1.593	0.548	0.	4.323

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	395	94	96	273	0	113
normalized size	1	1.	5.13	1.22	1.25	3.55	0.	1.47
time (sec)	N/A	0.097	1.146	0.054	1.462	0.521	0.	2.464

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	281	85	78	205	0	95
normalized size	1	1.	4.76	1.44	1.32	3.47	0.	1.61
time (sec)	N/A	0.093	0.817	0.05	1.497	0.504	0.	1.482

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	106	48	59	142	0	72
normalized size	1	1.	2.65	1.2	1.48	3.55	0.	1.8
time (sec)	N/A	0.029	0.373	0.031	1.002	0.482	0.	1.278

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	82	66	62	153	0	66
normalized size	1	1.	2.28	1.83	1.72	4.25	0.	1.83
time (sec)	N/A	0.08	0.692	0.049	1.476	0.488	0.	1.29

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	160	73	80	220	0	252
normalized size	1	1.	3.56	1.62	1.78	4.89	0.	5.6
time (sec)	N/A	0.087	0.831	0.053	1.51	0.492	0.	1.351

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	256	107	97	328	0	392
normalized size	1	1.	3.94	1.65	1.49	5.05	0.	6.03
time (sec)	N/A	0.093	1.061	0.059	1.472	0.499	0.	1.505

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	99	112	109	205	0	539
normalized size	1	1.	1.43	1.62	1.58	2.97	0.	7.81
time (sec)	N/A	0.1	0.293	0.061	1.016	0.698	0.	3.149

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	59	68	100	0	319
normalized size	1	1.	0.91	1.31	1.51	2.22	0.	7.09
time (sec)	N/A	0.077	0.114	0.058	1.025	0.597	0.	1.661

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	37	35	51	128	186
normalized size	1	1.	1.13	1.61	1.52	2.22	5.57	8.09
time (sec)	N/A	0.031	0.173	0.025	1.089	0.508	15.82	1.391

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	73	68	107	0	235
normalized size	1	1.	0.93	1.59	1.48	2.33	0.	5.11
time (sec)	N/A	0.08	0.105	0.078	0.981	0.663	0.	1.36

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	100	158	117	289	0	419
normalized size	1	1.	1.35	2.14	1.58	3.91	0.	5.66
time (sec)	N/A	0.11	0.235	0.087	1.006	0.868	0.	1.405

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	138	293	196	610	0	733
normalized size	1	1.	1.28	2.71	1.81	5.65	0.	6.79
time (sec)	N/A	0.148	0.654	0.089	0.995	1.321	0.	1.428

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	229	186	0	890	0	178
normalized size	1	1.	2.76	2.24	0.	10.72	0.	2.14
time (sec)	N/A	0.273	3.003	0.085	0.	0.596	0.	4.814

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	206	121	0	720	0	123
normalized size	1	1.	3.49	2.05	0.	12.2	0.	2.08
time (sec)	N/A	0.167	1.148	0.074	0.	0.574	0.	2.233

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	184	75	0	533	0	93
normalized size	1	1.	4.	1.63	0.	11.59	0.	2.02
time (sec)	N/A	0.126	0.309	0.081	0.	0.553	0.	1.489

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	48	0	544	0	92
normalized size	1	1.	4.04	1.07	0.	12.09	0.	2.04
time (sec)	N/A	0.045	0.26	0.069	0.	0.566	0.	1.389

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	204	73	0	757	0	123
normalized size	1	1.	3.29	1.18	0.	12.21	0.	1.98
time (sec)	N/A	0.174	1.354	0.086	0.	0.574	0.	1.39

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	390	110	0	1261	0	189
normalized size	1	1.	4.53	1.28	0.	14.66	0.	2.2
time (sec)	N/A	0.248	3.707	0.099	0.	0.621	0.	1.436

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	671	173	0	1972	0	300
normalized size	1	1.	5.59	1.44	0.	16.43	0.	2.5
time (sec)	N/A	0.344	2.93	0.112	0.	0.684	0.	1.472

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	109	126	132	262	0	770
normalized size	1	1.	1.42	1.64	1.71	3.4	0.	10.
time (sec)	N/A	0.106	0.455	0.087	1.014	0.744	0.	3.307

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	81	68	80	131	0	458
normalized size	1	1.	1.59	1.33	1.57	2.57	0.	8.98
time (sec)	N/A	0.082	0.732	0.079	0.994	0.524	0.	1.869

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	79	59	77	127	0	558
normalized size	1	1.	1.61	1.2	1.57	2.59	0.	11.39
time (sec)	N/A	0.056	0.506	0.036	0.994	0.524	0.	1.362

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	112	155	158	313	0	567
normalized size	1	1.	1.35	1.87	1.9	3.77	0.	6.83
time (sec)	N/A	0.114	0.362	0.092	1.006	0.996	0.	2.078

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	130	240	259	664	0	1165
normalized size	1	1.	1.17	2.16	2.33	5.98	0.	10.5
time (sec)	N/A	0.157	1.338	0.109	1.021	1.535	0.	1.943

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	162	374	377	1196	0	1305
normalized size	1	1.	1.16	2.67	2.69	8.54	0.	9.32
time (sec)	N/A	0.197	2.004	0.126	1.011	2.697	0.	1.57

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	286	242	0	1197	0	220
normalized size	1	1.	2.4	2.03	0.	10.06	0.	1.85
time (sec)	N/A	0.267	4.873	0.09	0.	0.637	0.	3.982

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	249	168	0	933	0	170
normalized size	1	1.	2.77	1.87	0.	10.37	0.	1.89
time (sec)	N/A	0.176	2.983	0.096	0.	0.609	0.	2.253

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	346	108	0	1052	0	134
normalized size	1	1.	4.07	1.27	0.	12.38	0.	1.58
time (sec)	N/A	0.156	7.709	0.096	0.	0.597	0.	1.588

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	240	127	0	1027	0	161
normalized size	1	1.	2.61	1.38	0.	11.16	0.	1.75
time (sec)	N/A	0.085	1.954	0.08	0.	0.622	0.	1.264

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	288	149	0	1384	0	247
normalized size	1	1.	2.38	1.23	0.	11.44	0.	2.04
time (sec)	N/A	0.254	3.929	0.104	0.	0.724	0.	1.375

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	1896	186	0	2182	0	297
normalized size	1	1.	11.85	1.16	0.	13.64	0.	1.86
time (sec)	N/A	0.354	6.932	0.118	0.	0.76	0.	1.361

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	3028	248	0	3366	0	419
normalized size	1	1.	14.63	1.2	0.	16.26	0.	2.02
time (sec)	N/A	0.437	7.328	0.125	0.	0.848	0.	1.464

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	136	138	151	271	0	772
normalized size	1	1.	1.74	1.77	1.94	3.47	0.	9.9
time (sec)	N/A	0.108	2.194	0.089	1.02	0.575	0.	2.915

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	131	115	153	262	0	952
normalized size	1	1.	1.62	1.42	1.89	3.23	0.	11.75
time (sec)	N/A	0.112	0.936	0.083	0.996	0.571	0.	1.994

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	129	81	138	242	0	1133
normalized size	1	1.	1.74	1.09	1.86	3.27	0.	15.31
time (sec)	N/A	0.074	1.509	0.038	0.993	0.573	0.	1.797

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	158	304	328	670	0	1100
normalized size	1	1.	1.22	2.34	2.52	5.15	0.	8.46
time (sec)	N/A	0.165	1.098	0.102	1.014	1.922	0.	1.532

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	176	389	464	1234	0	1530
normalized size	1	1.	1.14	2.53	3.01	8.01	0.	9.94
time (sec)	N/A	0.209	1.934	0.115	1.04	3.34	0.	1.601

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	208	522	613	1867	0	2751
normalized size	1	1.	1.08	2.72	3.19	9.72	0.	14.33
time (sec)	N/A	0.269	5.422	0.126	1.052	5.708	0.	1.645

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	760	356	0	1519	0	285
normalized size	1	1.	5.17	2.42	0.	10.33	0.	1.94
time (sec)	N/A	0.292	6.524	0.106	0.	0.678	0.	4.879

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	1473	264	0	1686	0	227
normalized size	1	1.	10.75	1.93	0.	12.31	0.	1.66
time (sec)	N/A	0.257	15.037	0.099	0.	0.706	0.	2.278

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	1473	263	0	1910	0	244
normalized size	1	1.	10.67	1.91	0.	13.84	0.	1.77
time (sec)	N/A	0.227	13.228	0.096	0.	0.713	0.	1.645

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	332	321	0	1854	0	277
normalized size	1	1.	2.31	2.23	0.	12.88	0.	1.92
time (sec)	N/A	0.178	5.826	0.089	0.	0.723	0.	1.42

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	2089	337	0	2372	0	347
normalized size	1	1.	11.54	1.86	0.	13.1	0.	1.92
time (sec)	N/A	0.38	6.999	0.113	0.	0.811	0.	1.447

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	3340	374	0	3640	0	424
normalized size	1	1.	14.52	1.63	0.	15.83	0.	1.84
time (sec)	N/A	0.461	7.619	0.122	0.	0.949	0.	1.443

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	285	285	976	437	0	5045	0	545
normalized size	1	1.	3.42	1.53	0.	17.7	0.	1.91
time (sec)	N/A	0.607	8.21	0.131	0.	1.109	0.	1.64

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	0	924	0	1123	0	0
normalized size	1	1.	0.	8.32	0.	10.12	0.	0.
time (sec)	N/A	0.141	2.418	0.439	0.	6.68	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	648	0	969	0	0
normalized size	1	1.	0.	8.1	0.	12.11	0.	0.
time (sec)	N/A	0.105	1.073	0.36	0.	1.888	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	119	61	0	779	0	537
normalized size	1	1.	2.2	1.13	0.	14.43	0.	9.94
time (sec)	N/A	0.065	0.486	0.079	0.	0.835	0.	1.501

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-2)	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	570	0	2398	0	0
normalized size	1	1.	0.	8.14	0.	34.26	0.	0.
time (sec)	N/A	0.11	1.798	0.408	0.	1.096	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	527	2528	0	3353	0	0
normalized size	1	1.	4.83	23.19	0.	30.76	0.	0.
time (sec)	N/A	0.149	6.139	0.454	0.	1.686	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	0	7346	0	4782	0	0
normalized size	1	1.	0.	45.63	0.	29.7	0.	0.
time (sec)	N/A	0.229	5.319	0.447	0.	4.825	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	263	2756	0	4315	0	0
normalized size	1	1.	1.2	12.58	0.	19.7	0.	0.
time (sec)	N/A	0.436	3.77	0.612	0.	14.227	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	208	2005	0	3947	0	0
normalized size	1	1.	1.26	12.15	0.	23.92	0.	0.
time (sec)	N/A	0.314	2.817	0.447	0.	4.309	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	526	1331	0	3605	0	0
normalized size	1	1.	4.46	11.28	0.	30.55	0.	0.
time (sec)	N/A	0.223	4.517	0.345	0.	1.759	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-2)	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	588	0	2984	0	0
normalized size	1	1.	0.	7.44	0.	37.77	0.	0.
time (sec)	N/A	0.051	0.095	0.429	0.	1.172	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	130	1004	0	1211	0	0
normalized size	1	1.	1.88	14.55	0.	17.55	0.	0.
time (sec)	N/A	0.179	0.634	0.495	0.	0.946	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	176	3855	0	1526	0	0
normalized size	1	1.	1.54	33.82	0.	13.39	0.	0.
time (sec)	N/A	0.25	0.792	0.416	0.	2.209	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	178	8605	0	2071	0	0
normalized size	1	1.	1.07	51.53	0.	12.4	0.	0.
time (sec)	N/A	0.333	1.77	0.556	0.	7.952	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	0	2606	0	1289	0	0
normalized size	1	1.	0.	19.3	0.	9.55	0.	0.
time (sec)	N/A	0.164	3.204	0.5	0.	24.082	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	0	2150	0	1095	0	0
normalized size	1	1.	0.	20.67	0.	10.53	0.	0.
time (sec)	N/A	0.127	2.055	0.407	0.	6.626	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	84	81	0	944	0	0
normalized size	1	1.	1.08	1.04	0.	12.1	0.	0.
time (sec)	N/A	0.082	0.279	0.056	0.	1.86	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	506	2424	0	2700	0	0
normalized size	1	1.	5.56	26.64	0.	29.67	0.	0.
time (sec)	N/A	0.136	5.687	0.339	0.	1.832	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	622	1609	0	3213	0	0
normalized size	1	1.	5.46	14.11	0.	28.18	0.	0.
time (sec)	N/A	0.17	5.844	0.458	0.	1.941	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	684	4955	0	4382	0	0
normalized size	1	1.	4.3	31.16	0.	27.56	0.	0.
time (sec)	N/A	0.243	6.067	0.456	0.	5.703	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	353	3583	0	4788	0	0
normalized size	1	1.	1.22	12.36	0.	16.51	0.	0.
time (sec)	N/A	0.571	6.58	0.89	0.	46.795	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	258	2757	0	4290	0	0
normalized size	1	1.	1.21	12.88	0.	20.05	0.	0.
time (sec)	N/A	0.478	4.33	0.578	0.	14.48	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	703	2002	0	3929	0	0
normalized size	1	1.	4.23	12.06	0.	23.67	0.	0.
time (sec)	N/A	0.365	6.587	0.402	0.	4.391	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	527	1557	0	3602	0	0
normalized size	1	1.	4.47	13.19	0.	30.53	0.	0.
time (sec)	N/A	0.096	1.988	0.313	0.	1.784	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	410	1952	0	3586	0	0
normalized size	1	1.	3.69	17.59	0.	32.31	0.	0.
time (sec)	N/A	0.228	6.643	0.534	0.	2.027	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	100	2015	0	1428	0	0
normalized size	1	1.	0.89	17.99	0.	12.75	0.	0.
time (sec)	N/A	0.275	0.356	0.351	0.	2.665	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	139	5850	0	1882	0	0
normalized size	1	1.	0.84	35.45	0.	11.41	0.	0.
time (sec)	N/A	0.355	1.505	0.494	0.	10.344	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	0	358	0	1006	0	0
normalized size	1	1.	0.	4.02	0.	11.3	0.	0.
time (sec)	N/A	0.133	2.122	0.44	0.	2.079	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	0	303	0	807	0	541
normalized size	1	1.	0.	5.41	0.	14.41	0.	9.66
time (sec)	N/A	0.095	1.656	0.389	0.	0.902	0.	2.511

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	B	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	0	42	0	652	0	167
normalized size	1	1.	0.	1.27	0.	19.76	0.	5.06
time (sec)	N/A	0.055	0.123	0.072	0.	0.684	0.	1.85

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	376	0	2527	0	0
normalized size	1	1.	0.	5.37	0.	36.1	0.	0.
time (sec)	N/A	0.106	2.356	0.407	0.	1.172	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	0	4245	0	3742	0	0
normalized size	1	1.	0.	36.59	0.	32.26	0.	0.
time (sec)	N/A	0.164	5.029	0.48	0.	1.959	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	0	10441	0	5370	0	0
normalized size	1	1.	0.	62.9	0.	32.35	0.	0.
time (sec)	N/A	0.244	8.485	0.564	0.	4.974	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	230	1993	0	4045	0	0
normalized size	1	1.	1.33	11.52	0.	23.38	0.	0.
time (sec)	N/A	0.319	5.213	0.504	0.	4.891	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	196	1320	0	3686	0	0
normalized size	1	1.	1.63	11.	0.	30.72	0.	0.
time (sec)	N/A	0.23	3.297	0.41	0.	2.062	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	404	0	3036	0	0
normalized size	1	1.	0.	5.05	0.	37.95	0.	0.
time (sec)	N/A	0.197	2.979	0.435	0.	1.31	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	380	0	976	0	0
normalized size	1	1.	2.23	9.74	0.	25.03	0.	0.
time (sec)	N/A	0.029	0.122	0.418	0.	0.767	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	127	1865	0	1272	0	0
normalized size	1	1.	1.72	25.2	0.	17.19	0.	0.
time (sec)	N/A	0.192	0.231	0.518	0.	0.978	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	168	5619	0	1705	0	0
normalized size	1	1.	1.41	47.22	0.	14.33	0.	0.
time (sec)	N/A	0.249	1.956	0.489	0.	2.076	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	199	11267	0	2334	0	0
normalized size	1	1.	1.16	65.51	0.	13.57	0.	0.
time (sec)	N/A	0.345	4.593	0.641	0.	6.611	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	6593	0	1095	0	0
normalized size	1	1.	0.	74.92	0.	12.44	0.	0.
time (sec)	N/A	0.151	5.264	0.529	0.	2.832	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	187	2371	0	1022	0	0
normalized size	1	1.	2.97	37.63	0.	16.22	0.	0.
time (sec)	N/A	0.116	4.269	0.372	0.	1.034	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	382	64	0	971	53	336
normalized size	1	1.	6.7	1.12	0.	17.04	0.93	5.89
time (sec)	N/A	0.072	7.16	0.056	0.	0.952	11.073	2.071

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	0	9693	0	3750	0	0
normalized size	1	1.	0.	96.93	0.	37.5	0.	0.
time (sec)	N/A	0.147	6.818	0.512	0.	2.166	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	B	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	0	19968	0	5342	0	0
normalized size	1	1.	0.	130.51	0.	34.92	0.	0.
time (sec)	N/A	0.242	10.772	0.967	0.	5.62	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	0	32565	0	7887	0	0
normalized size	1	1.	0.	152.89	0.	37.03	0.	0.
time (sec)	N/A	0.334	14.271	1.522	0.	17.283	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	247	3860	0	4514	0	0
normalized size	1	1.	1.44	22.44	0.	26.24	0.	0.
time (sec)	N/A	0.349	10.061	0.433	0.	6.322	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	201	2662	0	3985	0	0
normalized size	1	1.	1.73	22.95	0.	34.35	0.	0.
time (sec)	N/A	0.249	4.69	0.529	0.	2.314	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	169	569	0	1322	0	0
normalized size	1	1.	2.38	8.01	0.	18.62	0.	0.
time (sec)	N/A	0.209	2.64	0.434	0.	1.066	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	168	1007	0	1438	0	0
normalized size	1	1.	2.18	13.08	0.	18.68	0.	0.
time (sec)	N/A	0.046	1.452	0.433	0.	1.052	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	182	2836	0	1728	0	0
normalized size	1	1.	1.53	23.83	0.	14.52	0.	0.
time (sec)	N/A	0.272	4.44	0.556	0.	2.462	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	224	7541	0	2388	0	0
normalized size	1	1.	1.29	43.34	0.	13.72	0.	0.
time (sec)	N/A	0.362	5.66	0.489	0.	7.525	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	237	14137	0	3425	0	0
normalized size	1	1.	0.98	58.66	0.	14.21	0.	0.
time (sec)	N/A	0.471	11.144	0.697	0.	24.112	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	187	10947	0	1332	0	0
normalized size	1	1.	1.93	112.86	0.	13.73	0.	0.
time (sec)	N/A	0.166	8.613	2.303	0.	3.294	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	613	10839	0	1264	0	0
normalized size	1	1.	6.89	121.79	0.	14.2	0.	0.
time (sec)	N/A	0.128	10.446	2.165	0.	2.812	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	613	86	0	1208	78	613
normalized size	1	1.	7.39	1.04	0.	14.55	0.94	7.39
time (sec)	N/A	0.088	7.608	0.056	0.	2.365	66.105	2.041

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	0	68989	0	5253	0	0
normalized size	1	1.	0.	503.57	0.	38.34	0.	0.
time (sec)	N/A	0.207	9.145	2.566	0.	5.584	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	0	105237	0	7761	0	0
normalized size	1	1.	0.	526.18	0.	38.8	0.	0.
time (sec)	N/A	0.32	19.862	3.917	0.	18.009	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	0	145925	0	10710	0	0
normalized size	1	1.	0.	544.5	0.	39.96	0.	0.
time (sec)	N/A	0.449	30.519	15.336	0.	64.468	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	316	2256	0	4811	0	0
normalized size	1	1.	2.01	14.37	0.	30.64	0.	0.
time (sec)	N/A	0.345	11.658	0.469	0.	7.531	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	409	1142	0	1584	0	0
normalized size	1	1.	3.41	9.52	0.	13.2	0.	0.
time (sec)	N/A	0.267	6.548	0.401	0.	3.112	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	410	2112	0	1804	0	0
normalized size	1	1.	3.45	17.75	0.	15.16	0.	0.
time (sec)	N/A	0.257	4.839	0.418	0.	2.683	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	1927	3024	0	2021	0	0
normalized size	1	1.	15.42	24.19	0.	16.17	0.	0.
time (sec)	N/A	0.099	6.472	0.421	0.	2.291	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	247	7586	0	2473	0	0
normalized size	1	1.	1.42	43.6	0.	14.21	0.	0.
time (sec)	N/A	0.378	7.829	0.595	0.	7.656	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	234	15128	0	3449	0	0
normalized size	1	1.	0.99	64.1	0.	14.61	0.	0.
time (sec)	N/A	0.482	14.58	0.708	0.	24.494	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	272	22712	0	4632	0	0
normalized size	1	1.	0.86	72.1	0.	14.7	0.	0.
time (sec)	N/A	0.604	26.695	0.924	0.	74.047	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	259	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	3.801	0.804	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	94	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.6	0.487	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	61	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.167	0.396	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.053	0.477	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	115	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	2.152	0.442	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	139	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	3.536	0.354	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	2777	0	0	0	0	0
normalized size	1	1.	31.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	18.532	0.425	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	2465	0	0	0	0	0
normalized size	1	1.	28.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	16.919	0.356	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0
normalized size	1	1.	25.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	6.269	0.007	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	2469	0	0	0	0	0
normalized size	1	1.	29.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	17.294	0.366	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	3033	0	0	0	0	0
normalized size	1	1.	34.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	18.835	0.4	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	87	183	99	227	119	495
normalized size	1	1.	0.95	1.99	1.08	2.47	1.29	5.38
time (sec)	N/A	0.069	0.27	0.054	0.995	0.544	16.771	3.083

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	126	69	157	82	394
normalized size	1	1.	0.97	2.07	1.13	2.57	1.34	6.46
time (sec)	N/A	0.054	0.136	0.05	1.393	0.527	4.426	1.84

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	28	38	93	42	262
normalized size	1	1.	1.	0.93	1.27	3.1	1.4	8.73
time (sec)	N/A	0.023	0.013	0.02	1.036	0.509	2.322	1.303

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	65	48	61	177	0	122
normalized size	1	1.	1.2	0.89	1.13	3.28	0.	2.26
time (sec)	N/A	0.07	0.063	0.05	0.999	0.514	0.	1.222

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	114	69	84	254	0	244
normalized size	1	1.	1.58	0.96	1.17	3.53	0.	3.39
time (sec)	N/A	0.063	1.092	0.054	1.004	0.518	0.	1.417

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	251	274	0	10531	0	0
normalized size	1	1.	1.15	1.25	0.	48.09	0.	0.
time (sec)	N/A	0.321	0.366	0.063	0.	3.621	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	242	141	0	5415	0	1077
normalized size	1	1.	1.46	0.85	0.	32.62	0.	6.49
time (sec)	N/A	0.147	0.244	0.066	0.	72.243	0.	1.884

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	28	51	170	258
normalized size	1	1.	1.	1.61	1.22	2.22	7.39	11.22
time (sec)	N/A	0.031	0.019	0.026	0.998	0.552	82.6	1.229

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	290	393	0	12783	0	0
normalized size	1	1.	0.98	1.33	0.	43.33	0.	0.
time (sec)	N/A	0.517	0.389	0.081	0.	3.763	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	336	676	0	21913	0	0
normalized size	1	1.	0.85	1.72	0.	55.76	0.	0.
time (sec)	N/A	0.632	2.116	0.098	0.	7.896	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	3.251	2.434	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	221	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.524	6.892	0.573	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	171	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	4.125	0.546	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.09	0.553	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	3.823	0.584	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	2.346	0.537	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	1.944	0.493	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.901	0.484	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	1.875	0.541	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [255] had the largest ratio of [0.6]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	21	0.095
2	A	3	2	1.	21	0.095
3	A	3	2	1.	21	0.095
4	A	3	2	1.	19	0.105
5	A	3	3	1.	19	0.158
6	A	4	4	1.	21	0.19
7	A	5	4	1.	21	0.19
8	A	6	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	5	5	1.	21	0.238
10	A	4	4	1.	21	0.19
11	A	3	2	1.	12	0.167
12	A	3	2	1.	21	0.095
13	A	3	2	1.	21	0.095
14	A	3	2	1.	21	0.095
15	A	3	2	1.	23	0.087
16	A	3	2	1.	23	0.087
17	A	3	2	1.	21	0.095
18	A	4	3	1.	21	0.143
19	A	5	5	1.	23	0.217
20	A	6	5	1.	23	0.217
21	A	7	6	1.	23	0.261
22	A	6	5	1.	23	0.217
23	A	5	5	1.	23	0.217
24	A	4	3	1.	14	0.214
25	A	3	2	1.	23	0.087
26	A	3	2	1.	23	0.087
27	A	3	2	1.	23	0.087
28	A	4	3	1.	23	0.13
29	A	4	4	1.	23	0.174
30	A	3	3	1.	21	0.143
31	A	4	4	1.	21	0.19
32	A	5	5	1.	23	0.217
33	A	6	6	1.	23	0.261
34	A	7	7	1.	23	0.304
35	A	6	6	1.	23	0.261
36	A	5	5	1.	23	0.217
37	A	3	3	1.	14	0.214
38	A	3	3	1.	23	0.13
39	A	4	4	1.	23	0.174
40	A	4	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	6	5	1.	23	0.217
42	A	5	4	1.	23	0.174
43	A	4	4	1.	21	0.19
44	A	5	5	1.	21	0.238
45	A	6	6	1.	23	0.261
46	A	7	6	1.	23	0.261
47	A	8	7	1.	23	0.304
48	A	7	6	1.	23	0.261
49	A	6	6	1.	23	0.261
50	A	5	5	1.	14	0.357
51	A	4	4	1.	23	0.174
52	A	5	4	1.	23	0.174
53	A	6	5	1.	23	0.217
54	A	6	5	1.	23	0.217
55	A	6	5	1.	23	0.217
56	A	5	4	1.	21	0.19
57	A	6	6	1.	21	0.286
58	A	7	7	1.	23	0.304
59	A	8	7	1.	23	0.304
60	A	9	7	1.	23	0.304
61	A	8	6	1.	23	0.261
62	A	7	6	1.	23	0.261
63	A	6	6	1.	14	0.429
64	A	5	4	1.	23	0.174
65	A	6	5	1.	23	0.217
66	A	7	6	1.	23	0.261
67	A	6	6	1.	25	0.24
68	A	5	5	1.	25	0.2
69	A	4	4	1.	23	0.174
70	A	6	6	1.	23	0.261
71	A	7	7	1.	25	0.28
72	A	8	8	1.	25	0.32

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	9	8	1.	25	0.32
74	A	8	8	1.	25	0.32
75	A	7	7	1.	25	0.28
76	A	6	6	1.	16	0.375
77	A	4	4	1.	25	0.16
78	A	5	5	1.	25	0.2
79	A	6	6	1.	25	0.24
80	A	7	7	1.	25	0.28
81	A	6	6	1.	25	0.24
82	A	5	5	1.	23	0.217
83	A	7	7	1.	23	0.304
84	A	8	8	1.	25	0.32
85	A	9	9	1.	25	0.36
86	A	10	10	1.	25	0.4
87	A	9	9	1.	25	0.36
88	A	8	8	1.	25	0.32
89	A	7	7	1.	16	0.438
90	A	5	5	1.	25	0.2
91	A	6	6	1.	25	0.24
92	A	7	7	1.	25	0.28
93	A	4	4	1.	25	0.16
94	A	3	3	1.	25	0.12
95	A	2	2	1.	23	0.087
96	A	3	3	1.	23	0.13
97	A	5	5	1.	25	0.2
98	A	6	6	1.	25	0.24
99	A	7	7	1.	25	0.28
100	A	6	6	1.	25	0.24
101	A	5	5	1.	25	0.2
102	A	3	3	1.	16	0.188
103	A	2	2	1.	25	0.08
104	A	3	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	4	4	1.	25	0.16
106	A	5	5	1.	25	0.2
107	A	4	4	1.	25	0.16
108	A	3	3	1.	23	0.13
109	A	4	4	1.	23	0.174
110	A	6	6	1.	25	0.24
111	A	7	6	1.	25	0.24
112	A	8	7	1.	25	0.28
113	A	7	6	1.	25	0.24
114	A	6	6	1.	25	0.24
115	A	4	4	1.	16	0.25
116	A	3	3	1.	25	0.12
117	A	4	4	1.	25	0.16
118	A	5	5	1.	25	0.2
119	A	6	6	1.	25	0.24
120	A	5	5	1.	25	0.2
121	A	4	4	1.	23	0.174
122	A	6	6	1.	23	0.261
123	A	7	6	1.	25	0.24
124	A	8	6	1.	25	0.24
125	A	9	7	1.	25	0.28
126	A	8	6	1.	25	0.24
127	A	7	6	1.	25	0.24
128	A	6	6	1.	16	0.375
129	A	4	4	1.	25	0.16
130	A	5	5	1.	25	0.2
131	A	6	6	1.	25	0.24
132	F	0	0	N/A	0	N/A
133	A	5	5	1.	23	0.217
134	A	4	4	1.	23	0.174
135	A	3	3	1.	21	0.143
136	A	3	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	3	3	1.	23	0.13
138	A	3	3	1.	23	0.13
139	A	3	3	1.	23	0.13
140	A	3	3	1.	14	0.214
141	A	3	3	1.	23	0.13
142	A	4	4	1.	23	0.174
143	A	5	5	1.	23	0.217
144	A	6	3	1.	15	0.2
145	A	5	3	1.	15	0.2
146	A	4	3	1.	15	0.2
147	A	3	2	1.	13	0.154
148	A	3	3	1.	15	0.2
149	A	4	3	1.	15	0.2
150	A	5	3	1.	15	0.2
151	A	6	3	1.	15	0.2
152	A	4	3	1.	21	0.143
153	A	3	3	1.	21	0.143
154	A	2	2	1.	19	0.105
155	A	2	2	1.	19	0.105
156	A	3	2	1.	21	0.095
157	A	4	3	1.	21	0.143
158	A	3	2	1.	21	0.095
159	A	3	2	1.	21	0.095
160	A	3	3	1.	21	0.143
161	A	3	2	1.	12	0.167
162	A	2	2	1.	21	0.095
163	A	3	3	1.	21	0.143
164	A	4	3	1.	21	0.143
165	A	6	5	1.	23	0.217
166	A	5	5	1.	23	0.217
167	A	4	4	1.	21	0.19
168	A	5	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
169	A	4	3	1.	23	0.13
170	A	3	2	1.	23	0.087
171	A	3	2	1.	23	0.087
172	A	3	2	1.	23	0.087
173	A	3	2	1.	23	0.087
174	A	4	3	1.	14	0.214
175	A	5	4	1.	23	0.174
176	A	4	4	1.	23	0.174
177	A	5	5	1.	23	0.217
178	A	4	3	1.	14	0.214
179	A	4	3	1.	14	0.214
180	A	5	5	1.	23	0.217
181	A	4	4	1.	23	0.174
182	A	2	2	1.	21	0.095
183	A	3	3	1.	21	0.143
184	A	4	3	1.	23	0.13
185	A	4	3	1.	23	0.13
186	A	4	3	1.	23	0.13
187	A	3	3	1.	23	0.13
188	A	2	2	1.	23	0.087
189	A	3	3	1.	14	0.214
190	A	5	5	1.	23	0.217
191	A	6	6	1.	23	0.261
192	A	7	6	1.	23	0.261
193	A	5	5	1.	23	0.217
194	A	3	3	1.	23	0.13
195	A	3	3	1.	21	0.143
196	A	5	4	1.	21	0.19
197	A	5	4	1.	23	0.174
198	A	5	4	1.	23	0.174
199	A	5	4	1.	23	0.174
200	A	3	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	3	3	1.	23	0.13
202	A	5	5	1.	14	0.357
203	A	6	6	1.	23	0.261
204	A	7	6	1.	23	0.261
205	A	8	6	1.	23	0.261
206	A	4	3	1.	23	0.13
207	A	4	4	1.	23	0.174
208	A	4	4	1.	21	0.19
209	A	6	5	1.	21	0.238
210	A	6	5	1.	23	0.217
211	A	6	5	1.	23	0.217
212	A	4	4	1.	23	0.174
213	A	4	4	1.	23	0.174
214	A	4	3	1.	23	0.13
215	A	6	6	1.	14	0.429
216	A	7	6	1.	23	0.261
217	A	8	6	1.	23	0.261
218	A	9	6	1.	23	0.261
219	A	7	6	1.	14	0.429
220	A	6	4	1.	17	0.235
221	A	5	4	1.	17	0.235
222	A	4	4	1.	17	0.235
223	A	3	3	1.	17	0.176
224	A	3	3	1.	17	0.176
225	A	4	4	1.	17	0.235
226	A	5	4	1.	17	0.235
227	A	6	4	1.	17	0.235
228	A	11	10	1.27	25	0.4
229	A	10	10	1.26	25	0.4
230	A	10	10	1.24	23	0.435
231	A	5	5	1.29	23	0.217
232	A	9	9	1.22	25	0.36

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
233	A	10	10	1.18	25	0.4
234	A	6	6	1.	25	0.24
235	A	5	5	1.	25	0.2
236	A	4	4	1.	25	0.16
237	A	6	6	1.	16	0.375
238	A	4	4	1.	25	0.16
239	A	5	5	1.	25	0.2
240	A	7	6	1.	25	0.24
241	A	12	10	1.27	25	0.4
242	A	11	10	1.27	25	0.4
243	A	10	10	1.26	23	0.435
244	A	9	9	1.24	23	0.391
245	A	9	9	1.22	25	0.36
246	A	10	10	1.24	25	0.4
247	A	7	6	1.	25	0.24
248	A	6	5	1.	25	0.2
249	A	5	4	1.	25	0.16
250	A	7	7	1.	16	0.438
251	A	7	7	1.	25	0.28
252	A	5	4	1.	25	0.16
253	A	6	5	1.	25	0.2
254	A	8	8	1.	16	0.5
255	A	6	6	1.	10	0.6
256	A	5	5	1.	10	0.5
257	A	10	10	1.15	25	0.4
258	A	7	7	1.19	25	0.28
259	A	5	5	1.29	23	0.217
260	A	5	5	1.22	23	0.217
261	A	9	9	1.16	25	0.36
262	A	10	10	1.14	25	0.4
263	A	5	5	1.	25	0.2
264	A	4	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	3	3	1.	25	0.12
266	A	3	3	1.	16	0.188
267	A	4	4	1.	25	0.16
268	A	6	6	1.	25	0.24
269	A	7	6	1.	25	0.24
270	A	10	10	1.27	25	0.4
271	A	7	7	1.21	25	0.28
272	A	9	9	1.24	23	0.391
273	A	9	9	1.23	23	0.391
274	A	10	10	1.19	25	0.4
275	A	11	10	1.17	25	0.4
276	A	5	5	1.	25	0.2
277	A	4	4	1.	25	0.16
278	A	2	2	1.	25	0.08
279	A	4	4	1.	16	0.25
280	A	6	6	1.	25	0.24
281	A	7	6	1.	25	0.24
282	A	8	6	1.	25	0.24
283	A	10	10	1.19	25	0.4
284	A	10	10	1.19	25	0.4
285	A	10	10	1.19	23	0.435
286	A	10	10	1.18	23	0.435
287	A	11	11	1.16	25	0.44
288	A	12	11	1.14	25	0.44
289	A	5	5	1.	25	0.2
290	A	3	3	1.	25	0.12
291	A	3	3	1.	25	0.12
292	A	6	6	1.	16	0.375
293	A	7	6	1.	25	0.24
294	A	8	6	1.	25	0.24
295	A	9	6	1.	25	0.24
296	A	7	6	1.	16	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	3	3	1.	10	0.3
298	F	0	0	N/A	0	N/A
299	A	5	5	1.2	23	0.217
300	A	5	5	1.2	21	0.238
301	A	5	5	1.21	21	0.238
302	A	5	5	1.2	23	0.217
303	A	5	5	1.2	23	0.217
304	A	5	5	1.	23	0.217
305	A	4	4	1.	23	0.174
306	A	3	3	1.	23	0.13
307	A	3	3	1.	14	0.214
308	A	3	3	1.	23	0.13
309	A	3	3	1.	23	0.13
310	A	3	3	1.	23	0.13
311	A	4	3	1.	21	0.143
312	A	4	3	1.	21	0.143
313	A	3	2	1.	19	0.105
314	A	4	3	1.	19	0.158
315	A	4	3	1.	21	0.143
316	A	4	3	1.	21	0.143
317	A	4	3	1.	21	0.143
318	A	4	3	1.	21	0.143
319	A	4	3	1.	21	0.143
320	A	3	2	1.	12	0.167
321	A	4	3	1.	21	0.143
322	A	4	3	1.	21	0.143
323	A	4	3	1.	21	0.143
324	A	4	3	1.	23	0.13
325	A	4	3	1.	23	0.13
326	A	4	3	1.	21	0.143
327	A	4	3	1.	21	0.143
328	A	4	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	4	3	1.	23	0.13
330	A	4	3	1.	23	0.13
331	A	4	3	1.	23	0.13
332	A	4	3	1.	23	0.13
333	A	4	3	1.	14	0.214
334	A	4	3	1.	23	0.13
335	A	4	3	1.	23	0.13
336	A	4	3	1.	23	0.13
337	A	4	3	1.	23	0.13
338	A	4	3	1.	23	0.13
339	A	2	2	1.	21	0.095
340	A	4	3	1.	21	0.143
341	A	4	3	1.	23	0.13
342	A	4	3	1.	23	0.13
343	A	7	7	1.	23	0.304
344	A	6	6	1.	23	0.261
345	A	5	5	1.	23	0.217
346	A	3	3	1.	14	0.214
347	A	6	6	1.	23	0.261
348	A	7	7	1.	23	0.304
349	A	8	7	1.	23	0.304
350	A	4	3	1.	23	0.13
351	A	4	3	1.	23	0.13
352	A	4	3	1.	21	0.143
353	A	4	3	1.	21	0.143
354	A	4	3	1.	23	0.13
355	A	4	3	1.	23	0.13
356	A	7	7	1.	23	0.304
357	A	6	6	1.	23	0.261
358	A	6	6	1.	23	0.261
359	A	5	5	1.	14	0.357
360	A	7	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	8	7	1.	23	0.304
362	A	9	7	1.	23	0.304
363	A	4	3	1.	23	0.13
364	A	4	3	1.	23	0.13
365	A	4	3	1.	21	0.143
366	A	4	3	1.	21	0.143
367	A	4	3	1.	23	0.13
368	A	4	3	1.	23	0.13
369	A	7	7	1.	23	0.304
370	A	7	7	1.	23	0.304
371	A	7	7	1.	23	0.304
372	A	6	6	1.	14	0.429
373	A	8	8	1.	23	0.348
374	A	9	8	1.	23	0.348
375	A	10	8	1.	23	0.348
376	A	7	6	1.	25	0.24
377	A	6	6	1.	25	0.24
378	A	5	5	1.	23	0.217
379	A	7	5	1.	23	0.217
380	A	8	6	1.	25	0.24
381	A	9	7	1.	25	0.28
382	A	10	9	1.	25	0.36
383	A	9	9	1.	25	0.36
384	A	8	8	1.	25	0.32
385	A	6	6	1.	16	0.375
386	A	6	6	1.	25	0.24
387	A	7	7	1.	25	0.28
388	A	8	7	1.	25	0.28
389	A	8	6	1.	25	0.24
390	A	7	6	1.	25	0.24
391	A	6	5	1.	23	0.217
392	A	8	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
393	A	8	6	1.	25	0.24
394	A	9	7	1.	25	0.28
395	A	11	9	1.	25	0.36
396	A	10	9	1.	25	0.36
397	A	9	9	1.	25	0.36
398	A	7	7	1.	16	0.438
399	A	8	8	1.	25	0.32
400	A	7	7	1.	25	0.28
401	A	8	7	1.	25	0.28
402	A	6	5	1.	25	0.2
403	A	5	5	1.	25	0.2
404	A	4	4	1.	23	0.174
405	A	7	5	1.	23	0.217
406	A	8	6	1.	25	0.24
407	A	9	7	1.	25	0.28
408	A	9	9	1.	25	0.36
409	A	8	8	1.	25	0.32
410	A	7	7	1.	25	0.28
411	A	3	3	1.	16	0.188
412	A	6	6	1.	25	0.24
413	A	7	7	1.	25	0.28
414	A	8	7	1.	25	0.28
415	A	6	5	1.	25	0.2
416	A	5	5	1.	25	0.2
417	A	5	5	1.	23	0.217
418	A	8	6	1.	23	0.261
419	A	9	7	1.	25	0.28
420	A	10	8	1.	25	0.32
421	A	9	9	1.	25	0.36
422	A	8	8	1.	25	0.32
423	A	5	5	1.	25	0.2
424	A	4	4	1.	16	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
425	A	7	7	1.	25	0.28
426	A	8	7	1.	25	0.28
427	A	9	7	1.	25	0.28
428	A	6	5	1.	25	0.2
429	A	6	6	1.	25	0.24
430	A	6	5	1.	23	0.217
431	A	9	7	1.	23	0.304
432	A	10	7	1.	25	0.28
433	A	11	8	1.	25	0.32
434	A	9	9	1.	25	0.36
435	A	7	7	1.	25	0.28
436	A	7	7	1.	25	0.28
437	A	6	6	1.	16	0.375
438	A	8	8	1.	25	0.32
439	A	9	8	1.	25	0.32
440	A	10	8	1.	25	0.32
441	A	4	4	1.	25	0.16
442	A	5	4	1.	23	0.174
443	A	4	4	1.	23	0.174
444	A	3	3	1.	21	0.143
445	A	5	5	1.	21	0.238
446	A	6	6	1.	23	0.261
447	A	4	4	1.	23	0.174
448	A	4	4	1.	23	0.174
449	A	3	3	1.	14	0.214
450	A	4	4	1.	23	0.174
451	A	4	4	1.	23	0.174
452	A	3	2	1.	21	0.095
453	A	3	2	1.	21	0.095
454	A	3	2	1.	19	0.105
455	A	3	2	1.	19	0.105
456	A	5	4	1.	21	0.19

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
457	A	11	10	1.	23	0.435
458	A	9	9	1.	23	0.391
459	A	2	2	1.	21	0.095
460	A	11	10	1.	21	0.476
461	A	11	10	1.	23	0.435
462	A	0	0	0.	0	0.
463	A	15	8	1.	25	0.32
464	A	11	8	1.	25	0.32
465	A	5	5	1.	23	0.217
466	A	0	0	0.	0	0.
467	A	0	0	0.	0	0.
468	A	0	0	0.	0	0.
469	A	0	0	0.	0	0.
470	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int \left(a+b \sec^2(e+fx)\right) \sin^7(e+fx)\, dx$

Optimal. Leaf size=83

$$-\frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/f - ((3*a - b)*Cos
[e + f*x]^5)/(5*f) + (a*Cos[e + f*x]^7)/(7*f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.0614125, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 448}

$$-\frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(a-3b)\cos(e+fx)}{f}+\frac{a\cos^7(e+fx)}{7f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

```
[Out] -(((a - 3*b)*Cos[e + f*x])/f) + ((a - b)*Cos[e + f*x]^3)/f - ((3*a - b)*Cos
[e + f*x]^5)/(5*f) + (a*Cos[e + f*x]^7)/(7*f) + (b*Sec[e + f*x])/f
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3\left(b+ax^2\right)}{x^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{3b}{a}\right)+\frac{b}{x^2}-3(a-b)x^2+(3a-b)x^4-ax^6\right)dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a-3b)\cos(e+fx)}{f}+\frac{(a-b)\cos^3(e+fx)}{f}-\frac{(3a-b)\cos^5(e+fx)}{5f}+\frac{a\cos^7(\ldots}{7\ldots}$$

Mathematica [A] time = 0.0836027, size = 120, normalized size = 1.45

$$-\frac{35a\cos(e+fx)}{64f}+\frac{7a\cos(3(e+fx))}{64f}-\frac{7a\cos(5(e+fx))}{320f}+\frac{a\cos(7(e+fx))}{448f}+\frac{19b\cos(e+fx)}{8f}-\frac{3b\cos(3(e+fx))}{16f}+\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

```
[Out] (-35*a*Cos[e + f*x])/(64*f) + (19*b*Cos[e + f*x])/(8*f) + (7*a*Cos[3*(e + f
*x)])/(64*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (7*a*Cos[5*(e + f*x)])/(320*
f) + (b*Cos[5*(e + f*x)])/(80*f) + (a*Cos[7*(e + f*x)])/(448*f) + (b*Sec[e
+ f*x])/f
```

Maple [A] time = 0.047, size = 102, normalized size = 1.2

$$\frac{1}{f}\left(-\frac{a\cos\left(fx+e\right)}{7}\left(\frac{16}{5}+\left(\sin\left(fx+e\right)\right)^{6}+\frac{6\left(\sin\left(fx+e\right)\right)^{4}}{5}+\frac{8\left(\sin\left(fx+e\right)\right)^{2}}{5}\right)+b\left(\frac{\left(\sin\left(fx+e\right)\right)^{8}}{\cos\left(fx+e\right)}+\left(\frac{16}{5}+\left(\sin\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x)
```

```
[Out] 1/f*(-1/7*a*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e
)+b*(sin(f*x+e)^8/cos(f*x+e)+(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*
x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 1.00339, size = 99, normalized size = 1.19

$$\frac{5\,a\cos\left(fx+e\right)^{7}-7\left(3\,a-b\right)\cos\left(fx+e\right)^{5}+35\left(a-b\right)\cos\left(fx+e\right)^{3}-35\left(a-3\,b\right)\cos\left(fx+e\right)+\frac{35\,b}{\cos\left(fx+e\right)}}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="maxima")
```

```
[Out] 1/35*(5*a*cos(f*x + e)^7 - 7*(3*a - b)*cos(f*x + e)^5 + 35*(a - b)*cos(f*x
+ e)^3 - 35*(a - 3*b)*cos(f*x + e) + 35*b/cos(f*x + e))/f
```

Fricas [A] time = 0.928366, size = 186, normalized size = 2.24

$$\frac{5\,a\cos\left(fx+e\right)^{8}-7\left(3\,a-b\right)\cos\left(fx+e\right)^{6}+35\left(a-b\right)\cos\left(fx+e\right)^{4}-35\left(a-3\,b\right)\cos\left(fx+e\right)^{2}+35\,b}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*cos(f*x + e)^8 - 7*(3*a - b)*cos(f*x + e)^6 + 35*(a - b)*cos(f*x
+ e)^4 - 35*(a - 3*b)*cos(f*x + e)^2 + 35*b)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19947, size = 389, normalized size = 4.69

$$\frac{2\left(\frac{35\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}+\frac{16\,a-77\,b-\frac{112\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{504\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{336\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{1337\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{560\,a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{1680\,b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{1015}{\ldots}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^7}\right.}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="giac")
```

```
[Out] 2/35*(35*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (16*a - 77*b - 112
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 504*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 336*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 1337*b*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 560*a*(cos(f*x + e) - 1)^3/(cos(f*x
 + e) + 1)^3 + 1680*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 1015*b*(c
os(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 280*b*(cos(f*x + e) - 1)^5/(cos(f
*x + e) + 1)^5 - 35*b*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 1)^7)/f
```

3.2 $\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx$

Optimal. Leaf size=66

$$\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((2*a - b)*Cos[e + f*x]^3)/(3*f) - (a*Cos[e
 + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.051205, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 448}

$$\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{(a-2b)\cos(e+fx)}{f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

```
[Out] -(((a - 2*b)*Cos[e + f*x])/f) + ((2*a - b)*Cos[e + f*x]^3)/(3*f) - (a*Cos[e
 + f*x]^5)/(5*f) + (b*Sec[e + f*x])/f
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2(b+ax^2)}{x^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{2b}{a}\right)+\frac{b}{x^2}-(2a-b)x^2+ax^4\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{(a-2b)\cos(e+fx)}{f}+\frac{(2a-b)\cos^3(e+fx)}{3f}-\frac{a\cos^5(e+fx)}{5f}+\frac{b\sec(e+fx)}{f}$$

Mathematica [A] time = 0.0445724, size = 88, normalized size = 1.33

$$-\frac{5a\cos(e+fx)}{8f}+\frac{5a\cos(3(e+fx))}{48f}-\frac{a\cos(5(e+fx))}{80f}+\frac{7b\cos(e+fx)}{4f}-\frac{b\cos(3(e+fx))}{12f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

```
[Out] (-5*a*Cos[e + f*x])/(8*f) + (7*b*Cos[e + f*x])/(4*f) + (5*a*Cos[3*(e + f*x)
])/(48*f) - (b*Cos[3*(e + f*x)])/(12*f) - (a*Cos[5*(e + f*x)])/(80*f) + (b*
Sec[e + f*x])/f
```

Maple [A] time = 0.043, size = 82, normalized size = 1.2

$$\frac{1}{f}\left(-\frac{a\cos(fx+e)}{5}\left(\frac{8}{3}+(\sin(fx+e))^4+\frac{4(\sin(fx+e))^2}{3}\right)+b\left(\frac{(\sin(fx+e))^6}{\cos(fx+e)}+\left(\frac{8}{3}+(\sin(fx+e))^4+\frac{4(\sin(fx+e))^2}{3}\right)\cos(fx+e)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x)
```

```
[Out] 1/f*(-1/5*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^6/
cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 1.01269, size = 78, normalized size = 1.18

$$-\frac{3\,a\cos\left(fx+e\right)^{5}-5\,(2\,a-b)\cos\left(fx+e\right)^{3}+15\,(a-2\,b)\cos\left(fx+e\right)-\frac{15\,b}{\cos\left(fx+e\right)}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*a*cos(f*x + e)^5 - 5*(2*a - b)*cos(f*x + e)^3 + 15*(a - 2*b)*cos(f
*x + e) - 15*b/cos(f*x + e))/f
```

Fricas [A] time = 0.839111, size = 150, normalized size = 2.27

$$-\frac{3\,a\cos\left(fx+e\right)^{6}-5\,(2\,a-b)\cos\left(fx+e\right)^{4}+15\,(a-2\,b)\cos\left(fx+e\right)^{2}-15\,b}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*a*cos(f*x + e)^6 - 5*(2*a - b)*cos(f*x + e)^4 + 15*(a - 2*b)*cos(f
*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22106, size = 288, normalized size = 4.36

$$\frac{2\left(\frac{15\,b}{\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1}+\frac{8\,a-25\,b-\frac{40\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{110\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{80\,a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{160\,b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{90\,b\left(\cos\left(fx+e\right)-1\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{15\,b\left(\cos\left(fx+e\right)-1\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}{\left(\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-1\right)^{5}}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/15*(15*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (8*a - 25*b - 40*a
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 110*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 80*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 160*b*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3 - 15*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) -
1)/(cos(f*x + e) + 1) - 1)^5)/f
```

3.3 $\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx$

Optimal. Leaf size=44

$$-\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -(((a - b)*Cos[e + f*x])/f) + (a*Cos[e + f*x]^3)/(3*f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.0389122, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 448}

$$-\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

```
[Out] -(((a - b)*Cos[e + f*x])/f) + (a*Cos[e + f*x]^3)/(3*f) + (b*Sec[e + f*x])/f
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{b}{a}\right)+\frac{b}{x^2}-ax^2\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{(a-b)\cos(e+fx)}{f}+\frac{a\cos^3(e+fx)}{3f}+\frac{b\sec(e+fx)}{f}$$

Mathematica [A] time = 0.0324777, size = 53, normalized size = 1.2

$$-\frac{3a\cos(e+fx)}{4f}+\frac{a\cos(3(e+fx))}{12f}+\frac{b\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

```
[Out] (-3*a*Cos[e + f*x])/(4*f) + (b*Cos[e + f*x])/f + (a*Cos[3*(e + f*x)])/(12*f
) + (b*Sec[e + f*x])/f
```

Maple [A] time = 0.041, size = 62, normalized size = 1.4

$$\frac{1}{f}\left(-\frac{a\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+b\left(\frac{\left(\sin\left(fx+e\right)\right)^4}{\cos\left(fx+e\right)}+\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x)
```

```
[Out] 1/f*(-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(f
*x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 0.975774, size = 54, normalized size = 1.23

$$\frac{a\cos\left(fx+e\right)^{3}-3\left(a-b\right)\cos\left(fx+e\right)+\frac{3b}{\cos\left(fx+e\right)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a*cos(f*x + e)^3 - 3*(a - b)*cos(f*x + e) + 3*b/cos(f*x + e))/f
```

Fricas [A] time = 1.01928, size = 100, normalized size = 2.27

$$\frac{a\cos\left(fx+e\right)^{4}-3\left(a-b\right)\cos\left(fx+e\right)^{2}+3b}{3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^4 - 3*(a - b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27419, size = 82, normalized size = 1.86

$$\frac{b}{f\cos\left(fx+e\right)}+\frac{af^{5}\cos\left(fx+e\right)^{3}-3af^{5}\cos\left(fx+e\right)+3bf^{5}\cos\left(fx+e\right)}{3f^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - 3*a*f^5*cos(f*x + e) + 3*b
*f^5*cos(f*x + e))/f^6
```

3.4 $\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx$

Optimal. Leaf size=24

$$\frac{b\sec(e+fx)}{f}-\frac{a\cos(e+fx)}{f}$$

```
[Out] -((a*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.0202495, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4133, 14}

$$\frac{b\sec(e+fx)}{f}-\frac{a\cos(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

```
[Out] -((a*Cos[e + f*x])/f) + (b*Sec[e + f*x])/f
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a+\frac{b}{x^2}\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{a\cos(e+fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Mathematica [A] time = 0.0175184, size = 35, normalized size = 1.46

$$\frac{a\sin(e)\sin(fx)}{f}-\frac{a\cos(e)\cos(fx)}{f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

```
[Out] -((a*Cos[e]*Cos[f*x])/f) + (b*Sec[e + f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

Maple [A] time = 0.019, size = 25, normalized size = 1.

$$\frac{1}{f}\left(b\sec\left(fx+e\right)-\frac{a}{\sec\left(fx+e\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e),x)
```

```
[Out] 1/f*(b*sec(f*x+e)-1/sec(f*x+e)*a)
```

Maxima [A] time = 1.0071, size = 34, normalized size = 1.42

$$-\frac{a\cos\left(fx+e\right)-\frac{b}{\cos\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] -(a*cos(f*x + e) - b/cos(f*x + e))/f
```

Fricas [A] time = 0.900601, size = 57, normalized size = 2.38

$$-\frac{a\cos\left(fx+e\right)^{2}-b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] -(a*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

Giac [A] time = 1.29776, size = 38, normalized size = 1.58

$$-\frac{a\cos\left(fx+e\right)}{f}+\frac{b}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] -a*cos(f*x + e)/f + b/(f*cos(f*x + e))
```

3.5 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)dx$

Optimal. Leaf size=27

$$\frac{b\sec(e+fx)}{f}-\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f}$$

```
[Out] -(((a + b)*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.0308487, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4133, 453, 206}

$$\frac{b\sec(e+fx)}{f}-\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + b)*ArcTanh[Cos[e + f*x]])/f) + (b*Sec[e + f*x])/f
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f}\\ &= \frac{b\sec(e+fx)}{f}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{(a+b)\tanh^{-1}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

Mathematica [B] time = 0.039863, size = 84, normalized size = 3.11

$$\frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}+\frac{b\sec(e+fx)}{f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Log[Cos[e/2 + (f*x)/2]])/f) - (b*Log[Cos[(e + f*x)/2]])/f + (a*Log[Sin
[e/2 + (f*x)/2]])/f + (b*Log[Sin[(e + f*x)/2]])/f + (b*Sec[e + f*x])/f
```

Maple [B] time = 0.04, size = 57, normalized size = 2.1

$$\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{b}{f\cos\left(fx+e\right)}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*a*ln(csc(f*x+e)-cot(f*x+e))+1/f*b/cos(f*x+e)+1/f*b*ln(csc(f*x+e)-cot(f*
x+e))
```

Maxima [A] time = 1.02554, size = 59, normalized size = 2.19

$$-\frac{(a+b)\log\left(\cos\left(fx+e\right)+1\right)-(a+b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2b}{\cos\left(fx+e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)*log(cos(f*x + e) + 1) - (a + b)*log(cos(f*x + e) - 1) - 2*b/c
os(f*x + e))/f
```

Fricas [B] time = 0.836694, size = 178, normalized size = 6.59

$$-\frac{(a+b)\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-(a+b)\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-2b}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a + b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - (a + b)*cos(f*x +
e)*log(-1/2*cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\csc\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

Giac [B] time = 1.3427, size = 82, normalized size = 3.04

$$\frac{(a+b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+\frac{4\,b}{\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 4*b/((cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

3.6 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)dx$

Optimal. Leaf size=53

$$-\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] -((a + 3*b)*ArcTanh[Cos[e + f*x]])/(2*f) - ((a + b)*Cot[e + f*x]*Csc[e + f*
x])/(2*f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.0522348, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 456, 453, 206}

$$-\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f}-\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + 3*b)*ArcTanh[Cos[e + f*x]])/(2*f) - ((a + b)*Cot[e + f*x]*Csc[e + f*
x])/(2*f) + (b*Sec[e + f*x])/f
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2\left(1-x^2\right)^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f} + \frac{\text{Subst}\left(\int \frac{-2b-(a+b)x^2}{x^2\left(1-x^2\right)}dx, x, \cos(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+fx)}{f} - \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}dx, \ldots\right.}{2f}$$

$$= -\frac{(a+3b)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+f\ldots}{f}$$

Mathematica [B] time = 0.387665, size = 236, normalized size = 4.45

$$-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} - \frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} + \frac{b\,\text{s}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(a*Csc[(e + f*x)/2]^2)/(8*f) - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[(
e + f*x)/2]])/(2*f) - (3*b*Log[Cos[(e + f*x)/2]])/(2*f) + (a*Log[Sin[(e + f
```

```
*x)/2]])/(2*f) + (3*b*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]^2)
/(8*f) + (b*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])) - (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2]))
```

Maple [B] time = 0.05, size = 100, normalized size = 1.9

$$-\frac{a\csc\left(fx+e\right)\cot\left(fx+e\right)}{2f}+\frac{a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}-\frac{b}{2f\left(\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)}+\frac{3b}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2/f*a*csc(f*x+e)*cot(f*x+e)+1/2/f*a*ln(csc(f*x+e)-cot(f*x+e))-1/2/f*b/si
n(f*x+e)^2/cos(f*x+e)+3/2/f*b/cos(f*x+e)+3/2/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.01338, size = 103, normalized size = 1.94

$$-\frac{(a+3b)\log\left(\cos\left(fx+e\right)+1\right)-(a+3b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left((a+3b)\cos\left(fx+e\right)^{2}-2b\right)}{\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 3*b)*log(cos(f*x + e) + 1) - (a + 3*b)*log(cos(f*x + e) - 1) - 2
*((a + 3*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

Fricas [B] time = 0.815347, size = 325, normalized size = 6.13

$$\frac{2(a+3b)\cos\left(fx+e\right)^{2}-\left((a+3b)\cos\left(fx+e\right)^{3}-(a+3b)\cos\left(fx+e\right)\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\left((a+3b)\cos\left(f\right.\right.}{4\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a + 3*b)*cos(f*x + e)^2 - ((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*cos
(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 3*b)*cos(f*x + e)^3 - (a + 3
*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3 - f
*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26612, size = 278, normalized size = 5.25

$$\frac{2\,(a+3\,b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-\frac{a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a+b+\frac{14\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{3\,b\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}}{\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+\frac{\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 3*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - a*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) - b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + (a
+ b + 14*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 - 3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + (cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2))/f
```

3.7 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=81

$$-\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] (-3*(a + 5*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((3*a + 7*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - ((a + b)*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f) + (b*Sec[e +
f*x])/f
```

Rubi [A] time = 0.074872, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 456, 453, 206}

$$-\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f}-\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-3*(a + 5*b)*ArcTanh[Cos[e + f*x]])/(8*f) - ((3*a + 7*b)*Cot[e + f*x]*Csc[
e + f*x])/(8*f) - ((a + b)*Cot[e + f*x]*Csc[e + f*x]^3)/(4*f) + (b*Sec[e +
f*x])/f
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)dx = -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^2\left(1-x^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{\text{Subst}\left(\int \frac{-4b-3(a+b)x^2}{x^2\left(1-x^2\right)^2}\,dx, x, \cos(e+fx)\right)}{4f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} - \frac{\text{Subst}}{}$$

$$= -\frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{b\sec(}{}$$

$$= -\frac{3(a+5b)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)}{}$$

Mathematica [B] time = 1.84283, size = 198, normalized size = 2.44

$$\frac{-(a+b)\csc^4\left(\frac{1}{2}(e+fx)\right) - 2(3a+7b)\csc^2\left(\frac{1}{2}(e+fx)\right) + \frac{-(a+b)\sec^4\left(\frac{1}{2}(e+fx)\right)+\tan^2\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)((3a+7b)\cos(e+fx)+4(a+2b}{}}{64f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-2*(3*a + 7*b)*Csc[(e + f*x)/2]^2 - (a + b)*Csc[(e + f*x)/2]^4 + (2*(-3*(a
 + 13*b) + 4*Cos[e + f*x]*(8*b + 3*(a + 5*b)*Log[Cos[(e + f*x)/2]] - 3*(a +
 5*b)*Log[Sin[(e + f*x)/2]]))*Sec[(e + f*x)/2]^2 - (a + b)*Sec[(e + f*x)/2]
^4 + (4*(a + 2*b) + (3*a + 7*b)*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Tan[(e + f
*x)/2]^2)/(-1 + Tan[(e + f*x)/2]^2))/(64*f)
```

Maple [A] time = 0.049, size = 142, normalized size = 1.8

$$-\frac{\cot\left(fx+e\right)a\left(\csc\left(fx+e\right)\right)^{3}}{4f}-\frac{3a\csc\left(fx+e\right)\cot\left(fx+e\right)}{8f}+\frac{3a\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-\frac{}{4f\left(\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/4/f*a*cot(f*x+e)*csc(f*x+e)^3-3/8/f*a*csc(f*x+e)*cot(f*x+e)+3/8/f*a*ln(c
sc(f*x+e)-cot(f*x+e))-1/4/f*b/sin(f*x+e)^4/cos(f*x+e)-5/8/f*b/sin(f*x+e)^2/
cos(f*x+e)+15/8/f*b/cos(f*x+e)+15/8/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.01691, size = 136, normalized size = 1.68

$$-\frac{3\,(a+5\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+5\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+5\,b)\cos\left(fx+e\right)^{4}-5\,(a+5\,b)\cos\left(fx+e\right)^{2}+8\,b\right)}{\cos\left(fx+e\right)^{5}-2\,\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a + 5*b)*log(cos(f*x + e) + 1) - 3*(a + 5*b)*log(cos(f*x + e) - 1
) - 2*(3*(a + 5*b)*cos(f*x + e)^4 - 5*(a + 5*b)*cos(f*x + e)^2 + 8*b)/(cos(
f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

Fricas [B] time = 0.772039, size = 481, normalized size = 5.94

$$\frac{6\,(a+5\,b)\cos\left(fx+e\right)^{4}-10\,(a+5\,b)\cos\left(fx+e\right)^{2}-3\left((a+5\,b)\cos\left(fx+e\right)^{5}-2\,(a+5\,b)\cos\left(fx+e\right)^{3}+(a+5\,b)\cos\right.}{16\left(f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(a + 5*b)*cos(f*x + e)^4 - 10*(a + 5*b)*cos(f*x + e)^2 - 3*((a + 5*
b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f*x + e)^3 + (a + 5*b)*cos(f*x + e))*lo
g(1/2*cos(f*x + e) + 1/2) + 3*((a + 5*b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f
*x + e)^3 + (a + 5*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b)/(f
*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23797, size = 379, normalized size = 4.68

$$\frac{12\,(a+5\,b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-\frac{\left(a+b-\frac{8\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{16\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{18\,a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{90\,b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{\left(\cos\left(fx+e\right)-1\right)^{2}}-\frac{8\,a\left(\cos\left(fx+e\right)-\right.}{\cos\left(fx+e\right)+1}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(12*(a + 5*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - (a + b - 8
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 18*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^
2 - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + 128*b/((cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) + 1))/f
```

3.8 $\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$

Optimal. Leaf size=98

$$\frac{(13a-6b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-18b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x(a-6b)-\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{b}{}$$

```
[Out] (5*(a - 6*b)*x)/16 - ((11*a - 18*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a*Cos[e + f*x]^5*Sin[e +
f*x])/(6*f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.104163, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4132, 455, 1814, 1157, 388, 203}

$$\frac{(13a-6b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{(11a-18b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x(a-6b)-\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```


```
[Out] (5*(a - 6*b)*x)/16 - ((11*a - 18*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((1
3*a - 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a*Cos[e + f*x]^5*Sin[e +
f*x])/(6*f) + (b*Tan[e + f*x])/f
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6(a+b+bx^2)}{(1+x^2)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\cos^5(e+fx)\sin(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-a+6ax^2-6ax^4-6bx^6}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{6f}$$

$$= \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{\text{Subst}\left(\int \frac{-3}{}\right.}{}$$

$$= -\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a}{}$$

$$= -\frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a}{}$$

$$= \frac{5}{16}(a-6b)x - \frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\sin}{24f}$$

Mathematica [A] time = 0.308186, size = 78, normalized size = 0.8

$$\frac{(96b-45a)\sin(2(e+fx))+(9a-6b)\sin(4(e+fx))-a\sin(6(e+fx))+60ae+60afx+192b\tan(e+fx)-360be-36}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

```
[Out] (60*a*e - 360*b*e + 60*a*f*x - 360*b*f*x + (-45*a + 96*b)*Sin[2*(e + f*x)]
+ (9*a - 6*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + 192*b*Tan[e + f*x])/(
192*f)
```

Maple [A] time = 0.046, size = 112, normalized size = 1.1

$$\frac{1}{f}\left(a\left(-\frac{\cos(fx+e)}{6}\left((\sin(fx+e))^5+\frac{5(\sin(fx+e))^3}{4}+\frac{15\sin(fx+e)}{8}\right)+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{(\sin(fx+e))^7}{\cos(fx+e)}+\left((\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x)
```

```
[Out] 1/f*(a*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/1
6*f*x+5/16*e)+b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/
8*sin(f*x+e))*cos(f*x+e)-15/8*f*x-15/8*e))
```

Maxima [A] time = 1.4814, size = 150, normalized size = 1.53

$$\frac{15\left(fx+e\right)(a-6b)+48b\tan\left(fx+e\right)-\frac{3(11a-18b)\tan\left(fx+e\right)^5+8(5a-12b)\tan\left(fx+e\right)^3+3(5a-14b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^6+3\tan\left(fx+e\right)^4+3\tan\left(fx+e\right)^2+1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (3*(11*a - 18*b)*tan(f*x
 + e)^5 + 8*(5*a - 12*b)*tan(f*x + e)^3 + 3*(5*a - 14*b)*tan(f*x + e))/(tan
(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.502159, size = 220, normalized size = 2.24

$$\frac{15(a-6b)fx\cos\left(fx+e\right)-\left(8a\cos\left(fx+e\right)^6-2(13a-6b)\cos\left(fx+e\right)^4+3(11a-18b)\cos\left(fx+e\right)^2-48b\right)\sin\left(\right.}{48f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a - 6*b)*f*x*cos(f*x + e) - (8*a*cos(f*x + e)^6 - 2*(13*a - 6*b)*
cos(f*x + e)^4 + 3*(11*a - 18*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e))/(f*co
s(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2429, size = 153, normalized size = 1.56

$$\frac{15\left(fx+e\right)(a-6b)+48b\tan\left(fx+e\right)-\frac{33a\tan\left(fx+e\right)^{5}-54b\tan\left(fx+e\right)^{5}+40a\tan\left(fx+e\right)^{3}-96b\tan\left(fx+e\right)^{3}+15a\tan\left(fx+e\right)-42b\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{3}}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (33*a*tan(f*x + e)^5 - 5
4*b*tan(f*x + e)^5 + 40*a*tan(f*x + e)^3 - 96*b*tan(f*x + e)^3 + 15*a*tan(f
*x + e) - 42*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

3.9 $\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx$

Optimal. Leaf size=70

$$-\frac{(5a-4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0638165, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4132, 455, 1157, 388, 203}

$$-\frac{(5a-4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}x(a-4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(a - 4*b)*x)/8 - ((5*a - 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f) + (b*Tan[e + f*x])/f
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{a-4ax^2-4bx^4}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= -\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{\text{Subst}\left(\int \frac{3a}{}\right.}{}$$

$$= -\frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{b\tan(e+fx}{f}$$

$$= \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} +$$

Mathematica [A] time = 0.303466, size = 54, normalized size = 0.77

$$\frac{12(a-4b)(e+fx)-8(a-b)\sin(2(e+fx))+a\sin(4(e+fx))+32b\tan(e+fx)}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

```
[Out] (12*(a - 4*b)*(e + f*x) - 8*(a - b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)] +
 32*b*Tan[e + f*x])/(32*f)
```

Maple [A] time = 0.046, size = 92, normalized size = 1.3

$$\frac{1}{f}\left(a\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^{3}+\frac{3\sin\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\left(\sin\left(fx+e\right)\right)^{5}}{\cos\left(fx+e\right)}+\left(\left(\sin\left(fx+e\right)\right)^{3}+\frac{3\sin(}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x)
```

```
[Out] 1/f*(a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+b*(sin
(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e
))
```

Maxima [A] time = 1.50531, size = 111, normalized size = 1.59

$$\frac{3\left(fx+e\right)(a-4b)+8b\tan\left(fx+e\right)-\frac{(5a-4b)\tan\left(fx+e\right)^{3}+(3a-4b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - ((5*a - 4*b)*tan(f*x + e)^3
 + (3*a - 4*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.489777, size = 167, normalized size = 2.39

$$\frac{3\,(a-4\,b)fx\cos\left(fx+e\right)+\left(2\,a\cos\left(fx+e\right)^{4}-(5\,a-4\,b)\cos\left(fx+e\right)^{2}+8\,b\right)\sin\left(fx+e\right)}{8\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a - 4*b)*f*x*cos(f*x + e) + (2*a*cos(f*x + e)^4 - (5*a - 4*b)*cos(f
*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15206, size = 120, normalized size = 1.71

$$\frac{3\left(fx+e\right)(a-4\,b)+8\,b\tan\left(fx+e\right)-\frac{5\,a\tan\left(fx+e\right)^{3}-4\,b\tan\left(fx+e\right)^{3}+3\,a\tan\left(fx+e\right)-4\,b\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - (5*a*tan(f*x + e)^3 - 4*b*t
an(f*x + e)^3 + 3*a*tan(f*x + e) - 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2
)/f
```

3.10 $\int \left(a+b \sec^2(e+f x)\right) \sin^2(e+f x) \, dx$

Optimal. Leaf size=42

$$\frac{1}{2}x(a-2b)-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] ((a - 2*b)*x)/2 - (a*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0438048, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4132, 455, 388, 203}

$$\frac{1}{2}x(a-2b)-\frac{a\sin(e+fx)\cos(e+fx)}{2f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

```
[Out] ((a - 2*b)*x)/2 - (a*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b*Tan[e + f*x])/f
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a-2bx^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}\\
&= -\frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\right.}{2f}\\
&= \frac{1}{2}(a-2b)x - \frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.096271, size = 54, normalized size = 1.29

$$\frac{a(e+fx)}{2f} - \frac{a\sin(2(e+fx))}{4f} - \frac{b\tan^{-1}(\tan(e+fx))}{f} + \frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

```
[Out] (a*(e + f*x))/(2*f) - (b*ArcTan[Tan[e + f*x]])/f - (a*Sin[2*(e + f*x)])/(4*
f) + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.038, size = 46, normalized size = 1.1

$$\frac{1}{f}\left(a\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b\left(\tan\left(fx+e\right)-fx-e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x)
```

```
[Out] 1/f*(a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b*(tan(f*x+e)-f*x-e))
```

Maxima [A] time = 1.48669, size = 63, normalized size = 1.5

$$\frac{\left(fx+e\right)(a-2\,b)+2\,b\tan\left(fx+e\right)-\frac{a\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{2}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)^
2 + 1))/f
```

Fricas [A] time = 0.477747, size = 123, normalized size = 2.93

$$\frac{(a-2\,b)fx\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)^{2}-2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a - 2*b)*f*x*cos(f*x + e) - (a*cos(f*x + e)^2 - 2*b)*sin(f*x + e))/(f
*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\sin^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [A] time = 1.15883, size = 69, normalized size = 1.64

$$\frac{\left(fx + e\right)(a - 2\,b) + 2\,b\tan\left(fx + e\right) - \frac{a\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)^
2 + 1))/f
```

3.11 $\int \left(a+b \sec^2(e+f x)\right) \, dx$

Optimal. Leaf size=15

$$a x+\frac{b \tan (e+f x)}{f}$$

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.01241, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767, 8}

$$a x+\frac{b \tan (e+f x)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \left(a+b \sec^2(e+f x)\right) \, dx &= a x+b \int \sec^2(e+f x) \, dx\\ &= a x-\frac{b \operatorname{Subst}\left(\int 1 \, dx,x,-\tan (e+f x)\right)}{f}\\ &= a x+\frac{b \tan (e+f x)}{f}\\ \end{aligned}$$

Mathematica [A] time = 0.00264, size = 15, normalized size = 1.

$$ax + \frac{b\tan(e + fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.015, size = 16, normalized size = 1.1

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

Maxima [A] time = 0.977491, size = 20, normalized size = 1.33

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Fricas [B] time = 0.463286, size = 76, normalized size = 5.07

$$\frac{afx\cos\left(fx + e\right) + b\sin\left(fx + e\right)}{f\cos\left(fx + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Giac [A] time = 1.27023, size = 22, normalized size = 1.47

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

3.12 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx$

Optimal. Leaf size=26

$$\frac{b\tan(e+fx)}{f}-\frac{(a+b)\cot(e+fx)}{f}$$

```
[Out] -(((a + b)*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0335316, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 14}

$$\frac{b\tan(e+fx)}{f}-\frac{(a+b)\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x])/f) + (b*Tan[e + f*x])/f
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a+b)\cot(e+fx)}{f} + \frac{b\tan(e+fx)}{f}$$

Mathematica [A] time = 0.0644044, size = 36, normalized size = 1.38

$$-\frac{a\cot(e+fx)}{f} + \frac{b\tan(e+fx)}{f} - \frac{b\cot(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Cot[e + f*x])/f) - (b*Cot[e + f*x])/f + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.039, size = 43, normalized size = 1.7

$$\frac{1}{f}\left(-\cot\left(fx+e\right)a + b\left(\frac{1}{\sin\left(fx+e\right)\cos\left(fx+e\right)} - 2\cot\left(fx+e\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(-cot(f*x+e)*a+b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e)))
```

Maxima [A] time = 0.966755, size = 35, normalized size = 1.35

$$\frac{b\tan\left(fx+e\right) - \frac{a+b}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] (b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

Fricas [A] time = 0.454164, size = 85, normalized size = 3.27

$$-\frac{(a+2b)\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -((a + 2*b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [A] time = 1.31167, size = 38, normalized size = 1.46

$$\frac{b\tan\left(fx+e\right)-\frac{a+b}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

3.13 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx$

Optimal. Leaf size=46

$$-\frac{(a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((a + b)*Cot[e + f*x]^3)/(3*f) + (b*Tan[e +
 f*x])/f
```

Rubi [A] time = 0.0461213, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 448}

$$-\frac{(a+b)\cot^3(e+fx)}{3f}-\frac{(a+2b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + 2*b)*Cot[e + f*x])/f) - ((a + b)*Cot[e + f*x]^3)/(3*f) + (b*Tan[e +
 f*x])/f
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)}{x^4}dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^4}+\frac{a+2b}{x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a+2b)\cot(e+fx)}{f} - \frac{(a+b)\cot^3(e+fx)}{3f} + \frac{b\tan(e+fx)}{f}$$

Mathematica [A] time = 0.0463745, size = 84, normalized size = 1.83

$$-\frac{2a\cot(e+fx)}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)}{3f} + \frac{b\tan(e+fx)}{f} - \frac{5b\cot(e+fx)}{3f} - \frac{b\cot(e+fx)\csc^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-2*a*Cot[e + f*x])/(3*f) - (5*b*Cot[e + f*x])/(3*f) - (a*Cot[e + f*x]*Csc[
e + f*x]^2)/(3*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) + (b*Tan[e + f*x]
)/f
```

Maple [A] time = 0.048, size = 73, normalized size = 1.6

$$\frac{1}{f}\left(a\left(-\frac{2}{3}-\frac{\left(\csc\left(fx+e\right)\right)^2}{3}\right)\cot\left(fx+e\right)+b\left(-\frac{1}{3\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)}+\frac{4}{3\sin\left(fx+e\right)\cos\left(fx+e\right)}-\frac{8\cot\left(\right.}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+4
/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

Maxima [A] time = 1.00292, size = 58, normalized size = 1.26

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,(a+2\,b)\tan\left(fx+e\right)^{2}+a+b}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*(a + 2*b)*tan(f*x + e)^2 + a + b)/tan(f*x + e)^3
)/f
```

Fricas [A] time = 0.456512, size = 163, normalized size = 3.54

$$-\frac{2\,(a+4\,b)\cos\left(fx+e\right)^{4}-3\,(a+4\,b)\cos\left(fx+e\right)^{2}+3\,b}{3\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a + 4*b)*cos(f*x + e)^4 - 3*(a + 4*b)*cos(f*x + e)^2 + 3*b)/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29862, size = 73, normalized size = 1.59

$$\frac{3\,b\tan\left(fx+e\right)-\frac{3\,a\tan\left(fx+e\right)^{2}+6\,b\tan\left(fx+e\right)^{2}+a+b}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 + a + b)/t
an(f*x + e)^3)/f
```

3.14 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=68

$$-\frac{(a+b)\cot^5(e+fx)}{5f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+3b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

```
[Out] -(((a + 3*b)*Cot[e + f*x])/f) - ((2*a + 3*b)*Cot[e + f*x]^3)/(3*f) - ((a +
b)*Cot[e + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0560181, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4132, 448}

$$-\frac{(a+b)\cot^5(e+fx)}{5f}-\frac{(2a+3b)\cot^3(e+fx)}{3f}-\frac{(a+3b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(((a + 3*b)*Cot[e + f*x])/f) - ((2*a + 3*b)*Cot[e + f*x]^3)/(3*f) - ((a +
b)*Cot[e + f*x]^5)/(5*f) + (b*Tan[e + f*x])/f
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b+\frac{a+b}{x^6}+\frac{2a+3b}{x^4}+\frac{a+3b}{x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a+3b)\cot(e+fx)}{f} - \frac{(2a+3b)\cot^3(e+fx)}{3f} - \frac{(a+b)\cot^5(e+fx)}{5f} + \frac{b\,\text{ta}}{}$$

Mathematica [A] time = 0.0459084, size = 128, normalized size = 1.88

$$-\frac{8a\cot(e+fx)}{15f} - \frac{a\cot(e+fx)\csc^4(e+fx)}{5f} - \frac{4a\cot(e+fx)\csc^2(e+fx)}{15f} + \frac{b\tan(e+fx)}{f} - \frac{11b\cot(e+fx)}{5f} - \frac{b\cot}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-8*a*Cot[e + f*x])/(15*f) - (11*b*Cot[e + f*x])/(5*f) - (4*a*Cot[e + f*x]*
Csc[e + f*x]^2)/(15*f) - (3*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*f) - (a*Cot[e
 + f*x]*Csc[e + f*x]^4)/(5*f) - (b*Cot[e + f*x]*Csc[e + f*x]^4)/(5*f) + (b*
Tan[e + f*x])/f
```

Maple [A] time = 0.053, size = 101, normalized size = 1.5

$$\frac{1}{f}\left(a\left(-\frac{8}{15}-\frac{(\csc(fx+e))^4}{5}-\frac{4(\csc(fx+e))^2}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5(\sin(fx+e))^5\cos(fx+e)}-\frac{2}{5(\sin(fx+e))}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/5/sin(f*
x+e)^5/cos(f*x+e)-2/5/sin(f*x+e)^3/cos(f*x+e)+8/5/sin(f*x+e)/cos(f*x+e)-16/
5*cot(f*x+e)))
```

Maxima [A] time = 1.01311, size = 86, normalized size = 1.26

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,(a+3\,b)\tan\left(fx+e\right)^4+5\,(2\,a+3\,b)\tan\left(fx+e\right)^2+3\,a+3\,b}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*(a + 3*b)*tan(f*x + e)^4 + 5*(2*a + 3*b)*tan(
f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

Fricas [A] time = 0.469611, size = 236, normalized size = 3.47

$$-\frac{8\,(a+6\,b)\cos\left(fx+e\right)^6-20\,(a+6\,b)\cos\left(fx+e\right)^4+15\,(a+6\,b)\cos\left(fx+e\right)^2-15\,b}{15\left(f\cos\left(fx+e\right)^5-2\,f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a + 6*b)*cos(f*x + e)^6 - 20*(a + 6*b)*cos(f*x + e)^4 + 15*(a + 6
*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(
f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3392, size = 111, normalized size = 1.63

$$\frac{15\,b\tan\left(fx+e\right)-\frac{15\,a\tan\left(fx+e\right)^4+45\,b\tan\left(fx+e\right)^4+10\,a\tan\left(fx+e\right)^2+15\,b\tan\left(fx+e\right)^2+3\,a+3\,b}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 45*b*tan(f*x + e)^4 + 10*a
*tan(f*x + e)^2 + 15*b*tan(f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

3.15 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx$

Optimal. Leaf size=97

$$-\frac{\left(a^2-4ab+b^2\right)\cos(e+fx)}{f}-\frac{a^2\cos^5(e+fx)}{5f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(((a^2 - 4*a*b + b^2)*Cos[e + f*x])/f) + (2*a*(a - b)*Cos[e + f*x]^3)/(3*f
) - (a^2*Cos[e + f*x]^5)/(5*f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e
+ f*x]^3)/(3*f)
```

Rubi [A] time = 0.0897856, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4133, 448}

$$-\frac{\left(a^2-4ab+b^2\right)\cos(e+fx)}{f}-\frac{a^2\cos^5(e+fx)}{5f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}+\frac{2b(a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

```
[Out] -(((a^2 - 4*a*b + b^2)*Cos[e + f*x])/f) + (2*a*(a - b)*Cos[e + f*x]^3)/(3*f
) - (a^2*Cos[e + f*x]^5)/(5*f) + (2*(a - b)*b*Sec[e + f*x])/f + (b^2*Sec[e
+ f*x]^3)/(3*f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^2\right)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(-4a+b)}{a^2}\right)+\frac{b^2}{x^4}+\frac{2(a-b)b}{x^2}-2a(a-b)x^2+a^2x^4\right)dx, x, \cos(\right.}{f}$$

$$= -\frac{\left(a^2-4ab+b^2\right)\cos(e+fx)}{f}+\frac{2a(a-b)\cos^3(e+fx)}{3f}-\frac{a^2\cos^5(e+fx)}{5f}+$$

Mathematica [A] time = 0.632687, size = 118, normalized size = 1.22

$$-\frac{\sec^3(e+fx)\left(24\left(22a^2-215ab+120b^2\right)\cos(2(e+fx))+12\left(7a^2-60ab+20b^2\right)\cos(4(e+fx))-16a^2\cos(6(e+fx)\right.}{1920f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

```
[Out] -((425*a^2 - 4400*a*b + 2000*b^2 + 24*(22*a^2 - 215*a*b + 120*b^2)*Cos[2*(e
 + f*x)] + 12*(7*a^2 - 60*a*b + 20*b^2)*Cos[4*(e + f*x)] - 16*a^2*Cos[6*(e
+ f*x)] + 40*a*b*Cos[6*(e + f*x)] + 3*a^2*Cos[8*(e + f*x)])*Sec[e + f*x]^3)
/(1920*f)
```

Maple [A] time = 0.053, size = 155, normalized size = 1.6

$$\frac{1}{f}\left(-\frac{a^2\cos(fx+e)}{5}\left(\frac{8}{3}+\left(\sin(fx+e)\right)^4+\frac{4\left(\sin(fx+e)\right)^2}{3}\right)+2ab\left(\frac{\left(\sin(fx+e)\right)^6}{\cos(fx+e)}+\left(8/3+\left(\sin(fx+e)\right)^4+4/3\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x)
```

```
[Out] 1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x
+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*s
in(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f*x+e)^4+4/3*sin(
```

```
f*x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 1.00513, size = 120, normalized size = 1.24

$$-\frac{3\,a^2\cos\left(fx+e\right)^5-10\,\left(a^2-ab\right)\cos\left(fx+e\right)^3+15\,\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)-\frac{5\left(6\,\left(ab-b^2\right)\cos\left(fx+e\right)^2+b^2\right)}{\cos\left(fx+e\right)^3}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^5 - 10*(a^2 - a*b)*cos(f*x + e)^3 + 15*(a^2 - 4*a
*b + b^2)*cos(f*x + e) - 5*(6*(a*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e
)^3)/f
```

Fricas [A] time = 0.511764, size = 217, normalized size = 2.24

$$-\frac{3\,a^2\cos\left(fx+e\right)^8-10\,\left(a^2-ab\right)\cos\left(fx+e\right)^6+15\,\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^4-30\,\left(ab-b^2\right)\cos\left(fx+e\right)^2-5\,b^2}{15\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^8 - 10*(a^2 - a*b)*cos(f*x + e)^6 + 15*(a^2 - 4*a
*b + b^2)*cos(f*x + e)^4 - 30*(a*b - b^2)*cos(f*x + e)^2 - 5*b^2)/(f*cos(f*
x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3028, size = 602, normalized size = 6.21

$$2\left(\frac{5\left(6\,ab-5\,b^2+\frac{12\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{12\,b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{6\,ab\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{3\,b^2\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)}{\left(\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)^3}+\frac{8\,a^2-50\,ab+15\,b^2-\frac{40\,a^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{220\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/15*(5*(6*a*b - 5*b^2 + 12*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*
b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3 + (8*a^2 - 50*a*b + 15*b^2 - 40*a^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 220*a*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 - 320*a*b*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 180*a*b*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3 - 30*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*b^2*
(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) - 1)/(cos(f*x + e
) + 1) - 1)^5)/f
```

3.16 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^3(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{a^2\cos^3(e+fx)}{3f}-\frac{a(a-2b)\cos(e+fx)}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -((a*(a - 2*b)*Cos[e + f*x])/f) + (a^2*Cos[e + f*x]^3)/(3*f) + ((2*a - b)*b
*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.072152, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4133, 448}

$$\frac{a^2\cos^3(e+fx)}{3f}-\frac{a(a-2b)\cos(e+fx)}{f}+\frac{b(2a-b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

```
[Out] -((a*(a - 2*b)*Cos[e + f*x])/f) + (a^2*Cos[e + f*x]^3)/(3*f) + ((2*a - b)*b
*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^3(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a(a-2b)+\frac{b^2}{x^4}+\frac{(2a-b)b}{x^2}-a^2x^2\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{a(a-2b)\cos(e+fx)}{f}+\frac{a^2\cos^3(e+fx)}{3f}+\frac{(2a-b)b\sec(e+fx)}{f}+\frac{b^2\sec^3(}{3}$$

Mathematica [A] time = 0.464596, size = 83, normalized size = 1.15

$$\frac{\sec^3(e+fx)\left(-3\left(11a^2-64ab+16b^2\right)\cos(2(e+fx))+a^2\cos(6(e+fx))-26a^2-6a(a-4b)\cos(4(e+fx))+168ab-\right.}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

```
[Out] ((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cos[2*(e + f*x)
] - 6*a*(a - 4*b)*Cos[4*(e + f*x)] + a^2*Cos[6*(e + f*x)])*Sec[e + f*x]^3)/
(96*f)
```

Maple [A] time = 0.053, size = 125, normalized size = 1.7

$$\frac{1}{f}\left(-\frac{a^2\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+2\,ab\left(\frac{\left(\sin\left(fx+e\right)\right)^4}{\cos\left(fx+e\right)}+\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)\right)+b^2\left(\frac{(\sin(fx}{3\,(\cos(f}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x)
```

```
[Out] 1/f*(-1/3*a^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*x+e)^4/cos(f*x+e)+(2
+sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3*sin(f*x+e)^4/cos(f*x+e)^3-1/3*sin(f*x+e
)^4/cos(f*x+e)-1/3*(2+sin(f*x+e)^2)*cos(f*x+e)))
```

Maxima [A] time = 1.01258, size = 90, normalized size = 1.25

$$\frac{a^2\cos\left(fx+e\right)^3-3\left(a^2-2\,ab\right)\cos\left(fx+e\right)+\frac{3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+b^2}{\cos\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^3 - 3*(a^2 - 2*a*b)*cos(f*x + e) + (3*(2*a*b - b^2)*c
os(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

Fricas [A] time = 0.496206, size = 158, normalized size = 2.19

$$\frac{a^2\cos\left(fx+e\right)^6-3\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^6 - 3*(a^2 - 2*a*b)*cos(f*x + e)^4 + 3*(2*a*b - b^2)*
cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.33672, size = 131, normalized size = 1.82

$$\frac{6\,ab\cos\left(fx+e\right)^{2}-3\,b^{2}\cos\left(fx+e\right)^{2}+b^{2}}{3\,f\cos\left(fx+e\right)^{3}}+\frac{a^{2}f^{11}\cos\left(fx+e\right)^{3}-3\,a^{2}f^{11}\cos\left(fx+e\right)+6\,abf^{11}\cos\left(fx+e\right)}{3\,f^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/3*(6*a*b*cos(f*x + e)^2 - 3*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
+ 1/3*(a^2*f^11*cos(f*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 6*a*b*f^11*cos(f
*x + e))/f^12
```

3.17 $\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -((a^2*Cos[e + f*x])/f) + (2*a*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*
f)
```

Rubi [A] time = 0.0354167, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4133, 270}

$$-\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

```
[Out] -((a^2*Cos[e + f*x])/f) + (2*a*b*Sec[e + f*x])/f + (b^2*Sec[e + f*x]^3)/(3*
f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^4}+\frac{2ab}{x^2}\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{a^2\cos(e+fx)}{f}+\frac{2ab\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Mathematica [A] time = 0.114327, size = 75, normalized size = 1.63

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(-3a^2\cos^4(e+fx)+6ab\cos^2(e+fx)+b^2\right)}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*(b^2 + 6*a*b*Cos[e + f*x]^2 - 3*a^2*Cos[e + f*x
]^4)*Sec[e + f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.028, size = 42, normalized size = 0.9

$$\frac{1}{f}\left(\frac{\left(\sec\left(fx+e\right)\right)^3b^2}{3}+2\,ab\sec\left(fx+e\right)-\frac{a^2}{\sec\left(fx+e\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x)
```

```
[Out] 1/f*(1/3*sec(f*x+e)^3*b^2+2*a*b*sec(f*x+e)-a^2/sec(f*x+e))
```

Maxima [A] time = 0.975928, size = 57, normalized size = 1.24

$$-\frac{3\,a^2\cos\left(fx+e\right)-\frac{6\,ab}{\cos\left(fx+e\right)}-\frac{b^2}{\cos\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e) - 6*a*b/cos(f*x + e) - b^2/cos(f*x + e)^3)/f
```

Fricas [A] time = 0.484566, size = 104, normalized size = 2.26

$$-\frac{3\,a^2\cos\left(fx+e\right)^4-6\,ab\cos\left(fx+e\right)^2-b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e)^4 - 6*a*b*cos(f*x + e)^2 - b^2)/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2744, size = 63, normalized size = 1.37

$$-\frac{a^2\cos\left(fx+e\right)}{f}+\frac{6\,ab\cos\left(fx+e\right)^2+b^2}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="giac")
```

```
[Out] -a^2*cos(f*x + e)/f + 1/3*(6*a*b*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

3.18 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$

Optimal. Leaf size=52

$$\frac{b(2a+b)\sec(e+fx)}{f}-\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

```
[Out] -(((a + b)^2*ArcTanh[Cos[e + f*x]])/f) + (b*(2*a + b)*Sec[e + f*x])/f + (b^
2*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0657563, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 461, 207}

$$\frac{b(2a+b)\sec(e+fx)}{f}-\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b^2\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*ArcTanh[Cos[e + f*x]])/f) + (b*(2*a + b)*Sec[e + f*x])/f + (b^
2*Sec[e + f*x]^3)/(3*f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{b(2a+b)}{x^2}-\frac{(a+b)^2}{-1+x^2}\right)dx, x, \cos(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(\right.}{f}\\
&= -\frac{(a+b)^2\tanh^{-1}(\cos(e+fx))}{f}+\frac{b(2a+b)\sec(e+fx)}{f}+\frac{b^2\sec^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [B] time = 0.531735, size = 108, normalized size = 2.08

$$-\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(-3b(2a+b)\cos^2(e+fx)+3(a+b)^2\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\right.\right.\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cos[e + f*x]^2 + 3*(a +
b)^2*Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e
+ f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [B] time = 0.05, size = 117, normalized size = 2.3

$$\frac{a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+2\,\frac{ab}{f\cos\left(fx+e\right)}+2\,\frac{ab\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}+\frac{b^2}{3\,f\left(\cos\left(fx+e\right)\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*a^2*ln(csc(f*x+e)-cot(f*x+e))+2/f*a*b/cos(f*x+e)+2/f*a*b*ln(csc(f*x+e)-
cot(f*x+e))+1/3/f*b^2/cos(f*x+e)^3+1/f*b^2/cos(f*x+e)+1/f*b^2*ln(csc(f*x+e)
-cot(f*x+e))
```

Maxima [A] time = 1.02123, size = 111, normalized size = 2.13

$$-\frac{3\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(2\,ab+b^2\right)\cos\left(fx+e\right)^2+b^2\right)}{\cos\left(fx+e\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 2*a*b + b^2)*l
og(cos(f*x + e) - 1) - 2*(3*(2*a*b + b^2)*cos(f*x + e)^2 + b^2)/cos(f*x + e
)^3)/f
```

Fricas [A] time = 0.516601, size = 271, normalized size = 5.21

$$-\frac{3\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-3\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3*
(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*b
+ b^2)*cos(f*x + e)^2 - 2*b^2)/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29312, size = 244, normalized size = 4.69

$$\frac{3\left(a^2+2\,ab+b^2\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+\frac{8\left(3\,ab+2\,b^2+\frac{6\,ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{3\,b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{3\,ab\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{3\,b^2\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)}{\left(\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(a^2 + 2*a*b + b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 8*
(3*a*b + 2*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^2*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 1)^3)/f
```

3.19 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=104

$$-\frac{\left(3a^2+6ab+5b^2\right)\cot(e+fx)\csc(e+fx)}{6f}+\frac{b(6a+5b)\sec(e+fx)}{3f}-\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2\csc^2(e$$

```
[Out] -((a + b)*(a + 5*b)*ArcTanh[Cos[e + f*x]])/(2*f) - ((3*a^2 + 6*a*b + 5*b^2)
*Cot[e + f*x]*Csc[e + f*x])/(6*f) + (b*(6*a + 5*b)*Sec[e + f*x])/(3*f) + (b
^2*Csc[e + f*x]^2*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.110187, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 462, 456, 453, 206}

$$-\frac{\left(3a^2+6ab+5b^2\right)\cot(e+fx)\csc(e+fx)}{6f}+\frac{b(6a+5b)\sec(e+fx)}{3f}-\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f}+\frac{b^2\csc^2(e$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + b)*(a + 5*b)*ArcTanh[Cos[e + f*x]])/(2*f) - ((3*a^2 + 6*a*b + 5*b^2)
*Cot[e + f*x]*Csc[e + f*x])/(6*f) + (b*(6*a + 5*b)*Sec[e + f*x])/(3*f) + (b
^2*Csc[e + f*x]^2*Sec[e + f*x]^3)/(3*f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{(b+ax^2)^2}{x^4(1-x^2)^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f} - \frac{\text{Subst}\left(\int \frac{b(6a+5b)+3a^2x^2}{x^2(1-x^2)^2}dx, x, \cos(e+fx)\right)}{3f}$$

$$= -\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f} +$$

$$= -\frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b(6a+5b)\sec(e+fx)}{3f} + \frac{b^2\csc}{}$$

$$= -\frac{(a+b)(a+5b)\tanh^{-1}(\cos(e+fx))}{2f} - \frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+}{6f}$$

Mathematica [B] time = 6.57456, size = 1021, normalized size = 9.82

$$\frac{(-a^2-2ba-b^2)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(b\sec^2(e+fx)+a\right)^2\cos^4(e+fx)}{2f(\cos(2e+2fx)a+a+2b)^2} + \frac{(a^2+2ba+b^2)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(b\sec^2(e+fx)+a\right)^2 c}{2f(\cos(2e+2fx)a+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((-a^2 - 2*a*b - b^2)*Cos[e + f*x]^4*Csc[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*
x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) - (2*(a^2 + 6*a*b + 5*b^2)*
Cos[e + f*x]^4*Log[Cos[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*
b + a*Cos[2*e + 2*f*x])^2) + (2*(a^2 + 6*a*b + 5*b^2)*Cos[e + f*x]^4*Log[Si
n[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*b + a*Cos[2*e + 2*f*x
])^2) + (2*b*(12*a + 13*b)*Cos[e + f*x]^4*Sec[e]*(a + b*Sec[e + f*x]^2)^2)/
(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + ((a^2 + 2*a*b + b^2)*Cos[e + f*x]^
4*Sec[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e
+ 2*f*x])^2) + (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*Sin[(f*x)/2])
/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*
x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(
b^2*Cos[e/2] + b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/
2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) + (2*Cos[e + f*
x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*Sin[(f*x)/2]))/
```

```
(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x
)/2] - Sin[e/2 + (f*x)/2])) - (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^
2*Sin[(f*x)/2])/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])
*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[
e + f*x]^2)^2*(b^2*Cos[e/2] - b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*
f*x])^2*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^2)
- (2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*
Sin[(f*x)/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])*
(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]))
```

Maple [B] time = 0.063, size = 195, normalized size = 1.9

$$-\frac{a^2\csc\left(fx+e\right)\cot\left(fx+e\right)}{2f}+\frac{a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}-\frac{ab}{f\left(\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)}+3\frac{ab}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2*csc(f*x+e)*cot(f*x+e)+1/2/f*a^2*ln(csc(f*x+e)-cot(f*x+e))-1/f*a*
b/sin(f*x+e)^2/cos(f*x+e)+3/f*a*b/cos(f*x+e)+3/f*a*b*ln(csc(f*x+e)-cot(f*x+
e))+1/3/f*b^2/sin(f*x+e)^2/cos(f*x+e)^3-5/6/f*b^2/sin(f*x+e)^2/cos(f*x+e)+5
/2/f*b^2/cos(f*x+e)+5/2/f*b^2*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.04664, size = 170, normalized size = 1.63

$$-\frac{3\left(a^2+6ab+5b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+6ab+5b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(a^2+6ab+5b^2\right)\cos\left(fx+e\right)^4-2\left(6a\right.\right.}{\cos\left(fx+e\right)^5-\cos\left(f\right.}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 6*a*b + 5*b
^2)*log(cos(f*x + e) - 1) - 2*(3*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 2*(
6*a*b + 5*b^2)*cos(f*x + e)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3))/f
```

Fricas [B] time = 0.540695, size = 471, normalized size = 4.53

$$\frac{6\left(a^2+6ab+5b^2\right)\cos\left(fx+e\right)^4-4\left(6ab+5b^2\right)\cos\left(fx+e\right)^2-4b^2-3\left(\left(a^2+6ab+5b^2\right)\cos\left(fx+e\right)^5-\left(a^2+6ab\right.\right.}{12\left(f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 4*(6*a*b + 5*b^2)*cos(f*x +
e)^2 - 4*b^2 - 3*((a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b
^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^2 + 6*a*b + 5*b^2)*
cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x + e
) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29211, size = 491, normalized size = 4.72

$$-\frac{\frac{3a^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{6ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{3b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-6\left(a^2+6ab+5b^2\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-\frac{3\left(a^2+2ab+b^2-\frac{2a^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\right.}{}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 3*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 6*(a^
2 + 6*a*b + 5*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 3*(a^2 + 2
*a*b + b^2 - 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*a*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 10*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
)*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - 16*(6*a*b + 7*b^2 + 12*a*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 12*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*b^2*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3
)/f
```

3.20 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=141

$$-\frac{(3a^2+30ab+35b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f}+\frac{b(6a+7b)\sec(e+fx)}{3f}-\frac{(3a\ldots}{}$$

```
[Out] -((3*a^2 + 30*a*b + 35*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - ((3*a + 7*b)^2*C
ot[e + f*x]*Csc[e + f*x])/(24*f) - ((3*a^2 + 6*a*b + 7*b^2)*Cot[e + f*x]*Cs
c[e + f*x]^3)/(12*f) + (b*(6*a + 7*b)*Sec[e + f*x])/(3*f) + (b^2*Csc[e + f*
x]^4*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.138497, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 462, 456, 453, 206}

$$-\frac{(3a^2+30ab+35b^2)\tanh^{-1}(\cos(e+fx))}{8f}-\frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f}+\frac{b(6a+7b)\sec(e+fx)}{3f}-\frac{(3a\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] -((3*a^2 + 30*a*b + 35*b^2)*ArcTanh[Cos[e + f*x]])/(8*f) - ((3*a + 7*b)^2*C
ot[e + f*x]*Csc[e + f*x])/(24*f) - ((3*a^2 + 6*a*b + 7*b^2)*Cot[e + f*x]*Cs
c[e + f*x]^3)/(12*f) + (b*(6*a + 7*b)*Sec[e + f*x])/(3*f) + (b^2*Csc[e + f*
x]^4*Sec[e + f*x]^3)/(3*f)
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
```

```
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^4\left(1-x^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f} - \frac{\text{Subst}\left(\int \frac{b(6a+7b)+3a^2x^2}{x^2\left(1-x^2\right)^3}\,dx, x, \cos(e+fx)\right)}{3f}$$

$$= -\frac{\left(3a^2+6ab+7b^2\right)\cot(e+fx)\csc^3(e+fx)}{12f} + \frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f} + \cdots$$

$$= -\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f} - \frac{\left(3a^2+6ab+7b^2\right)\cot(e+fx)\csc^3(e+\cdots}{12f}$$

$$= -\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f} - \frac{\left(3a^2+6ab+7b^2\right)\cot(e+fx)\csc^3(e+\cdots}{12f}$$

$$= -\frac{\left(3a^2+30ab+35b^2\right)\tanh^{-1}(\cos(e+fx))}{8f} - \frac{(3a+7b)^2\cot(e+fx)\csc(e+\cdots}{24f}$$

Mathematica [A] time = 1.89722, size = 218, normalized size = 1.55

$$-\frac{\sec^4(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(\frac{1}{2}\left(105a^2+282ab+329b^2\right)(\cos(e+fx)+\cos(3(e+fx)))\csc^4(e+fx)+96\left(3a^2+\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((b + a*Cos[e + f*x]^2)^2*((90*a^2 + 132*a*b - 102*b^2 + (6*a^2 + 60*a*b +
 70*b^2)*Cos[4*(e + f*x)] - 3*(3*a^2 + 30*a*b + 35*b^2)*Cos[6*(e + f*x)])*C
ot[e + f*x]*Csc[e + f*x]^3 + ((105*a^2 + 282*a*b + 329*b^2)*(Cos[e + f*x] +
 Cos[3*(e + f*x)])*Csc[e + f*x]^4)/2 + 96*(3*a^2 + 30*a*b + 35*b^2)*Cos[e +
 f*x]^4*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e + f*x]^4)/(1
92*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [B] time = 0.069, size = 264, normalized size = 1.9

$$-\frac{a^2\cot\left(fx+e\right)\left(\csc\left(fx+e\right)\right)^3}{4f}-\frac{3a^2\csc\left(fx+e\right)\cot\left(fx+e\right)}{8f}+\frac{3a^2\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{8f}-\frac{}{2f\left(\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/4/f*a^2*cot(f*x+e)*csc(f*x+e)^3-3/8/f*a^2*csc(f*x+e)*cot(f*x+e)+3/8/f*a^
2*ln(csc(f*x+e)-cot(f*x+e))-1/2/f*a*b/sin(f*x+e)^4/cos(f*x+e)-5/4/f*a*b/sin
(f*x+e)^2/cos(f*x+e)+15/4/f*a*b/cos(f*x+e)+15/4/f*a*b*ln(csc(f*x+e)-cot(f*x
+e))-1/4/f*b^2/sin(f*x+e)^4/cos(f*x+e)^3+7/12/f*b^2/sin(f*x+e)^2/cos(f*x+e)
^3-35/24/f*b^2/sin(f*x+e)^2/cos(f*x+e)+35/8/f*b^2/cos(f*x+e)+35/8/f*b^2*ln(
csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.03574, size = 223, normalized size = 1.58

$$-\frac{3\left(3a^2+30ab+35b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(3a^2+30ab+35b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(3a^2+30ab+35b^2\right)\cos\right.}{}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*(3*a^2 + 30*a*b + 35*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 30*a*
b + 35*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x
+ e)^6 - 5*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^4 + 8*(6*a*b + 7*b^2)*cos
(f*x + e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^3))/
f
```

Fricas [B] time = 0.537309, size = 713, normalized size = 5.06

$$\frac{6\left(3a^2+30ab+35b^2\right)\cos\left(fx+e\right)^6-10\left(3a^2+30ab+35b^2\right)\cos\left(fx+e\right)^4+16\left(6ab+7b^2\right)\cos\left(fx+e\right)^2+16b^2-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(6*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 30*a*b + 35*
b^2)*cos(f*x + e)^4 + 16*(6*a*b + 7*b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*a^
2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x +
 e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/
2) + 3*((3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b
^2)*cos(f*x + e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(-1/2*cos
(f*x + e) + 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x + e)^3
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31271, size = 713, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/192*(24*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 96*a*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 72*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3
*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 6*a*b*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 1
2*(3*a^2 + 30*a*b + 35*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 3
*(a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 32*a*b*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*b^2*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 180*a*b*(cos
```

```
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 210*b^2*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2 - 256*(3*a*b + 5*
b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 9*b^2*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 6*
b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 1)^3)/f
```

3.21 $\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$

Optimal. Leaf size=148

$$-\frac{\left(a^2-12ab+12b^2\right)\tan(e+fx)}{6f}-\frac{\left(3a^2-36ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x\left(a^2-12ab+8b^2\right)+\frac{a^2\sin^6(e+fx}{6}$$

```
[Out] (5*(a^2 - 12*a*b + 8*b^2)*x)/16 - ((3*a^2 - 36*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(a - 12*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((
a^2 - 12*a*b + 12*b^2)*Tan[e + f*x])/(6*f) + (a^2*Sin[e + f*x]^6*Tan[e + f*
x])/(6*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.176964, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 463, 455, 1814, 1153, 203}

$$-\frac{\left(a^2-12ab+12b^2\right)\tan(e+fx)}{6f}-\frac{\left(3a^2-36ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}x\left(a^2-12ab+8b^2\right)+\frac{a^2\sin^6(e+fx}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```



```
[Out] (5*(a^2 - 12*a*b + 8*b^2)*x)/16 - ((3*a^2 - 36*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(a - 12*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - ((
a^2 - 12*a*b + 12*b^2)*Tan[e + f*x])/(6*f) + (a^2*Sin[e + f*x]^6*Tan[e + f*
x])/(6*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
```

```
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^4}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{x^6\left(7a^2-6(a+b)^2-6b^2x^2\right)}{\left(1+x^2\right)^3}\,dx,x,\tan(e+fx)\right)}{6f}$$

$$= \frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx)\sin(e+}{24f}$$

$$= -\frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx)\sin(e+}{24f}$$

$$= -\frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-12b)\cos^3(e+fx)\sin(e+}{24f}$$

$$= \frac{5}{16}\left(a^2-12ab+8b^2\right)x - \frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(a-1}{}$$

Mathematica [B] time = 1.50546, size = 499, normalized size = 3.37

$$\frac{\sec(e)\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(360fx\left(a^2-12ab+8b^2\right)\cos(2e+fx)+360fx\left(a^2-12ab+8b^2\right)\cos(fx)-81a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

```
[Out] ((b + a*Cos[e + f*x]^2)^2*Sec[e]*Sec[e + f*x]^3*(360*(a^2 - 12*a*b + 8*b^2)
*f*x*Cos[f*x] + 360*(a^2 - 12*a*b + 8*b^2)*f*x*Cos[2*e + f*x] + 120*a^2*f*x
*Cos[2*e + 3*f*x] - 1440*a*b*f*x*Cos[2*e + 3*f*x] + 960*b^2*f*x*Cos[2*e + 3
*f*x] + 120*a^2*f*x*Cos[4*e + 3*f*x] - 1440*a*b*f*x*Cos[4*e + 3*f*x] + 960*
b^2*f*x*Cos[4*e + 3*f*x] - 81*a^2*Sin[f*x] + 3444*a*b*Sin[f*x] - 3168*b^2*S
in[f*x] - 81*a^2*Sin[2*e + f*x] - 1164*a*b*Sin[2*e + f*x] + 2208*b^2*Sin[2*
e + f*x] - 109*a^2*Sin[2*e + 3*f*x] + 2076*a*b*Sin[2*e + 3*f*x] - 1936*b^2*
Sin[2*e + 3*f*x] - 109*a^2*Sin[4*e + 3*f*x] + 540*a*b*Sin[4*e + 3*f*x] - 14
```

```
4*b^2*Sin[4*e + 3*f*x] - 21*a^2*Sin[4*e + 5*f*x] + 156*a*b*Sin[4*e + 5*f*x]
 - 48*b^2*Sin[4*e + 5*f*x] - 21*a^2*Sin[6*e + 5*f*x] + 156*a*b*Sin[6*e + 5*
f*x] - 48*b^2*Sin[6*e + 5*f*x] + 6*a^2*Sin[6*e + 7*f*x] - 12*a*b*Sin[6*e +
7*f*x] + 6*a^2*Sin[8*e + 7*f*x] - 12*a*b*Sin[8*e + 7*f*x] - a^2*Sin[8*e + 9
*f*x] - a^2*Sin[10*e + 9*f*x]))/(768*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.056, size = 199, normalized size = 1.3

$$\frac{1}{f}\left(a^2\left(-\frac{\cos\left(fx+e\right)}{6}\left(\left(\sin\left(fx+e\right)\right)^5+\frac{5\left(\sin\left(fx+e\right)\right)^3}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\sin\left(fx+e\right)\right)^7}{\cos\left(fx+e\right)}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x)
```

```
[Out] 1/f*(a^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)+2*a*b*(sin(f*x+e)^7/cos(f*x+e)+(sin(f*x+e)^5+5/4*sin(f*x+e)
^3+15/8*sin(f*x+e))*cos(f*x+e)-15/8*f*x-15/8*e)+b^2*(1/3*sin(f*x+e)^7/cos(f
*x+e)^3-4/3*sin(f*x+e)^7/cos(f*x+e)-4/3*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8
*sin(f*x+e))*cos(f*x+e)+5/2*f*x+5/2*e))
```

Maxima [A] time = 1.5586, size = 221, normalized size = 1.49

$$\frac{16b^2\tan\left(fx+e\right)^3+15\left(a^2-12ab+8b^2\right)\left(fx+e\right)+96\left(ab-b^2\right)\tan\left(fx+e\right)-\frac{3\left(11a^2-36ab+8b^2\right)\tan\left(fx+e\right)^5+8\left(5a^2-24ab+\right.}{\tan\left(fx+e\right)^6+3\tan\left(f\right.}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/48*(16*b^2*tan(f*x + e)^3 + 15*(a^2 - 12*a*b + 8*b^2)*(f*x + e) + 96*(a*b
 - b^2)*tan(f*x + e) - (3*(11*a^2 - 36*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a
^2 - 24*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(5*a^2 - 28*a*b + 8*b^2)*tan(f*x +
e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.550109, size = 320, normalized size = 2.16

$$\frac{15\left(a^2-12\,ab+8\,b^2\right)fx\cos\left(fx+e\right)^3-\left(8\,a^2\cos\left(fx+e\right)^8-2\left(13\,a^2-12\,ab\right)\cos\left(fx+e\right)^6+3\left(11\,a^2-36\,ab+8\,b^2\right)\cos\cdots}{48\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*(a^2 - 12*a*b + 8*b^2)*f*x*cos(f*x + e)^3 - (8*a^2*cos(f*x + e)^8
- 2*(13*a^2 - 12*a*b)*cos(f*x + e)^6 + 3*(11*a^2 - 36*a*b + 8*b^2)*cos(f*x
+ e)^4 - 16*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 16*b^2)*sin(f*x + e))/(f*cos(f
*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26573, size = 266, normalized size = 1.8

$$\frac{16\,b^2\tan\left(fx+e\right)^3+96\,ab\tan\left(fx+e\right)-96\,b^2\tan\left(fx+e\right)+15\left(a^2-12\,ab+8\,b^2\right)\left(fx+e\right)-\frac{33\,a^2\tan\left(fx+e\right)^5-108\,ab\tan\left(f\cdots\right.}{\cdots}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/48*(16*b^2*tan(f*x + e)^3 + 96*a*b*tan(f*x + e) - 96*b^2*tan(f*x + e) + 1
5*(a^2 - 12*a*b + 8*b^2)*(f*x + e) - (33*a^2*tan(f*x + e)^5 - 108*a*b*tan(f
*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 192*a*b*tan(f*x
```

```
+ e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) - 84*a*b*tan(f*x + e)
+ 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

3.22 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^4(e+fx)\,dx$

Optimal. Leaf size=114

$$-\frac{\left(a^2-8ab+4b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-24ab+8b^2\right)+\frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{a(a-8b)\sin(e+fx)\cos(e+fx)}{8f}$$

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8
*f) - ((a^2 - 8*a*b + 4*b^2)*Tan[e + f*x])/(4*f) + (a^2*Sin[e + f*x]^4*Tan[
e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.120714, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 463, 455, 1153, 203}

$$-\frac{\left(a^2-8ab+4b^2\right)\tan(e+fx)}{4f}+\frac{1}{8}x\left(3a^2-24ab+8b^2\right)+\frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f}-\frac{a(a-8b)\sin(e+fx)\cos(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

```
[Out] ((3*a^2 - 24*a*b + 8*b^2)*x)/8 - (a*(a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8
*f) - ((a^2 - 8*a*b + 4*b^2)*Tan[e + f*x])/(4*f) + (a^2*Sin[e + f*x]^4*Tan[
e + f*x])/(4*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
```

```
IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{x^4\left(5a^2-4(a+b)^2-4b^2x^2\right)}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{4f}$$

$$= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{\left(a^2-8ab+4b^2\right)\tan(e+fx)}{4f} + \frac{a^2\sin^4(e}{}$$

$$= \frac{1}{8}\left(3a^2-24ab+8b^2\right)x - \frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{\left(a^2-8ab+4b^2\right)}{4f}$$

Mathematica [A] time = 1.72079, size = 153, normalized size = 1.34

$$\frac{\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3\cos^3(e+fx)\left(4fx\left(3a^2-24ab+8b^2\right)+a^2\sin(4(e+fx))-8a(a-2b)\sin(2(e+fx)\right.\right.}{24f(a\cos(2(e+fx))+a+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

```
[Out] ((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(32*b^2*Sec[e]*Sin[f*x] + 64*(3*a
- 2*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*Cos[e + f*x]^3*(4*(3*a^2 - 24*a
*b + 8*b^2)*f*x - 8*a*(a - 2*b)*Sin[2*(e + f*x)] + a^2*Sin[4*(e + f*x)]) +
32*b^2*Cos[e + f*x]*Tan[e]))/(24*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.054, size = 123, normalized size = 1.1

$$\frac{1}{f}\left(a^2\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\left(\sin\left(fx+e\right)\right)^5}{\cos\left(fx+e\right)}+\left(\left(\sin\left(fx+e\right)\right)^3+3/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x)
```

```
[Out] 1/f*(a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(sin(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x
-3/2*e)+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

Maxima [A] time = 1.52969, size = 162, normalized size = 1.42

$$\frac{8\,b^2\tan\left(fx+e\right)^3+3\left(3\,a^2-24\,ab+8\,b^2\right)\left(fx+e\right)+24\left(2\,ab-b^2\right)\tan\left(fx+e\right)-\frac{3\left(\left(5\,a^2-8\,ab\right)\tan\left(fx+e\right)^3+\left(3\,a^2-8\,ab\right)\tan\left(fx\right.\right.}{\tan\left(fx+e\right)^4+2\,\tan\left(fx+e\right)^2+1}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) + 24*(2*a
*b - b^2)*tan(f*x + e) - 3*((5*a^2 - 8*a*b)*tan(f*x + e)^3 + (3*a^2 - 8*a*b
)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.521652, size = 257, normalized size = 2.25

$$\frac{3\left(3\,a^2-24\,ab+8\,b^2\right)fx\cos\left(fx+e\right)^3+\left(6\,a^2\cos\left(fx+e\right)^6-3\left(5\,a^2-8\,ab\right)\cos\left(fx+e\right)^4+16\left(3\,ab-2\,b^2\right)\cos\left(fx+\right.\right.}{24\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 - 24*a*b + 8*b^2)*f*x*cos(f*x + e)^3 + (6*a^2*cos(f*x + e)^6
 - 3*(5*a^2 - 8*a*b)*cos(f*x + e)^4 + 16*(3*a*b - 2*b^2)*cos(f*x + e)^2 + 8
*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29904, size = 178, normalized size = 1.56

$$\frac{8\,b^2\tan\left(fx+e\right)^3+48\,ab\tan\left(fx+e\right)-24\,b^2\tan\left(fx+e\right)+3\left(3\,a^2-24\,ab+8\,b^2\right)\left(fx+e\right)-\frac{3\left(5\,a^2\tan\left(fx+e\right)^3-8\,ab\tan\left(fx+\right.\right.}{\left(\tan\left(\right.\right.}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/24*(8*b^2*tan(f*x + e)^3 + 48*a*b*tan(f*x + e) - 24*b^2*tan(f*x + e) + 3*
(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) - 3*(5*a^2*tan(f*x + e)^3 - 8*a*b*tan(f*
x + e)^3 + 3*a^2*tan(f*x + e) - 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)
/f
```

3.23 $\int \left(a+b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx$

Optimal. Leaf size=73

$$\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{1}{2}ax(a-4b)+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] (a*(a - 4*b)*x)/2 - (a*(a - 4*b)*Tan[e + f*x])/(2*f) + (a^2*Sin[e + f*x]^2*
Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0989284, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 463, 459, 321, 203}

$$\frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f}-\frac{a(a-4b)\tan(e+fx)}{2f}+\frac{1}{2}ax(a-4b)+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

```
[Out] (a*(a - 4*b)*x)/2 - (a*(a - 4*b)*Tan[e + f*x])/(2*f) + (a^2*Sin[e + f*x]^2*
Tan[e + f*x])/(2*f) + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a^2-2(a+b)^2-2b^2x^2\right)}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2\tan^3(e+fx)}{3f} - \frac{(a(a-4b))\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,\right.}{2f}$$

$$= -\frac{a(a-4b)\tan(e+fx)}{2f} + \frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2\tan^3(e+fx)}{3f} + \frac{(a(}{}$$

$$= \frac{1}{2}a(a-4b)x - \frac{a(a-4b)\tan(e+fx)}{2f} + \frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2\tan^3(}{3}$$

Mathematica [A] time = 0.988913, size = 126, normalized size = 1.73

$$-\frac{\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3a\cos^3(e+fx)(a\sin(2(e+fx))-2fx(a-4b))-4b(6a-b)\sec(e)\sin(fx)\cos^2(e+\dots\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

```
[Out] -((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(-4*b^2*Sec[e]*Sin[f*x] - 4*(6*a
- b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*a*Cos[e + f*x]^3*(-2*(a - 4*b)*f*
x + a*Sin[2*(e + f*x)]) - 4*b^2*Cos[e + f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos
[2*(e + f*x)])^2)
```

Maple [A] time = 0.049, size = 71, normalized size = 1.

$$\frac{1}{f}\left(a^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab\left(\tan(fx+e)-fx-e\right)+\frac{b^2\left(\sin(fx+e)\right)^3}{3\left(\cos(fx+e)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x)
```

```
[Out] 1/f*(a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(tan(f*x+e)-f*x-e
)+1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3)
```

Maxima [A] time = 1.48454, size = 90, normalized size = 1.23

$$\frac{2b^2\tan(fx+e)^3+12ab\tan(fx+e)+3\left(a^2-4ab\right)(fx+e)-\frac{3a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e)
 - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.503966, size = 189, normalized size = 2.59

$$\frac{3\left(a^2-4\,ab\right)fx\cos\left(fx+e\right)^3-\left(3\,a^2\cos\left(fx+e\right)^4-2\left(6\,ab-b^2\right)\cos\left(fx+e\right)^2-2\,b^2\right)\sin\left(fx+e\right)}{6\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2 - 4*a*b)*f*x*cos(f*x + e)^3 - (3*a^2*cos(f*x + e)^4 - 2*(6*a*b
- b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21921, size = 97, normalized size = 1.33

$$\frac{2\,b^2\tan\left(fx+e\right)^3+12\,ab\tan\left(fx+e\right)+3\left(a^2-4\,ab\right)\left(fx+e\right)-\frac{3\,a^2\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e)
 - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

3.24 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0286617, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right) dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$
$$= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [B] time = 0.357621, size = 106, normalized size = 2.65

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3a^2fx\cos^3(e+fx)+2b(3a+b)\sec(e)\sin(fx)\cos^2(e+fx)+b^2\tan(e)\cos(e+fx)\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.032, size = 48, normalized size = 1.2

$$\frac{1}{f}\left(a^2\left(fx+e\right)+2\,ab\tan\left(fx+e\right)-b^2\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 1.01277, size = 59, normalized size = 1.48

$$a^2x + \frac{\left(\tan\left(fx+e\right)^3 + 3\,\tan\left(fx+e\right)\right)b^2}{3\,f} + \frac{2\,ab\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Fricas [A] time = 0.486627, size = 142, normalized size = 3.55

$$\frac{3\,a^2fx\cos\left(fx+e\right)^3 + \left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2 + b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.15509, size = 72, normalized size = 1.8

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

3.25 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{2b(a+b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(((a + b)^2*Cot[e + f*x])/f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e +
 f*x]^3)/(3*f)
```

Rubi [A] time = 0.0583803, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 270}

$$\frac{2b(a+b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((a + b)^2*Cot[e + f*x])/f) + (2*b*(a + b)*Tan[e + f*x])/f + (b^2*Tan[e +
 f*x]^3)/(3*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(2b(a+b)+\frac{(a+b)^2}{x^2}+b^2x^2\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a+b)^2\cot(e+fx)}{f}+\frac{2b(a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [B] time = 1.01941, size = 109, normalized size = 2.18

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(\sin(fx)\cos^2(e+fx)\left(3(a+b)^2\csc(e)\cot(e+fx)+b(6a+5b)\sec(e)\right)+b^2\tan(e)\,\text{c}\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(b^2*Sec[e]*Sin[f*x] + Cos[e + f
*x]^2*(3*(a + b)^2*Cot[e + f*x]*Csc[e] + b*(6*a + 5*b)*Sec[e])*Sin[f*x] + b
^2*Cos[e + f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.053, size = 96, normalized size = 1.9

$$\frac{1}{f}\left(-a^2\cot\left(fx+e\right)+2\,ab\left(\frac{1}{\sin\left(fx+e\right)\cos\left(fx+e\right)}-2\,\cot\left(fx+e\right)\right)+b^2\left(\frac{1}{3\,\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3}+\frac{}{3\,\sin\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*cot(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+b^2*(1/3/
sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

Maxima [A] time = 1.01374, size = 73, normalized size = 1.46

$$\frac{b^2\tan\left(fx+e\right)^3+6\left(ab+b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2+2\,ab+b^2\right)}{\tan\left(fx+e\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*(a*b + b^2)*tan(f*x + e) - 3*(a^2 + 2*a*b + b^2
)/tan(f*x + e))/f
```

Fricas [A] time = 0.482707, size = 163, normalized size = 3.26

$$-\frac{\left(3\,a^2+12\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+2\,b^2\right)\cos\left(fx+e\right)^2-b^2}{3\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 2*b^2)*cos(f*x +
 e)^2 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26379, size = 86, normalized size = 1.72

$$\frac{b^2\tan\left(fx+e\right)^3+6\,ab\tan\left(fx+e\right)+6\,b^2\tan\left(fx+e\right)-\frac{3\left(a^2+2\,ab+b^2\right)}{\tan\left(fx+e\right)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 6*b^2*tan(f*x + e) - 3*(a^2
+ 2*a*b + b^2)/tan(f*x + e))/f
```

3.26 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=76

$$\frac{b(2a+3b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}-\frac{(a+b)(a+3b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(((a + b)*(a + 3*b)*Cot[e + f*x])/f) - ((a + b)^2*Cot[e + f*x]^3)/(3*f) +
(b*(2*a + 3*b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0756403, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 448}

$$\frac{b(2a+3b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}-\frac{(a+b)(a+3b)\cot(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] -(((a + b)*(a + 3*b)*Cot[e + f*x])/f) - ((a + b)^2*Cot[e + f*x]^3)/(3*f) +
(b*(2*a + 3*b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^2}{x^4} dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(b(2a+3b)+\frac{(a+b)^2}{x^4}+\frac{(a+b)(a+3b)}{x^2}+b^2x^2\right) dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{(a+b)(a+3b)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f} + \frac{b(2a+3b)\tan(e+fx}{f}$$

Mathematica [A] time = 1.30718, size = 151, normalized size = 1.99

$$-\frac{\csc(2e)\csc^3(2(e+fx))\left(-3a^2\sin(2(e+fx))+a^2\sin(6(e+fx))+3a^2\sin(4e+2fx)+a^2\sin(4e+6fx)-6ab\sin(2(e+\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(Csc[2*e]*Csc[2*(e + f*x)]^3*(8*a*(a + 2*b)*Sin[2*e] - 6*(a + 2*b)^2*Sin[2
*f*x] - 3*a^2*Sin[2*(e + f*x)] - 6*a*b*Sin[2*(e + f*x)] + a^2*Sin[6*(e + f*
x)] + 2*a*b*Sin[6*(e + f*x)] + 3*a^2*Sin[4*e + 2*f*x] + a^2*Sin[4*e + 6*f*x
] + 8*a*b*Sin[4*e + 6*f*x] + 8*b^2*Sin[4*e + 6*f*x]))/(6*f)
```

Maple [A] time = 0.061, size = 144, normalized size = 1.9

$$\frac{1}{f}\left(a^2\left(-\frac{2}{3}-\frac{(\csc(fx+e))^2}{3}\right)\cot(fx+e)+2ab\left(-1/3\frac{1}{(\sin(fx+e))^3\cos(fx+e)}+4/3\frac{1}{\sin(fx+e)\cos(fx+e)}-8/3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(-1/3/sin(f*x+e)^3/cos(f*
x+e)+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e))+b^2*(1/3/sin(f*x+e)^3/cos(f*
x+e)^3-2/3/sin(f*x+e)^3/cos(f*x+e)+8/3/sin(f*x+e)/cos(f*x+e)-16/3*cot(f*x+e
)))
```

Maxima [A] time = 0.987961, size = 108, normalized size = 1.42

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(2\,ab+3\,b^2\right)\tan\left(fx+e\right)-\frac{3\left(a^2+4\,ab+3\,b^2\right)\tan\left(fx+e\right)^2+a^2+2\,ab+b^2}{\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + 3*b^2)*tan(f*x + e) - (3*(a^2 + 4*a*b
+ 3*b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2)/tan(f*x + e)^3)/f
```

Fricas [A] time = 0.477801, size = 240, normalized size = 3.16

$$-\frac{2\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^6-3\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+6\left(ab+b^2\right)\cos\left(fx+e\right)^2+b^2}{3\left(f\cos\left(fx+e\right)^5-f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^6 - 3*(a^2 + 8*a*b + 8*b^2)*cos(
f*x + e)^4 + 6*(a*b + b^2)*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*cos
(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2773, size = 142, normalized size = 1.87

$$\frac{b^2\tan\left(fx+e\right)^3+6\,ab\tan\left(fx+e\right)+9\,b^2\tan\left(fx+e\right)-\frac{3\,a^2\tan\left(fx+e\right)^2+12\,ab\tan\left(fx+e\right)^2+9\,b^2\tan\left(fx+e\right)^2+a^2+2\,ab+b^2}{\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e) - (3*a^2*
tan(f*x + e)^2 + 12*a*b*tan(f*x + e)^2 + 9*b^2*tan(f*x + e)^2 + a^2 + 2*a*b
 + b^2)/tan(f*x + e)^3)/f
```

3.27 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=103

$$-\frac{\left(a^2+6ab+6b^2\right)\cot(e+fx)}{f}+\frac{2b(a+2b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] -(((a^2 + 6*a*b + 6*b^2)*Cot[e + f*x])/f) - (2*(a + b)*(a + 2*b)*Cot[e + f*
x]^3)/(3*f) - ((a + b)^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x
])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0968982, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4132, 448}

$$-\frac{\left(a^2+6ab+6b^2\right)\cot(e+fx)}{f}+\frac{2b(a+2b)\tan(e+fx)}{f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] -(((a^2 + 6*a*b + 6*b^2)*Cot[e + f*x])/f) - (2*(a + b)*(a + 2*b)*Cot[e + f*
x]^3)/(3*f) - ((a + b)^2*Cot[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x
])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a+b+bx^2\right)^2}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(2b(a+2b)+\frac{(a+b)^2}{x^6}+\frac{2(a+b)(a+2b)}{x^4}+\frac{a^2+6ab+6b^2}{x^2}+b^2x^2\right)dx, x, \tan(e-\right.}{f}$$
$$= -\frac{\left(a^2+6ab+6b^2\right)\cot(e+fx)}{f}-\frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^{}}{5f}$$

Mathematica [B] time = 1.77154, size = 353, normalized size = 3.43

$$-\frac{\csc(e)\sec(e)\csc^5(e+fx)\sec^3(e+fx)\left(-32\left(2a^2+9ab+12b^2\right)\sin(2fx)-24a^2\sin(2(e+fx))+8a^2\sin(4(e+fx))+8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(Csc[e]*Csc[e + f*x]^5*Sec[e]*Sec[e + f*x]^3*(20*a*(5*a + 12*b)*Sin[2*e] -
 32*(2*a^2 + 9*a*b + 12*b^2)*Sin[2*f*x] - 24*a^2*Sin[2*(e + f*x)] - 108*a*b
*Sin[2*(e + f*x)] - 54*b^2*Sin[2*(e + f*x)] + 8*a^2*Sin[4*(e + f*x)] + 36*a
*b*Sin[4*(e + f*x)] + 18*b^2*Sin[4*(e + f*x)] + 8*a^2*Sin[6*(e + f*x)] + 36
*a*b*Sin[6*(e + f*x)] + 18*b^2*Sin[6*(e + f*x)] - 4*a^2*Sin[8*(e + f*x)] -
18*a*b*Sin[8*(e + f*x)] - 9*b^2*Sin[8*(e + f*x)] + 8*a^2*Sin[2*(e + 2*f*x)]
 + 96*a*b*Sin[2*(e + 2*f*x)] + 128*b^2*Sin[2*(e + 2*f*x)] + 40*a^2*Sin[4*e
+ 2*f*x] + 8*a^2*Sin[4*e + 6*f*x] + 96*a*b*Sin[4*e + 6*f*x] + 128*b^2*Sin[4
*e + 6*f*x] - 4*a^2*Sin[6*e + 8*f*x] - 48*a*b*Sin[6*e + 8*f*x] - 64*b^2*Sin
[6*e + 8*f*x]))/(1920*f)
```

Maple [A] time = 0.061, size = 190, normalized size = 1.8

$$\frac{1}{f}\left(a^2\left(-\frac{8}{15}-\frac{\left(\csc\left(fx+e\right)\right)^4}{5}-\frac{4\left(\csc\left(fx+e\right)\right)^2}{15}\right)\cot\left(fx+e\right)+2\,ab\left(-1/5\,\frac{1}{\left(\sin\left(fx+e\right)\right)^5\cos\left(fx+e\right)}-2/5\,\frac{}{\left(\sin\left(fx\right.\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-8/15-1/5*csc(f*x+e)^4-4/15*csc(f*x+e)^2)*cot(f*x+e)+2*a*b*(-1/5/
sin(f*x+e)^5/cos(f*x+e)-2/5/sin(f*x+e)^3/cos(f*x+e)+8/5/sin(f*x+e)/cos(f*x+
e)-16/5*cot(f*x+e))+b^2*(-1/5/sin(f*x+e)^5/cos(f*x+e)^3+8/15/sin(f*x+e)^3/c
os(f*x+e)^3-16/15/sin(f*x+e)^3/cos(f*x+e)+64/15/sin(f*x+e)/cos(f*x+e)-128/1
5*cot(f*x+e)))
```

Maxima [A] time = 0.999698, size = 144, normalized size = 1.4

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\left(ab+2\,b^2\right)\tan\left(fx+e\right)-\frac{15\left(a^2+6\,ab+6\,b^2\right)\tan\left(fx+e\right)^4+10\left(a^2+3\,ab+2\,b^2\right)\tan\left(fx+e\right)^2+3\,a^2+6\,ab+3\,b^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + 2*b^2)*tan(f*x + e) - (15*(a^2 + 6*a
*b + 6*b^2)*tan(f*x + e)^4 + 10*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^
2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Fricas [A] time = 0.497218, size = 346, normalized size = 3.36

$$-\frac{8\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^8-20\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^4-10\cdots}{15\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^8 - 20*(a^2 + 12*a*b + 16*b^2
)*cos(f*x + e)^6 + 15*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^4 - 10*(3*a*b +
4*b^2)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*
cos(f*x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16885, size = 204, normalized size = 1.98

$$\frac{5\,b^2\tan\left(fx+e\right)^3+30\,ab\tan\left(fx+e\right)+60\,b^2\tan\left(fx+e\right)-\frac{15\,a^2\tan\left(fx+e\right)^4+90\,ab\tan\left(fx+e\right)^4+90\,b^2\tan\left(fx+e\right)^4+10\,a^2\tan\left(fx+e\right)^2+3}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 60*b^2*tan(f*x + e) - (1
5*a^2*tan(f*x + e)^4 + 90*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 + 10*a
^2*tan(f*x + e)^2 + 30*a*b*tan(f*x + e)^2 + 20*b^2*tan(f*x + e)^2 + 3*a^2 +
 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

3.28 $\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{(2a+b)\cos^3(e+fx)}{3a^2f}-\frac{(a+b)^2\cos(e+fx)}{a^3f}+\frac{\sqrt{b}(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f}-\frac{\cos^5(e+fx)}{5af}$$

```
[Out] (Sqrt[b]*(a + b)^2*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(7/2)*f) - ((
a + b)^2*Cos[e + f*x])/(a^3*f) + ((2*a + b)*Cos[e + f*x]^3)/(3*a^2*f) - Cos
[e + f*x]^5/(5*a*f)
```

Rubi [A] time = 0.105306, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4133, 461, 205}

$$\frac{(2a+b)\cos^3(e+fx)}{3a^2f}-\frac{(a+b)^2\cos(e+fx)}{a^3f}+\frac{\sqrt{b}(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f}-\frac{\cos^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*(a + b)^2*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(7/2)*f) - ((
a + b)^2*Cos[e + f*x])/(a^3*f) + ((2*a + b)*Cos[e + f*x]^3)/(3*a^2*f) - Cos
[e + f*x]^5/(5*a*f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

```
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^3} - \frac{(2a+b)x^2}{a^2} + \frac{x^4}{a} + \frac{-a^2b-2ab^2-b^3}{a^3\left(b+ax^2\right)}\right)dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos(e+fx)}{a^3f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2f} - \frac{\cos^5(e+fx)}{5af} + \frac{\left(b(a+b)^2\right)\text{Subst}\left(\int \frac{1}{b+ax^2}\,d\right.}{a^3f}$$

$$= \frac{\sqrt{b}(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f} - \frac{(a+b)^2\cos(e+fx)}{a^3f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2f} - \frac{\cos^5(e+fx)}{5af}$$

Mathematica [C] time = 3.51777, size = 425, normalized size = 4.34

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-8\sqrt{a}\sqrt{b}\cos(e+fx)\left(3a^2\cos(4(e+fx))+89a^2-4a(7a+5b)\cos(2(e+fx))+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)
*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[
(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f
*x)/2]))/Sqrt[b]] + 15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)*ArcTan[((-Sq
rt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Co
s[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]] - 75*a^3*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 75
*a^3*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 8*Sqrt[a]*S
```

```
qrt[b]*Cos[e + f*x]*(89*a^2 + 220*a*b + 120*b^2 - 4*a*(7*a + 5*b)*Cos[2*(e
+ f*x)] + 3*a^2*Cos[4*(e + f*x)]))*Sec[e + f*x]^2)/(1920*a^(7/2)*Sqrt[b]*f*
(a + b*Sec[e + f*x]^2))
```

Maple [B] time = 0.066, size = 183, normalized size = 1.9

$$-\frac{(\cos(fx+e))^5}{5af}+\frac{2(\cos(fx+e))^3}{3af}+\frac{(\cos(fx+e))^3 b}{3fa^2}-\frac{\cos(fx+e)}{af}-2\frac{b\cos(fx+e)}{fa^2}-\frac{b^2\cos(fx+e)}{fa^3}+\frac{b}{af}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/5*cos(f*x+e)^5/a/f+2/3*cos(f*x+e)^3/a/f+1/3/f/a^2*cos(f*x+e)^3*b-cos(f*x
+e)/a/f-2/f/a^2*b*cos(f*x+e)-1/f/a^3*b^2*cos(f*x+e)+1/f*b/a/(a*b)^(1/2)*arc
tan(a*cos(f*x+e)/(a*b)^(1/2))+2/f*b^2/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(
a*b)^(1/2))+1/f*b^3/a^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.569868, size = 555, normalized size = 5.66

$$\left[-\frac{6a^2\cos(fx+e)^5-10(2a^2+ab)\cos(fx+e)^3-15(a^2+2ab+b^2)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+30(}{30a^3f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*a^2*cos(f*x + e)^5 - 10*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2 +
2*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e
) - b)/(a*cos(f*x + e)^2 + b)) + 30*(a^2 + 2*a*b + b^2)*cos(f*x + e))/(a^3*
f), -1/15*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2
+ 2*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(a^2 + 2*a
*b + b^2)*cos(f*x + e))/(a^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15467, size = 504, normalized size = 5.14

$$-\frac{\frac{15\left(a^2b+2ab^2+b^3\right)\arctan\left(-\frac{a\cos\left(fx+e\right)-b}{\sqrt{ab}\cos\left(fx+e\right)+\sqrt{ab}}\right)}{\sqrt{ab}a^3}-\frac{2\left(8a^2+25ab+15b^2-\frac{40a^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{110ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{60b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{80a^2\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{160a\cdots}{(\cdots}\cdots}{a^3(\cdots}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*c
os(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^3) - 2*(8*a^2 + 25*a*b + 15*b^2 - 40
*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 110*a*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 90*a*b
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 15*
```

```
b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^3*((cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 1)^5))/f
```

3.29 $\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=71

$$-\frac{(a+b)\cos(e+fx)}{a^2f}+\frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f}+\frac{\cos^3(e+fx)}{3af}$$

```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(5/2)*f) - ((a
+ b)*Cos[e + f*x])/(a^2*f) + Cos[e + f*x]^3/(3*a*f)
```

Rubi [A] time = 0.0840075, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 459, 321, 205}

$$-\frac{(a+b)\cos(e+fx)}{a^2f}+\frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f}+\frac{\cos^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```



```
[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(5/2)*f) - ((a
+ b)*Cos[e + f*x])/(a^2*f) + Cos[e + f*x]^3/(3*a*f)
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{x^2(1-x^2)}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f} \\
&= \frac{\cos^3(e+fx)}{3af} - \frac{(a+b)\,\mathrm{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{af} \\
&= -\frac{(a+b)\cos(e+fx)}{a^2 f} + \frac{\cos^3(e+fx)}{3af} + \frac{(b(a+b))\,\mathrm{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{a^2 f} \\
&= \frac{\sqrt{b}(a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2} f} - \frac{(a+b)\cos(e+fx)}{a^2 f} + \frac{\cos^3(e+fx)}{3af}
\end{aligned}$$

Mathematica [C] time = 1.62599, size = 376, normalized size = 5.3

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(3\left(a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{\ldots}\right.}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a]
- I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(
Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] +
```

```
 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I
*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] - Sqrt[a
+ b]*Tan[(e + f*x)/2])/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(
e + f*x)/2])/Sqrt[b]] + 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(-5*a - 6*b + a*Cos[
2*(e + f*x)]))*Sec[e + f*x]^2)/(48*a^(5/2)*Sqrt[b]*f*(a + b*Sec[e + f*x]^2)
)
```

Maple [A] time = 0.064, size = 103, normalized size = 1.5

$$\frac{\left(\cos\left(fx+e\right)\right)^{3}}{3\,af}-\frac{\cos\left(fx+e\right)}{af}-\frac{b\cos\left(fx+e\right)}{fa^{2}}+\frac{b}{af}\arctan\left(a\cos\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^{2}}{fa^{2}}\arctan\left(a\cos\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/3*cos(f*x+e)^3/a/f-cos(f*x+e)/a/f-1/f/a^2*b*cos(f*x+e)+1/f*b/a/(a*b)^(1/2
)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/f*b^2/a^2/(a*b)^(1/2)*arctan(a*cos(f*x
+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.551744, size = 381, normalized size = 5.37

$$\left[\frac{2\,a\cos\left(fx+e\right)^{3}+3\,(a+b)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\,a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^{2}+b}\right)-6\,(a+b)\cos\left(fx+e\right)}{6\,a^{2}f},\frac{a\cos\left(fx+e\right)^{3}+3\,(a+b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*cos(f*x + e)^3 + 3*(a + b)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*
a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(a + b)*cos(f*x
+ e))/(a^2*f), 1/3*(a*cos(f*x + e)^3 + 3*(a + b)*sqrt(b/a)*arctan(a*sqrt(b/
a)*cos(f*x + e)/b) - 3*(a + b)*cos(f*x + e))/(a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18679, size = 120, normalized size = 1.69

$$\frac{\left(ab+b^2\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2f}+\frac{a^2f^5\cos\left(fx+e\right)^3-3\,a^2f^5\cos\left(fx+e\right)-3\,abf^5\cos\left(fx+e\right)}{3\,a^3f^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) + 1/3*(a^2*f
^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) - 3*a*b*f^5*cos(f*x + e))/(a^3*f
^6)
```

3.30 $\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(3/2)*f) - Cos[e + f*x]
/(a*f)
```

Rubi [A] time = 0.0408325, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4133, 321, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a^(3/2)*f) - Cos[e + f*x]
/(a*f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)}{af}+\frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{af}\\ &= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f}-\frac{\cos(e+fx)}{af}\end{aligned}$$

Mathematica [C] time = 0.957014, size = 329, normalized size = 7.

$$\frac{(a\cos(2(e+fx))+a+2b)\left(-4\sqrt{a}\sqrt{b}\cos(e+fx)-a\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)-a\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}}{\sqrt{b}}\right)+(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (((a + 4*b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*
Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Tan[(f*x)/2]))/Sqrt[b]] + (a + 4*b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*
Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a
 + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - a*ArcTan[(Sqrt[
a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - a*ArcTan[(Sqrt[a] + Sqrt[a +
b]*Tan[(e + f*x)/2])/Sqrt[b]] - 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x])*(a + 2*b +
a*Cos[2*(e + f*x)]))/(8*a^(3/2)*Sqrt[b]*f*(b + a*Cos[e + f*x]^2))
```

Maple [A] time = 0.033, size = 46, normalized size = 1.

$$-\frac{b}{fa}\arctan\left(b\sec\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{fa\sec\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f*b/a/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2))-1/f/a/sec(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.534208, size = 265, normalized size = 5.64

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\,a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^{2}+b}\right)-2\,\cos\left(fx+e\right)}{2\,af},\ \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos\left(fx+e\right)}{b}\right)-\cos\left(fx+e\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/
(a*cos(f*x + e)^2 + b)) - 2*cos(f*x + e))/(a*f), (sqrt(b/a)*arctan(a*sqrt(b
/a)*cos(f*x + e)/b) - cos(f*x + e))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.21357, size = 59, normalized size = 1.26

$$\frac{b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{ab}af}-\frac{\cos\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a*f) - cos(f*x + e)/(a*f)
```

3.31 $\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}f(a+b)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*f) - ArcT
anh[Cos[e + f*x]]/((a + b)*f)
```

Rubi [A] time = 0.0705629, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 481, 206, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}f(a+b)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b)*f) - ArcT
anh[Cos[e + f*x]]/((a + b)*f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)f} + \frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{(a+b)f}\\ &= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)f} - \frac{\tanh^{-1}(\cos(e+fx))}{(a+b)f}\end{aligned}$$

Mathematica [C] time = 0.623957, size = 239, normalized size = 4.35

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos\ldots}{\sqrt{b}}\right)}{\sqrt{a}} \ldots}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a
 + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] +
Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] -
Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]])/((a + b)*f)
```

Maple [A] time = 0.066, size = 76, normalized size = 1.4

$$-\frac{\ln\left(1+\cos\left(fx+e\right)\right)}{f\left(2a+2b\right)}+\frac{b}{f\left(a+b\right)}\arctan\left(a\cos\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{\ln\left(-1+\cos\left(fx+e\right)\right)}{f\left(2a+2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/(2*a+2*b)*ln(1+cos(f*x+e))+1/f*b/(a+b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)
/(a*b)^(1/2))+1/f/(2*a+2*b)*ln(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.564539, size = 410, normalized size = 7.45

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^{2}+b}\right)-\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)}{2\left(a+b\right)f},\frac{2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos\left(fx+e\right)}{b}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/
(a*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x +
e) + 1/2))/((a + b)*f), 1/2*(2*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b)
```

```
 - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/((a + b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.14841, size = 116, normalized size = 2.11

$$-\frac{\frac{2\,b\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}-\frac{\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)}{a+b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b))
)/(sqrt(a*b)*(a + b)) - log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a + b)
)/f
```

3.32 $\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^2}-\frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^2}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)}$$

```
[Out] (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^2*f) - ((
a - b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x])
/(2*(a + b)*f)
```

Rubi [A] time = 0.099954, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 471, 522, 206, 205}

$$\frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^2}-\frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^2}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^2*f) - ((
a - b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x])
/(2*(a + b)*f)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{b-ax^2}{\left(1-x^2\right)\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)^2f} + \frac{(ab)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\right.}{(a+b}$$

$$= \frac{\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^2f} - \frac{(a-b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^2f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f}$$

Mathematica [C] time = 1.86616, size = 371, normalized size = 4.31

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-8\sqrt{a}\sqrt{b}\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\,s}\right.}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(-8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a] - I*
Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt
[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - 8*S
qrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2
])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin
[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + a*Csc[(e + f*x)/2]^2 + b*Csc[(e + f*x)/2]
^2 + 4*a*Log[Cos[(e + f*x)/2]] - 4*b*Log[Cos[(e + f*x)/2]] - 4*a*Log[Sin[(e
 + f*x)/2]] + 4*b*Log[Sin[(e + f*x)/2]] - a*Sec[(e + f*x)/2]^2 - b*Sec[(e +
 f*x)/2]^2)*Sec[e + f*x]^2)/(16*(a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [B] time = 0.081, size = 158, normalized size = 1.8

$$\frac{1}{f\left(4a+4b\right)\left(1+\cos\left(fx+e\right)\right)}-\frac{\ln\left(1+\cos\left(fx+e\right)\right)a}{4f\left(a+b\right)^{2}}+\frac{\ln\left(1+\cos\left(fx+e\right)\right)b}{4f\left(a+b\right)^{2}}+\frac{ab}{f\left(a+b\right)^{2}}\arctan\left(a\cos\left(fx+e\right)\frac{1}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/(4*a+4*b)/(1+cos(f*x+e))-1/4/f/(a+b)^2*ln(1+cos(f*x+e))*a+1/4/f/(a+b)^2
*ln(1+cos(f*x+e))*b+1/f*a*b/(a+b)^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(
1/2))+1/f/(4*a+4*b)/(-1+cos(f*x+e))+1/4/f/(a+b)^2*ln(-1+cos(f*x+e))*a-1/4/f
/(a+b)^2*ln(-1+cos(f*x+e))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.674584, size = 848, normalized size = 9.86

$$\left[\frac{2\sqrt{-ab}\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{-ab}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^{2}+b}\right)+2\left(a+b\right)\cos\left(fx+e\right)-\left(\left(a-b\right)\cos\left(fx+e\right)^{2}-a+b\right.}{4\left(\left(a^{2}+2ab+b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{2}+2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*
b)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(a + b)*cos(f*x + e) - ((a
 - b)*cos(f*x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*
x + e)^2 - a + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(
f*x + e)^2 - (a^2 + 2*a*b + b^2)*f), 1/4*(4*sqrt(a*b)*(cos(f*x + e)^2 - 1)*
arctan(sqrt(a*b)*cos(f*x + e)/b) + 2*(a + b)*cos(f*x + e) - ((a - b)*cos(f*
x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*x + e)^2 - a
 + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 -
 (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{3}\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Giac [B] time = 1.22227, size = 294, normalized size = 3.42

$$-\frac{\frac{8ab\arctan\left(-\frac{a\cos\left(fx+e\right)-b}{\sqrt{ab}\cos\left(fx+e\right)+\sqrt{ab}}\right)}{\left(a^{2}+2ab+b^{2}\right)\sqrt{ab}}-\frac{2\left(a-b\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a^{2}+2ab+b^{2}}-\frac{\left(a+b-\frac{2a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{2b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)\left(\cos\left(fx+e\right)+1\right)}{\left(a^{2}+2ab+b^{2}\right)\left(\cos\left(fx+e\right)-1\right)}+\frac{\cos\left(fx+e\right)-1}{\left(a+b\right)\left(\cos\left(fx+e\right)+1\right)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b
)))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(a - b)*log(-(cos(f*x + e) - 1)/(co
s(f*x + e) + 1))/(a^2 + 2*a*b + b^2) - (a + b - 2*a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) +
 1)/((a^2 + 2*a*b + b^2)*(cos(f*x + e) - 1)) + (cos(f*x + e) - 1)/((a + b)*
(cos(f*x + e) + 1)))/f
```

3.33 $\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=129

$$-\frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^3}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^3}-\frac{\cot(e+fx)\csc^3(e+fx)}{4f(a+b)}-\frac{(3a-b)\cot(e+fx)}{8f(a+b}$$

```
[Out] (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^3*f) - ((
3*a^2 - 6*a*b - b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^3*f) - ((3*a - b)*Co
t[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4
*(a + b)*f)
```

Rubi [A] time = 0.153726, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4133, 471, 527, 522, 206, 205}

$$-\frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^3}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{f(a+b)^3}-\frac{\cot(e+fx)\csc^3(e+fx)}{4f(a+b)}-\frac{(3a-b)\cot(e+fx)}{8f(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```


```
[Out] (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/((a + b)^3*f) - ((
3*a^2 - 6*a*b - b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^3*f) - ((3*a - b)*Co
t[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f) - (Cot[e + f*x]*Csc[e + f*x]^3)/(4
*(a + b)*f)
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^3(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\text{Subst}\left(\int \frac{b-3ax^2}{(1-x^2)^2(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{\text{Subst}\left(\int \frac{b(5a+b)-a(3a-b)x^2}{(1-x^2)(b+ax^2)}\,dx\right.}{8(a+b)^2f}$$

$$= -\frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f} + \frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x,\right.}{(a+b)^3f}$$

$$= \frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^3f} - \frac{(3a^2-6ab-b^2)\tanh^{-1}(\cos(e+fx))}{8(a+b)^3f} - \frac{(3a-b)\cot(e+fx)\,\text{c}}{8(a+b)^2f}$$

Mathematica [C] time = 5.59755, size = 549, normalized size = 4.26

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-64a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-\right.}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(-64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[a] - I
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqr
t[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - 64
*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + 6*a^2*Csc[(e + f*x)/2]^2 + 4*a*b*Csc[(e
 + f*x)/2]^2 - 2*b^2*Csc[(e + f*x)/2]^2 + a^2*Csc[(e + f*x)/2]^4 + 2*a*b*Cs
c[(e + f*x)/2]^4 + b^2*Csc[(e + f*x)/2]^4 + 24*a^2*Log[Cos[(e + f*x)/2]] -
48*a*b*Log[Cos[(e + f*x)/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] - 24*a^2*Log[Sin
[(e + f*x)/2]] + 48*a*b*Log[Sin[(e + f*x)/2]] + 8*b^2*Log[Sin[(e + f*x)/2]]
 - 6*a^2*Sec[(e + f*x)/2]^2 - 4*a*b*Sec[(e + f*x)/2]^2 + 2*b^2*Sec[(e + f*x
)/2]^2 - a^2*Sec[(e + f*x)/2]^4 - 2*a*b*Sec[(e + f*x)/2]^4 - b^2*Sec[(e + f
*x)/2]^4)*Sec[e + f*x]^2)/(128*(a + b)^3*f*(a + b*Sec[e + f*x]^2))
```

Maple [B] time = 0.084, size = 296, normalized size = 2.3

$$\frac{1}{2 f (8 a+8 b) \left(1+\cos \left(f x+e\right)\right)^{2}}+\frac{3 a}{16 f (a+b)^{2} \left(1+\cos \left(f x+e\right)\right)}-\frac{b}{16 f (a+b)^{2} \left(1+\cos \left(f x+e\right)\right)}-\frac{3 \ln \left(1+\cos \left(f\right.\right.}{16 f (a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2/f/(8*a+8*b)/(1+cos(f*x+e))^2+3/16/f/(a+b)^2/(1+cos(f*x+e))*a-1/16/f/(a+
b)^2/(1+cos(f*x+e))*b-3/16/f/(a+b)^3*ln(1+cos(f*x+e))*a^2+3/8/f/(a+b)^3*ln(
1+cos(f*x+e))*a*b+1/16/f/(a+b)^3*ln(1+cos(f*x+e))*b^2+1/f*a^2*b/(a+b)^3/(a*
b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))-1/2/f/(8*a+8*b)/(-1+cos(f*x+e))^2
+3/16/f/(a+b)^2/(-1+cos(f*x+e))*a-1/16/f/(a+b)^2/(-1+cos(f*x+e))*b+3/16/f/(
a+b)^3*ln(-1+cos(f*x+e))*a^2-3/8/f/(a+b)^3*ln(-1+cos(f*x+e))*a*b-1/16/f/(a+
b)^3*ln(-1+cos(f*x+e))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.799531, size = 1643, normalized size = 12.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3 + 8*(a*cos(f*x + e)^4 - 2*a*c
os(f*x + e)^2 + a)*sqrt(-a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*b)*cos(f*x
```

```
 + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) -
 ((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x +
e)^2 + 3*a^2 - 6*a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b -
 b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a
*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*
cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*f), 1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3
 + 16*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 + a)*sqrt(a*b)*arctan(sqrt(a*b
)*cos(f*x + e)/b) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) - ((3*a^2 - 6*a*b
- b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*
a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b - b^2)*cos(f*x + e
)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a*b - b^2)*log(-1/
2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28211, size = 551, normalized size = 4.27

$$-\frac{\frac{64\,a^2b\arctan\left(-\frac{a\cos\left(fx+e\right)-b}{\sqrt{ab}\cos\left(fx+e\right)+\sqrt{ab}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{ab}}-\frac{4\left(3\,a^2-6\,ab-b^2\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{\frac{8\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{b\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}}{a^2+2\,ab+b^2}+\frac{\left(a^2+2\,ab+b^2-\frac{8\,a^2(\cos}{\cos(}\right.}{}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*a^2*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt
(a*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) - 4*(3*a^2 - 6*a*b - b^
```

```
2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) + (8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 - b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^2 + 2
*a*b + b^2) + (a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 36*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 - 6*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) - 1)^2))/f
```

3.34 $\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(11a^2+18ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16a^3f}+\frac{x\left(30a^2b+5a^3+40ab^2+16b^3\right)}{16a^4}-\frac{\sqrt{b}(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f}+$$

```
[Out] ((5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*x)/(16*a^4) - (Sqrt[b]*(a + b)^(5/2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*f) - ((11*a^2 + 18*a*b +
 8*b^2)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((3*a + 2*b)*Cos[e + f*x]^3
*Sin[e + f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f)
```

Rubi [A] time = 0.335182, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4132, 470, 578, 527, 522, 203, 205}

$$-\frac{\left(11a^2+18ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16a^3f}+\frac{x\left(30a^2b+5a^3+40ab^2+16b^3\right)}{16a^4}-\frac{\sqrt{b}(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```



```
[Out] ((5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*x)/(16*a^4) - (Sqrt[b]*(a + b)^(5/2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*f) - ((11*a^2 + 18*a*b +
 8*b^2)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((3*a + 2*b)*Cos[e + f*x]^3
*Sin[e + f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)-3(2a+b)x^2\right)}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{3(a+b)(3a+2b)-3(8a}{(1+x^2)^2(a+}\right.}{2}$$

$$= -\frac{\left(11a^2+18ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e}{}$$

$$= -\frac{\left(11a^2+18ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e}{}$$

$$= \frac{\left(5a^3+30a^2b+40ab^2+16b^3\right)x}{16a^4} - \frac{\sqrt{b}(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f} - \frac{\left(11a^2+18ab+8b^2\right)\text{c}}{16}$$

Mathematica [C] time = 4.34908, size = 357, normalized size = 2.15

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{b(\cos(e)-i\sin(e))^4}\left(2\sqrt{b}\sqrt{a+b}\left(-3a\left(15a^2+32ab+16b^2\right)\sin(2(e+fx))+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*Sqrt[b]*(9*a^4 + 136*a^3*
b + 384*a^2*b^2 + 384*a*b^3 + 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2
*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos
[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4
]*(3*a^3*(9*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + 2*Sqrt[b]
*Sqrt[a + b]*(-12*a^3*e + 60*a^3*f*x + 360*a^2*b*f*x + 480*a*b^2*f*x + 192*
b^3*f*x - 3*a*(15*a^2 + 32*a*b + 16*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a + 2*
b)*Sin[4*(e + f*x)] - a^3*Sin[6*(e + f*x)]))))/(768*a^4*Sqrt[b]*Sqrt[a + b]
*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [B] time = 0.102, size = 460, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -9/8/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b-1/2/f/a^3/(tan(f*x+e)^2+1)^3*t
an(f*x+e)^5*b^2-11/16/f/a/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5-2/f/a^2/(tan(f*x+
e)^2+1)^3*tan(f*x+e)^3*b-1/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b^2-5/6/f/
a/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-5/16/f/a/(tan(f*x+e)^2+1)^3*tan(f*x+e)-7/
8/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b-1/2/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*
x+e)*b^2+15/8/f/a^2*arctan(tan(f*x+e))*b+5/2/f/a^3*arctan(tan(f*x+e))*b^2+1
/f/a^4*arctan(tan(f*x+e))*b^3+5/16/f/a*arctan(tan(f*x+e))-1/f*b/a/((a+b)*b)
^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-3/f*b^2/a^2/((a+b)*b)^(1/2)*arc
tan(tan(f*x+e)*b/((a+b)*b)^(1/2))-3/f*b^3/a^3/((a+b)*b)^(1/2)*arctan(tan(f*
x+e)*b/((a+b)*b)^(1/2))-1/f*b^4/a^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a
+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.624802, size = 1019, normalized size = 6.14

$$\left[\frac{3\left(5a^3+30a^2b+40ab^2+16b^3\right)fx+12\left(a^2+2ab+b^2\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2+4\left((a+\right.}{a^2\cos\left(fx+e\right)^4+2ab\,c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 12*(a^2 + 2*a*b + b^2
)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4
*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x + e)^3 + 3*(1
1*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), 1/48*(3*(5
*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 24*(a^2 + 2*a*b + b^2)*sqrt(a*b
+ b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x +
 e)*sin(f*x + e))) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x +
 e)^3 + 3*(11*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27881, size = 338, normalized size = 2.04

$$\frac{\frac{3\left(5a^3+30a^2b+40ab^2+16b^3\right)(fx+e)}{a^4} - \frac{48\left(a^3b+3a^2b^2+3ab^3+b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^4} - \frac{33a^2\tan(fx+e)^5+54ab\tan(fx+e)^5+24b^2}{}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 - 48*(a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*
```

```
tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^4) - (33*a^2*tan(f*x + e)
^5 + 54*a*b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3
+ 96*a*b*tan(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 42*
a*b*tan(f*x + e) + 24*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

3.35 $\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{x\left(3a^2+12ab+8b^2\right)}{8a^3}-\frac{\sqrt{b}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f}-\frac{(5a+4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]])/(a^3*f) - ((5*a + 4*b)*Cos[e + f*x]*Sin[e +
f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rubi [A] time = 0.168949, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 203, 205}

$$\frac{x\left(3a^2+12ab+8b^2\right)}{8a^3}-\frac{\sqrt{b}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f}-\frac{(5a+4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```


```
[Out] ((3*a^2 + 12*a*b + 8*b^2)*x)/(8*a^3) - (Sqrt[b]*(a + b)^(3/2)*ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]])/(a^3*f) - ((5*a + 4*b)*Cos[e + f*x]*Sin[e +
f*x])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{a+b+(b-4(a+b))x^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+4b)-b(5a+}{(1+x^2)(a+b+bx^2}\right.}{8a^2}$$

$$= -\frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\left(b(a+b)^2\right)\text{Subst}\left(\int \frac{1}{a+b+b}\right.}{a^3f}$$

$$= \frac{\left(3a^2+12ab+8b^2\right)x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f} - \frac{(5a+4b)\cos(e+fx)\sin(e+fx}{8a^2f}$$

Mathematica [C] time = 2.13793, size = 303, normalized size = 2.59

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{b(\cos(e)-i\sin(e))^4}\left(\sqrt{b}\sqrt{a+b}\left(a^2\sin(4(e+fx))-2a^2e+12a^2fx-8a(a+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[b]*(3*a^3 + 34*a^2*b +
 64*a*b^2 + 32*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4]*(a^2*(3*a + 2*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]*(-2*a^2*e
+ 12*a^2*f*x + 48*a*b*f*x + 32*b^2*f*x - 8*a*(a + b)*Sin[2*(e + f*x)] + a^2
*Sin[4*(e + f*x)]))))/(64*a^3*Sqrt[b]*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*
Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [B] time = 0.097, size = 260, normalized size = 2.2

$$-\frac{\left(\tan\left(fx+e\right)\right)^{3} b}{2 f a^{2} \left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{5 \left(\tan\left(fx+e\right)\right)^{3}}{8 f a \left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{3 \tan\left(fx+e\right)}{8 f a \left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{\tan\left(fx+e\right) b}{2 f a^{2} \left(\left(\tan\left(fx+e\right)\right)^{2}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3*b-5/8/f/a/(tan(f*x+e)^2+1)^2*tan
(f*x+e)^3-3/8/f/a/(tan(f*x+e)^2+1)^2*tan(f*x+e)-1/2/f/a^2/(tan(f*x+e)^2+1)^
2*tan(f*x+e)*b+3/2/f/a^2*arctan(tan(f*x+e))*b+1/f/a^3*arctan(tan(f*x+e))*b^
2+3/8/f/a*arctan(tan(f*x+e))-1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((
a+b)*b)^(1/2))-2/f*b^2/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1
/2))-1/f*b^3/a^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.592449, size = 792, normalized size = 6.77

$$\left[\frac{\left(3 a^{2}+12 ab+8 b^{2}\right) fx+2 \sqrt{-ab-b^{2}}(a+b) \log\left(\frac{\left(a^{2}+8 ab+8 b^{2}\right) \cos\left(fx+e\right)^{4}-2 \left(3 ab+4 b^{2}\right) \cos\left(fx+e\right)^{2}+4 \left(\left(a+2 b\right) \cos\left(fx+e\right)^{3}-b \cos\left(fx+e\right)\right)}{a^{2} \cos\left(fx+e\right)^{4}+2 ab \cos\left(fx+e\right)^{2}+b^{2}}\right.}{8 a^{3} f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^2 + 12*a*b + 8*b^2)*f*x + 2*sqrt(-a*b - b^2)*(a + b)*log(((a^2 +
 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a +
 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)
/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^2*cos(f*x + e)^3
 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/8*((3*a^2 + 12*a*
b + 8*b^2)*f*x + 4*sqrt(a*b + b^2)*(a + b)*arctan(1/2*((a + 2*b)*cos(f*x +
e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + (2*a^2*cos(f*x + e
)^3 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28923, size = 216, normalized size = 1.85

$$\frac{\frac{(3a^2+12ab+8b^2)(fx+e)}{a^3} - \frac{8(a^2b+2ab^2+b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^3} - \frac{5a\tan(fx+e)^3+4b\tan(fx+e)^3+3a\tan(fx+e)+4b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a^3 - 8*(a^2*b + 2*a*b^2 + b^3)*(pi
*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))
/(sqrt(a*b + b^2)*a^3) - (5*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 3*a*tan
(f*x + e) + 4*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

3.36 $\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f}+\frac{x(a+2b)}{2a^2}-\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

```
[Out] ((a + 2*b)*x)/(2*a^2) - (Sqrt[b]*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(a^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rubi [A] time = 0.0983991, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 471, 522, 203, 205}

$$-\frac{\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f}+\frac{x(a+2b)}{2a^2}-\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/(2*a^2) - (Sqrt[b]*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(a^2*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af} + \frac{\text{Subst}\left(\int \frac{a+b-bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af} - \frac{(b(a+b))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a^2 f} + \frac{(a+2b)\,\text{Subs}}{}$$

$$= \frac{(a+2b)x}{2a^2} - \frac{\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2 f} - \frac{\cos(e+fx)\sin(e+fx)}{2af}$$

Mathematica [C] time = 0.89639, size = 245, normalized size = 3.22

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}f\sqrt{a+b}}-\frac{-\frac{(a^2+8ab+8b^2)(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec(fx)(a\sin(2e+fx)-(a+2b)\sin}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{f\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}}{a^2}\right.}{16\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f) - (-4*(a + 2*b)*x - ((a^2 + 8*a*b +
 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a
*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e]
- I*Sin[2*e]))/(Sqrt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*a*Cos[2*f
*x]*Sin[2*e])/f + (2*a*Cos[2*e]*Sin[2*f*x])/f)/a^2))/(16*(a + b*Sec[e + f*x
]^2))
```

Maple [A] time = 0.085, size = 124, normalized size = 1.6

$$-\frac{\tan\left(fx+e\right)}{2fa\left(\left(\tan\left(fx+e\right)\right)^2+1\right)}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{2fa}+\frac{\arctan\left(\tan\left(fx+e\right)\right)b}{fa^2}-\frac{b}{fa}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2/f/a*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2/f/a*arctan(tan(f*x+e))+1/f/a^2*arc
tan(tan(f*x+e))*b-1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/
2))-1/f*b^2/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.563316, size = 625, normalized size = 8.22

$$\left[\frac{2(a+2b)fx-2a\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-2\left(3ab+4b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a+2b\right)\cos\left(fx+\right.\right.}{a^2\cos\left(fx+e\right)^4+2ab\cos\left(fx+e\right)^2+b}\right.}{4a^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + 2*b)*f*x - 2*a*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b - b^2)*lo
g(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
+ 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^2*f), 1/2*
((a + 2*b)*f*x - a*cos(f*x + e)*sin(f*x + e) + sqrt(a*b + b^2)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))))
/(a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.2368, size = 131, normalized size = 1.72

$$\frac{\frac{(fx+e)(a+2b)}{a^2} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\sqrt{ab+b^2}}{a^2} - \frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b)/a^2 - 2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))*sqrt(a*b + b^2)/a^2 - tan(f*x + e)/((t
an(f*x + e)^2 + 1)*a))/f
```

3.37 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rubi [A] time = 0.0444918, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4127, 3181, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 4127

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sec^2(e+fx)}\,dx &= \frac{x}{a}-\frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}\\ &= \frac{x}{a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx,x,\cot(e+fx)\right)}{af}\\ &= \frac{x}{a}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}\end{aligned}$$

Mathematica [C] time = 0.292686, size = 182, normalized size = 4.04

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(fx\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}+b(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec}{2\sqrt{a+b}}\right.\right.}{2af\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.07, size = 48, normalized size = 1.1

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}-\frac{b}{fa}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/a*arctan(tan(f*x+e))-1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.556943, size = 544, normalized size = 12.09

$$\left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)}{4\,af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.25687, size = 92, normalized size = 2.04

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

3.38 $\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(3/2)*f)) -
 Cot[e + f*x]/((a + b)*f)
```

Rubi [A] time = 0.0729386, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4132, 325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(3/2)*f)) -
 Cot[e + f*x]/((a + b)*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot(e+fx)}{(a+b)f}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)f}\\ &= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f}-\frac{\cot(e+fx)}{(a+b)f}\end{aligned}$$

Mathematica [C] time = 0.670154, size = 189, normalized size = 3.5

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\csc(e)\sin(fx)\sqrt{b(\cos(e)-i\sin(e))^4}\csc(e+fx)+b(\cos(2e)-i\sin(2e))\,\mathrm{t}\right.}{2f(a+b)^{3/2}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b*ArcTan[(Sec[f*x]*(Cos[2*e
] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*
Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Csc[e
]*Csc[e + f*x]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*Sin[f*x]))/(2*(a + b)^(3/2)*f*
(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.084, size = 54, normalized size = 1.

$$-\frac{1}{f\,(a+b)\tan\left(fx+e\right)}-\frac{b}{f\,(a+b)}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/(a+b)/tan(f*x+e)-1/f*b/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b
)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.555723, size = 655, normalized size = 12.13

$$\left[\frac{\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a^2+3ab+2b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(fx+e)+b^2}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right)\sin(f}{4(a+b)f\sin(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b +
 b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^
4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/((a + b)*f*
sin(f*x + e)), 1/2*(sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 2*cos(f*x
+ e))/((a + b)*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.2738, size = 100, normalized size = 1.85

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}(a+b)}+\frac{1}{(a+b)\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*(a + b)) + 1/((a + b)*tan(f*x + e)))/f
```

3.39 $\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}-\frac{a\cot(e+fx)}{f(a+b)^2}$$

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(5/2)*f))
 - (a*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rubi [A] time = 0.0944464, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 453, 325, 205}

$$-\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}-\frac{a\cot(e+fx)}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(5/2)*f))
 - (a*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot^3(e+fx)}{3(a+b)f}+\frac{a\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{(a+b)f}\\ &= -\frac{a\cot(e+fx)}{(a+b)^2f}-\frac{\cot^3(e+fx)}{3(a+b)f}-\frac{(ab)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{(a+b)^2f}\\ &= -\frac{a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f}-\frac{a\cot(e+fx)}{(a+b)^2f}-\frac{\cot^3(e+fx)}{3(a+b)f}\end{aligned}$$

Mathematica [C] time = 2.13727, size = 226, normalized size = 2.97

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{1}{4}\sqrt{a+b}\csc(e)\sqrt{b(\cos(e)-i\sin(e))^4}\csc^3(e+fx)((b-2a)\sin(2e+3fx)+6a\sin\right.}{6f(a+b)^{5/2}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*a*b*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a + b]*
Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(6*a*Sin[f*x] - 3*b*Sin
[2*e + f*x] + (-2*a + b)*Sin[2*e + 3*f*x]))/4))/(6*(a + b)^(5/2)*f*(a + b*S
ec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.097, size = 74, normalized size = 1.

$$-\frac{1}{3f(a+b)\left(\tan\left(fx+e\right)\right)^{3}}-\frac{a}{f(a+b)^{2}\tan\left(fx+e\right)}-\frac{ab}{f(a+b)^{2}}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)b}}\right)\frac{1}{\sqrt{(a+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/3/f/(a+b)/tan(f*x+e)^3-1/f*a/(a+b)^2/tan(f*x+e)-1/f*a*b/(a+b)^2/((a+b)*b
)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.573484, size = 954, normalized size = 12.55

$$\left[-\frac{4\,(2\,a-b)\cos\left(fx+e\right)^{3}-3\left(a\cos\left(fx+e\right)^{2}-a\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a^{2}+3\,ab+2\,b^{2}\right)\right.}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(f\right.}\right.}{12\left(\left(a^{2}+2\,ab+b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{2}+2\,ab+b^{2}\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2))*sin(f*x + e) - 12*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos
(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), -1/6*(2*(2*a - b)*cos(f
*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*
cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x
+ e) - 6*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*
a*b + b^2)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30426, size = 144, normalized size = 1.89

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)ab}{\left(a^2+2\,ab+b^2\right)\sqrt{ab+b^2}}+\frac{3\,a\tan(fx+e)^2+a+b}{\left(a^2+2\,ab+b^2\right)\tan(fx+e)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*a*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*a*tan(f*x + e)^2
 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

3.40 $\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{7/2}}-\frac{a^2\cot(e+fx)}{f(a+b)^3}-\frac{\cot^5(e+fx)}{5f(a+b)}-\frac{(2a+b)\cot^3(e+fx)}{3f(a+b)^2}$$

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(7/2)*f
)) - (a^2*Cot[e + f*x])/((a + b)^3*f) - ((2*a + b)*Cot[e + f*x]^3)/(3*(a +
b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```

Rubi [A] time = 0.139397, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4132, 461, 205}

$$-\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{f(a+b)^{7/2}}-\frac{a^2\cot(e+fx)}{f(a+b)^3}-\frac{\cot^5(e+fx)}{5f(a+b)}-\frac{(2a+b)\cot^3(e+fx)}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```


```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/((a + b)^(7/2)*f
)) - (a^2*Cot[e + f*x])/((a + b)^3*f) - ((2*a + b)*Cot[e + f*x]^3)/(3*(a +
b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

```
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)x^6} + \frac{2a+b}{(a+b)^2x^4} + \frac{a^2}{(a+b)^3x^2} - \frac{a^2b}{(a+b)^3(a+b+bx^2)}\right) dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a^2\cot(e+fx)}{(a+b)^3f} - \frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} - \frac{(a^2b)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e\right.}{(a+b)^3f}$$

$$= -\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}f} - \frac{a^2\cot(e+fx)}{(a+b)^3f} - \frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f}$$

Mathematica [C] time = 1.82896, size = 318, normalized size = 3.03

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\csc(e)\sqrt{b(\cos(e)-i\sin(e))^4}\csc^5(e+fx)\left(10\left(8a^2+b^2\right)\sin(fx)-40a^2\text{si}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(240*a^2*b*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Csc[e]*Csc[e + f*x]^5*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(10*(8*a^2 + b^2)*Si
n[f*x] - 30*b*(3*a + b)*Sin[2*e + f*x] - 40*a^2*Sin[2*e + 3*f*x] + 30*a*b*S
in[2*e + 3*f*x] + 10*b^2*Sin[2*e + 3*f*x] + 15*a*b*Sin[4*e + 3*f*x] + 8*a^2
*Sin[4*e + 5*f*x] - 9*a*b*Sin[4*e + 5*f*x] - 2*b^2*Sin[4*e + 5*f*x])))/(480
*(a + b)^(7/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.115, size = 116, normalized size = 1.1

$$-\frac{1}{5 f (a+b) \left(\tan \left(f x+e\right)\right)^{5}}-\frac{a^{2}}{f (a+b)^{3} \tan \left(f x+e\right)}-\frac{b}{3 f (a+b)^{2} \left(\tan \left(f x+e\right)\right)^{3}}-\frac{2 a}{3 f (a+b)^{2} \left(\tan \left(f x+e\right)\right)^{3}}-\frac{\ }{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/5/f/(a+b)/tan(f*x+e)^5-1/f*a^2/(a+b)^3/tan(f*x+e)-1/3/f/(a+b)^2/tan(f*x+
e)^3*b-2/3/f/(a+b)^2/tan(f*x+e)^3*a-1/f*a^2*b/(a+b)^3/((a+b)*b)^(1/2)*arcta
n(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.606366, size = 1397, normalized size = 13.3

$$\left[-\frac{4 \left(8 a^{2}-9 a b-2 b^{2}\right) \cos \left(f x+e\right)^{5}-20 \left(4 a^{2}-3 a b-b^{2}\right) \cos \left(f x+e\right)^{3}-15 \left(a^{2} \cos \left(f x+e\right)^{4}-2 a^{2} \cos \left(f x+e\right)^{2}+\right.}{60 \left(\left(a^{3}+3 a^{2} b+3 a b^{2}+b^{3}\right) f \cos \left(f x+e\right)^{4}-2 \left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 3*a*b - b^2)
*cos(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt
```

```
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*a^2*cos(f*x + e))/(((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*co
s(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)), -1/30*(2*(
8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 10*(4*a^2 - 3*a*b - b^2)*cos(f*x +
e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(b/(a + b))
*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*
sin(f*x + e)))*sin(f*x + e) + 30*a^2*cos(f*x + e))/(((a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x +
e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32298, size = 243, normalized size = 2.31

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2b}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab+b^2}}+\frac{15a^2\tan(fx+e)^4+10a^2\tan(fx+e)^2+15ab\tan(fx+e)^2+5b^2\tan(fx+e)^2+3a^2+6ab+3b^2}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*a^2*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (15
*a^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 15*a*b*tan(f*x + e)^2 + 5*b^2
*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*t
an(f*x + e)^5))/f
```

3.41 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(a\cos^2(e+fx)+b\right)}+\frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf}-\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f}+\frac{\sqrt{b}(a+b)(3a+7b)\tan^{-}}{2a^{9/2}f}$$

```
[Out] (Sqrt[b]*(a + b)*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(
9/2)*f) - ((a + b)*(3*a + 7*b)*Cos[e + f*x])/(2*a^4*f) + ((a + b)*(3*a + 7*
b)*Cos[e + f*x]^3)/(6*a^3*b*f) - Cos[e + f*x]^5/(5*a^2*f) - ((a + b)^2*Cos[
e + f*x]^5)/(2*a^2*b*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.177968, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 463, 459, 302, 205}

$$-\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(a\cos^2(e+fx)+b\right)}+\frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf}-\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f}+\frac{\sqrt{b}(a+b)(3a+7b)\tan^{-}}{2a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(a + b)*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(
9/2)*f) - ((a + b)*(3*a + 7*b)*Cos[e + f*x])/(2*a^4*f) + ((a + b)*(3*a + 7*
b)*Cos[e + f*x]^3)/(6*a^3*b*f) - Cos[e + f*x]^5/(5*a^2*f) - ((a + b)^2*Cos[
e + f*x]^5)/(2*a^2*b*f*(b + a*Cos[e + f*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4\left(1-x^2\right)^2}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{x^4\left(-2a^2+5(a+b)^2-2abx^2\right)}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^2bf}$$

$$= -\frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)} + \frac{((a+b)(3a+7b))\,\text{Subst}\left(\int \frac{x^4}{b+ax^2}\,dx, x, \text{co}\right.}{2a^2bf}$$

$$= -\frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2bf\left(b+a\cos^2(e+fx)\right)} + \frac{((a+b)(3a+7b))\,\text{Subst}\left(\int\left(-\frac{b}{a^2}+\frac{x^2}{a}+\right.\right.}{2a^2bf}$$

$$= -\frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f} + \frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3bf} - \frac{\cos^5(e+fx)}{5a^2f} - \frac{(a+}{2a^2bf\,(}$$

$$= \frac{\sqrt{b}(a+b)(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f} - \frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4f} + \frac{(a+b)(3a+7b}{6a^3}$$

Mathematica [C] time = 6.82331, size = 454, normalized size = 2.82

$$-\frac{\frac{16\sqrt{a}\cos(e+fx)\left(a\left(125a^2+688ab+560b^2\right)\cos(2(e+fx))-2a^2(11a+14b)\cos(4(e+fx))+1436a^2b+3a^3\cos(6(e+fx))+150a^3+2960ab^2+1680b^3\right)}{a\cos(2(e+fx))+a+2b} - \frac{45a^4\tan^{-1}\left(\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] - I*Sqr
t[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a]
 - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2)
 + (15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[
a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/
2) - (45*a^4*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3
/2) - (45*a^4*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(
```

```
3/2) - (16*Sqrt[a]*Cos[e + f*x]*(150*a^3 + 1436*a^2*b + 2960*a*b^2 + 1680*b
^3 + a*(125*a^2 + 688*a*b + 560*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11*a + 14*b)
*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])
)/(3840*a^(9/2)*f)
```

Maple [A] time = 0.097, size = 276, normalized size = 1.7

$$-\frac{\left(\cos\left(fx+e\right)\right)^{5}}{5a^{2}f}+\frac{2\left(\cos\left(fx+e\right)\right)^{3}}{3a^{2}f}+\frac{2\left(\cos\left(fx+e\right)\right)^{3}b}{3fa^{3}}-\frac{\cos\left(fx+e\right)}{a^{2}f}-4\frac{b\cos\left(fx+e\right)}{fa^{3}}-3\frac{b^{2}\cos\left(fx+e\right)}{fa^{4}}-\frac{}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/5*cos(f*x+e)^5/a^2/f+2/3*cos(f*x+e)^3/a^2/f+2/3/f/a^3*cos(f*x+e)^3*b-cos
(f*x+e)/a^2/f-4/f/a^3*b*cos(f*x+e)-3/f/a^4*b^2*cos(f*x+e)-1/2/f*b/a^2*cos(f
*x+e)/(b+a*cos(f*x+e)^2)-1/f*b^2/a^3*cos(f*x+e)/(b+a*cos(f*x+e)^2)-1/2/f*b^
3/a^4*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2/f*b/a^2/(a*b)^(1/2)*arctan(a*cos(f*
x+e)/(a*b)^(1/2))+5/f*b^2/a^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+
7/2/f*b^3/a^4/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.650045, size = 941, normalized size = 5.84

$$\left[-\frac{12a^{3}\cos\left(fx+e\right)^{7}-4\left(10a^{3}+7a^{2}b\right)\cos\left(fx+e\right)^{5}+20\left(3a^{3}+10a^{2}b+7ab^{2}\right)\cos\left(fx+e\right)^{3}-15\left(3a^{2}b+10ab^{2}+7\right.}{60\left(a^{5}f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(12*a^3*cos(f*x + e)^7 - 4*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 20*(3
*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3
+ (3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x +
 e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(3*a^
2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/
30*(6*a^3*cos(f*x + e)^7 - 2*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 10*(3*a^3
+ 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^3 + (3*
a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(
f*x + e)/b) + 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x
+ e)^2 + a^4*b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16877, size = 736, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a
*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^4) + 30*(a^2*b + 2*a*b^2 + b^3
+ a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - b^3*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1))/((a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^4) - 4*(8*a^2 + 50
```

```
*a*b + 45*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 220*a*b*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) - 180*b^2*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 320*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 270*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 180*b^2*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a*b*(cos(f*x + e) - 1)^4/(cos
(f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^4*(
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

3.42 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=114

$$-\frac{b(a+b)\cos(e+fx)}{2a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{(a+2b)\cos(e+fx)}{a^3f}+\frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f}+\frac{\cos^3(e+fx)}{3a^2f}$$

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(7/2)*f)
- ((a + 2*b)*Cos[e + f*x])/(a^3*f) + Cos[e + f*x]^3/(3*a^2*f) - (b*(a + b)*
Cos[e + f*x])/(2*a^3*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.112188, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4133, 455, 1153, 205}

$$-\frac{b(a+b)\cos(e+fx)}{2a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{(a+2b)\cos(e+fx)}{a^3f}+\frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f}+\frac{\cos^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(7/2)*f)
- ((a + 2*b)*Cos[e + f*x])/(a^3*f) + Cos[e + f*x]^3/(3*a^2*f) - (b*(a + b)*
Cos[e + f*x])/(2*a^3*f*(b + a*Cos[e + f*x]^2))
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4\left(1-x^2\right)}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b(a+b)-2a(a+b)x^2+2a^2x^4}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^3f}$$

$$= -\frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \left(-2(a+2b)+2ax^2+\frac{3ab+5b^2}{b+ax^2}\right)dx, x, \cos(e+fx)\right)}{2a^3f}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f} - \frac{b(a+b)\cos(e+fx)}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{(b(3a+5b))\,\text{Subst}\left(\right.}{}$$

$$= \frac{\sqrt{b}(3a+5b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f} - \frac{(a+2b)\cos(e+fx)}{a^3f} + \frac{\cos^3(e+fx)}{3a^2f} - \frac{b(a+b)\cos(e}{2a^3f\left(b+a\cos^2\right.}$$

Mathematica [C] time = 4.21356, size = 403, normalized size = 3.54

$$-\frac{\frac{32\sqrt{a}\cos(e+fx)\left(a^2(-\cos(4(e+fx)))+9a^2+4a(2a+5b)\cos(2(e+fx))+56ab+60b^2\right)}{a\cos(2(e+fx))+a+2b}-\frac{9a^3\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{9a^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}}{\sqrt{b}}\right)}{b^{3/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*(3*a^3 + 192*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) + (3*(3*a^3 + 1
92*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] - Sq
rt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a] + Sq
rt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (32*Sqrt[a]*Cos[e + f*x]*(9
*a^2 + 56*a*b + 60*b^2 + 4*a*(2*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e +
f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))/(384*a^(7/2)*f)
```

Maple [A] time = 0.082, size = 165, normalized size = 1.5

$$\frac{\left(\cos\left(fx+e\right)\right)^3}{3\,a^2f}-\frac{\cos\left(fx+e\right)}{a^2f}-2\,\frac{b\cos\left(fx+e\right)}{fa^3}-\frac{b\cos\left(fx+e\right)}{2\,a^2f\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}-\frac{b^2\cos\left(fx+e\right)}{2\,fa^3\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}+\frac{3}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/3*cos(f*x+e)^3/a^2/f-cos(f*x+e)/a^2/f-2/f/a^3*b*cos(f*x+e)-1/2/f*b/a^2*co
s(f*x+e)/(b+a*cos(f*x+e)^2)-1/2/f*b^2/a^3*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2
/f*b/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+5/2/f*b^2/a^3/(a*b)^(
1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.588781, size = 684, normalized size = 6.

$$\left[\frac{4\,a^2\cos\left(fx+e\right)^5 - 4\left(3\,a^2+5\,ab\right)\cos\left(fx+e\right)^3 + 3\left(\left(3\,a^2+5\,ab\right)\cos\left(fx+e\right)^2 + 3\,ab + 5\,b^2\right)\sqrt{-\frac{b}{a}}\,\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,a}{a\cos(f}\right.}{12\left(a^4f\cos\left(fx+e\right)^2 + a^3bf\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*cos(f*x + e)^5 - 4*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2
+ 5*a*b)*cos(f*x + e)^2 + 3*a*b + 5*b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2
+ 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(3*a*b + 5*b
^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), 1/6*(2*a^2*cos(f*x + e)
^5 - 2*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2 + 5*a*b)*cos(f*x + e)^2 +
 3*a*b + 5*b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 3*(3*a*b + 5
*b^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17243, size = 193, normalized size = 1.69

$$\frac{\left(3\,ab+5\,b^2\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3f}-\frac{\frac{ab\cos\left(fx+e\right)}{f}+\frac{b^2\cos\left(fx+e\right)}{f}}{2\left(a\cos\left(fx+e\right)^2+b\right)a^3}+\frac{a^4f^{11}\cos\left(fx+e\right)^3-3\,a^4f^{11}\cos\left(fx+e\right)-6\,a^3bf^{11}\mathrm{co}}{3\,a^6f^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b + 5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - 1/
2*(a*b*cos(f*x + e)/f + b^2*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)*a^3) +
1/3*(a^4*f^11*cos(f*x + e)^3 - 3*a^4*f^11*cos(f*x + e) - 6*a^3*b*f^11*cos(f
*x + e))/(a^6*f^12)
```

3.43 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f}-\frac{3\cos(e+fx)}{2a^2f}+\frac{\cos^3(e+fx)}{2af\left(a\cos^2(e+fx)+b\right)}$$

```
[Out] (3*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(5/2)*f) - (3*Cos[e
 + f*x])/(2*a^2*f) + Cos[e + f*x]^3/(2*a*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.0505626, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 288, 321, 205}

$$\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f}-\frac{3\cos(e+fx)}{2a^2f}+\frac{\cos^3(e+fx)}{2af\left(a\cos^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(5/2)*f) - (3*Cos[e
 + f*x])/(2*a^2*f) + Cos[e + f*x]^3/(2*a*f*(b + a*Cos[e + f*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^4}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= \frac{\cos^3(e+fx)}{2af\left(b+a\cos^2(e+fx)\right)}-\frac{3\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2af}\\ &= -\frac{3\cos(e+fx)}{2a^2f}+\frac{\cos^3(e+fx)}{2af\left(b+a\cos^2(e+fx)\right)}+\frac{(3b)\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{2a^2f}\\ &= \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f}-\frac{3\cos(e+fx)}{2a^2f}+\frac{\cos^3(e+fx)}{2af\left(b+a\cos^2(e+fx)\right)}\end{aligned}$$

Mathematica [C] time = 3.87624, size = 393, normalized size = 4.68

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(-\frac{a^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{b}}\right)}{b^{3/2}}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{a}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{\left(a^2+24b^2\right)\tan^{-1}\left(\frac{\sin(e)\,\text{ta}}{}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*(((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] - I*Sq
rt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a
] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2
) + ((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] - Sqrt
[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] + Sqrt[a
 + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (16*Sqrt[a]*Cos[e + f*x]*(a + 3
*b + a*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))*Sec[e + f*x]^4)/(
64*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.042, size = 75, normalized size = 0.9

$$-\frac{b\sec\left(fx+e\right)}{2\,fa^{2}\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)}-\frac{3\,b}{2\,fa^{2}}\arctan\left(b\sec\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{fa^{2}\sec\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*b/a^2*sec(f*x+e)/(a+b*sec(f*x+e)^2)-3/2/f*b/a^2/(a*b)^(1/2)*arctan(s
ec(f*x+e)*b/(a*b)^(1/2))-1/f/a^2/sec(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.552978, size = 478, normalized size = 5.69

$$\left[-\frac{4\,a\cos\left(fx+e\right)^{3}-3\left(a\cos\left(fx+e\right)^{2}+b\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\,a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^{2}+b}\right)+6\,b\cos\left(fx+e\right)}{4\left(a^{3}f\cos\left(fx+e\right)^{2}+a^{2}bf\right)},-\frac{2\,a\cos\left(fx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/a)*log(-(a*cos
(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 6*
b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), -1/2*(2*a*cos(f*x + e)^3
- 3*(a*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 3
*b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20922, size = 103, normalized size = 1.23

$$\frac{3\,b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^{2}f}-\frac{\cos\left(fx+e\right)}{a^{2}f}-\frac{b\cos\left(fx+e\right)}{2\left(a\cos\left(fx+e\right)^{2}+b\right)a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) - cos(f*x + e)/(a^
2*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 + b)*a^2*f)
```

3.44 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}f(a+b)^2}-\frac{b\cos(e+fx)}{2af(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)^2}$$

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(3/2)*(a +
b)^2*f) - ArcTanh[Cos[e + f*x]]/((a + b)^2*f) - (b*Cos[e + f*x])/(2*a*(a +
b)*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.106569, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4133, 470, 522, 206, 205}

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}f(a+b)^2}-\frac{b\cos(e+fx)}{2af(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\tanh^{-1}(\cos(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*a^(3/2)*(a +
b)^2*f) - ArcTanh[Cos[e + f*x]]/((a + b)^2*f) - (b*Cos[e + f*x])/(2*a*(a +
b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b+(-2a-b)x^2}{(1-x^2)\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(e+fx)\right)}{(a+b)^2 f} + \frac{(b(3a+b))\,\text{Subs}}{}$$

$$= \frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2 f} - \frac{\tanh^{-1}(\cos(e+fx))}{(a+b)^2 f} - \frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)}$$

Mathematica [C] time = 1.13839, size = 384, normalized size = 3.88

$$\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\sqrt{b}(3a+b)\sec(e+fx)(a\cos(2(e+fx))+a+2b)\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{}\right.}{\sqrt{b}}\right.}{a^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*b*(a + b))/a + (Sqrt[b]
*(3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])/a^
(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] -
 I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[e + f*x])/a^(3/2) - 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/
2]]*Sec[e + f*x] + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Sin[(e + f*x)/2]]*S
ec[e + f*x]))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.085, size = 172, normalized size = 1.7

$$-\frac{\ln\left(1+\cos\left(fx+e\right)\right)}{2f\left(a+b\right)^2}-\frac{b\cos\left(fx+e\right)}{2f\left(a+b\right)^2\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}-\frac{b^2\cos\left(fx+e\right)}{2f\left(a+b\right)^2a\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}+\frac{3b}{2f\left(a+b\right)^2}\operatorname{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/(a+b)^2*ln(1+cos(f*x+e))-1/2/f/(a+b)^2*b*cos(f*x+e)/(b+a*cos(f*x+e)^
2)-1/2/f/(a+b)^2*b^2/a*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2/f/(a+b)^2*b/(a*b)^
(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/2/f/(a+b)^2*b^2/a/(a*b)^(1/2)*arct
an(a*cos(f*x+e)/(a*b)^(1/2))+1/2/f/(a+b)^2*ln(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.72682, size = 932, normalized size = 9.41

$$\left[\frac{\left(\left(3\,a^2+ab\right)\cos\left(fx+e\right)^2+3\,ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,a\sqrt{-\frac{b}{a}}\cos\left(fx+e\right)-b}{a\cos\left(fx+e\right)^2+b}\right)-2\left(ab+b^2\right)\cos\left(fx+e\right)-2\left(a^2\cos\right.}{4\left(\left(a^4+2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f
*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(a
*b + b^2)*cos(f*x + e) - 2*(a^2*cos(f*x + e)^2 + a*b)*log(1/2*cos(f*x + e)
+ 1/2) + 2*(a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e) + 1/2))/((a^4 +
 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f), 1/2*
(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*
cos(f*x + e)/b) - (a*b + b^2)*cos(f*x + e) - (a^2*cos(f*x + e)^2 + a*b)*log
(1/2*cos(f*x + e) + 1/2) + (a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e)
 + 1/2))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 +
 a*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16464, size = 385, normalized size = 3.89

$$-\frac{\frac{\left(3\,ab+b^2\right)\arctan\left(-\frac{a\cos\left(fx+e\right)-b}{\sqrt{ab}\cos\left(fx+e\right)+\sqrt{ab}}\right)}{\left(a^3+2\,a^2b+ab^2\right)\sqrt{ab}}-\frac{\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a^2+2\,ab+b^2}+\frac{2\left(ab+b^2+\frac{ab\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{b^2\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)}{\left(a^3+2\,a^2b+ab^2\right)\left(a+b+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{b\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b + b^2)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) +
sqrt(a*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) - log(-(cos(f*x + e) - 1)/(
cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2) + 2*(a*b + b^2 + a*b*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) - b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/((a^3 +
 2*a^2*b + a*b^2)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e
) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)))/f
```

3.45 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=147

$$\frac{(a-b)\cos(e+fx)}{2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}f(a+b)^3}-\frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^3}-\frac{\cot(e+fx)\csc}{2f(a+b)\left(a\cos^2(e\right.}$$

```
[Out] ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*Sqrt[a]*(a +
b)^3*f) - ((a - 3*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^3*f) + ((a - b)*Cos[
e + f*x])/(2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (Cot[e + f*x]*Csc[e + f*
x])/(2*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.172685, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4133, 470, 527, 522, 206, 205}

$$\frac{(a-b)\cos(e+fx)}{2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}f(a+b)^3}-\frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2f(a+b)^3}-\frac{\cot(e+fx)\csc}{2f(a+b)\left(a\cos^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*Sqrt[a]*(a +
b)^3*f) - ((a - 3*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^3*f) + ((a - b)*Cos[
e + f*x])/(2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (Cot[e + f*x]*Csc[e + f*
x])/(2*(a + b)*f*(b + a*Cos[e + f*x]^2))
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b+(-a+2b)x^2}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-4b^2+2(a-b)bx}{(1-x^2)(b+ax^2}\right.}{4b(a+}$$

$$= \frac{(a-b)\cos(e+fx)}{2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-3b)\,\text{Subst}\left(\int \frac{1}{1-x}\right.}{2(a+}$$

$$= \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}(a+b)^3f} - \frac{(a-3b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^3f} + \frac{(a-b)\cos(e+f}{2(a+b)^2f\left(b+a\cos^2(\right.}$$

Mathematica [C] time = 1.90282, size = 468, normalized size = 3.18

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left((a+b)\sec^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(a\cos(2(e+fx))+a+2b)-4(a-3b)\sec\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(-8*b*(a + b) - (4*Sqrt[b]*(
-3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])/Sqr
t[a] - (4*Sqrt[b]*(-3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)]
)*Sec[e + f*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[(e + f
*x)/2]^2*Sec[e + f*x] - 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[
```

```
(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Log
[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Se
c[(e + f*x)/2]^2*Sec[e + f*x]))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.11, size = 250, normalized size = 1.7

$$\frac{1}{4f(a+b)^2(1+\cos(fx+e))} - \frac{\ln(1+\cos(fx+e))a}{4f(a+b)^3} + \frac{3\ln(1+\cos(fx+e))b}{4f(a+b)^3} - \frac{b\cos(fx+e)a}{2f(a+b)^3(b+a(\cos(fx+e))^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/4/f/(a+b)^2/(1+cos(f*x+e))-1/4/f/(a+b)^3*ln(1+cos(f*x+e))*a+3/4/f/(a+b)^3
*ln(1+cos(f*x+e))*b-1/2/f/(a+b)^3*b*cos(f*x+e)/(b+a*cos(f*x+e)^2)*a-1/2/f/(
a+b)^3*b^2*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2/f/(a+b)^3*b/(a*b)^(1/2)*arctan
(a*cos(f*x+e)/(a*b)^(1/2))*a-1/2/f/(a+b)^3*b^2/(a*b)^(1/2)*arctan(a*cos(f*x
+e)/(a*b)^(1/2))+1/4/f/(a+b)^2/(-1+cos(f*x+e))+1/4/f/(a+b)^3*ln(-1+cos(f*x+
e))*a-3/4/f/(a+b)^3*ln(-1+cos(f*x+e))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.822236, size = 1613, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 - ((3*a^2 - a*b)*cos(f*x + e)^4 - (3*a^2
 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(-b/a)*log((a*cos(f*x + e
)^2 - 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 4*(a*b + b
^2)*cos(f*x + e) - ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*co
s(f*x + e)^2 - a*b + 3*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*co
s(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2
*cos(f*x + e) + 1/2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4
 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 +
3*a*b^3 + b^4)*f), 1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 + 2*((3*a^2 - a*b)*cos
(f*x + e)^4 - (3*a^2 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(b/a)
*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 4*(a*b + b^2)*cos(f*x + e) - ((a^2 -
3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)
*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b
 + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3
 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24147, size = 787, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(6*(a - 3*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) - 12*(3*a*b - b^2)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*
```

```
b)*cos(f*x + e) + sqrt(a*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) +
 (3*a^2 + 6*a*b + 3*b^2 + 4*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*
a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*b^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*a*b*(cos(
f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 15*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 - 2*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 4*a*b*(cos
(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 6*b^2*(cos(f*x + e) - 1)^3/(cos(f*x
 + e) + 1)^3)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x + e) - 1)^3/(co
s(f*x + e) + 1)^3)) - 3*(cos(f*x + e) - 1)/((a^2 + 2*a*b + b^2)*(cos(f*x +
e) + 1)))/f
```

3.46 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=197

$$-\frac{3\left(a^2-6ab+b^2\right)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^4}+\frac{3a(a-3b)\cos(e+fx)}{8f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{3\sqrt{a}\sqrt{b}(a-b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2f(a+b)^4}-\frac{\text{co}}{4f(a}$$

```
[Out] (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*(a +
b)^4*f) - (3*(a^2 - 6*a*b + b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^4*f) + (
3*a*(a - 3*b)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) - ((a -
5*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (C
ot[e + f*x]*Csc[e + f*x]^3)/(4*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.246437, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4133, 470, 527, 522, 206, 205}

$$-\frac{3\left(a^2-6ab+b^2\right)\tanh^{-1}(\cos(e+fx))}{8f(a+b)^4}+\frac{3a(a-3b)\cos(e+fx)}{8f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{3\sqrt{a}\sqrt{b}(a-b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2f(a+b)^4}-\frac{\text{co}}{4f(a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(2*(a +
b)^4*f) - (3*(a^2 - 6*a*b + b^2)*ArcTanh[Cos[e + f*x]])/(8*(a + b)^4*f) + (
3*a*(a - 3*b)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) - ((a -
5*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - (C
ot[e + f*x]*Csc[e + f*x]^3)/(4*(a + b)*f*(b + a*Cos[e + f*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^3\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b+(-a+4b)x^2}{\left(1-x^2\right)^2\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{3(a-b)b-3a}{\left(1-x^2\right)(b}\right.}{8(}$$

$$= \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc^3(}{4(a+b)f\left(b+a\cos\right.}$$

$$= \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\cot(e+fx)\csc^3(}{4(a+b)f\left(b+a\cos\right.}$$

$$= \frac{3\sqrt{a}(a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2(a+b)^4f} - \frac{3\left(a^2-6ab+b^2\right)\tanh^{-1}(\cos(e+fx))}{8(a+b)^4f} + \frac{3a(a-}{8(a+b)^3f}$$

Mathematica [C] time = 2.32487, size = 450, normalized size = 2.28

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(-24\left(a^2-6ab+b^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)(a\cos(2(e+fx))+a+2b)+24\left(a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqrt[
a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
]*(a + 2*b + a*Cos[2*(e + f*x)]) + 96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqr
t[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos
[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a + b)*(11*a^2 + 43*a*b - 4*b^2 + 4
*(2*a^2 - 5*a*b + 5*b^2)*Cos[2*(e + f*x)] - 3*a*(a - 3*b)*Cos[4*(e + f*x)])
```

```
*Cot[e + f*x]*Csc[e + f*x]^3 - 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a*Cos[2*(e
 + f*x)])*Log[Cos[(e + f*x)/2]] + 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a*Cos[2
*(e + f*x)])*Log[Sin[(e + f*x)/2]])*Sec[e + f*x]^4)/(256*(a + b)^4*f*(a + b
*Sec[e + f*x]^2)^2)
```

Maple [B] time = 0.119, size = 390, normalized size = 2.

$$\frac{1}{16 f (a+b)^2 \left(1+\cos \left(f x+e\right)\right)^2}+\frac{3 a}{16 f (a+b)^3 \left(1+\cos \left(f x+e\right)\right)}-\frac{5 b}{16 f (a+b)^3 \left(1+\cos \left(f x+e\right)\right)}-\frac{3 \ln \left(1+\cos \left(f x\right.\right.}{16 f (a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/16/f/(a+b)^2/(1+cos(f*x+e))^2+3/16/f/(a+b)^3/(1+cos(f*x+e))*a-5/16/f/(a+b
)^3/(1+cos(f*x+e))*b-3/16/f/(a+b)^4*ln(1+cos(f*x+e))*a^2+9/8/f/(a+b)^4*ln(1
+cos(f*x+e))*a*b-3/16/f/(a+b)^4*ln(1+cos(f*x+e))*b^2-1/2/f*a^2/(a+b)^4*b*co
s(f*x+e)/(b+a*cos(f*x+e)^2)-1/2/f*a/(a+b)^4*b^2*cos(f*x+e)/(b+a*cos(f*x+e)^
2)+3/2/f*a^2/(a+b)^4*b/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))-3/2/f*a
/(a+b)^4*b^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))-1/16/f/(a+b)^2/(-
1+cos(f*x+e))^2+3/16/f/(a+b)^3/(-1+cos(f*x+e))*a-5/16/f/(a+b)^3/(-1+cos(f*x
+e))*b+3/16/f/(a+b)^4*ln(-1+cos(f*x+e))*a^2-9/8/f/(a+b)^4*ln(-1+cos(f*x+e))
*a*b+3/16/f/(a+b)^4*ln(-1+cos(f*x+e))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.973963, size = 2747, normalized size = 13.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9*
a*b^2 + 5*b^3)*cos(f*x + e)^3 - 12*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 - 3
*a*b + b^2)*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b - b
^2)*sqrt(-a*b)*log((a*cos(f*x + e)^2 - 2*sqrt(-a*b)*cos(f*x + e) - b)/(a*co
s(f*x + e)^2 + b)) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e) - 3*((a^3 - 6
*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x
 + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*
x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x
+ e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^
2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x
+ e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*
x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(
f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f), 1/16*(6*(a
^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9*a*b^2 + 5*b
^3)*cos(f*x + e)^3 + 24*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 - 3*a*b + b^2)
*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b - b^2)*sqrt(a*
b)*arctan(sqrt(a*b)*cos(f*x + e)/b) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x +
 e) - 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b
^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^
2 - 2*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b
+ a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4
 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^
2)*log(-1/2*cos(f*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^
4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^
5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32657, size = 950, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 6*a*b + b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 96*(a^2*b - a*b^2)*arctan(-(a*c
os(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) - (8*a^2*(cos(f*x + e) - 1)/(cos(f*x +
e) + 1) - 8*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 - 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 - b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 8*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 - 108*a*b*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 18*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e
) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1)^2)
 - 64*(a^2*b + a*b^2 + a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*b^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)))/f
```

3.47 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=267

$$-\frac{b\left(19a^2+52ab+32b^2\right)\tan(e+fx)}{16a^4f\left(a+b\tan^2(e+fx)+b\right)}-\frac{\left(33a^2+82ab+48b^2\right)\sin(e+fx)\cos(e+fx)}{48a^3f\left(a+b\tan^2(e+fx)+b\right)}+\frac{x\left(60a^2b+5a^3+120ab^2+64b\right.}{16a^5}$$

```
[Out] ((5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*x)/(16*a^5) - (Sqrt[b]*(a + b)^(3/
2)*(3*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*f) - ((33
*a^2 + 82*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan
[e + f*x]^2)) + ((9*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b
+ b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Ta
n[e + f*x]^2)) - (b*(19*a^2 + 52*a*b + 32*b^2)*Tan[e + f*x])/(16*a^4*f*(a +
 b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.426426, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4132, 470, 578, 527, 522, 203, 205}

$$-\frac{b\left(19a^2+52ab+32b^2\right)\tan(e+fx)}{16a^4f\left(a+b\tan^2(e+fx)+b\right)}-\frac{\left(33a^2+82ab+48b^2\right)\sin(e+fx)\cos(e+fx)}{48a^3f\left(a+b\tan^2(e+fx)+b\right)}+\frac{x\left(60a^2b+5a^3+120ab^2+64b\right.}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] ((5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*x)/(16*a^5) - (Sqrt[b]*(a + b)^(3/
2)*(3*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*f) - ((33
*a^2 + 82*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan
[e + f*x]^2)) + ((9*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b
+ b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Ta
n[e + f*x]^2)) - (b*(19*a^2 + 52*a*b + 32*b^2)*Tan[e + f*x])/(16*a^4*f*(a +
 b + b*Tan[e + f*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(b-6(a+b))x^2\right)}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{(a+b)(9a+8b}{(1+}\right.}{}$$

$$= -\frac{\left(33a^2+82ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{}{6}$$

$$= -\frac{\left(33a^2+82ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{}{6}$$

$$= -\frac{\left(33a^2+82ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{}{6}$$

$$= \frac{\left(5a^3+60a^2b+120ab^2+64b^3\right)x}{16a^5} - \frac{\sqrt{b}(a+b)^{3/2}(3a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f} - \frac{\left(33a^2+\right.}{4}$$

Mathematica [C] time = 24.0212, size = 2738, normalized size = 10.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]

[Out] -((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*a^2*b
+ 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*
Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]
)]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4
]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)
*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))))/(5
12*a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[
e + f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^3 -
128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) +
a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e]
 - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (16*a*C
os[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a^2*b + 4
8*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*
b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(4096*a^3*(a
 + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^
4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (
a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(204
8*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*Cos[2*e + 2*f*x])^2*S
ec[e + f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/
2)) + (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e +
 f*x)]))))/(2048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*
e + 2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b
^3 - 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4
])) + (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^4)
*f*x*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f*x*Cos[2*f*x
] + 80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e + 2*f*x] + 768*
a^2*b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x] + a^5*Sin[2*e
] + 34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*Sin[2*e] + 640*a
*b^4*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a^4*b*Sin[2*f*x] - 3
18*a^3*b^2*Sin[2*f*x] - 512*a^2*b^3*Sin[2*f*x] - 256*a*b^4*Sin[2*f*x] - 12*
a^4*b*Sin[2*(e + 2*f*x)] - 36*a^3*b^2*Sin[2*(e + 2*f*x)] - 24*a^2*b^3*Sin[2
*(e + 2*f*x)] - 30*a^4*b*Sin[4*e + 2*f*x] - 158*a^3*b^2*Sin[4*e + 2*f*x] -
256*a^2*b^3*Sin[4*e + 2*f*x] - 128*a*b^4*Sin[4*e + 2*f*x] - 12*a^4*b*Sin[6*
e + 4*f*x] - 36*a^3*b^2*Sin[6*e + 4*f*x] - 24*a^2*b^3*Sin[6*e + 4*f*x] + 2*
a^4*b*Sin[4*e + 6*f*x] + 2*a^3*b^2*Sin[4*e + 6*f*x] + 2*a^4*b*Sin[8*e + 6*f
*x] + 2*a^3*b^2*Sin[8*e + 6*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)]))))/(2048*a
^4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^
2*Sec[e + f*x]^4*(-(((a^6 - 48*a^5*b - 1200*a^4*b^2 - 6400*a^3*b^3 - 13440*
a^2*b^4 - 12288*a*b^5 - 4096*b^6)*((ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b
]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*C
```

```
os[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])
]*Cos[2*e])/(8*a^5*b*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/8
)*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])
 - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin
[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*b*Sqrt[a + b]*f*S
qrt[b*Cos[4*e] - I*b*Sin[4*e]])))/(a + b)) - (Sec[2*e]*(-960*a^5*b*f*x*Cos[
2*e] - 10944*a^4*b^2*f*x*Cos[2*e] - 44544*a^3*b^3*f*x*Cos[2*e] - 83712*a^2*
b^4*f*x*Cos[2*e] - 73728*a*b^5*f*x*Cos[2*e] - 24576*b^6*f*x*Cos[2*e] - 480*
a^5*b*f*x*Cos[2*f*x] - 4512*a^4*b^2*f*x*Cos[2*f*x] - 13248*a^3*b^3*f*x*Cos[
2*f*x] - 15360*a^2*b^4*f*x*Cos[2*f*x] - 6144*a*b^5*f*x*Cos[2*f*x] - 480*a^5
*b*f*x*Cos[4*e + 2*f*x] - 4512*a^4*b^2*f*x*Cos[4*e + 2*f*x] - 13248*a^3*b^3
*f*x*Cos[4*e + 2*f*x] - 15360*a^2*b^4*f*x*Cos[4*e + 2*f*x] - 6144*a*b^5*f*x
*Cos[4*e + 2*f*x] - 3*a^6*Sin[2*e] - 156*a^5*b*Sin[2*e] - 1500*a^4*b^2*Sin[
2*e] - 5760*a^3*b^3*Sin[2*e] - 10560*a^2*b^4*Sin[2*e] - 9216*a*b^5*Sin[2*e]
 - 3072*b^6*Sin[2*e] + 3*a^6*Sin[2*f*x] + 366*a^5*b*Sin[2*f*x] + 3000*a^4*b
^2*Sin[2*f*x] + 8400*a^3*b^3*Sin[2*f*x] + 9600*a^2*b^4*Sin[2*f*x] + 3840*a*
b^5*Sin[2*f*x] + 216*a^5*b*Sin[4*e + 2*f*x] + 1800*a^4*b^2*Sin[4*e + 2*f*x]
 + 5040*a^3*b^3*Sin[4*e + 2*f*x] + 5760*a^2*b^4*Sin[4*e + 2*f*x] + 2304*a*b
^5*Sin[4*e + 2*f*x] + 76*a^5*b*Sin[2*e + 4*f*x] + 460*a^4*b^2*Sin[2*e + 4*f
*x] + 768*a^3*b^3*Sin[2*e + 4*f*x] + 384*a^2*b^4*Sin[2*e + 4*f*x] + 76*a^5*
b*Sin[6*e + 4*f*x] + 460*a^4*b^2*Sin[6*e + 4*f*x] + 768*a^3*b^3*Sin[6*e + 4
*f*x] + 384*a^2*b^4*Sin[6*e + 4*f*x] - 16*a^5*b*Sin[4*e + 6*f*x] - 48*a^4*b
^2*Sin[4*e + 6*f*x] - 32*a^3*b^3*Sin[4*e + 6*f*x] - 16*a^5*b*Sin[8*e + 6*f*
x] - 48*a^4*b^2*Sin[8*e + 6*f*x] - 32*a^3*b^3*Sin[8*e + 6*f*x] + 4*a^5*b*Si
n[6*e + 8*f*x] + 4*a^4*b^2*Sin[6*e + 8*f*x] + 4*a^5*b*Sin[10*e + 8*f*x] + 4
*a^4*b^2*Sin[10*e + 8*f*x]))/(24*a^5*b*(a + b)*f*(a + 2*b + a*Cos[2*e + 2*f
*x]))))/(512*(a + b*Sec[e + f*x]^2)^2)
```

Maple [B] time = 0.112, size = 555, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -9/4/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b-3/2/f/a^4/(tan(f*x+e)^2+1)^3*t
an(f*x+e)^5*b^2-11/16/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5-4/f/a^3/(tan(f*
x+e)^2+1)^3*tan(f*x+e)^3*b-3/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b^2-5/6/
f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-5/16/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x
+e)-7/4/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b-3/2/f/a^4/(tan(f*x+e)^2+1)^3*
tan(f*x+e)*b^2+15/4/f/a^3*arctan(tan(f*x+e))*b+15/2/f/a^4*arctan(tan(f*x+e)
```

```
)*b^2+4/f/a^5*arctan(tan(f*x+e))*b^3+5/16/f/a^2*arctan(tan(f*x+e))-1/2*b*ta
n(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)-1/f*b^2/a^3*tan(f*x+e)/(a+b+b*tan(f*x+e
)^2)-1/2/f*b^3/a^4*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)-3/2/f*b/a^2/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-7/f*b^2/a^3/((a+b)*b)^(1/2)*arctan
(tan(f*x+e)*b/((a+b)*b)^(1/2))-19/2/f*b^3/a^4/((a+b)*b)^(1/2)*arctan(tan(f*
x+e)*b/((a+b)*b)^(1/2))-4/f*b^4/a^5/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a
+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.740129, size = 1577, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^4 + 60*a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3
*(5*a^3*b + 60*a^2*b^2 + 120*a*b^3 + 64*b^4)*f*x + 6*(3*a^2*b + 11*a*b^2 +
8*b^3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e)
+ b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - (8*a^4*cos(f*x
+ e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b + 48*a^2*
b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x + e))*si
n(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), 1/48*(3*(5*a^4 + 60*a^3*b + 1
20*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3*(5*a^3*b + 60*a^2*b^2 + 120*a
*b^3 + 64*b^4)*f*x + 12*(3*a^2*b + 11*a*b^2 + 8*b^3 + (3*a^3 + 11*a^2*b + 8
*a*b^2)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - (8*a^4*cos(f*x + e)^7
 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b + 48*a^2*b^2)*c
```

```
os(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x + e))*sin(f*x
+ e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26216, size = 420, normalized size = 1.57

$$\frac{\frac{3\left(5a^3+60a^2b+120ab^2+64b^3\right)\left(fx+e\right)}{a^5} - \frac{24\left(3a^3b+14a^2b^2+19ab^3+8b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^5} - \frac{24\left(a^2b\tan\left(fx+e\right)+2ab^2\tan\left(fx+e\right)\right.}{\left(b\tan\left(fx+e\right)^2+a+b\right)a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 - 24*(3*a^3*b
 + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^5) - 24*(a^2*b*tan
(f*x + e) + 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a
 + b)*a^4) - (33*a^2*tan(f*x + e)^5 + 108*a*b*tan(f*x + e)^5 + 72*b^2*tan(f
*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 192*a*b*tan(f*x + e)^3 + 144*b^2*tan(f*
x + e)^3 + 15*a^2*tan(f*x + e) + 84*a*b*tan(f*x + e) + 72*b^2*tan(f*x + e))
/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

3.48 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=191

$$\frac{3x\left(a^2+8ab+8b^2\right)}{8a^4}-\frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f}-\frac{3b(3a+4b)\tan(e+fx)}{8a^3f\left(a+b\tan^2(e+fx)+b\right)}-\frac{(5a+6b)\sin(e+fx)\,\mathrm{c}}{8a^2f\left(a+b\tan^2(e+\right.}$$

```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
an[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*f) - ((5*a + 6*b)*Cos[e + f*
x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) - (3*b*(3*a + 4*b)*Tan[e + f*
x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.255016, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 203, 205}

$$\frac{3x\left(a^2+8ab+8b^2\right)}{8a^4}-\frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f}-\frac{3b(3a+4b)\tan(e+fx)}{8a^3f\left(a+b\tan^2(e+fx)+b\right)}-\frac{(5a+6b)\sin(e+fx)\,\mathrm{c}}{8a^2f\left(a+b\tan^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] (3*(a^2 + 8*a*b + 8*b^2)*x)/(8*a^4) - (3*Sqrt[b]*Sqrt[a + b]*(a + 2*b)*ArcT
an[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*f) - ((5*a + 6*b)*Cos[e + f*
x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*Sin
[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) - (3*b*(3*a + 4*b)*Tan[e + f*
x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{a+b+(-4a-5b)x^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{3(a+b)(a+2b)}{(1+x^2)(a}\right.}{}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{3b(3a+4b)\tan(e}{8a^3f\left(a+b+b\tan^2\right.}$$

$$= -\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{3b(3a+4b)\tan(e}{8a^3f\left(a+b+b\tan^2\right.}$$

$$= \frac{3\left(a^2+8ab+8b^2\right)x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f} - \frac{(5a+6b)\cos(e+fx)\sin}{8a^2f\left(a+b+b\tan^2(e\right.}$$

Mathematica [C] time = 14.2768, size = 1105, normalized size = 5.79

$$-\frac{(\cos(2e+2fx)a+a+2b)^2\left(16x+\frac{\left(-a^3+6ba^2+24b^2a+16b^3\right)\tan^{-1}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(a\sin(2e+fx)-(a+2b)\sin(fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}+\frac{}{b(a+b}\right.}{256a^2\left(b\sec^2(e+fx)+a\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*a^2*b
+ 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*
Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]
)]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4
]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)
*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(2
```

```
56*a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[
e + f*x]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^
(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]
))))/(1024*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*
f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 1600*a^2*b^3 - 1
920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*
Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]
)]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) +
(Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^3 + 48*b^4)*f*x*C
os[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f*x*Cos[2*f*x] + 80
*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e + 2*f*x] + 768*a^2*b^
3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x] + a^5*Sin[2*e] + 34
*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*Sin[2*e] + 640*a*b^4*S
in[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a^4*b*Sin[2*f*x] - 318*a^3
*b^2*Sin[2*f*x] - 512*a^2*b^3*Sin[2*f*x] - 256*a*b^4*Sin[2*f*x] - 12*a^4*b*
Sin[2*(e + 2*f*x)] - 36*a^3*b^2*Sin[2*(e + 2*f*x)] - 24*a^2*b^3*Sin[2*(e +
2*f*x)] - 30*a^4*b*Sin[4*e + 2*f*x] - 158*a^3*b^2*Sin[4*e + 2*f*x] - 256*a^
2*b^3*Sin[4*e + 2*f*x] - 128*a*b^4*Sin[4*e + 2*f*x] - 12*a^4*b*Sin[6*e + 4*
f*x] - 36*a^3*b^2*Sin[6*e + 4*f*x] - 24*a^2*b^3*Sin[6*e + 4*f*x] + 2*a^4*b*
Sin[4*e + 6*f*x] + 2*a^3*b^2*Sin[4*e + 6*f*x] + 2*a^4*b*Sin[8*e + 6*f*x] +
2*a^3*b^2*Sin[8*e + 6*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])))/(1024*a^4*b*(
a + b)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.102, size = 323, normalized size = 1.7

$$-\frac{\left(\tan\left(fx+e\right)\right)^{3}b}{fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{5\left(\tan\left(fx+e\right)\right)^{3}}{8fa^{2}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{3\tan\left(fx+e\right)}{8fa^{2}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{\tan\left(fx+e\right)b}{fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3*b-5/8/f/a^2/(tan(f*x+e)^2+1)^2*tan
(f*x+e)^3-3/8/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)-1/f/a^3/(tan(f*x+e)^2+1)^
2*tan(f*x+e)*b+3/f/a^3*arctan(tan(f*x+e))*b+3/f/a^4*arctan(tan(f*x+e))*b^2+
3/8/f/a^2*arctan(tan(f*x+e))-1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)-3/
2/f*b/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-9/2/f*b^2/a^
3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/2/f*b^2/a^3*tan(f*
x+e)/(a+b+b*tan(f*x+e)^2)-3/f*b^3/a^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/(
(a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.664461, size = 1211, normalized size = 6.34

$$\left[\frac{3\left(a^3+8a^2b+8ab^2\right)fx\cos\left(fx+e\right)^2+3\left(a^2b+8ab^2+8b^3\right)fx+3\left(\left(a^2+2ab\right)\cos\left(fx+e\right)^2+ab+2b^2\right)\sqrt{-ab-b^2}\,\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 +
 8*b^3)*f*x + 3*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^3*
cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*a*b^2)*c
os(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), 1/8*(3*(a^3 +
8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 + 8*b^3)*f*x + 6
*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((
a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) +
 (2*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*
a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25044, size = 275, normalized size = 1.44

$$\frac{\frac{3\left(a^2+8ab+8b^2\right)\left(fx+e\right)}{a^4}-\frac{12\left(a^2b+3ab^2+2b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^4}-\frac{4\left(ab\tan\left(fx+e\right)+b^2\tan\left(fx+e\right)\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)a^3}-\frac{5a\tan\left(fx+e\right)^3+8b\tan(f}{(\tan}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 - 12*(a^2*b + 3*a*b^2 + 2*b^3)*(
pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)
))/(sqrt(a*b + b^2)*a^4) - 4*(a*b*tan(f*x + e) + b^2*tan(f*x + e))/((b*tan(
f*x + e)^2 + a + b)*a^3) - (5*a*tan(f*x + e)^3 + 8*b*tan(f*x + e)^3 + 3*a*t
an(f*x + e) + 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

3.49 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f\sqrt{a+b}}-\frac{b\tan(e+fx)}{a^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{x(a+4b)}{2a^3}-\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] ((a + 4*b)*x)/(2*a^3) - (Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a
 + b + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/(a^2*f*(a + b + b*Tan[e + f*x]
^2))
```

Rubi [A] time = 0.168873, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 471, 527, 522, 203, 205}

$$-\frac{\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f\sqrt{a+b}}-\frac{b\tan(e+fx)}{a^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{x(a+4b)}{2a^3}-\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 4*b)*x)/(2*a^3) - (Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a
 + b + b*Tan[e + f*x]^2)) - (b*Tan[e + f*x])/(a^2*f*(a + b + b*Tan[e + f*x]
^2))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2(a+b)(a+2b)-4b(a-}{(1+x^2)(a+b+bx^2}\right.}{4a^2(a+}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+4b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.}{2a^3f}$$

$$= \frac{(a+4b)x}{2a^3} - \frac{\sqrt{b}(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan}{a^2f\left(a+b\right.}$$

Mathematica [C] time = 11.8335, size = 825, normalized size = 6.35

$$-\frac{(\cos(2e+2fx)a+a+2b)^2\left(16x + \frac{\left(-a^3+6ba^2+24b^2a+16b^3\right)\tan^{-1}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(a\sin(2e+fx)-(a+2b)\sin(fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i\sin(2e))}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}} + \frac{}{b(a+b}\right.}{128a^2\left(b\sec^2(e+fx)+a\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6*a^2*b
+ 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*
Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]
)]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4
]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)
*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(1
28*a^2*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e
+ f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^3 - 12
8*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*
Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] -
```

```
 I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (16*a*Cos
[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a^2*b + 48*
a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b
+ a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(256*a^3*(a +
b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(((
a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a*Sqr
t[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))/(128*b^(3
/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e +
 f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)) +
(Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]
))))/(256*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.098, size = 155, normalized size = 1.2

$$-\frac{\tan\left(fx+e\right)}{2\,fa^{2}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{2\,fa^{2}}+2\,\frac{\arctan\left(\tan\left(fx+e\right)\right)b}{fa^{3}}-\frac{b\tan\left(fx+e\right)}{2\,fa^{2}\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/a^2*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2/f/a^2*arctan(tan(f*x+e))+2/f/a^3
*arctan(tan(f*x+e))*b-1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)-3/2/f*b/a
^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-2/f*b^2/a^3/((a+b)*
b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.626608, size = 1046, normalized size = 8.05

$$\left[\frac{4\left(a^2+4ab\right)fx\cos\left(fx+e\right)^2+4\left(ab+4b^2\right)fx+\left(\left(3a^2+4ab\right)\cos\left(fx+e\right)^2+3ab+4b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos(\ldots}{\ldots}\right)}{8\left(a^4\ldots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 4*(a*b + 4*b^2)*f*x + ((3*a^2 +
4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b
 + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a^2*
cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f*x + e)^2 +
a^3*b*f), 1/4*(2*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 2*(a*b + 4*b^2)*f*x + (
(3*a^2 + 4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(b/(a + b))*arctan(1/2*
((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)
)) - 2*(a^2*cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f
*x + e)^2 + a^3*b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2681, size = 213, normalized size = 1.64

$$\frac{\frac{(fx+e)(a+4b)}{a^3}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3ab+4b^2\right)}{\sqrt{ab+b^2}\,a^3}-\frac{2b\tan\left(fx+e\right)^3+a\tan\left(fx+e\right)+2b\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^4+a\tan\left(fx+e\right)^2+2b\tan\left(fx+e\right)^2+a+b\right)a^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 4*b)/a^3 - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arcta
n(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a*b + 4*b^2)/(sqrt(a*b + b^2)*a^3) -
(2*b*tan(f*x + e)^3 + a*tan(f*x + e) + 2*b*tan(f*x + e))/((b*tan(f*x + e)^4
 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)*a^2))/f
```

3.50 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rubi [A] time = 0.085927, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4128, 414, 522, 203, 205}

$$-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```


```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} - \frac{(b(3a+2b))}{}$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 2.01834, size = 240, normalized size = 2.61

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(2x(a\cos(2(e+fx))+a+2b)+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{f(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))}+\frac{b(3a+2b)(\cos(2e)-i\sin\ldots}{\ldots}\right.}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

Maple [A] time = 0.083, size = 127, normalized size = 1.4

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^2}-\frac{b\tan\left(fx+e\right)}{2a\left(a+b\right)f\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)}-\frac{3b}{2a\left(a+b\right)f}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{\left(a+b\right)b}}\right)\frac{1}{\sqrt{(a\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f/a^2*arctan(tan(f*x+e))-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)-
3/2/f*b/a/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^
2/a^2/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.607849, size = 1027, normalized size = 11.16

$$\left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2-4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+8\left(ab+b^2\right)fx+\left(\left(3\,a^2+2\,ab\right)\cos\left(fx+e\right)^2+3\,ab+2\,b\right.}{8\left(\left(a^4+a^3b\right)f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Giac [A] time = 1.12255, size = 161, normalized size = 1.75

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

3.51 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}-\frac{3\cot(e+fx)}{2f(a+b)^2}+\frac{\cot(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*f)
 - (3*Cot[e + f*x])/(2*(a + b)^2*f) + Cot[e + f*x]/(2*(a + b)*f*(a + b + b*
Tan[e + f*x]^2))
```

Rubi [A] time = 0.0850206, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}-\frac{3\cot(e+fx)}{2f(a+b)^2}+\frac{\cot(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a + b)^(5/2)*f)
 - (3*Cot[e + f*x])/(2*(a + b)^2*f) + Cot[e + f*x]/(2*(a + b)*f*(a + b + b*
Tan[e + f*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{3\cot(e+fx)}{2(a+b)^2 f} + \frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e-\right.}{2(a+b)^2 f}$$

$$= -\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{3\cot(e+fx)}{2(a+b)^2 f} + \frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 2.32557, size = 242, normalized size = 2.66

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{a(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + 2\csc(e)\sin(fx)\csc(e+fx)(a\cos(2(e+fx))+a+2b\right.}{8f(a+b)^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((3*b*ArcTan[(Sec[f*x]*(Cos[
2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e]
 - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2*(a + 2*b +
a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + (b*((a + 2*b)*Sin[2*e] -
 a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a + b)^2*f*(a
 + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.11, size = 86, normalized size = 1.

$$-\frac{1}{f(a+b)^2\tan\left(fx+e\right)}-\frac{b\tan\left(fx+e\right)}{2f(a+b)^2\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)}-\frac{3b}{2f(a+b)^2}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{(a+b)b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/f/(a+b)^2/tan(f*x+e)-1/2/f*b/(a+b)^2*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)-3/2
/f*b/(a+b)^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.600493, size = 975, normalized size = 10.71

$$\left[-\frac{4\,(2\,a-b)\cos\left(fx+e\right)^{3}-3\left(a\cos\left(fx+e\right)^{2}+b\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a^{2}+3\,ab+2\,b^{2}\right)\mathrm{c}\right.}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+\right.}\right.}{8\left(\left(a^{3}+2\,a^{2}b+ab^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b+2\,ab^{2}+b^{3}\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/(a + b
))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*
sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
e)^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*
cos(f*x + e)^2 + (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e)), -1/4*(2*(2*a - b
)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(b/(a + b))*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*s
in(f*x + e) + 6*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2
+ (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27012, size = 180, normalized size = 1.98

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)b}{\left(a^{2}+2\,ab+b^{2}\right)\sqrt{ab+b^{2}}}+\frac{3\,b\tan\left(fx+e\right)^{2}+2\,a+2\,b}{\left(b\tan\left(fx+e\right)^{3}+a\tan\left(fx+e\right)+b\tan\left(fx+e\right)\right)\left(a^{2}+2\,ab+b^{2}\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^2 +
 2*a + 2*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^2 + 2*
a*b + b^2)))/f
```

3.52 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}-\frac{ab\tan(e+fx)}{2f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}-\frac{\cot^3(e+fx)}{3f(a+b)^2}-\frac{(a-b)\cot(e+fx)}{f(a+b)^3}$$

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a + b
)^(7/2)*f) - ((a - b)*Cot[e + f*x])/((a + b)^3*f) - Cot[e + f*x]^3/(3*(a +
b)^2*f) - (a*b*Tan[e + f*x])/(2*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.174616, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 456, 1261, 205}

$$-\frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}-\frac{ab\tan(e+fx)}{2f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}-\frac{\cot^3(e+fx)}{3f(a+b)^2}-\frac{(a-b)\cot(e+fx)}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a + b
)^(7/2)*f) - ((a - b)*Cot[e + f*x])/((a + b)^3*f) - Cot[e + f*x]^3/(3*(a +
b)^2*f) - (a*b*Tan[e + f*x])/(2*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{2}{b(a+b)}-\frac{2ax^2}{b(a+b)^2}+\frac{ax^4}{(a+b)^3}}{x^4\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= -\frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2}{b(a+b)^2x^4} - \frac{2(a-b)}{b(a+b)^3x^2} + \frac{3a-2b}{(a+b)^3\left(a+b+bx^2\right)}\right.\right.}{2f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} - \frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{((3a-2b)b)}{}$$

$$= -\frac{(3a-2b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} - \frac{ab\,\mathrm{t}}{2(a+b)^3 f\left(a-\right.}$$

Mathematica [C] time = 6.76797, size = 637, normalized size = 5.18

$$\frac{\sec^4(e+fx)\left(ab\sin(2e)-ab\sin(2fx)+2b^2\sin(2e)\right)(a\cos(2e+2fx)+a+2b)}{8f(a+b)^3(\cos(e)-\sin(e))(\sin(e)+\cos(e))\left(a+b\sec^2(e+fx)\right)^2}-\frac{\cot(e)\csc^2(e+fx)\sec^4(e+fx)(a\cos}{12f(a+b)^2\left(a+b\sec^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + 2*b + a*Cos[2*e + 2*f*x])^2*Cot[e]*Csc[e + f*x]^2*Sec[e + f*x]^4)/(1
2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2) + ((3*a - 2*b)*(a + 2*b + a*Cos[2*e
 + 2*f*x])^2*Sec[e + f*x]^4*((b*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sq
rt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4
*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Co
s[2*e])/(8*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/8)*b*ArcTan
[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2
)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) -
 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(Sqrt[a + b]*f*Sqrt[b*Cos[4*e]
 - I*b*Sin[4*e]])))/((a + b)^3*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Co
s[2*e + 2*f*x])^2*Csc[e]*Csc[e + f*x]^3*Sec[e + f*x]^4*Sin[f*x])/(12*(a + b
)^2*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Csc[e]*
Csc[e + f*x]*Sec[e + f*x]^4*(a*Sin[f*x] - 2*b*Sin[f*x]))/(6*(a + b)^3*f*(a
+ b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^4*(a*
b*Sin[2*e] + 2*b^2*Sin[2*e] - a*b*Sin[2*f*x]))/(8*(a + b)^3*f*(a + b*Sec[e
+ f*x]^2)^2*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
```

Maple [A] time = 0.106, size = 160, normalized size = 1.3

$$-\frac{1}{3f(a+b)^2\left(\tan\left(fx+e\right)\right)^3}-\frac{a}{f(a+b)^3\tan\left(fx+e\right)}+\frac{b}{f(a+b)^3\tan\left(fx+e\right)}-\frac{ab\tan\left(fx+e\right)}{2f(a+b)^3\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/3/f/(a+b)^2/tan(f*x+e)^3-1/f/(a+b)^3/tan(f*x+e)*a+1/f/(a+b)^3/tan(f*x+e)
*b-1/2*a*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)-3/2/f/(a+b)^3*b/((a+b)
*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a+1/f/(a+b)^3*b^2/((a+b)*b)^
```

```
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 0.688201, size = 1534, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")

[Out] [-1/24*(4*(4*a^2 - 11*a*b)*cos(f*x + e)^5 - 8*(3*a^2 - 8*a*b + 4*b^2)*cos(f
*x + e)^3 + 3*((3*a^2 - 2*a*b)*cos(f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos
(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*
cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) +
b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(
3*a*b - 2*b^2)*cos(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x + e)), -1/12*(2*(4*a^2 - 11*a*b)*cos(f*x
+ e)^5 - 4*(3*a^2 - 8*a*b + 4*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - 2*a*b)*cos(
f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(b
/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(
f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b - 2*b^2)*cos(f*x + e))/(((a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b
^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x
 + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28925, size = 259, normalized size = 2.11

$$-\frac{\frac{3\,ab\tan\left(fx+e\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,ab-2\,b^2\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{ab+b^2}}+\frac{2\left(3\,a\tan\left(fx+e\right)^2-3\,b\tan\left(fx+e\right)^2+a+b\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\tan\left(fx+e\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*tan(f*x + e)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*tan(f*x + e)^2
 + a + b)) + 3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)
/sqrt(a*b + b^2)))*(3*a*b - 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*
b + b^2)) + 2*(3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^2 + a + b)/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

3.53 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=188

$$-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5f(a+b)^4}-\frac{a\sqrt{b}(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{9/2}}-\frac{(10a+3b}{15f}$$

```
[Out] -(a*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a +
 b)^(9/2)*f) - ((5*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(5*(a + b)^4*f) - ((10
*a + 3*b)*Cot[e + f*x]^3)/(15*(a + b)^3*f) - Cot[e + f*x]^5/(5*(a + b)*f*(a
 + b + b*Tan[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Tan[e + f*x])/(10*(a + b)^4*
f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.26434, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 462, 456, 1261, 205}

$$-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5f(a+b)^4}-\frac{a\sqrt{b}(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2f(a+b)^{9/2}}-\frac{(10a+3b}{15f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] -(a*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*(a +
 b)^(9/2)*f) - ((5*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(5*(a + b)^4*f) - ((10
*a + 3*b)*Cot[e + f*x]^3)/(15*(a + b)^3*f) - Cot[e + f*x]^5/(5*(a + b)*f*(a
 + b + b*Tan[e + f*x]^2)) - (b*(5*a^2 + 2*b^2)*Tan[e + f*x])/(10*(a + b)^4*
f*(a + b + b*Tan[e + f*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{10a+3b+5(a+b)x^2}{x^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{10(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5(a+b)^4f} - \frac{(10a+3b)\cot^3(e+fx)}{15(a+b)^3f} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e\right.}$$

$$= -\frac{a(3a-4b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{9/2}f} - \frac{\left(5a^2-10ab-b^2\right)\cot(e+fx)}{5(a+b)^4f} - \frac{(10a+3b)\cot^3(e}{15(a+b)^3f}$$

Mathematica [C] time = 3.53815, size = 777, normalized size = 4.13

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\csc(e)\sec(2e)\csc^5(e+fx)\left(10a\left(16a^2+34ab+123b^2\right)\sin(fx)-a\left(16a^2-2\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((960*a*(3*a - 4*b)*b*ArcTan
[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x
]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
f*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 - Csc[e]*Csc[e + f*x]^5*Sec[2*e]*(10*a*(16*a^2 + 34*a*b + 123*b^2)*Sin[f*x
```

```
] - a*(16*a^2 - 223*a*b + 1336*b^2)*Sin[3*f*x] + 240*a^3*Sin[2*e - f*x] + 6
40*a^2*b*Sin[2*e - f*x] - 1460*a*b^2*Sin[2*e - f*x] + 240*b^3*Sin[2*e - f*x
] - 240*a^3*Sin[2*e + f*x] - 715*a^2*b*Sin[2*e + f*x] + 860*a*b^2*Sin[2*e +
 f*x] - 240*b^3*Sin[2*e + f*x] + 160*a^3*Sin[4*e + f*x] + 415*a^2*b*Sin[4*e
 + f*x] + 1830*a*b^2*Sin[4*e + f*x] + 165*a^2*b*Sin[2*e + 3*f*x] - 30*a*b^2
*Sin[2*e + 3*f*x] + 120*b^3*Sin[2*e + 3*f*x] - 16*a^3*Sin[4*e + 3*f*x] + 20
8*a^2*b*Sin[4*e + 3*f*x] - 1036*a*b^2*Sin[4*e + 3*f*x] + 180*a^2*b*Sin[6*e
+ 3*f*x] - 330*a*b^2*Sin[6*e + 3*f*x] + 120*b^3*Sin[6*e + 3*f*x] + 48*a^3*S
in[2*e + 5*f*x] - 268*a^2*b*Sin[2*e + 5*f*x] + 290*a*b^2*Sin[2*e + 5*f*x] -
 24*b^3*Sin[2*e + 5*f*x] + 48*a^3*Sin[6*e + 5*f*x] - 223*a^2*b*Sin[6*e + 5*
f*x] + 230*a*b^2*Sin[6*e + 5*f*x] - 24*b^3*Sin[6*e + 5*f*x] - 45*a^2*b*Sin[
8*e + 5*f*x] + 60*a*b^2*Sin[8*e + 5*f*x] - 16*a^3*Sin[4*e + 7*f*x] + 83*a^2
*b*Sin[4*e + 7*f*x] - 6*a*b^2*Sin[4*e + 7*f*x] - 15*a^2*b*Sin[6*e + 7*f*x]
- 16*a^3*Sin[8*e + 7*f*x] + 68*a^2*b*Sin[8*e + 7*f*x] - 6*a*b^2*Sin[8*e + 7
*f*x])))/(7680*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.118, size = 189, normalized size = 1.

$$-\frac{1}{5 f (a+b)^2 \left(\tan \left(f x+e\right)\right)^5}-\frac{a^2}{f (a+b)^4 \tan \left(f x+e\right)}+2 \frac{ab}{f (a+b)^4 \tan \left(f x+e\right)}-\frac{2 a}{3 f (a+b)^3 \left(\tan \left(f x+e\right)\right)^3}-\frac{}{2 f (a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/5/f/(a+b)^2/tan(f*x+e)^5-1/f*a^2/(a+b)^4/tan(f*x+e)+2/f*a/(a+b)^4/tan(f*
x+e)*b-2/3/f*a/(a+b)^3/tan(f*x+e)^3-1/2/f*a^2/(a+b)^4*b*tan(f*x+e)/(a+b+b*t
an(f*x+e)^2)-3/2/f*a^2/(a+b)^4*b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)
*b)^(1/2))+2/f*a/(a+b)^4*b^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.746461, size = 2260, normalized size = 12.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(40*a^3 - 201*a
^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 29*a^2*b + 28*a*b^2)*
cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11*a^2*b +
 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b + 8*a*b^2)
*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)
^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^2*b - 4*
a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*co
s(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*
cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*
f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f
*x + e)), -1/60*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 2*(40*a^3
 - 201*a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 10*(6*a^3 - 29*a^2*b + 28
*a*b^2)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11
*a^2*b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b +
8*a*b^2)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)
^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(3
*a^2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^
4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^
5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3106, size = 355, normalized size = 1.89

$$-\frac{\frac{15\,a^2 b\tan\left(fx+e\right)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{15\left(3\,a^2b-4\,ab^2\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\sqrt{ab+b^2}}+\frac{2\left(15\,a^2\tan\left(fx+e\right)^4-30\,ab\tan\left(fx+e\right)^4+\cdots\right.}{\left(a^4+4\,a^3b+6\cdots\right.}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^2*b*tan(f*x + e)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(
b*tan(f*x + e)^2 + a + b)) + 15*(3*a^2*b - 4*a*b^2)*(pi*floor((f*x + e)/pi
+ 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b + b^2)) + 2*(15*a^2*tan(f*x + e)^4 - 30*
a*b*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3*a^2
+ 6*a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^
5))/f
```

3.54 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(3a^2+14ab+13b^2\right)\cos(e+fx)}{2a^5f}+\frac{\sqrt{b}\left(15a^2+70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f}-\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(a\cos^2(e+fx)+b\right)^2}+\frac{(a\cdots}{\cdots}$$

```
[Out] (Sqrt[b]*(15*a^2 + 70*a*b + 63*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])
/(8*a^(11/2)*f) - ((3*a^2 + 14*a*b + 13*b^2)*Cos[e + f*x])/(2*a^5*f) + ((a
+ 3*b)*(3*a + 5*b)*Cos[e + f*x]^3)/(12*a^4*b*f) - Cos[e + f*x]^5/(5*a^3*f)
- ((a + b)^2*Cos[e + f*x]^7)/(4*a^2*b*f*(b + a*Cos[e + f*x]^2)^2) - (b*(a +
 b)*(3*a + 11*b)*Cos[e + f*x])/(8*a^5*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.25457, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 463, 455, 1810, 205}

$$-\frac{\left(3a^2+14ab+13b^2\right)\cos(e+fx)}{2a^5f}+\frac{\sqrt{b}\left(15a^2+70ab+63b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f}-\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(a\cos^2(e+fx)+b\right)^2}+\frac{(a\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] (Sqrt[b]*(15*a^2 + 70*a*b + 63*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])
/(8*a^(11/2)*f) - ((3*a^2 + 14*a*b + 13*b^2)*Cos[e + f*x])/(2*a^5*f) + ((a
+ 3*b)*(3*a + 5*b)*Cos[e + f*x]^3)/(12*a^4*b*f) - Cos[e + f*x]^5/(5*a^3*f)
- ((a + b)^2*Cos[e + f*x]^7)/(4*a^2*b*f*(b + a*Cos[e + f*x]^2)^2) - (b*(a +
 b)*(3*a + 11*b)*Cos[e + f*x])/(8*a^5*f*(b + a*Cos[e + f*x]^2))
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)^2}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^6(-4a^2+7(a+b)^2-4abx^2)}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4a^2bf}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-ab^2(a+b)(3a+11}{}\right.}{}$$

$$= -\frac{(a+b)^2\cos^7(e+fx)}{4a^2bf\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(4ab\left(3a^2+1\right.\right.\right.}{}$$

$$= -\frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f} + \frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4bf} - \frac{\cos^5(e+fx)}{5a^3f} - \frac{}{4a}$$

$$= \frac{\sqrt{b}(15a^2+70ab+63b^2)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f} - \frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5f} + \frac{(a+3}{}$$

Mathematica [C] time = 12.255, size = 1641, normalized size = 7.67

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-900*a^(11/2)*b^(3/2)*Cos[e
 + f*x] - 109000*a^(9/2)*b^(5/2)*Cos[e + f*x] - 936000*a^(7/2)*b^(7/2)*Cos[
e + f*x] - 2803072*a^(5/2)*b^(9/2)*Cos[e + f*x] - 3763200*a^(3/2)*b^(11/2)*
Cos[e + f*x] - 1935360*Sqrt[a]*b^(13/2)*Cos[e + f*x] - 900*a^(11/2)*b^(3/2)
*Cos[e + f*x]*Cos[2*(e + f*x)] + 900*a^(9/2)*b^(3/2)*Cos[e + f*x]*(a + 2*b
+ a*Cos[2*(e + f*x)]) + 24000*a^(7/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b + a*Cos
[2*(e + f*x)]) + 43200*a^(5/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e +
 f*x)]) + 225*a^5*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 11520
0*a^2*b^3*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
```

```
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 537600*a*b^4*
ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(
f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*
x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 483840*b^5*ArcTan[((-Sq
rt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Co
s[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 225*a^5*ArcTan[((-Sqrt[a] + I*Sqrt[
a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] +
 Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b +
 a*Cos[2*(e + f*x)])^2 + 115200*a^2*b^3*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a
+ b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2
*(e + f*x)])^2 + 537600*a*b^4*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)
])^2 + 483840*b^5*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 225*a
^5*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Co
s[2*(e + f*x)])^2 - 225*a^5*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 19200*a^(5/2)*b^(5/2)*Cos[e]*C
os[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 20352*a^(9/2)*b^(5/2)*Cos[e + f*
x]*Cos[4*(e + f*x)] - 115712*a^(7/2)*b^(7/2)*Cos[e + f*x]*Cos[4*(e + f*x)]
- 129024*a^(5/2)*b^(9/2)*Cos[e + f*x]*Cos[4*(e + f*x)] + 2048*a^(9/2)*b^(5/
2)*Cos[e + f*x]*Cos[6*(e + f*x)] + 4608*a^(7/2)*b^(7/2)*Cos[e + f*x]*Cos[6*
(e + f*x)] - 384*a^(9/2)*b^(5/2)*Cos[e + f*x]*Cos[8*(e + f*x)] - 19200*a^(5
/2)*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x] - 32496*a^(9/2
)*b^(5/2)*Csc[e + f*x]*Sin[4*(e + f*x)] - 252080*a^(7/2)*b^(7/2)*Csc[e + f*
x]*Sin[4*(e + f*x)] - 577024*a^(5/2)*b^(9/2)*Csc[e + f*x]*Sin[4*(e + f*x)]
- 403200*a^(3/2)*b^(11/2)*Csc[e + f*x]*Sin[4*(e + f*x)]))/(491520*a^(11/2)*
b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.098, size = 374, normalized size = 1.8

$$-\frac{\left(\cos\left(fx+e\right)\right)^{5}}{5\,a^{3}f}+\frac{2\,\left(\cos\left(fx+e\right)\right)^{3}}{3\,a^{3}f}+\frac{\left(\cos\left(fx+e\right)\right)^{3}b}{fa^{4}}-\frac{\cos\left(fx+e\right)}{a^{3}f}-6\,\frac{b\cos\left(fx+e\right)}{fa^{4}}-6\,\frac{b^{2}\cos\left(fx+e\right)}{fa^{5}}-\frac{9}{8\,fa^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/5*cos(f*x+e)^5/a^3/f+2/3*cos(f*x+e)^3/a^3/f+1/f/a^4*cos(f*x+e)^3*b-cos(f
*x+e)/a^3/f-6/f/a^4*b*cos(f*x+e)-6/f/a^5*b^2*cos(f*x+e)-9/8/f*b/a^2/(b+a*co
s(f*x+e)^2)^2*cos(f*x+e)^3-13/4/f*b^2/a^3/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3
-17/8/f*b^3/a^4/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3-7/8/f*b^2/a^3/(b+a*cos(f*
x+e)^2)^2*cos(f*x+e)-11/4/f*b^3/a^4/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)-15/8/f*
b^4/a^5/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)+15/8/f*b/a^3/(a*b)^(1/2)*arctan(a*c
os(f*x+e)/(a*b)^(1/2))+35/4/f*b^2/a^4/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)
^(1/2))+63/8/f*b^3/a^5/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.740858, size = 1361, normalized size = 6.36

$$\left[-\frac{48\,a^4\cos\left(fx+e\right)^9-16\left(10\,a^4+9\,a^3b\right)\cos\left(fx+e\right)^7+16\left(15\,a^4+70\,a^3b+63\,a^2b^2\right)\cos\left(fx+e\right)^5+50\left(15\,a^3b+70\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(48*a^4*cos(f*x + e)^9 - 16*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 16*
(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 50*(15*a^3*b + 70*a^2*b^2
 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x
+ e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63*a*b
^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(
f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 70*a*b^3 + 63*b^4)
*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f
), -1/120*(24*a^4*cos(f*x + e)^9 - 8*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 8*
(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 25*(15*a^3*b + 70*a^2*b^2
```

```
 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x
+ e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63*a*b
^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(15*a
^2*b^2 + 70*a*b^3 + 63*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f
*cos(f*x + e)^2 + a^5*b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24367, size = 1130, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqr
t(a*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^5) + 30*(9*a^3*b + 33*a^2*b^
2 + 39*a*b^3 + 15*b^4 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 49
*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 23*a*b^3*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) - 45*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^
3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^2*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2 + 45*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x +
e) - 1)^3/(cos(f*x + e) + 1)^3 + 11*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 15*b^4*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)^2*a^5) - 16*(8*a^2 + 75*a*b + 90*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 330*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 360*b^2*(co
```

```
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 480*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 540*b^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 270*a*b*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 - 360*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 45*a
*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 90*b^2*(cos(f*x + e) - 1)^4/
(cos(f*x + e) + 1)^4)/(a^5*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/
f
```

3.55 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(a\cos^2(e+fx)+b\right)}-\frac{(a+3b)\cos(e+fx)}{a^4f}+\frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f}+\text{co}$$

```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(9/2)*f
) - ((a + 3*b)*Cos[e + f*x])/(a^4*f) + Cos[e + f*x]^3/(3*a^3*f) + (b^2*(a +
 b)*Cos[e + f*x])/(4*a^4*f*(b + a*Cos[e + f*x]^2)^2) - (b*(9*a + 13*b)*Cos[
e + f*x])/(8*a^4*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.188327, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4133, 455, 1814, 1153, 205}

$$\frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(a\cos^2(e+fx)+b\right)}-\frac{(a+3b)\cos(e+fx)}{a^4f}+\frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f}+\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(9/2)*f
) - ((a + 3*b)*Cos[e + f*x])/(a^4*f) + Cos[e + f*x]^3/(3*a^3*f) + (b^2*(a +
 b)*Cos[e + f*x])/(4*a^4*f*(b + a*Cos[e + f*x]^2)^2) - (b*(9*a + 13*b)*Cos[
e + f*x])/(8*a^4*f*(b + a*Cos[e + f*x]^2))
```


Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6(1-x^2)}{(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-b^2(a+b)+4ab(a+b)x^2-4a^2(a+b)x^4+4a^3x^6}{(b+ax^2)^2}\,dx, x, \cos(e+fx)\right.}{4a^4f}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-b^2(7a+11b)+8ab(a+2b)x^2-8}{b+ax^2}\right.}{8a^4bf}$$

$$= \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \left(8b(a+3b) - 8abx^2 - \right.\right.}{8a^4}$$

$$= -\frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b)\cos(e+}{8a^4f\left(b+a\cos^2(e+\right.}$$

$$= \frac{5\sqrt{b}(3a+7b)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f} - \frac{(a+3b)\cos(e+fx)}{a^4f} + \frac{\cos^3(e+fx)}{3a^3f} + \frac{b^2(a+b)\,\text{co}}{4a^4f\left(b+a\,\text{co}\right.}$$

Mathematica [C] time = 11.6896, size = 1392, normalized size = 9.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*((-3*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/Sqrt[a] -
 (3*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/Sqrt[a] - (2*
Sqrt[b]*Cos[e + f*x]*(3*a + 10*b + 3*a*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[
2*(e + f*x)])^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(8192*b^(
5/2)*f*(a + b*Sec[e + f*x]^2)^3) + (((3*a - 4*b)*ArcTan[((-Sqrt[a] - I*Sqrt
[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a]
- Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (3*a -
4*b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*
Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Ta
n[(f*x)/2]))/Sqrt[b]] + (2*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(3*a^2 + 6*a*b + 8*
```

```
b^2 + a*(3*a - 4*b)*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])^2)*(a
 + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(2048*a^(3/2)*b^(5/2)*f*(a +
 b*Sec[e + f*x]^2)^3) - ((-3*(3*a^4 - 40*a^3*b + 720*a^2*b^2 + 6720*a*b^3 +
 8960*b^4)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2]*Tan[(f*x)/2]))/Sqrt[b]] - 3*(3*a^4 - 40*a^3*b + 720*a^2*b^2 + 6720*a*b^
3 + 8960*b^4)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
)*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - (2*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(9*a^5 - 9
0*a^4*b - 10144*a^3*b^2 - 48672*a^2*b^3 - 85120*a*b^4 - 53760*b^5 + a*(9*a^
4 - 120*a^3*b - 12432*a^2*b^2 - 47936*a*b^3 - 44800*b^4)*Cos[2*(e + f*x)] -
 128*a^2*b^2*(15*a + 28*b)*Cos[4*(e + f*x)] + 128*a^3*b^2*Cos[6*(e + f*x)])
)/(a + 2*b + a*Cos[2*(e + f*x)])^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6)/(49152*a^(9/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - (3*(a + 2*b
+ a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*
b^3)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*
Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Ta
n[(f*x)/2]))/Sqrt[b]])/b^(5/2) + (3*(a^3 - 8*a^2*b + 80*a*b^2 + 320*b^3)*Ar
cTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*
x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)
/2]))/Sqrt[b]])/b^(5/2) - 512*Sqrt[a]*Cos[e]*Cos[f*x] + (8*Sqrt[a]*(a^3 + 2
4*a^2*b + 80*a*b^2 + 64*b^3)*Cos[e + f*x])/(b*(a + 2*b + a*Cos[2*(e + f*x)]
)^2) + (2*Sqrt[a]*(3*a^3 - 24*a^2*b - 400*a*b^2 - 576*b^3)*Cos[e + f*x])/(b
^2*(a + 2*b + a*Cos[2*(e + f*x)])) + 512*Sqrt[a]*Sin[e]*Sin[f*x]))/(16384*a
^(7/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.093, size = 231, normalized size = 1.5

$$\frac{\left(\cos\left(fx+e\right)\right)^{3}}{3\,a^{3}f}-\frac{\cos\left(fx+e\right)}{a^{3}f}-3\,\frac{b\cos\left(fx+e\right)}{fa^{4}}-\frac{9\,b\left(\cos\left(fx+e\right)\right)^{3}}{8\,fa^{2}\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)^{2}}-\frac{13\,b^{2}\left(\cos\left(fx+e\right)\right)^{3}}{8\,a^{3}f\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)^{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/3*cos(f*x+e)^3/a^3/f-cos(f*x+e)/a^3/f-3/f/a^4*b*cos(f*x+e)-9/8/f*b/a^2/(b
+a*cos(f*x+e)^2)^2*cos(f*x+e)^3-13/8/f*b^2/a^3/(b+a*cos(f*x+e)^2)^2*cos(f*x
+e)^3-7/8/f*b^2/a^3/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)-11/8/f*b^3/a^4/(b+a*cos
(f*x+e)^2)^2*cos(f*x+e)+15/8/f*b/a^3/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^
(1/2))+35/8/f*b^2/a^4/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.67883, size = 1007, normalized size = 6.54

$$\left[\frac{16\,a^3\cos\left(fx+e\right)^7-16\left(3\,a^3+7\,a^2b\right)\cos\left(fx+e\right)^5-50\left(3\,a^2b+7\,ab^2\right)\cos\left(fx+e\right)^3+15\left(\left(3\,a^3+7\,a^2b\right)\cos\left(fx+e\right)^4\right.}{48\left(a^6f\cos\left(fx+e\right)^4+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*cos(f*x + e)^7 - 16*(3*a^3 + 7*a^2*b)*cos(f*x + e)^5 - 50*(3*
a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*cos(f*x + e)^4 + 3*
a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*co
s(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 3
0*(3*a*b^2 + 7*b^3)*cos(f*x + e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x
 + e)^2 + a^4*b^2*f), 1/24*(8*a^3*cos(f*x + e)^7 - 8*(3*a^3 + 7*a^2*b)*cos(
f*x + e)^5 - 25*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*
cos(f*x + e)^4 + 3*a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sq
rt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 15*(3*a*b^2 + 7*b^3)*cos(f*x +
 e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 + a^4*b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21004, size = 247, normalized size = 1.6

$$\frac{5\left(3\,ab+7\,b^2\right)\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4f}-\frac{\frac{9\,a^2b\cos\left(fx+e\right)^3}{f}+\frac{13\,ab^2\cos\left(fx+e\right)^3}{f}+\frac{7\,ab^2\cos\left(fx+e\right)}{f}+\frac{11\,b^3\cos\left(fx+e\right)}{f}}{8\left(a\cos\left(fx+e\right)^2+b\right)^2a^4}+\frac{a^6f^{17}\cos\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 5/8*(3*a*b + 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4*f) - 1/
8*(9*a^2*b*cos(f*x + e)^3/f + 13*a*b^2*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x +
 e)/f + 11*b^3*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)^2*a^4) + 1/3*(a^6*f^
17*cos(f*x + e)^3 - 3*a^6*f^17*cos(f*x + e) - 9*a^5*b*f^17*cos(f*x + e))/(a
^9*f^18)
```

3.56 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=116

$$\frac{5\cos^3(e+fx)}{8a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f}-\frac{15\cos(e+fx)}{8a^3f}+\frac{\cos^5(e+fx)}{4af\left(a\cos^2(e+fx)+b\right)^2}$$

```
[Out] (15*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(7/2)*f) - (15*Cos
[e + f*x])/(8*a^3*f) + Cos[e + f*x]^5/(4*a*f*(b + a*Cos[e + f*x]^2)^2) + (5
*Cos[e + f*x]^3)/(8*a^2*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.0681461, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4133, 288, 321, 205}

$$\frac{5\cos^3(e+fx)}{8a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f}-\frac{15\cos(e+fx)}{8a^3f}+\frac{\cos^5(e+fx)}{4af\left(a\cos^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (15*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8*a^(7/2)*f) - (15*Cos
[e + f*x])/(8*a^3*f) + Cos[e + f*x]^5/(4*a*f*(b + a*Cos[e + f*x]^2)^2) + (5
*Cos[e + f*x]^3)/(8*a^2*f*(b + a*Cos[e + f*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{4af}$$

$$= \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \cos(e+\right.}{8a^2f}$$

$$= -\frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2f\left(b+a\cos^2(e+fx)\right)} + \frac{(15b)\,\text{Subst}}{}$$

$$= \frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f} - \frac{15\cos(e+fx)}{8a^3f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e}{8a^2f\left(b+a\,\text{co}\right.}$$

Mathematica [C] time = 7.14912, size = 656, normalized size = 5.66

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(15\left(a^3+64b^3\right)\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e}\right.}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(15*(a^3 + 64*b^3)*ArcTan[
((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]] + 15*(a^3 + 64*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (Sqrt[a]*(24*a^4*Sqrt[b]*C
os[e + f*x] - 24*a^3*b^(3/2)*Cos[e + f*x] - 144*a^2*b^(5/2)*Cos[e + f*x] +
512*b^(9/2)*Cos[e + f*x] - 72*a^3*b^(3/2)*Cos[e + f*x]*Cos[2*(e + f*x)] - 2
4*a^3*Sqrt[b]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) + 72*a^2*b^(3/2)*
Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 1152*b^(7/2)*Cos[e + f*x]*(a
+ 2*b + a*Cos[2*(e + f*x)]) - 15*a^(5/2)*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[
(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 15*a^(5/2)*ArcTan
[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e +
f*x)])^2 - 512*b^(5/2)*Cos[e]*Cos[f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 5
12*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x] + 6*a^4*Sqrt[b]
*Csc[e + f*x]*Sin[4*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4096*a
^(7/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.046, size = 108, normalized size = 0.9

$$-\frac{7b^2\left(\sec\left(fx+e\right)\right)^3}{8fa^3\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^2}-\frac{9b\sec\left(fx+e\right)}{8fa^2\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^2}-\frac{15b}{8fa^3}\arctan\left(b\sec\left(fx+e\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{fa^3\sec\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -7/8/f/a^3*b^2/(a+b*sec(f*x+e)^2)^2*sec(f*x+e)^3-9/8/f/a^2*b/(a+b*sec(f*x+e
)^2)^2*sec(f*x+e)-15/8/f/a^3*b/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2))
-1/f/a^3/sec(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.603093, size = 713, normalized size = 6.15

$$\left[-\frac{16\,a^2\cos\left(fx+e\right)^5+50\,ab\cos\left(fx+e\right)^3+30\,b^2\cos\left(fx+e\right)-15\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-\frac{b}{a}}\,\mathrm{l}}{16\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^2*cos(f*x + e)^5 + 50*a*b*cos(f*x + e)^3 + 30*b^2*cos(f*x + e)
 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/a)*log(-(a*
cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)))/
(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), -1/8*(8*a^2*
cos(f*x + e)^5 + 25*a*b*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) - 15*(a^2*cos(
f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f
*x + e)/b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33127, size = 131, normalized size = 1.13

$$\frac{15\,b\arctan\left(\frac{a\cos\left(fx+e\right)}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3f}-\frac{\cos\left(fx+e\right)}{a^3f}-\frac{\frac{9\,ab\cos\left(fx+e\right)^3}{f}+\frac{7\,b^2\cos\left(fx+e\right)}{f}}{8\left(a\cos\left(fx+e\right)^2+b\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 15/8*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - cos(f*x + e)/(a
^3*f) - 1/8*(9*a*b*cos(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a*cos(f*x + e
)^2 + b)^2*a^3)
```

3.57 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}f(a+b)^3}-\frac{b(7a+3b)\cos(e+fx)}{8a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}-\frac{b\cos^3(e+fx)}{4af(a+b)\left(a\cos^2(e+fx)+b\right)^2}-\text{t}$$

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/
(8*a^(5/2)*(a + b)^3*f) - ArcTanh[Cos[e + f*x]]/((a + b)^3*f) - (b*Cos[e +
f*x]^3)/(4*a*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) - (b*(7*a + 3*b)*Cos[e + f
*x])/(8*a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2))
```

Rubi [A] time = 0.196444, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4133, 470, 578, 522, 206, 205}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}f(a+b)^3}-\frac{b(7a+3b)\cos(e+fx)}{8a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}-\frac{b\cos^3(e+fx)}{4af(a+b)\left(a\cos^2(e+fx)+b\right)^2}-\text{t}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/
(8*a^(5/2)*(a + b)^3*f) - ArcTanh[Cos[e + f*x]]/((a + b)^3*f) - (b*Cos[e +
f*x]^3)/(4*a*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) - (b*(7*a + 3*b)*Cos[e + f
*x])/(8*a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2))
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(b+ax^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3b+(-4a-3b)x^2)}{(1-x^2)(b+ax^2)^2}\,dx, x, \cos(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{b(7a+3\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\cdots\right.}{(a+\cdots}$$

$$= \frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3f} - \frac{\tanh^{-1}(\cos(e+fx))}{(a+b)^3f} - \frac{b\cos^3(e+\cdots}{4a(a+b)f\left(b+a\,\mathrm{c}\cdots\right.}$$

Mathematica [C] time = 2.61918, size = 447, normalized size = 2.9

$$\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\sec(e+fx)(a\cos(2(e+fx))+a+2b)^2\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))\cdots}\right.}{\cdots}\right.}{a^{5/2}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a^2 - (2*
b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a^2 + (Sqrt[b]*(15*a^
2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e
 + f*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] + I
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqr
t[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a +
```

```
 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(5/2) - 8*(a + 2*b + a*Cos[2*(
e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 8*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x]))/(64*(a + b)^3*f*(a + b*Sec[e
 + f*x]^2)^3)
```

Maple [B] time = 0.099, size = 352, normalized size = 2.3

$$-\frac{\ln\left(1+\cos\left(fx+e\right)\right)}{2f\left(a+b\right)^{3}}-\frac{9ab\left(\cos\left(fx+e\right)\right)^{3}}{8f\left(a+b\right)^{3}\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)^{2}}-\frac{7b^{2}\left(\cos\left(fx+e\right)\right)^{3}}{4f\left(a+b\right)^{3}\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)^{2}}-\frac{5b^{3}(\cos}{8f\left(a+b\right)^{3}(b+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/2/f/(a+b)^3*ln(1+cos(f*x+e))-9/8/f*b/(a+b)^3/(b+a*cos(f*x+e)^2)^2*a*cos(
f*x+e)^3-7/4/f*b^2/(a+b)^3/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3-5/8/f*b^3/(a+b
)^3/(b+a*cos(f*x+e)^2)^2/a*cos(f*x+e)^3-7/8/f*b^2/(a+b)^3/(b+a*cos(f*x+e)^2
)^2*cos(f*x+e)-5/4/f*b^3/(a+b)^3/(b+a*cos(f*x+e)^2)^2/a*cos(f*x+e)-3/8/f*b^
4/(a+b)^3/(b+a*cos(f*x+e)^2)^2/a^2*cos(f*x+e)+15/8/f*b/(a+b)^3/(a*b)^(1/2)*
arctan(a*cos(f*x+e)/(a*b)^(1/2))+5/4/f*b^2/(a+b)^3/a/(a*b)^(1/2)*arctan(a*c
os(f*x+e)/(a*b)^(1/2))+3/8/f*b^3/(a+b)^3/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e
)/(a*b)^(1/2))+1/2/f/(a+b)^3*ln(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.00985, size = 1773, normalized size = 11.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 10*a
^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15*a^
3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)
^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(7*a^2*b^
2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 8*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*
x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2) - 8*(a^4*cos(f*x + e)^4 + 2
*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 3*a^
6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^
5)*f), -1/8*((9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 1
0*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15
*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)
*cos(f*x + e)/b) + (7*a^2*b^2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 4*(a^4*cos
(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2)
 - 4*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f
*x + e) + 1/2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2
*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*
a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27685, size = 844, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)
*cos(f*x + e) + sqrt(a*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b
)) - 4*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2
 + b^3) + 2*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + 27*a^3*b*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 13*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) - 23*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 9*b^4*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 - 9*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 21*a*b^3*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - a^2*b^2*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f
*x + e) + 1)^3 - 3*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2))/f
```

3.58 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=213

$$\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{8af(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}f(a+b)^4}-\frac{b(2a-b)\cos(e+fx)}{4af(a+b)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{}{2f}$$

```
[Out] (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8
*a^(3/2)*(a + b)^4*f) - ((a - 5*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^4*f) -
 ((2*a - b)*b*Cos[e + f*x])/(4*a*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) + ((
4*a^2 - 9*a*b - b^2)*Cos[e + f*x])/(8*a*(a + b)^3*f*(b + a*Cos[e + f*x]^2))
 - (Cos[e + f*x]*Cot[e + f*x]^2)/(2*(a + b)*f*(b + a*Cos[e + f*x]^2)^2)
```

Rubi [A] time = 0.31166, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4133, 470, 578, 527, 522, 206, 205}

$$\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{8af(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}f(a+b)^4}-\frac{b(2a-b)\cos(e+fx)}{4af(a+b)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(8
*a^(3/2)*(a + b)^4*f) - ((a - 5*b)*ArcTanh[Cos[e + f*x]])/(2*(a + b)^4*f) -
 ((2*a - b)*b*Cos[e + f*x])/(4*a*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) + ((
4*a^2 - 9*a*b - b^2)*Cos[e + f*x])/(8*a*(a + b)^3*f*(b + a*Cos[e + f*x]^2))
 - (Cos[e + f*x]*Cot[e + f*x]^2)/(2*(a + b)*f*(b + a*Cos[e + f*x]^2)^2)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b+(-a+2b)x^2\right)}{\left(1-x^2\right)\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2(2a-b)b}{(1}\right.}{}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{8a(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\cos(e+fx)}{2(a+b)f\left(b+a\right.}$$

$$= -\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{8a(a+b)^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\cos(e+fx)}{2(a+b)f\left(b+a\right.}$$

$$= \frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4f} - \frac{(a-5b)\tanh^{-1}(\cos(e+fx))}{2(a+b)^4f} - \frac{(2a-}{4a(a+b)^2f}$$

Mathematica [C] time = 3.65137, size = 532, normalized size = 2.5

$$\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\frac{\sqrt{b}\left(-15a^2+10ab+b^2\right)\sec(e+fx)(a\cos(2(e+fx))+a+2b)^2\tan^{-1}\left(\frac{\sin(e)\tan\left(\frac{fx}{2}\right)\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(}\right.}{}\right)}{a^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a - (2*b*
(a + b)*(9*a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a - (Sqrt[b]*(-15*a^2 + 1
0*a*b + b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])
*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]
)/a^(3/2) - (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a
+ b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a
*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(3/2) - (a + b)*(a + 2*b + a*Cos[2*(e
+ f*x)])^2*Csc[(e + f*x)/2]^2*Sec[e + f*x] - 4*(a - 5*b)*(a + 2*b + a*Cos[2
*(e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 5*b)*(a + 2*b +
a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Sec[(e + f*x)/2]^2*Sec[e + f*x]))/(64*(a + b)^4*f*
(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.114, size = 430, normalized size = 2.

$$\frac{1}{4 f (a+b)^3 \left(1+\cos\left(fx+e\right)\right)}-\frac{\ln\left(1+\cos\left(fx+e\right)\right) a}{4 f (a+b)^4}+\frac{5 \ln\left(1+\cos\left(fx+e\right)\right) b}{4 f (a+b)^4}-\frac{9 b \left(\cos\left(fx+e\right)\right)^3 a^2}{8 f (a+b)^4 \left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/4/f/(a+b)^3/(1+cos(f*x+e))-1/4/f/(a+b)^4*ln(1+cos(f*x+e))*a+5/4/f/(a+b)^4
*ln(1+cos(f*x+e))*b-9/8/f/(a+b)^4*b/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3*a^2-5
/4/f/(a+b)^4*b^2/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3*a-1/8/f/(a+b)^4*b^3/(b+a
*cos(f*x+e)^2)^2*cos(f*x+e)^3-7/8/f/(a+b)^4*b^2/(b+a*cos(f*x+e)^2)^2*a*cos(
f*x+e)-3/4/f/(a+b)^4*b^3/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)+1/8/f/(a+b)^4*b^4/
(b+a*cos(f*x+e)^2)^2/a*cos(f*x+e)+15/8/f/(a+b)^4*b*a/(a*b)^(1/2)*arctan(a*c
os(f*x+e)/(a*b)^(1/2))-5/4/f/(a+b)^4*b^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a
*b)^(1/2))-1/8/f/(a+b)^4*b^3/a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))
+1/4/f/(a+b)^3/(-1+cos(f*x+e))+1/4/f/(a+b)^4*ln(-1+cos(f*x+e))*a-5/4/f/(a+b
)^4*ln(-1+cos(f*x+e))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.16509, size = 2946, normalized size = 13.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + 2*(17*a^3*
b + 11*a^2*b^2 - 5*a*b^3 + b^4)*cos(f*x + e)^3 - ((15*a^4 - 10*a^3*b - a^2*
b^2)*cos(f*x + e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x +
e)^4 - 15*a^2*b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 + b^4
)*cos(f*x + e)^2)*sqrt(-b/a)*log((a*cos(f*x + e)^2 - 2*a*sqrt(-b/a)*cos(f*x
 + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 + 10*a*b^3 - b^4)*cos(f*
x + e) - 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*c
os(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*
x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 -
 (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b
 - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^
7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*
a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (
2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x
+ e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/8*((4*
a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a^3*b + 11*a^2*b^2
 - 5*a*b^3 + b^4)*cos(f*x + e)^3 + ((15*a^4 - 10*a^3*b - a^2*b^2)*cos(f*x +
 e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 - 15*a^2*
b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 + b^4)*cos(f*x + e)
^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + (11*a^2*b^2 + 10*a*b^3 -
 b^4)*cos(f*x + e) - 2*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 1
0*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a
*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 2*((a^4 - 5*a^3*b)*cos(
f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^
3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e)
+ 1/2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*
x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6
```

```
)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*
f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49216, size = 1068, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 5*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(-(a*cos(
f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^5 + 4*a^4*b + 6*a^3
*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) + (a + b - 2*a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 10*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e)
+ 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1)) - (co
s(f*x + e) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) + 1)) - 2*(9
*a^3*b + 17*a^2*b^2 + 7*a*b^3 - b^4 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 23*a*b^3*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 21*a^2*b^2*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 29*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 - 3*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 5*a^2*b^2*(cos(f*x + e) - 1)^3/(
cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 +
b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 + 4*a^4*b + 6*a^3*b^2
+ 4*a^2*b^3 + a*b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2
*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2))/f
```

3.59 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=257

$$\frac{3\left(a^2-6ab+b^2\right)\cos(e+fx)}{8f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}f(a+b)^5}-\frac{3\left(a\right.}{}$$

```
[Out] (3*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(
8*Sqrt[a]*(a + b)^5*f) - (3*(a^2 - 10*a*b + 5*b^2)*ArcTanh[Cos[e + f*x]])/(
8*(a + b)^5*f) + ((a^2 - 9*a*b + 2*b^2)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a
*Cos[e + f*x]^2)^2) + (3*(a^2 - 6*a*b + b^2)*Cos[e + f*x])/(8*(a + b)^4*f*(
b + a*Cos[e + f*x]^2)) - ((a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2
*f*(b + a*Cos[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*(
b + a*Cos[e + f*x]^2)^2)
```

Rubi [A] time = 0.366037, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4133, 470, 578, 527, 522, 206, 205}

$$\frac{3\left(a^2-6ab+b^2\right)\cos(e+fx)}{8f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}f(a+b)^5}-\frac{3\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(
8*Sqrt[a]*(a + b)^5*f) - (3*(a^2 - 10*a*b + 5*b^2)*ArcTanh[Cos[e + f*x]])/(
8*(a + b)^5*f) + ((a^2 - 9*a*b + 2*b^2)*Cos[e + f*x])/(8*(a + b)^3*f*(b + a
*Cos[e + f*x]^2)^2) + (3*(a^2 - 6*a*b + b^2)*Cos[e + f*x])/(8*(a + b)^4*f*(
b + a*Cos[e + f*x]^2)) - ((a - 7*b)*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2
*f*(b + a*Cos[e + f*x]^2)^2) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*(
b + a*Cos[e + f*x]^2)^2)
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
```

```
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b+(-a+4b)x^2\right)}{\left(1-x^2\right)^2\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{(a-7b)b+(}{(1-}\right.}{}$$

$$= \frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(b+a\cos^2(e+fx)\right)^2} - \frac{\cot^3(e+fx)\,\text{cs}}{4(a+b)f\left(b+a\,\text{co}\right.}$$

$$= \frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3\left(a^2-6ab+b^2\right)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)\cot(e+fx}{8(a+b)^2f\left(b+a\,\text{co}\right.}$$

$$= \frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{8(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{3\left(a^2-6ab+b^2\right)\cos(e+fx)}{8(a+b)^4f\left(b+a\cos^2(e+fx)\right)} - \frac{(a-7b)\cot(e+fx}{8(a+b)^2f\left(b+a\,\text{co}\right.}$$

$$= \frac{3\sqrt{b}\left(5a^2-10ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}(a+b)^5f} - \frac{3\left(a^2-10ab+5b^2\right)\tanh^{-1}(\cos(e+fx))}{8(a+b)^5f} +$$

Mathematica [C] time = 5.23082, size = 549, normalized size = 2.14

$$\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(-48\left(a^2-10ab+5b^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)(a\cos(2(e+fx))+a+2b)^2+48\left(a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*((48*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[
((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] + (48*Sqrt[b]*(5*a^2 -
10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
)*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] -
2*(a + b)*(30*a^3 + 112*a^2*b + 182*a*b^2 - 140*b^3 + (35*a^3 + 78*a^2*b -
93*a*b^2 + 224*b^3)*Cos[2*(e + f*x)] + 2*(a^3 - 8*a^2*b + 53*a*b^2 - 10*b^3
)*Cos[4*(e + f*x)] - 3*a^3*Cos[6*(e + f*x)] + 18*a^2*b*Cos[6*(e + f*x)] - 3
*a*b^2*Cos[6*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 - 48*(a^2 - 10*a*b + 5
*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]] + 48*(a^2 - 10
*a*b + 5*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]])*Sec[e
 + f*x]^6)/(1024*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.123, size = 567, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/16/f/(a+b)^3/(1+cos(f*x+e))^2+3/16/f/(a+b)^4/(1+cos(f*x+e))*a-9/16/f/(a+b
)^4/(1+cos(f*x+e))*b-3/16/f/(a+b)^5*ln(1+cos(f*x+e))*a^2+15/8/f/(a+b)^5*ln(
1+cos(f*x+e))*a*b-15/16/f/(a+b)^5*ln(1+cos(f*x+e))*b^2-9/8/f/(a+b)^5*b/(b+a
*cos(f*x+e)^2)^2*cos(f*x+e)^3*a^3-3/4/f/(a+b)^5*b^2/(b+a*cos(f*x+e)^2)^2*co
s(f*x+e)^3*a^2+3/8/f/(a+b)^5*b^3/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)^3*a-7/8/f/
```

```
(a+b)^5*b^2/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)*a^2-1/4/f/(a+b)^5*b^3/(b+a*cos(
f*x+e)^2)^2*cos(f*x+e)*a+5/8/f/(a+b)^5*b^4/(b+a*cos(f*x+e)^2)^2*cos(f*x+e)+
15/8/f/(a+b)^5*b/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))*a^2-15/4/f/(a
+b)^5*b^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))*a+3/8/f/(a+b)^5*b^3/
(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))-1/16/f/(a+b)^3/(-1+cos(f*x+e))
^2+3/16/f/(a+b)^4/(-1+cos(f*x+e))*a-9/16/f/(a+b)^4/(-1+cos(f*x+e))*b+3/16/f
/(a+b)^5*ln(-1+cos(f*x+e))*a^2-15/8/f/(a+b)^5*ln(-1+cos(f*x+e))*a*b+15/16/f
/(a+b)^5*ln(-1+cos(f*x+e))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.36488, size = 4163, normalized size = 16.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(a^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 26
*a^3*b + 26*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15*a
*b^3 + 19*b^4)*cos(f*x + e)^3 + 3*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x + e
)^8 - 2*(5*a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4 - 3
0*a^3*b + 46*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10*a*b^
3 + b^4 + 2*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-b
/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x +
e)^2 + b)) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10*a^3*b + 5*a^2*b
^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x + e)
^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 + a^2*
b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*cos(f*x
+ e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x + e)^6 + (a^4
```

```
 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 + a^2*b^2 - 10*
a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*cos(f*x + e)^2)*l
og(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*
a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*
b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a
^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(a^6*b
+ 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^
5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f), 1/16*(6*(a
^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 26*a^3*b + 26
*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15*a*b^3 + 19*b
^4)*cos(f*x + e)^3 + 6*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x + e)^8 - 2*(5*
a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4 - 30*a^3*b + 4
6*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10*a*b^3 + b^4 + 2
*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a
*sqrt(b/a)*cos(f*x + e)/b) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10
*a^3*b + 5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b
^3)*cos(f*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f
*x + e)^4 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 -
 5*b^4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b +
5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f
*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4
 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*c
os(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*
a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b -
9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x +
e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(
f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7
)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39203, size = 1889, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 10*a*b + 5*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))
/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 24*(5*a^2*b -
10*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt
(a*b)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b
)) - (8*a^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a*b^2*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) - 16*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^
3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*a^2*b*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 - 3*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 -
b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 6*a^5*b + 15*a^4*b^2
+ 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4 - 4*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 24*a^2*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 32*a*b^3*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^4*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + 136*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2 + 224*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 40*a*b
^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 108*b^4*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 - 20*a^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 +
280*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 64*a^2*b^2*(cos(f*x +
 e) - 1)^3/(cos(f*x + e) + 1)^3 - 152*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 + 212*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 5*a^4*(cos
(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 84*a^3*b*(cos(f*x + e) - 1)^4/(cos(
f*x + e) + 1)^4 + 30*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 84
*a*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 - 123*b^4*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4 + 16*a^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^
5 - 104*a^3*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 24*a^2*b^2*(cos(f
*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 72*a*b^3*(cos(f*x + e) - 1)^5/(cos(f*
x + e) + 1)^5 - 24*b^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 6*a^4*(c
os(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 - 48*a^3*b*(cos(f*x + e) - 1)^6/(co
s(f*x + e) + 1)^6 - 84*a^2*b^2*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 +
30*b^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((a^5 + 5*a^4*b + 10*a^3*
b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
+ b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x
 + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
1)^3)^2))/f
```

3.60 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=314

$$-\frac{5b\left(5a^2+20ab+16b^2\right)\tan(e+fx)}{16a^5f\left(a+b\tan^2(e+fx)+b\right)}-\frac{5b\left(9a^2+32ab+24b^2\right)\tan(e+fx)}{48a^4f\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\left(33a^2+110ab+80b^2\right)\sin(e+fx)\cos(e+}{48a^3f\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] (5*(a + 2*b)*(a^2 + 16*a*b + 16*b^2)*x)/(16*a^6) - (5*Sqrt[b]*Sqrt[a + b]*(
a + 4*b)*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*f)
- ((33*a^2 + 110*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b
+ b*Tan[e + f*x]^2)^2) + ((9*a + 10*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2
*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(
a + b + b*Tan[e + f*x]^2)^2) - (5*b*(9*a^2 + 32*a*b + 24*b^2)*Tan[e + f*x])
/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (5*b*(5*a^2 + 20*a*b + 16*b^2)*T
an[e + f*x])/(16*a^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.504943, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4132, 470, 578, 527, 522, 203, 205}

$$-\frac{5b\left(5a^2+20ab+16b^2\right)\tan(e+fx)}{16a^5f\left(a+b\tan^2(e+fx)+b\right)}-\frac{5b\left(9a^2+32ab+24b^2\right)\tan(e+fx)}{48a^4f\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\left(33a^2+110ab+80b^2\right)\sin(e+fx)\cos(e+}{48a^3f\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] (5*(a + 2*b)*(a^2 + 16*a*b + 16*b^2)*x)/(16*a^6) - (5*Sqrt[b]*Sqrt[a + b]*(
a + 4*b)*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*f)
- ((33*a^2 + 110*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b
+ b*Tan[e + f*x]^2)^2) + ((9*a + 10*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2
*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(
a + b + b*Tan[e + f*x]^2)^2) - (5*b*(9*a^2 + 32*a*b + 24*b^2)*Tan[e + f*x])
/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (5*b*(5*a^2 + 20*a*b + 16*b^2)*T
an[e + f*x])/(16*a^5*f*(a + b + b*Tan[e + f*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(-6a-7b)x^2\right)}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{(a+b)(9a+1}{(1}\right.}{}$$

$$= -\frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{}{6}$$

$$= -\frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{}{6}$$

$$= -\frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{}{6}$$

$$= -\frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{}{6}$$

$$= \frac{5(a+2b)\left(a^2+16ab+16b^2\right)x}{16a^6} - \frac{5\sqrt{b}\sqrt{a+b}(a+4b)(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f} - \frac{\left(33a^2\right.}{}$$

Mathematica [C] time = 19.8016, size = 1639, normalized size = 5.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(65536*b^(5/2)*f*(a + b*Sec[
e + f*x]^2)^3) - (15*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-6*a
^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[
2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*S
in[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^
4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*
a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x)] + (3*a^4 - 64*a^2*b^2 - 128*a
*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2
*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2
)))/(262144*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + (3*(a + 2*b + a*Cos
[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2*b)*x - (3*(a^6 - 8*a^5*b + 12
0*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*b^6)*ArcTan[(Se
c[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b^
2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*(a^4 + 32*a^3*b + 160
*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (256*a*Sin[2*(e + f*x)]
)/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^2 + 2624*a^2*b^3 + 3200*a*b^4 + 128
0*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24*a^5*b - 920*a^4*b^2 - 4864*a^3*b^3
 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2
*b + a*Cos[2*(e + f*x)]))))/(65536*a^4*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*
b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-6144*(7*a^3 + 54*a^2*b + 120*a*b
^2 + 80*b^3)*x - (3*(3*a^8 - 64*a^7*b + 2240*a^6*b^2 + 53760*a^5*b^3 + 3136
00*a^4*b^4 + 802816*a^3*b^5 + 1032192*a^2*b^6 + 655360*a*b^7 + 163840*b^8)*
ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e
 + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[
2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (12*(a^6 + 72*
a^5*b + 840*a^4*b^2 + 3584*a^3*b^3 + 6912*a^2*b^4 + 6144*a*b^5 + 2048*b^6)*
Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos
[2*(e + f*x)])^2) + (1152*a*(7*a^2 + 32*a*b + 32*b^2)*((-I)*Cos[2*(e + f*x)
] + Sin[2*(e + f*x)]))/f + (1152*a*(7*a^2 + 32*a*b + 32*b^2)*(I*Cos[2*(e +
f*x)] + Sin[2*(e + f*x)]))/f + (192*a^2*(a + 2*b)*((-6*I)*Cos[4*(e + f*x)]
- 6*Sin[4*(e + f*x)]))/f + ((1152*I)*a^2*(a + 2*b)*(Cos[4*(e + f*x)] + I*Si
```

```
n[4*(e + f*x)]))/f + (256*a^3*Sin[6*(e + f*x)])/f + (3*(3*a*(-a^7 + 22*a^6*
b + 1352*a^5*b^2 + 11312*a^4*b^3 + 37120*a^3*b^4 + 57856*a^2*b^5 + 43008*a*
b^6 + 12288*b^7)*Sec[2*e]*Sin[2*f*x] + (3*a^8 - 64*a^7*b - 4480*a^6*b^2 - 4
5696*a^5*b^3 - 196928*a^4*b^4 - 438272*a^3*b^5 - 528384*a^2*b^6 - 327680*a*
b^7 - 81920*b^8)*Tan[2*e]))/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])
)))/(393216*a^6*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.126, size = 689, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -27/8/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b-3/f/a^5/(tan(f*x+e)^2+1)^3*ta
n(f*x+e)^5*b^2-11/16/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5-6/f/a^4/(tan(f*x
+e)^2+1)^3*tan(f*x+e)^3*b-6/f/a^5/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b^2-5/6/f
/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-5/16/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+
e)-21/8/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b-3/f/a^5/(tan(f*x+e)^2+1)^3*ta
n(f*x+e)*b^2+45/8/f/a^4*arctan(tan(f*x+e))*b+15/f/a^5*arctan(tan(f*x+e))*b^
2+10/f/a^6*arctan(tan(f*x+e))*b^3+5/16/f/a^3*arctan(tan(f*x+e))-7/8/f*b^2/a
^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-23/8/f*b^3/a^4/(a+b+b*tan(f*x+e)^2)^
2*tan(f*x+e)^3-9/8*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-17/4/f*b^2/a^3
/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-41/8/f*b^3/a^4/(a+b+b*tan(f*x+e)^2)^2*ta
n(f*x+e)-15/8/f*b/a^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-
95/8/f*b^2/a^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-20/f*b^
3/a^5/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-2/f*b^4/a^5/(a+b
+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-2/f*b^4/a^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+
e)-10/f*b^4/a^6/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.88904, size = 2167, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(30*(a^5 + 18*a^4*b + 48*a^3*b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 6
0*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3 + 32*a*b^4)*f*x*cos(f*x + e)^2 + 30*(a^3
*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2
*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*
b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x +
 e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 2*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5
+ 10*a^4*b)*cos(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x + e)
^5 + 20*(6*a^4*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5*a^3*b^2
+ 20*a^2*b^3 + 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(f*x + e)^4
+ 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*f), 1/48*(15*(a^5 + 18*a^4*b + 48*a^3*
b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 30*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3
+ 32*a*b^4)*f*x*cos(f*x + e)^2 + 15*(a^3*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b
^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 +
16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqr
t(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos
(f*x + e)*sin(f*x + e))) - (8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 10*a^4*b)*co
s(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x + e)^5 + 20*(6*a^4
*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5*a^3*b^2 + 20*a^2*b^3 +
 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*co
s(f*x + e)^2 + a^6*b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32462, size = 502, normalized size = 1.6

$$\frac{\frac{15\left(a^{3}+18a^{2}b+48ab^{2}+32b^{3}\right)(fx+e)}{a^{6}}-\frac{30\left(3a^{3}b+19a^{2}b^{2}+32ab^{3}+16b^{4}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^{2}}}\right)\right)}{\sqrt{ab+b^{2}}a^{6}}-\frac{6\left(7a^{2}b^{2}\tan(fx+e)^{3}+23ab^{3}\tan(fx+e\cdots\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(15*(a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*(f*x + e)/a^6 - 30*(3*a^3*b +
 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^6) - 6*(7*a^2*b^2*t
an(f*x + e)^3 + 23*a*b^3*tan(f*x + e)^3 + 16*b^4*tan(f*x + e)^3 + 9*a^3*b*t
an(f*x + e) + 34*a^2*b^2*tan(f*x + e) + 41*a*b^3*tan(f*x + e) + 16*b^4*tan(
f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^5) - (33*a^2*tan(f*x + e)^5 + 162
*a*b*tan(f*x + e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 288*
a*b*tan(f*x + e)^3 + 288*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 126*a*b
*tan(f*x + e) + 144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

3.61 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=238

$$-\frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f\sqrt{a+b}}+\frac{3x\left(a^2+12ab+16b^2\right)}{8a^5}-\frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b\tan^2(e+fx)+b\right)}-\frac{b(7a+1}{8a^3f\left(a+b\right.}$$

```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*f) - ((5*a
 + 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) +
 (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(7
*a + 12*b)*Tan[e + f*x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*(a +
 2*b)*Tan[e + f*x])/(2*a^4*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.340288, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 470, 527, 522, 203, 205}

$$-\frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f\sqrt{a+b}}+\frac{3x\left(a^2+12ab+16b^2\right)}{8a^5}-\frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b\tan^2(e+fx)+b\right)}-\frac{b(7a+1}{8a^3f\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] (3*(a^2 + 12*a*b + 16*b^2)*x)/(8*a^5) - (3*Sqrt[b]*(5*a^2 + 20*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*Sqrt[a + b]*f) - ((5*a
 + 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) +
 (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(7
*a + 12*b)*Tan[e + f*x])/(8*a^3*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*(a +
 2*b)*Tan[e + f*x])/(2*a^4*f*(a + b + b*Tan[e + f*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{a+b+(-4a-7b)x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+}{(1+x^2}\right.}{}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+12b)\tan}{8a^3f\left(a+b+b\tan\right.}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+12b)\tan}{8a^3f\left(a+b+b\tan\right.}$$

$$= -\frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+12b)\tan}{8a^3f\left(a+b+b\tan\right.}$$

$$= \frac{3\left(a^2+12ab+16b^2\right)x}{8a^5} - \frac{3\sqrt{b}\left(5a^2+20ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}f} - \frac{(5a+8b)\cos(e}{8a^2f\left(a+b+\right.}$$

Mathematica [C] time = 25.8581, size = 2469, normalized size = 10.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(16384*b^(5/2)*f*(a + b*Sec[
e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a
 + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b
```

]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(
e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))
/(16384*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - (3*(a + 2*b + a*Cos[2*e + 2*f
*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 64
0*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Si
n[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]
*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Se
c[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*
(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 2
56*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 51
2*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a
^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2
*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*
Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e
] + 2880*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1
536*b^6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin
[2*f*x] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Si
n[2*f*x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4
*b^2*Sin[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[
2*(e + 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a
^4*b^2*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[
4*e + 2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^
2))/(65536*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Co
s[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Si
n[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(
Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^
3 + 64*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e
+ 2*f*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (
9*a^5 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e]
)/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^2*(a + b)^2*f*(a + b*Sec
[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(
a + 2*b)*x - (3*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4
+ 3072*a*b^5 + 1024*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(Cos[2*e] - I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + (4*(a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]
*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e +
f*x)])^2) + (256*a*Sin[2*(e + f*x)])/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^
2 + 2624*a^2*b^3 + 3200*a*b^4 + 1280*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24
*a^5*b - 920*a^4*b^2 - 4864*a^3*b^3 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6
)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))))/(16384*a^4*(
a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
(768(7*a^2 + 32*a*b + 32*b^2)*x + (3*(a^7 - 14*a^6*b + 336*a^5*b^2 + 5600
*a^4*b^3 + 22400*a^3*b^4 + 37632*a^2*b^5 + 28672*a*b^6 + 8192*b^7)*ArcTan[(

```
Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x])
)/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(
b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*(a^5 + 50*a^4*b + 4
00*a^3*b^2 + 1120*a^2*b^3 + 1280*a*b^4 + 512*b^5)*Sec[2*e]*((a + 2*b)*Sin[2
*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) - ((768
*I)*a*(a + 2*b)*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)]))/f + ((768*I)*a*(a
+ 2*b)*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/f + (128*a^2*Sin[4*(e + f*x
)])/f + (a*(3*a^6 - 44*a^5*b - 1900*a^4*b^2 - 10880*a^3*b^3 - 23360*a^2*b^4
 - 21504*a*b^5 - 7168*b^6)*Sec[2*e]*Sin[2*f*x] + (-3*a^7 + 42*a^6*b + 2192*
a^5*b^2 + 16480*a^4*b^3 + 51200*a^3*b^4 + 77824*a^2*b^5 + 57344*a*b^6 + 163
84*b^7)*Tan[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])))))/(32768
*a^5*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.117, size = 423, normalized size = 1.8

$$-\frac{3\left(\tan\left(fx+e\right)\right)^{3}b}{2\,fa^{4}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{5\left(\tan\left(fx+e\right)\right)^{3}}{8\,fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{3\tan\left(fx+e\right)}{8\,fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{3\tan\left(fx+e\right)b}{2\,fa^{4}\left(\left(\tan\left(fx+e\right)\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -3/2/f/a^4/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3*b-5/8/f/a^3/(tan(f*x+e)^2+1)^2*t
an(f*x+e)^3-3/8/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e)-3/2/f/a^4/(tan(f*x+e)^2
+1)^2*tan(f*x+e)*b+9/2/f/a^4*arctan(tan(f*x+e))*b+6/f/a^5*arctan(tan(f*x+e)
)*b^2+3/8/f/a^3*arctan(tan(f*x+e))-7/8/f*b^2/a^3/(a+b+b*tan(f*x+e)^2)^2*tan
(f*x+e)^3-3/2/f*b^3/a^4/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8*b*tan(f*x+e
)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-21/8/f*b^2/a^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*
x+e)-3/2/f*b^3/a^4/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f*b/a^3/((a+b)*b)
^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-15/2/f*b^2/a^4/((a+b)*b)^(1/2)*
arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-6/f*b^3/a^5/((a+b)*b)^(1/2)*arctan(tan
(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.796025, size = 1859, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 24*(a^3*b + 12
*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 12*a*b^3 + 16*b^4)*
f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^2 + 20*a*
b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-b/
(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(
f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x
+ e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(
f*x + e)^2 + b^2)) + 4*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b)*cos(f*x +
e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2*a*b^3)*cos(
f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 +
a^5*b^2*f), 1/16*(6*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 12*(
a^3*b + 12*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2 + 12*a*b^3 +
 16*b^4)*f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^
2 + 20*a*b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b
)*cos(f*x + e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2
*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f
*x + e)^2 + a^5*b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36882, size = 439, normalized size = 1.84

$$\frac{3\left(a^2+12\,ab+16\,b^2\right)\left(fx+e\right)}{a^5}-\frac{3\left(5\,a^2b+20\,ab^2+16\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5}-\frac{12\,ab^2\tan\left(fx+e\right)^7+24\,b^3\tan\left(fx+e\right)^7+19\,a^2b\tan\left(fx+e\right)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 - 3*(5*a^2*b + 20*a*b^2 + 16*b
^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
 b^2)))/(sqrt(a*b + b^2)*a^5) - (12*a*b^2*tan(f*x + e)^7 + 24*b^3*tan(f*x +
 e)^7 + 19*a^2*b*tan(f*x + e)^5 + 72*a*b^2*tan(f*x + e)^5 + 72*b^3*tan(f*x
+ e)^5 + 5*a^3*tan(f*x + e)^3 + 46*a^2*b*tan(f*x + e)^3 + 108*a*b^2*tan(f*x
 + e)^3 + 72*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 27*a^2*b*tan(f*x + e
) + 48*a*b^2*tan(f*x + e) + 24*b^3*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan
(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2*a^4))/f
```

3.62 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal. Leaf size=184

$$-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{3/2}}-\frac{b(11a+12b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3b\tan(e+fx)}{4a^2f\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] ((a + 6*b)*x)/(2*a^4) - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*f) - (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*a^2*f*
(a + b + b*Tan[e + f*x]^2)^2) - (b*(11*a + 12*b)*Tan[e + f*x])/(8*a^3*(a +
b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.278306, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 471, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{3/2}}-\frac{b(11a+12b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{3b\tan(e+fx)}{4a^2f\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 6*b)*x)/(2*a^4) - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*f) - (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) - (3*b*Tan[e + f*x])/(4*a^2*f*
(a + b + b*Tan[e + f*x]^2)^2) - (b*(11*a + 12*b)*Tan[e + f*x])/(8*a^3*(a +
b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b-5bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2(a+b)(2a+3b)-}{(1+x^2)(a+b}\right.}{8a}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+12b)\tan}{8a^3(a+b)f\left(a+b+b\right.}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+12b)\tan}{8a^3(a+b)f\left(a+b+b\right.}$$

$$= \frac{(a+6b)x}{2a^4} - \frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2}$$

Mathematica [C] time = 19.2405, size = 1915, normalized size = 10.41

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/
((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^(5/2)*f*(a + b*Sec[e
 + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a
+ 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]
*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e
 + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/
(2048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])
^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*
```

```
b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*
x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Co
s[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*
e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a +
 b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a
^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^
4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b
^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x
*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[
6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] +
2880*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*
b^6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f
*x] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*
f*x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2
*Sin[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e
 + 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b
^2*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e
+ 2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/
(4096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e
 + 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e
])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 6
4*b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f
*x)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5
 + 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^
2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(4096*b^2*(a + b)^2*f*(a + b*Sec[e +
f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2
*b)*x - (3*(a^6 - 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 307
2*a*b^5 + 1024*b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
]*(Cos[2*e] - I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + (4*(a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a
+ 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]
)^2) + (256*a*Sin[2*(e + f*x)])/f + (a*(-3*a^5 + 26*a^4*b + 736*a^3*b^2 + 2
624*a^2*b^3 + 3200*a*b^4 + 1280*b^5)*Sec[2*e]*Sin[2*f*x] + (3*a^6 - 24*a^5*
b - 920*a^4*b^2 - 4864*a^3*b^3 - 10112*a^2*b^4 - 9216*a*b^5 - 3072*b^6)*Tan
[2*e])/(b^2*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])))))/(8192*a^4*(a + b*
Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.101, size = 314, normalized size = 1.7

$$-\frac{\tan\left(fx+e\right)}{2fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{2fa^{3}}+3\frac{\arctan\left(\tan\left(fx+e\right)\right)b}{fa^{4}}-\frac{7b^{2}\left(\tan\left(fx+e\right)\right)^{3}}{8fa^{2}\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/2/f/a^3*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2/f/a^3*arctan(tan(f*x+e))+3/f/a^4
*arctan(tan(f*x+e))*b-7/8/f/a^2*b^2/(a+b+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)
^3-1/f/a^3*b^3/(a+b+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)^3-9/8*b*tan(f*x+e)/a
^2/f/(a+b+b*tan(f*x+e)^2)^2-1/f*b^2/a^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-1
5/8/f/a^2*b/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-5/f/
a^3*b^2/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-3/f/a^4*
b^3/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.753736, size = 1872, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b + 7*a^
2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 7*a*b^3 + 6*b^4)*f*x +
((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3 +
```

```
24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)) - 4*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^3*b + 18*a^2*b^2)
*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7
 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^
2 + a^4*b^3)*f), 1/16*(8*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 1
6*(a^3*b + 7*a^2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2 + 7*a*b^3 +
 6*b^4)*f*x + ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 40*a*b^3 + 24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^
3*b + 18*a^2*b^2)*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*si
n(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x
 + e)^2 + (a^5*b^2 + a^4*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29612, size = 296, normalized size = 1.61

$$-\frac{\frac{\left(15\,a^2b+40\,ab^2+24\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+a^4b\right)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan\left(fx+e\right)^3+8\,b^3\tan\left(fx+e\right)^3+9\,a^2b\tan\left(fx+e\right)+17\,ab^2\tan\left(fx+e\right)+8\,b^3\tan\left(fx\right.}{\left(a^4+a^3b\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) +
```

```
(7*a*b^2*tan(f*x + e)^3 + 8*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) + 17*
a*b^2*tan(f*x + e) + 8*b^3*tan(f*x + e))/((a^4 + a^3*b)*(b*tan(f*x + e)^2 +
 a + b)^2) - 4*(f*x + e)*(a + 6*b)/a^4 + 4*tan(f*x + e)/((tan(f*x + e)^2 +
1)*a^3))/f
```

3.63 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e\right.}$$

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.188422, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```


```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\mathrm{Subst}}{}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\mathrm{Subst}}{}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}$$

Mathematica [C] time = 5.87615, size = 332, normalized size = 2.31

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b\left(\left(9a^2+28ab+16b^2\right)\sin(2e)-3a(3a+2b)\sin(2fx)\right)(a\cos(2(e+fx))+a+2b)}{f(a+b)^2(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + \frac{b\left(15a^2+20ab+8b^2\right)(\cos(2e)}{}\right.}{64a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.083, size = 321, normalized size = 2.2

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f a^{3}}-\frac{7 b^{2} \left(\tan\left(fx+e\right)\right)^{3}}{8 f a \left(a+b+b \left(\tan\left(fx+e\right)\right)^{2}\right)^{2} \left(a^{2}+2 a b+b^{2}\right)}-\frac{b^{3} \left(\tan\left(fx+e\right)\right)^{3}}{2 f a^{2} \left(a+b+b \left(\tan\left(fx+e\right)\right)^{2}\right)^{2} \left(a^{2}+2 a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f/a^3*arctan(tan(f*x+e))-7/8/f/a*b^2/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^
2)*tan(f*x+e)^3-1/2/f/a^2*b^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*
x+e)^3-9/8*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/2/f/a^2*b^2/(a+b
+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)-15/8/f/a*b/(a^2+2*a*b+b^2)/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-5/2/f/a^2*b^2/(a^2+2*a*b+b^2)/((a+
b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/a^3*b^3/(a^2+2*a*b+b^2
)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.712553, size = 1854, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
```

```
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13302, size = 277, normalized size = 1.92

$$-\frac{\frac{\left(15\,a^{2}b+20\,ab^{2}+8\,b^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\sqrt{ab+b^{2}}}+\frac{7\,ab^{2}\tan(fx+e)^{3}+4\,b^{3}\tan(fx+e)^{3}+9\,a^{2}b\tan(fx+e)+13\,ab^{2}\tan(fx+e)+4\,b^{3}\tan(fx+}{\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)\left(b\tan(fx+e)^{2}+a+b\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

3.64 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=124

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}-\frac{15\cot(e+fx)}{8f(a+b)^3}+\frac{5\cot(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\cot(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right.}$$

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*f
) - (15*Cot[e + f*x])/(8*(a + b)^3*f) + Cot[e + f*x]/(4*(a + b)*f*(a + b +
b*Tan[e + f*x]^2)^2) + (5*Cot[e + f*x])/(8*(a + b)^2*f*(a + b + b*Tan[e + f
*x]^2))
```

Rubi [A] time = 0.111021, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 290, 325, 205}

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}-\frac{15\cot(e+fx)}{8f(a+b)^3}+\frac{5\cot(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\cot(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a + b)^(7/2)*f
) - (15*Cot[e + f*x])/(8*(a + b)^3*f) + Cot[e + f*x]/(4*(a + b)*f*(a + b +
b*Tan[e + f*x]^2)^2) + (5*Cot[e + f*x])/(8*(a + b)^2*f*(a + b + b*Tan[e + f
*x]^2))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{15\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2f\left(a+b+b\tan^2(e+f\ldots\right.}$$

$$= -\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{15\cot(e+fx)}{8(a+b)^3f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\ldots}{8(a\ldots}$$

Mathematica [C] time = 6.79444, size = 987, normalized size = 7.96

$$\frac{(\cos(2e+2fx)a+a+2b)^3\left(\frac{15b\tan^{-1}\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right)(-a\sin(fx)-2b\sin(fx)+a\sin(2e+fx))\right)\cos(2e)}{64\sqrt{a+b}f\sqrt{b\cos(4e)-ib\sin(4e)}}\right.}{(a+b)^3\left(b\sec^2(e+f\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((15*b*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Cos[2*e])/(64*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin
[4*e]]) - (((15*I)/64)*b*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Co
s[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I
*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])
/(Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^3*(a + b*Sec[e
+ f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[e + f*x]*Sec[2*e]
*Sec[e + f*x]^6*(-32*a^4*Sin[f*x] - 64*a^3*b*Sin[f*x] + 22*a^2*b^2*Sin[f*x]
 + 80*a*b^3*Sin[f*x] + 16*b^4*Sin[f*x] + 32*a^4*Sin[3*f*x] + 46*a^3*b*Sin[3
*f*x] - 54*a^2*b^2*Sin[3*f*x] - 8*a*b^3*Sin[3*f*x] - 48*a^4*Sin[2*e - f*x]
- 128*a^3*b*Sin[2*e - f*x] - 106*a^2*b^2*Sin[2*e - f*x] + 80*a*b^3*Sin[2*e
- f*x] + 16*b^4*Sin[2*e - f*x] + 48*a^4*Sin[2*e + f*x] + 146*a^3*b*Sin[2*e
+ f*x] + 182*a^2*b^2*Sin[2*e + f*x] + 80*a*b^3*Sin[2*e + f*x] + 16*b^4*Sin[
2*e + f*x] - 32*a^4*Sin[4*e + f*x] - 82*a^3*b*Sin[4*e + f*x] - 54*a^2*b^2*S
in[4*e + f*x] - 80*a*b^3*Sin[4*e + f*x] - 16*b^4*Sin[4*e + f*x] - 8*a^4*Sin
[2*e + 3*f*x] + 18*a^3*b*Sin[2*e + 3*f*x] + 54*a^2*b^2*Sin[2*e + 3*f*x] + 8
*a*b^3*Sin[2*e + 3*f*x] + 32*a^4*Sin[4*e + 3*f*x] + 73*a^3*b*Sin[4*e + 3*f*
x] + 24*a^2*b^2*Sin[4*e + 3*f*x] + 8*a*b^3*Sin[4*e + 3*f*x] - 8*a^4*Sin[6*e
 + 3*f*x] - 9*a^3*b*Sin[6*e + 3*f*x] - 24*a^2*b^2*Sin[6*e + 3*f*x] - 8*a*b^
3*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x] - 9*a^3*b*Sin[2*e + 5*f*x] - 2*
a^2*b^2*Sin[2*e + 5*f*x] + 9*a^3*b*Sin[4*e + 5*f*x] + 2*a^2*b^2*Sin[4*e + 5
*f*x] + 8*a^4*Sin[6*e + 5*f*x]))/(512*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2
)^3)
```

Maple [A] time = 0.104, size = 157, normalized size = 1.3

$$-\frac{1}{f(a+b)^3\tan(fx+e)}-\frac{7b^2(\tan(fx+e))^3}{8f(a+b)^3\left(a+b+b(\tan(fx+e))^2\right)^2}-\frac{9ab\tan(fx+e)}{8f(a+b)^3\left(a+b+b(\tan(fx+e))^2\right)^2}-\frac{}{8f(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/f/(a+b)^3/tan(f*x+e)-7/8/f/(a+b)^3*b^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)
^3-9/8*a*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^2-9/8/f/(a+b)^3*b^2/(a
+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/(a+b)^3*b/((a+b)*b)^(1/2)*arctan(tan
(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.701863, size = 1430, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 20*(5*a*b - b^2)*cos(f*x
 + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/(a +
 b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
+ e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e)
)*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b + 3*a^3
*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4
)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x + e))
, -1/16*(2*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 10*(5*a*b - b^2)*cos(f*
x + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/(a +
 b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x +
 e)*sin(f*x + e)))*sin(f*x + e) + 30*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b + 3
```

```
*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a
*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f*x +
 e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28838, size = 248, normalized size = 2.

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab+b^2}}+\frac{7b^2\tan(fx+e)^3+9ab\tan(fx+e)+9b^2\tan(fx+e)}{(a^3+3a^2b+3ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}+\frac{8}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (7*b^2*t
an(f*x + e)^3 + 9*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e))/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2) + 8/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*tan(f*x + e)))/f
```

3.65 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{5\sqrt{b}(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}-\frac{b(7a-4b)\tan(e+fx)}{8f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{ab\tan(e+fx)}{4f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\cot^3(}{3f(}$$

```
[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a +
 b)^(9/2)*f) - ((a - 2*b)*Cot[e + f*x])/((a + b)^4*f) - Cot[e + f*x]^3/(3*(
a + b)^3*f) - (a*b*Tan[e + f*x])/(4*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^
2) - ((7*a - 4*b)*b*Tan[e + f*x])/(8*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)
)
```

Rubi [A] time = 0.25045, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 456, 1259, 1261, 205}

$$-\frac{5\sqrt{b}(3a-4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}-\frac{b(7a-4b)\tan(e+fx)}{8f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}-\frac{ab\tan(e+fx)}{4f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\cot^3(}{3f(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] (-5*(3*a - 4*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*(a +
 b)^(9/2)*f) - ((a - 2*b)*Cot[e + f*x])/((a + b)^4*f) - Cot[e + f*x]^3/(3*(
a + b)^3*f) - (a*b*Tan[e + f*x])/(4*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^
2) - ((7*a - 4*b)*b*Tan[e + f*x])/(8*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)
)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\left(\int \frac{-\frac{4}{b(a+b)}-\frac{4ax^2}{b(a+b)^2}+\frac{3ax^4}{(a+b)^3}}{x^4\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int\left(\right.\right.}{}$$

$$= -\frac{(a-2b)\cot(e+fx)}{(a+b)^4 f} - \frac{\cot^3(e+fx)}{3(a+b)^3 f} - \frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a}{8(a+b)^4 f}$$

$$= -\frac{5(3a-4b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2} f} - \frac{(a-2b)\cot(e+fx)}{(a+b)^4 f} - \frac{\cot^3(e+fx)}{3(a+b)^3 f} - \frac{ab}{4(a+b)^3 f\left(a\right.}$$

Mathematica [C] time = 4.42902, size = 994, normalized size = 6.06

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^6(e+fx)\left(\frac{480(3a-4b)b\tan^{-1}\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(a\sin(2e+fx)-(a+2b)\sin(fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2(e+fx))a+a+2b)^2(\cos(2e)-i\sin}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((480*(3*a - 4*b)*b*ArcTan[(
Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x])
)/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*
x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

```
 - (Csc[e]*Csc[e + f*x]^3*Sec[2*e]*(4*(44*a^4 + 122*a^3*b + 63*a^2*b^2 + 12
6*a*b^3 + 36*b^4)*Sin[f*x] + (-96*a^4 - 71*a^3*b + 344*a^2*b^2 - 1208*a*b^3
 + 48*b^4)*Sin[3*f*x] + 224*a^4*Sin[2*e - f*x] + 576*a^3*b*Sin[2*e - f*x] +
 124*a^2*b^2*Sin[2*e - f*x] - 2184*a*b^3*Sin[2*e - f*x] + 144*b^4*Sin[2*e -
 f*x] - 224*a^4*Sin[2*e + f*x] - 657*a^3*b*Sin[2*e + f*x] - 538*a^2*b^2*Sin
[2*e + f*x] + 984*a*b^3*Sin[2*e + f*x] + 144*b^4*Sin[2*e + f*x] + 176*a^4*S
in[4*e + f*x] + 569*a^3*b*Sin[4*e + f*x] + 666*a^2*b^2*Sin[4*e + f*x] + 170
4*a*b^3*Sin[4*e + f*x] - 144*b^4*Sin[4*e + f*x] + 48*a^4*Sin[2*e + 3*f*x] +
 111*a^3*b*Sin[2*e + 3*f*x] + 360*a^2*b^2*Sin[2*e + 3*f*x] + 312*a*b^3*Sin[
2*e + 3*f*x] - 48*b^4*Sin[2*e + 3*f*x] - 96*a^4*Sin[4*e + 3*f*x] - 152*a^3*
b*Sin[4*e + 3*f*x] + 146*a^2*b^2*Sin[4*e + 3*f*x] - 728*a*b^3*Sin[4*e + 3*f
*x] - 48*b^4*Sin[4*e + 3*f*x] + 48*a^4*Sin[6*e + 3*f*x] + 192*a^3*b*Sin[6*e
 + 3*f*x] + 558*a^2*b^2*Sin[6*e + 3*f*x] - 168*a*b^3*Sin[6*e + 3*f*x] + 48*
b^4*Sin[6*e + 3*f*x] + 16*a^4*Sin[2*e + 5*f*x] - 598*a^2*b^2*Sin[2*e + 5*f*
x] + 48*a*b^3*Sin[2*e + 5*f*x] + 72*a^3*b*Sin[4*e + 5*f*x] + 150*a^2*b^2*Si
n[4*e + 5*f*x] - 48*a*b^3*Sin[4*e + 5*f*x] + 16*a^4*Sin[6*e + 5*f*x] + 27*a
^3*b*Sin[6*e + 5*f*x] - 388*a^2*b^2*Sin[6*e + 5*f*x] + 45*a^3*b*Sin[8*e + 5
*f*x] - 60*a^2*b^2*Sin[8*e + 5*f*x] + 16*a^4*Sin[4*e + 7*f*x] - 83*a^3*b*Si
n[4*e + 7*f*x] + 6*a^2*b^2*Sin[4*e + 7*f*x] + 27*a^3*b*Sin[6*e + 7*f*x] - 6
*a^2*b^2*Sin[6*e + 7*f*x] + 16*a^4*Sin[8*e + 7*f*x] - 56*a^3*b*Sin[8*e + 7*
f*x]))/a))/(6144*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.122, size = 306, normalized size = 1.9

$$-\frac{1}{3f(a+b)^3(\tan(fx+e))^3}-\frac{a}{f(a+b)^4\tan(fx+e)}+2\frac{b}{f(a+b)^4\tan(fx+e)}-\frac{7b^2(\tan(fx+e))^3a}{8f(a+b)^4(a+b+b(\tan(fx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/3/f/(a+b)^3/tan(f*x+e)^3-1/f/(a+b)^4/tan(f*x+e)*a+2/f/(a+b)^4/tan(f*x+e)
*b-7/8/f/(a+b)^4*b^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a+1/2/f/(a+b)^4*b^
3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-9/8/f/(a+b)^4*b/(a+b+b*tan(f*x+e)^2)^
2*tan(f*x+e)*a^2-5/8/f/(a+b)^4*b^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)*a+1/2/
f/(a+b)^4*b^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-15/8/f/(a+b)^4*b/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a+5/2/f/(a+b)^4*b^2/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.787752, size = 2288, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(24*a^3 - 134*a^
2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 32*a*b^2 + 16*b^3
)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (3*a^3 - 10*a^2*b
 + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b - 11*a*b^2 + 4*b^3)
*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)
^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 60*(3*a*b^2 - 4*
b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*co
s(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5
)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5
 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6
)*f)*sin(f*x + e)), -1/48*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 -
 2*(24*a^3 - 134*a^2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 10*(15*a^2*b
- 32*a*b^2 + 16*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6
- (3*a^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b
- 11*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*c
os(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x +
 e) - 30*(3*a*b^2 - 4*b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a
^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3
 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3
 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a
^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31419, size = 371, normalized size = 2.26

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3ab-4b^2)}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\sqrt{ab+b^2}}+\frac{3\left(7ab^2\tan(fx+e)^3-4b^3\tan(fx+e)^3+9a^2b\tan(fx+e)+5ab^2\tan(fx+e)-4b^3\tan(fx+e)\right)}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}+}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*(3*a*b - 4*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*
sqrt(a*b + b^2)) + 3*(7*a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f*x + e)^3 + 9*a^2
*b*tan(f*x + e) + 5*a*b^2*tan(f*x + e) - 4*b^3*tan(f*x + e))/((a^4 + 4*a^3*
b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(3*a*tan(f
*x + e)^2 - 6*b*tan(f*x + e)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*tan(f*x + e)^3))/f
```

3.66 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{b}\left(15a^2-40ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{11/2}}-\frac{b\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{40f(a+b)^5\left(a+b\tan^2(e+fx)+b\right)}-\frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{20f(a+b)^4\left(a+b\tan^2(e+fx)+\cdots\right.}$$

```
[Out] -(Sqrt[b]*(15*a^2 - 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b]])/(8*(a + b)^(11/2)*f) - ((5*a^2 - 20*a*b + 2*b^2)*Cot[e + f*x])/(5*(a +
 b)^5*f) - ((10*a + b)*Cot[e + f*x]^3)/(15*(a + b)^4*f) - Cot[e + f*x]^5/(5
*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Tan[e + f*x])
/(20*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(35*a^2 - 40*a*b + 24*b
^2)*Tan[e + f*x])/(40*(a + b)^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.369588, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4132, 462, 456, 1259, 1261, 205}

$$-\frac{\sqrt{b}\left(15a^2-40ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8f(a+b)^{11/2}}-\frac{b\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{40f(a+b)^5\left(a+b\tan^2(e+fx)+b\right)}-\frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{20f(a+b)^4\left(a+b\tan^2(e+fx)+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b]])/(8*(a + b)^(11/2)*f) - ((5*a^2 - 20*a*b + 2*b^2)*Cot[e + f*x])/(5*(a +
 b)^5*f) - ((10*a + b)*Cot[e + f*x]^3)/(15*(a + b)^4*f) - Cot[e + f*x]^5/(5
*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(5*a^2 + 4*b^2)*Tan[e + f*x])
/(20*(a + b)^4*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(35*a^2 - 40*a*b + 24*b
^2)*Tan[e + f*x])/(40*(a + b)^5*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{10a+b+5(a+b)x^2}{x^4\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{20(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\,\text{Subst}\Big(}{}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{20(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\left(35a^2-\right.}{40(a+b)^5}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{20(a+b)^4f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b\left(35a^2-\right.}{40(a+b)^5}$$

$$= -\frac{\left(5a^2-20ab+2b^2\right)\cot(e+fx)}{5(a+b)^5f} - \frac{(10a+b)\cot^3(e+fx)}{15(a+b)^4f} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+\right.}$$

$$= -\frac{\sqrt{b}\left(15a^2-40ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{11/2}f} - \frac{\left(5a^2-20ab+2b^2\right)\cot(e+fx)}{5(a+b)^5f} - \frac{(10a+b}{15}$$

Mathematica [C] time = 6.06661, size = 479, normalized size = 1.98

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(8\left(8a^2-59ab+23b^2\right)\csc(e)\sin(fx)\csc(e+fx)(a\cos(2(e+fx))+a+2b)^2+1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-8*(4*a - 11*b)*(a + b)*(a
+ 2*b + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^2 - 24*(a + b)^2*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^4 + (15*b*(15*a^2 - 40*a*b + 8
*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*S
in[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a
*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4]) + 8*(8*a^2 - 59*a*b + 23*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])
^2*Csc[e]*Csc[e + f*x]*Sin[f*x] + 8*(4*a - 11*b)*(a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^3*Sin[f*x] + 24*(a + b)^2*(a + 2*b + a*C
os[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^5*Sin[f*x] - 60*b^2*(a + b)*Sec[2*e]
*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]) + 15*b*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[2*e]*((9*a^2 + 16*a*b - 8*b^2)*Sin[2*e] + 3*a*(-3*a + 2*b)*Sin[2*f*x]))
)/(960*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.136, size = 411, normalized size = 1.7

$$-\frac{1}{5\,f\,(a+b)^{3}\left(\tan\left(fx+e\right)\right)^{5}}-\frac{2\,a}{3\,f\,(a+b)^{4}\left(\tan\left(fx+e\right)\right)^{3}}+\frac{b}{3\,f\,(a+b)^{4}\left(\tan\left(fx+e\right)\right)^{3}}-\frac{a^{2}}{f\,(a+b)^{5}\tan\left(fx+e\right)}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/5/f/(a+b)^3/tan(f*x+e)^5-2/3/f/(a+b)^4/tan(f*x+e)^3*a+1/3/f/(a+b)^4/tan(
f*x+e)^3*b-1/f/(a+b)^5/tan(f*x+e)*a^2+4/f/(a+b)^5/tan(f*x+e)*a*b-1/f/(a+b)^
5/tan(f*x+e)*b^2-7/8/f/(a+b)^5*b^2/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a^2+
1/f/(a+b)^5*b^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3*a-9/8/f/(a+b)^5*b/(a+b+
b*tan(f*x+e)^2)^2*tan(f*x+e)*a^3-1/8/f/(a+b)^5*b^2/(a+b+b*tan(f*x+e)^2)^2*t
an(f*x+e)*a^2+1/f/(a+b)^5*b^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)*a-15/8/f/(a
+b)^5*b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a^2+5/f/(a+b)^
5*b^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a-1/f/(a+b)^5*b^
3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.898391, size = 3312, normalized size = 13.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*x + e)^9 - 4*(160*a^4 -
 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)^7 + 4*(120*a^4 - 1205*
a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x + e)^5 + 20*(75*a^3*b
- 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 40*a^3*b
 + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b + 48*a^2*b^2 - 8*a*b^3)
*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2 - 72*a*b^3 + 8*b^4)*cos
(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*a^3*b - 55*a^2*b^2 + 48
*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^
2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e)
 + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 6
0*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(((a^7 + 5*a^6*b + 10*a^5*b
^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b
+ 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*
b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*
x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*
cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
 b^7)*f)*sin(f*x + e)), -1/240*(2*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*
x + e)^9 - 2*(160*a^4 - 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)
^7 + 2*(120*a^4 - 1205*a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x
 + e)^5 + 10*(75*a^3*b - 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 -
 15*((15*a^4 - 40*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b
+ 48*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2
- 72*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*
a^3*b - 55*a^2*b^2 + 48*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arct
an(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f
*x + e)))*sin(f*x + e) + 30*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(
((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x
+ e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*co
s(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b
```

```
^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^
4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29214, size = 516, normalized size = 2.13

$$-\frac{\frac{15\left(15\,a^2b-40\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5\right)\sqrt{ab+b^2}}+\frac{15\left(7\,a^2b^2\tan\left(fx+e\right)^3-8\,ab^3\tan\left(fx+e\right)^3+9\,a^3b\tan\left(fx+e\right)+a^2b^2\tan\left(fx+e\right)-8\,ab^3\,\mathrm{t}\right.}{\left(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 +
 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b + b^2)) + 15*(7*a^2*b^2*tan(f*x + e)^
3 - 8*a*b^3*tan(f*x + e)^3 + 9*a^3*b*tan(f*x + e) + a^2*b^2*tan(f*x + e) -
8*a*b^3*tan(f*x + e))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
 b^5)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(15*a^2*tan(f*x + e)^4 - 60*a*b*tan
(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 5*a*b*tan(f*x
 + e)^2 - 5*b^2*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 1
0*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e)^5))/f
```

3.67 $\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$

Optimal. Leaf size=139

$$\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (2*(5*a + b)*Cos[e + f*x]^3*(a
+ b*Sec[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]
^2)^(3/2))/(5*a*f)
```

Rubi [A] time = 0.146946, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4134, 462, 451, 277, 217, 206}

$$\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```


```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (2*(5*a + b)*Cos[e + f*x]^3*(a
+ b*Sec[e + f*x]^2)^(3/2))/(15*a^2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]
^2)^(3/2))/(5*a*f)
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\sqrt{a+bx^2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af} + \frac{\text{Subst}\left(\int \frac{\left(-2(5a+b)+5ax^2\right)\sqrt{a+bx^2}}{x^4}\,dx, x, \text{se}\right.}{5af}$$

$$= \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx\right.}{5af}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)}{15a^2f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)}{15a^2f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3}{}$$

Mathematica [A] time = 0.889095, size = 152, normalized size = 1.09

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{2\left(a\cos^2(e+fx)+b\right)^{5/2}}{5a^2} - \frac{2(2a+b)\left(a\cos^2(e+fx)+b\right)^{3/2}}{3a^2} + 2\sqrt{a\cos^2(e+fx)+b} - 2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a\cos}}{}\right.\right.}{\sqrt{2}f\sqrt{a\cos(2e+2fx)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

```
[Out] -((Cos[e + f*x]*(-2*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]] + 2
*Sqrt[b + a*Cos[e + f*x]^2] - (2*(2*a + b)*(b + a*Cos[e + f*x]^2)^(3/2))/(3
*a^2) + (2*(b + a*Cos[e + f*x]^2)^(5/2))/(5*a^2))*Sqrt[a + b*Sec[e + f*x]^2
])/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]))
```

Maple [B] time = 0.745, size = 1840, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/30/f/b^(1/2)/(a+b)^(3/2)/a^2*(-1+cos(f*x+e))^2*(30*cos(f*x+e)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*(a+b)^(3/2)*b^(1/2)*a^2-4*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(3/2)-16*cos(f*x+e)^3*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(3/2)-24*cos(f*x+e)
^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(3/2)-15*cos(f
*x+e)*4^(1/2)*(a+b)^(3/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(
f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*a^2*b-16*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(3/2)*(a+b)^(3/2)*b^(3/2)+15*4^(1/2)*b^(5/2)*ln(-4/(a+b)^(1/2)*(-1+cos
(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/s
in(f*x+e)^2)*a^2-15*4^(1/2)*b^(5/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(
f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)
+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a
^2+10*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(1/2)*a+15*
4^(1/2)*b^(3/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3-15*4^(1/2)*b^(3/2
)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3-4*cos(f*x+e)^4*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(3/2)-15*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*4^(1/2)*(a+b)^(3/2)*b^(3/2)*a+15*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*4^(1/2)*(a+b)^(3/2)*b^(1/2)*a^2+6*cos(f*x+e)^6*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(1/2)*a+24*cos(f*x+e)^5
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(1/2)*a+16*cos(f
*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(1/2)*a-5
6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)*b^(1
/2)*a-84*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/
2)*b^(1/2)*a+15*cos(f*x+e)*4^(1/2)*b^(5/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e)
)*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+
e)^2)*a^2-15*cos(f*x+e)*4^(1/2)*b^(5/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(
cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^
2)*a^2-20*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2
)*b^(1/2)*a+15*cos(f*x+e)*4^(1/2)*b^(3/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))
*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e
)^2)*a^3-15*cos(f*x+e)*4^(1/2)*b^(3/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(c
```

```
os(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x
+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2
)*a^3-15*4^(1/2)*(a+b)^(3/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(c
os(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2))*a^2*b)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2
)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.18855, size = 734, normalized size = 5.28

$$\left[\frac{15\,a^2\sqrt{b}\log\left(\frac{a\cos\left(fx+e\right)^2+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^2}\right)-2\left(3\,a^2\cos\left(fx+e\right)^5-\left(10\,a^2-ab\right)\cos\left(fx+e\right)^3+\left(15\,a^2-10\,a\right.\right.}{30\,a^2 f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(3*a^2*cos
(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15*a^2 - 10*a*b - 2*b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f), -1/15*(15*
a^2*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e)/b) + (3*a^2*cos(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15
*a^2 - 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
```

```
e)^2))/(a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\sin\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^5, x)
```

3.68 $\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a + b*Sec[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rubi [A] time = 0.0938918, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4134, 451, 277, 217, 206}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f + (Cos[e + f*x]^3*(a + b*Sec[e +
f*x]^2)^(3/2))/(3*a*f)
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
```

```
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)\sqrt{a+bx^2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{b\,\text{S}}{}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{b\,\text{S}}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)}{}$$

Mathematica [A] time = 0.379185, size = 120, normalized size = 1.2

$$\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{a\cos^2(e+fx)+b}\left(a\cos^2(e+fx)-3a+b\right)+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)+b}}{\sqrt{b}}\right)\right)}{3af\sqrt{a\cos(2(e+fx))+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*(3*a*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[
b]] + Sqrt[b + a*Cos[e + f*x]^2]*(-3*a + b + a*Cos[e + f*x]^2))*Sqrt[a + b*
Sec[e + f*x]^2])/(3*a*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] time = 0.384, size = 1525, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/6/f/b^(1/2)/(a+b)^(3/2)/a*(-1+cos(f*x+e))^2*(-2*(a+b)^(3/2)*cos(f*x+e)^4
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)-8*(a+b)^(3/2)*cos(f*x+
e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)-6*(a+b)^(3/2)*cos(
f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)-3*4^(1/2)*cos(
f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a+3*4^(1/2)*cos(f*x+e)
*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a+6*(a+b)^(3/2)*4^(1/2)*cos(
f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*a-3*(a+b)^(3/2)*
4^(1/2)*cos(f*x+e)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*
4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2))*a*b-3*(a+b)^(3/2)*4^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(3/2)+4*(a+b)^(3/2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(3/2)*b^(1/2)-3*4^(1/2)*cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*
(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)
^2)*b^(3/2)*a^2+3*4^(1/2)*cos(f*x+e)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos
(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e
```

```
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*
b^(3/2)*a^2-3*4^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a+3*4^(
1/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a+3*(a+b)^(3/2)*4^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*a-3*(a+b)^(3/2)*4^(1/2)
*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+
e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b
+4*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)-3*4^(1/2
)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^(3/2)*a^2+3*4^(1/2)*ln(-4/(a+b)^(
1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)+b)/sin(f*x+e)^2)*b^(3/2)*a^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*
x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.12116, size = 578, normalized size = 5.78

$$\left[\frac{3\,a\sqrt{b}\log\left(\frac{a\cos\left(fx+e\right)^{2}+2\,\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^{2}}\right)+2\left(a\cos\left(fx+e\right)^{3}-(3\,a-b)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\quad 3\,a}{6\,af},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(a*cos(f*x + e
)^3 - (3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(a*f), -1/3*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/b) - (a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31934, size = 120, normalized size = 1.2

$$-\frac{\left(\frac{3\,b\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{\left(a\cos\left(fx+e\right)^2+b\right)^{\frac{3}{2}}a^2-3\,\sqrt{a\cos\left(fx+e\right)^2+b}a^3}{a^3}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) - ((a*cos(f*
x + e)^2 + b)^(3/2)*a^2 - 3*sqrt(a*cos(f*x + e)^2 + b)*a^3)/a^3)*sgn(cos(f*
x + e))/f
```

3.69 $\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$

Optimal. Leaf size=66

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f
```

Rubi [A] time = 0.0533864, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4134, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (C
os[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

Mathematica [A] time = 0.150015, size = 98, normalized size = 1.48

$$\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)+b}}{\sqrt{b}}\right)-\sqrt{a\cos^2(e+fx)+b}\right)}{f\sqrt{a\cos(2(e+fx))+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*(Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]]
- Sqrt[b + a*Cos[e + f*x]^2])*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b +
 a*Cos[2*(e + f*x)]])
```

Maple [A] time = 0.077, size = 93, normalized size = 1.4

$$-\frac{1}{fa\sec\left(fx+e\right)}\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{\frac{3}{2}}+\frac{b\sec\left(fx+e\right)}{fa}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^{2}}+\frac{1}{f}\sqrt{b}\ln\left(\sec\left(fx+e\right)\sqrt{b}+\sqrt{a+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)+1/f*b/a*sec(f*x+e)*(a+b*sec(f*x+
e)^2)^(1/2)+1/f*b^(1/2)*ln(sec(f*x+e)*b^(1/2)+(a+b*sec(f*x+e)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.07868, size = 468, normalized size = 7.09

$$\left[-\frac{2\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)-\sqrt{b}\log\left(\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^{2}}\right)}{2f},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos}{b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b)
*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*arctan(sqrt(-b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\sin\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

Giac [A] time = 1.25855, size = 78, normalized size = 1.18

$$-\frac{\left(\frac{b\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \sqrt{a\cos\left(fx+e\right)^2 + b}\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(a*cos(f*x +
 e)^2 + b))*sgn(cos(f*x + e))/f
```

3.70 $\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (S
qrt[a + b]*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/
f
```

Rubi [A] time = 0.0899001, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4134, 402, 217, 206, 377, 207}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - (S
qrt[a + b]*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/
f
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
```

```
GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{-1+x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \sec\right.}{f}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec}{\sqrt{a+b}}\right.}{f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.131028, size = 119, normalized size = 1.45

$$\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{b}}\right)-\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right)\right)}{f\sqrt{a\cos(2e+2fx)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(Sqrt[b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a
+ b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]])*Cos[e + f*x]*Sqrt
[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [B] time = 0.401, size = 688, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f/b^(1/2)/(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*4^(1/2)*
cos(f*x+e)*(2*(a+b)^(1/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(
f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*b-2*b^(3/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)+ln(
-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^(3/2)-ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e)
)*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+
e)^2)*a*b^(1/2)-ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+b)/(-1+cos(f*x+e)))*b^(1/2)*a-ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^(3/2))*(-1+cos(f*x+e)
)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e), x)
```

Fricas [A] time = 0.777769, size = 1281, normalized size = 15.62

$$\left[\frac{\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^{2}-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+a+2b\right)}{\cos\left(fx+e\right)^{2}-1}\right)+\sqrt{b}\log\left(\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^{2}}\right)}{2f}, \frac{2\sqrt{-a \cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) +
sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, 1/2*(2*sqrt(-a - b)*arct
an(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a
 + b)) + sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -1/2*(2*sqrt(-b
)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/f,
 (sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/(a + b)) - sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)/b))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\csc\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\csc\left(fx + e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e), x)
```

3.71 $\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f\sqrt{a+b}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
a + 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*Sqrt[a + b]*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2
*f)
```

Rubi [A] time = 0.135745, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4134, 467, 523, 217, 206, 377, 207}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f\sqrt{a+b}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```



```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
a + 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*Sqrt[a + b]*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2
*f)
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a+2bx^2}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \text{se}\right.}{2f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sec(e+\right.}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sec(e+}{\sqrt{a+b\sec^2}}\right.}{f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{a+b}f} - \frac{\cot(e+fx)\,\text{cs}}{}$$

Mathematica [A] time = 0.440221, size = 163, normalized size = 1.31

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{b}(a+b)\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{b}}\right) - \sqrt{a+b}(a+2b)\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right) - (a\right.}{\sqrt{2}f(a+b)\sqrt{a\cos(2(e+fx))+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[b]*(a + b)*ArcTanh[Sqrt[a
+ b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(a + 2*b)*ArcTanh[Sqrt[a + b
 - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2*Sqrt[a + b - a*S
in[e + f*x]^2]))/(Sqrt[2]*(a + b)*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] time = 0.447, size = 3015, normalized size = 24.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/(a+b)^(5/2)/b^(1/2)*(-1+cos(f*x+e))*(-2*(a+b)^(3/2)*4^(1/2)*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*a-8*(a+b)^(3/2)*cos(
f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)+2*(a+b)^(3/2)*
4^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)-16*(a+b)^(3/2)*
cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)+3*4^(1/2)*ln
(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^(5/2)*a-8*4^(1/2)*ln(-4/(a+b)^(1/2)*(
-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+b)/sin(f*x+e)^2)*b^(5/2)*a-4*4^(1/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(c
os(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x
+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2
)*b^(3/2)*a^2-4*4^(1/2)*b^(7/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)-2*4^(
1/2)*b^(7/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+b)/(-1+cos(f*x+e)))+2*4^(1/2)*b^(7/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e
))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*co
s(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x
+e)^2)-8*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*b^(1/2)+co
s(f*x+e)^2*4^(1/2)*b^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3+8*cos(
f*x+e)^2*4^(1/2)*b^(5/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a+5*cos(f*x+
e)^2*4^(1/2)*b^(5/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a-3*cos(f*x+e)^2*4^(1/2)*b^(5/2)*ln(-2/(a
+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a-2*(a+b)^(3/2)*4^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(1/2)*a+4*(a+b)^(3/2)*4^(1/2)*arctanh(1/8*b^(1/2)*
4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x
+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b+4*4^(1/2)*(a+b)^(3/2
)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x
+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^
2-4^(1/2)*b^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3-4^(1/2)*b^(1/2)*ln(-2/(a+b)^(1/2)*(-1+co
```

```
s(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/
sin(f*x+e)^2)*a^3+4*cos(f*x+e)^2*4^(1/2)*b^(7/2)*ln(-4/(a+b)^(1/2)*(-1+cos(
f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/si
n(f*x+e)^2)+2*cos(f*x+e)^2*4^(1/2)*b^(7/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))-2*cos(f*x+e)^2*4^(1
/2)*b^(7/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)-5*4^(1/2)*b^(5/2)*ln(-4*(
cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x
+e)))*a-4*4^(1/2)*b^(3/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2-2*cos(f*x+e)*4^(1/2)*b^(3/2)*(a+
b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+4*cos(f*x+e)^2*4^(1/2)
*b^(3/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2+4*cos(f*x+e)^2*4^(1/2)*b
^(3/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
b)/(-1+cos(f*x+e)))*a^2-4*cos(f*x+e)^2*4^(1/2)*(a+b)^(3/2)*arctanh(1/8*b^(1
/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin
(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^2+cos(f*x+e)^2*4^(
1/2)*b^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+b)/(-1+cos(f*x+e)))*a^3+4*cos(f*x+e)^2*4^(1/2)*b^(1/2)*(a+b)^(3/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-4*cos(f*x+e)^2*4^(1/2)*(a+b)^(3
/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f
*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*
a*b)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \csc\left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Fricas [A] time = 1.05077, size = 2234, normalized size = 18.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 -
2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a
+ 2*b)/(cos(f*x + e)^2 - 1)) + 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*l
og((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^2 - (a + b)*
f), 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a -
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a
+ b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + ((a + b)*co
s(f*x + e)^2 - a - b)*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^2 - (a + b)*f), -1/4*(4*((a + b)*cos(f*x + e)^2 - a - b)
*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/b) - 2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) - ((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x +
e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sqrt(-a - b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - 2*((a
 + b)*cos(f*x + e)^2 - a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + (a + b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*cos(f*x + e))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^2 + a} \csc \left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

3.72 $\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=183

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+}}{4f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e
 + f*x]^2]])/(8*(a + b)^(3/2)*f) - ((3*a + 4*b)*Cot[e + f*x]*Csc[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqr
t[a + b*Sec[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.22016, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4134, 467, 578, 523, 217, 206, 377, 207}

$$-\frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}-\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f - ((
3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e
 + f*x]^2]])/(8*(a + b)^(3/2)*f) - ((3*a + 4*b)*Cot[e + f*x]*Csc[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f) - (Cot[e + f*x]*Csc[e + f*x]^3*Sqr
t[a + b*Sec[e + f*x]^2])/(4*f)
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a+4bx^2\right)}{\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x\right.}{4f}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\csc^3(e+}{}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\csc^3(e+}{}$$

$$= -\frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot(e+fx)\csc^3(e+}{}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\left(3a^2+12ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{3/2}f} - \frac{(3}{}$$

Mathematica [A] time = 1.40722, size = 198, normalized size = 1.08

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(-\left(3a^2+12ab+8b^2\right)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right)+8\sqrt{b}(a+b)^2\tanh^{-1}\left(\frac{\sqrt{-a\sin}}{}\right.\right.}{4\sqrt{2}f(a+b)^2\sqrt{a\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(8*Sqrt[b]*(a + b)^2*ArcTanh[Sqrt[
a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(3*a^2 + 12*a*b + 8*b^2)*A
rcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2
*(3*a + 4*b + 2*(a + b)*Csc[e + f*x]^2)*Sqrt[a + b - a*Sin[e + f*x]^2]))/(4
```

```
*Sqrt[2]*(a + b)^2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] time = 0.472, size = 9758, normalized size = 53.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\csc\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Fricas [B] time = 2.16584, size = 3656, normalized size = 19.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2
)*cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e
)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 8*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4
- 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log((a*
cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
```

```
f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3
 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f
*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b +
 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)/(a + b)) + 4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2
*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log((a*cos
(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3 - (5
*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(
f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(16*((a^2 + 2*a*b + b^2)*cos(f*x
 + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b
)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/
b) - ((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*
cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^
2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
+ a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x + e)^
3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*
f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b + 8*b^2)*c
os(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b
+ 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)/(a + b)) - 8*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 -
2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(s
qrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((3*a
^2 + 7*a*b + 4*b^2)*cos(f*x + e)^3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*
x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a}\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

3.73 $\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$

Optimal. Leaf size=240

$$\frac{\left(-15a^2b+5a^3-5ab^2-b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}-\frac{(a-b)(5a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16a^2f}$$

```
[Out] ((5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
b + b*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f - ((a - b)*(5*a + b)*Cos[e + f*x]*
Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a^2*f) - ((5*a - b)*Cos[e
+ f*x]*Sin[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) - (Cos[e + f
*x]*Sin[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rubi [A] time = 0.384422, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4132, 467, 578, 523, 217, 206, 377, 203}

$$\frac{\left(-15a^2b+5a^3-5ab^2-b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}-\frac{(a-b)(5a+b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```


```
[Out] ((5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
b + b*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f - ((a - b)*(5*a + b)*Cos[e + f*x]*
Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a^2*f) - ((5*a - b)*Cos[e
+ f*x]*Sin[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) - (Cos[e + f
*x]*Sin[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b+bx^2}}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4\left(5(a+b)+6bx^2\right)}{\left(1+x^2\right)^3\sqrt{a+b+bx}}\right.}{6f}$$

$$= -\frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} - \frac{\cos(e+fx)\sin}{}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} - \frac{(5a-b)\,c}{}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} - \frac{(5a-b)\,c}{}$$

$$= -\frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} - \frac{(5a-b)\,c}{}$$

$$= -\frac{\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\,\text{ta}}{\sqrt{a+b+b}}\right.}{f}$$

Mathematica [F] time = 9.03791, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6, x]
```

Maple [C] time = 0.752, size = 2665, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/48/f/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(-3*2^(1/2)*(1/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-30*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin
(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e
)-15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-96*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+90*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
```

```
/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+3
0*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*b^3+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+15*2^(1/2)*(1/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^3-8*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^3-26*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+26*cos(f*
x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+33*cos(f*x+e)^3*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-33*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^3-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b
^3-33*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+14*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a*b^2+10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a^2*b-10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-
40*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-cos(f*x+e)^3*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+40*cos(f*x+e)^2*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b^2+33*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*
b-14*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2)*cos(f*x+e)*(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*cos(
f*x+e)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

Fricas [A] time = 9.96628, size = 4146, normalized size = 17.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(96*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b
^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
 e)^4) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x
+ e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5 - 2*
(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), 1/384
*(192*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
 b^2)*sin(f*x + e))) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f
*x + e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^
2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/
(a^3*f), 1/192*(48*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*
(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4) - 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8
*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*co
s(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos
(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))
) - 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 -
```

```
 14*a^2*b - 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^3*f), 1/192*(96*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(5*a^3 - 15*a^2*b -
 5*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3
*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^
3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*cos(f*x + e))*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

3.74 $\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$

Optimal. Leaf size=181

$$\frac{(3a^2-6ab-b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+f\cdots}}{4f}$$

```
[Out] ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/f - ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b +
b*Tan[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.22117, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4132, 467, 578, 523, 217, 206, 377, 203}

$$\frac{(3a^2-6ab-b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin^3(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+f\cdots}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

```
[Out] ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*a^(3/2)*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/f - ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a*f) - (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b +
b*Tan[e + f*x]^2])/(4*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{(1+x^2)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+4bx^2)}{(1+x^2)^2\sqrt{a+b+bx^2}}\right.}{4f}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e+fx)\sin^3(e}{}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e+fx)\sin^3(e}{}$$

$$= -\frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} - \frac{\cos(e+fx)\sin^3(e}{}$$

$$= \frac{(3a^2-6ab-b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(3}{}$$

Mathematica [F] time = 5.42471, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4, x]
```

Maple [C] time = 0.411, size = 1940, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(-2*cos(f*x+e)^5*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*a^2*sin(f*x+e)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a*b*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*si
n(f*x+e)-16*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f
*x+e)+12*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2
)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
```

```
+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x
+e)+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+5*cos(f*x+e)
^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^3*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^2+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5
*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*sin(f*x+e)*(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^{2}+a} \sin \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Fricas [A] time = 3.47421, size = 3776, normalized size = 20.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(16*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^
2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
e)^4) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 - a*b)*cos(
```

```
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f)
, 1/64*(32*a^2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e
)^2 + b^2)*sin(f*x + e))) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(
a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*
x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3
- (5*a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(a^2*f), 1/32*(8*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x
+ e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
 + 8*b^2)/cos(f*x + e)^4) - (3*a^2 - 6*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2
*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*
x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) +
4*(2*a^2*cos(f*x + e)^3 - (5*a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f), 1/32*(16*a^2*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - (3*
a^2 - 6*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)
*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
- 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*a^2*cos(f*x + e)^3 - (5*a^
2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \sin\left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

3.75 $\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx$

Optimal. Leaf size=123

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
2*f)
```

Rubi [A] time = 0.131126, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4132, 467, 523, 217, 206, 377, 203}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/f - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
2*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a+b+2bx^2}{(1+x^2)\sqrt{a+b+bx^2}}\right.}{2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a}}\right.}{2}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx\right.}{2f}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cos(e+f}{}$$

Mathematica [C] time = 5.75247, size = 432, normalized size = 3.51

$$\frac{e^{-i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{2e^{2i(e+fx)}\left(-i(a-b)\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+i(a-b)\log\left(\right.\right.}{}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]*(I*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a*f*x - 2*b*f*
x - I*(a - b)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + I*(a - b)*Log[a + a*E^((2
*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*Sqrt[a]*Sqrt[b]*Log[((-(Sqrt[b]*
(-1 + E^((2*I)*(e + f*x)))) + I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
```

```
*I)*(e + f*x)))^2])*f)/(2*b*(1 + E^((2*I)*(e + f*x)))]]))/(Sqrt[a]*Sqrt[4*b
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e +
f*x]^2])/(4*Sqrt[2]*E^(I*(e + f*x))*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [C] time = 0.321, size = 1290, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(-2*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin
(f*x+e)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*b*sin(f*x+e)-4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/c
```

```
os(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Fricas [B] time = 1.67062, size = 3478, normalized size = 28.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) - sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)) - 4*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(
a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/co
s(f*x + e)^4))/(a*f), -1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e)*sin(f*x + e) - 8*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*(a - b)*log(128*a^4
*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*c
os(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
```

```
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*f), -1/8*(4*a*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a - b
)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 +
 (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*
x + e)^2)*sin(f*x + e))) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4))/(a*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*a*
sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\sin^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\sin\left(fx + e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

3.76 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rubi [A] time = 0.0508236, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\ &= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [F] time = 1.80961, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.511, size = 589, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*2^(1/2)*(2*EllipticPi((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*a+2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b-
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a-EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*b)*cos(f*x+e)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)^2*((b
+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.18898, size = 2984, normalized size = 37.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

3.77 $\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rubi [A] time = 0.0808933, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4132, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+f\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+}}\right.}{f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

Mathematica [C] time = 0.227181, size = 61, normalized size = 0.9

$$-\frac{\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{b\sin^2(e+fx)}{-a\sin^2(e+fx)+a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, (b*Sin[e + f*x]^2)/(a + b -
 a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

Maple [C] time = 0.499, size = 1003, normalized size = 14.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^
2)^(1/2)*cos(f*x+e)*(2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*si
n(f*x+e)*cos(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elli
pticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)*cos(f*x+e)+2^(1/2)*(1/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*s
in(f*x+e)-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(b+a*cos(f*x+e)^2)/sin(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 0.700511, size = 780, normalized size = 11.47

$$\left[\frac{\sqrt{b}\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)+8\,b^2}{\cos\left(fx+e\right)^4}\right)\sin\left(fx+e\right)-4}{4\,f\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f
*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*si
n(f*x + e) - 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f
*sin(f*x + e)), 1/2*(sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f
*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec^2\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^2 + a} \csc \left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

3.78 $\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b
+ b*Tan[e + f*x]^2)^(3/2))/(3*(a + b)*f)
```

Rubi [A] time = 0.0971405, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4132, 451, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3f(a+b)}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (Cot[e + f*x]^3*(a + b
+ b*Tan[e + f*x]^2)^(3/2))/(3*(a + b)*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)\sqrt{a+b+bx^2}}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2}\,dx, x, \tan(e+f\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/}}{3(a+b)f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/}}{3(a+b)f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e}{}$$

Mathematica [C] time = 3.8762, size = 285, normalized size = 2.71

$$-\frac{\sqrt{2}\cot(e+fx)\csc^2(e+fx)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\sqrt{a+b\sec^2(e+fx)}\left(\frac{4b\tan^2(e+fx)\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)^2\sqrt{\frac{a+b\sec^2(e+fx)}{a+b}}\text{Hy}}{(a+b)^2}\right.}{3f\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(Sqrt[2]*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (a*Si
n[e + f*x]^2)/(a + b))*((4*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*(a + b -
a*Sin[e + f*x]^2)^2*Tan[e + f*x]^2)/(a + b)^2 + (a + b + 2*a*Sin[e + f*x]^2
)*(Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/
(a + b))]]*Sqrt[-((b*Tan[e + f*x]^2)/(a + b))])))/(3*f*Sqrt[a + 2*b + a*Cos
[2*e + 2*f*x]]*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e +
f*x]^2])
```

Maple [C] time = 0.415, size = 3847, normalized size = 36.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)*(3*sin(f*x+e)*cos(f*x+
e)^3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+3*sin(f*x+e)*cos
(f*x+e)^3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-6*sin(f*x+e
)*cos(f*x+e)^3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
```

```
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-6*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*b^2+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*a*b+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b^2-6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-6*sin(f
*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-3*sin(f*x+e)*cos(f*x+e)*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+6*sin(f*x+e)*cos(f*x+e)*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
```

```
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2))*a*b+6*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-3*2^(1/2)*(1/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin
(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2
)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a*b+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos
(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)^2*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*co
s(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.27468, size = 1092, normalized size = 10.4

$$\left[\frac{3\left((a+b)\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{b}\log\left(\frac{\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+8\left(ab-b^{2}\right)\cos\left(fx+e\right)^{2}+4\left((a-b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+}{\cos\left(fx+e\right)^{2}}}}{\cos\left(fx+e\right)^{4}}\right.}{12\left((a+b)f\cos\left(fx+e\right)^{2}-(a+b)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*
cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 +
 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e
)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
)/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), 1/6*(3*((a + b)*co
s(f*x + e)^2 - a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(
f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*((2*a + 3*b)*cos(f*x + e)
^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

3.79 $\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=149

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{5f(a+b)}-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+\right.}{15f(a+b)^2}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (2*(5*a + 4*b)*Cot[e +
f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(15*(a + b)^2*f) - (Cot[e + f*x]^5
*(a + b + b*Tan[e + f*x]^2)^(3/2))/(5*(a + b)*f)
```

Rubi [A] time = 0.14012, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4132, 462, 451, 277, 217, 206}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{5f(a+b)}-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+\right.}{15f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f - (2*(5*a + 4*b)*Cot[e +
f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(15*(a + b)^2*f) - (Cot[e + f*x]^5
*(a + b + b*Tan[e + f*x]^2)^(3/2))/(5*(a + b)*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2\sqrt{a+b+bx^2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}\left(2(5a+4b)+5(a+b)x^2\right)}{x^4}\right.}{5(a+b)f}$$

$$= -\frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2 f} - \frac{\cot^5(e+fx)\left(a+b+b\right.}{5(a+b)}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan\right.}{15(a+b)^2 f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan\right.}{15(a+b)^2 f}$$

$$= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{2(5a+}{}$$

Mathematica [C] time = 9.55979, size = 941, normalized size = 6.32

$$\frac{\sqrt{2}\csc^5(e+fx)\sec(e+fx)\sqrt{b\sec^2(e+fx)+a}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(\frac{24a^2b^2\text{Hypergeometric2F1}\left(2,2,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^2(e+fx)\sin^6(e+f}{(a+b)^3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*Csc[e + f*x]^5*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (a*Sin
[e + f*x]^2)/(a + b))*(-3*(a + b)*Cos[e + f*x]^2 - 4*a*Cos[e + f*x]^2*Sin[e
 + f*x]^2 - 16*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))
]*Sin[e + f*x]^2 + 8*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e +
f*x]^2)/(a + b))]*Sin[e + f*x]^2 - (8*a^2*Cos[e + f*x]^2*Sin[e + f*x]^4)/(a
 + b) - (8*a*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*
Sin[e + f*x]^4)/(a + b) - (16*a*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -(
```

```
(b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^4)/(a + b) + (24*a^2*b*Hypergeome
tric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6)/(a + b)^2
 + (8*a^2*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a
+ b))]*Sin[e + f*x]^6)/(a + b)^2 - (16*b^2*Hypergeometric2F1[2, 2, 3/2, -((
b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(a + b) + (8*b^2
*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[
e + f*x]^2*Tan[e + f*x]^2)/(a + b) - (8*a*b^2*Hypergeometric2F1[2, 2, 3/2,
-((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^4*Tan[e + f*x]^2)/(a + b)^2 - (
16*a*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(a + b
))]*Sin[e + f*x]^4*Tan[e + f*x]^2)/(a + b)^2 + (24*a^2*b^2*Hypergeometric2F
1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6*Tan[e + f*x]^2)/
(a + b)^3 + (8*a^2*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e +
f*x]^2)/(a + b))]*Sin[e + f*x]^6*Tan[e + f*x]^2)/(a + b)^3 - (4*a*ArcSin[Sq
rt[-((b*Tan[e + f*x]^2)/(a + b))]]*Cos[e + f*x]^2*Sin[e + f*x]^2*Sqrt[-((b*
Tan[e + f*x]^2)/(a + b))])/Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + (3*b*ArcS
in[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*Sin[e + f*x]^2)/Sqrt[-((b*Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)] + (8*a^2*b*Arc
Sin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*Sin[e + f*x]^6)/((a + b)^2*Sqrt[-(
(b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])))
/(15*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Maple [C] time = 0.58, size = 8587, normalized size = 57.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.45319, size = 1639, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f
*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x
+ e)^5 - (20*a^2 + 59*a*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*
b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*
a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2
 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/30*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e
)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b)*ar
ctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*
sin(f*x + e) - 2*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x + e)^5 - (20*a^2 + 59*a
*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*b^2)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b + b^2)*f*cos(f*x +
e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)*sin(
f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^{2}+a} \csc \left(fx+e\right)^{6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

3.80 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx$

Optimal. Leaf size=196

$$\frac{b(3a-4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}$$

```
[Out] ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 4*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 4*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (2*C
os[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f) - (Cos[e + f*x]^5*(a +
b*Sec[e + f*x]^2)^(5/2))/(5*a*f)
```

Rubi [A] time = 0.179943, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4134, 462, 453, 277, 195, 217, 206}

$$\frac{b(3a-4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}-\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}+\frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```


```
[Out] ((3*a - 4*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 4*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 4*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (2*C
os[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f) - (Cos[e + f*x]^5*(a +
b*Sec[e + f*x]^2)^(5/2))/(5*a*f)
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)^2(a+bx^2)^{3/2}}{x^6}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af} + \frac{\text{Subst}\left(\int \frac{(-10a+5ax^2)(a+bx^2)^{3/2}}{x^4}\,dx, x, \text{se}\right.}{5af}$$

$$= \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}$$

$$= -\frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)\left(a+b\sec^2(e\right.}{3af}$$

$$= \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(a+b\sec\right.}{3af}$$

$$= \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(a+b\sec\right.}{3af}$$

$$= \frac{(3a-4b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+\right.}{2af}$$

Mathematica [A] time = 1.47831, size = 188, normalized size = 0.96

$$\frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-5(3a-4b)\left(\sqrt{-a\sin^2(e+fx)+a+b}\left(-a\sin^2(e+fx)+a+4b\right)-3b^{3/2}\tanh^{-}\right.\right.}{15bf(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-6*b*(a + b - a*Sin[
e + f*x]^2)^(5/2))/a + 15*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)^(5/2) -
 5*(3*a - 4*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]
+ Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*Sin[e + f*x]^2))))/(15*b*f*(a
 + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.624, size = 2537, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x)
```

```
[Out] 1/60/f/a/b^(1/2)/(a+b)^(9/2)*(-1+cos(f*x+e))^3*(45*cos(f*x+e)^2*b^(5/2)*ln(
-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^5-45*cos(f*x+e)^2*b^(5/2)*ln(-4/(a+b)^
(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+b)/sin(f*x+e)^2)*a^5+225*cos(f*x+e)^2*b^(11/2)*ln(-2/(a+b)^(1/2)*(
-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+b)/sin(f*x+e)^2)*a^2-225*cos(f*x+e)^2*b^(11/2)*ln(-4/(a+b)^(1/2)*(-1+cos(
f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/si
n(f*x+e)^2)*a^2+315*cos(f*x+e)^2*b^(9/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*
(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)
^2)*a^3-315*cos(f*x+e)^2*b^(9/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3+
195*cos(f*x+e)^2*b^(7/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4-195*cos(
f*x+e)^2*b^(7/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4+60*cos(f*x+e)^2*
b^(13/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a-60*cos(f*x+e)^2*b^(13/2)*l
n(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a-15*(a+b)^(7/2)*b^(5/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*b^(3/2)*(a+b)^(7/2)*a^2+6*cos(f*x+e)^3*(a+b)^(7/2)*b^(7/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+6*cos(f*x+e)^2*b^(7/2)*(a+b)^(7/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+6*cos(f*x+e)^7*b^(3/2)*(a+b)^(7/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+6*(a+b)^(7/2)*cos(f*x+e)^7*a^3
*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+6*(a+b)^(7/2)*cos(f*x+
```

```
e)^6*a^2*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+12*cos(f*x+e)^
5*b^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+6*cos(f
*x+e)^6*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3
-8*cos(f*x+e)^5*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^2+12*cos(f*x+e)^4*b^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a-20*(a+b)^(7/2)*cos(f*x+e)^5*a^3*b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-8*cos(f*x+e)^4*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-74*cos(f*x+e)^3*b^(5/2)*(a+b)^(7/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-20*cos(f*x+e)^4*b^(1/2)*(a+b)^(7/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3-50*cos(f*x+e)^3*b^(3/2)*(a
+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-74*cos(f*x+e)^2*b
^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+30*cos(f*x
+e)^3*(a+b)^(7/2)*a^3*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-5
0*cos(f*x+e)^2*b^(3/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a^2-15*cos(f*x+e)*b^(5/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a+30*cos(f*x+e)^2*b^(1/2)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^3-45*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*b^(1/2)*4^(1/
2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2
/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3*b+15*cos(f*x+e)^2*(a+b)^(
7/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(
f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))
*a^2*b^2+60*cos(f*x+e)^2*(a+b)^(7/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*
x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^3-15*cos(f*x+e)*b^(3/2)*(a+b)^(7/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2)*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(3/2)*4^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/
sin(f*x+e)^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72373, size = 876, normalized size = 4.47

$$\left[-\frac{15\left(3a^{2}-4ab\right)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2b}{\cos\left(fx+e\right)^{2}}\right)+2\left(6a^{2}\cos\left(fx+e\right)^{6}-4\left(5a^{2}-3ab\right)\right.}{60af\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(3*a^2 - 4*a*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(
f*x + e)^2) + 2*(6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4 +
2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 4*a*b)*sqrt(-
b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
/b)*cos(f*x + e) + (6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4
 + 2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^5, x)
```

3.81 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=162

$$\frac{b(3a-2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}-\frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e\right.}{3af}$$

```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 2*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 2*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (Cos
[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f)
```

Rubi [A] time = 0.142408, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4134, 453, 277, 195, 217, 206}

$$\frac{b(3a-2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}-\frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e\right.}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) + ((3*a - 2*b)*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*a*
f) - ((3*a - 2*b)*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a*f) + (Cos
[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a*f)
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^{3/2}}{x^4}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} + \frac{(3a-2b)\,\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx, x, \text{se}\right.}{3af}$$

$$= -\frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e\right.}{3af}$$

$$= \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(a+b\,\text{se}\right.}{3af}$$

$$= \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(a+b\,\text{se}\right.}{3af}$$

$$= \frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e\right.}{2af}$$

Mathematica [A] time = 0.790841, size = 164, normalized size = 1.01

$$\frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(3\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)^{5/2}-(3a-2b)\left(\sqrt{-a\sin^2(e+fx)+a+\right.\right.}{3bf(a\cos(2(e+fx))+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*(3*Sec[e + f*x]^2*(a +
 b - a*Sin[e + f*x]^2)^(5/2) - (3*a - 2*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b -
 a*Sin[e + f*x]^2]/Sqrt[b]] + Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*S
in[e + f*x]^2))))/(3*b*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.42, size = 1913, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x)
```

```
[Out] -1/12/f/b^(1/2)/(a+b)^(9/2)*(-1+cos(f*x+e))^3*(6*cos(f*x+e)^2*b^(13/2)*ln(-
4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)+18*cos(f*x+e)^2*b^(11/2)*ln(-4/(a+b)^(1/2
)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)+b)/sin(f*x+e)^2)*a-18*cos(f*x+e)^2*b^(11/2)*ln(-2/(a+b)^(1/2)*(-1+cos(
f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/si
n(f*x+e)^2)*a+18*cos(f*x+e)^2*b^(9/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(co
s(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+
e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)
*a^2-18*cos(f*x+e)^2*b^(9/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2+6*co
s(f*x+e)^2*b^(7/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3-6*cos(f*x+e)^2
*b^(7/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3-6*cos(f*x+e)^2*arctanh(1
/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)
-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(7/2)*b
^3+8*cos(f*x+e)^3*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(7/2)+8*cos(f*x+e)^2*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(7/2)+3*cos(f*x+e)*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(7/2)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)*(a+b)^(7
/2)*a+2*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(3/2)*(a
+b)^(7/2)*a+2*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(3
/2)*(a+b)^(7/2)*a-6*cos(f*x+e)^2*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(7/2)*a^2+3*cos(f*x+e)*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(7/2)*a+9*cos(f*x+e)^2*arctanh(1/8*b^(1/2)*4^(1/2)*(-
1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(7/2)*a^2*b+3*cos(f*x+e)^2*a
rctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)
-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)
^(7/2)*a*b^2+2*cos(f*x+e)^5*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(7/2)*a^2+2*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*b^(3/2)*(a+b)^(7/2)*a+2*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^(1/2)*(a+b)^(7/2)*a^2+2*cos(f*x+e)^3*b^(3/2)*((b+a*cos(f*x+e)^
```

```
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a-6*cos(f*x+e)^3*b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^2-6*cos(f*x+e)^2*b^(13/2)*ln
(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^(5/2)*(a+b)^(7/2))*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3
/2)*4^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)/sin(f*x+e)^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58215, size = 710, normalized size = 4.38

$$\left[-\frac{3\,(3\,a-2\,b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos\left(fx+e\right)^{2}-2\,\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^{2}}\right)-2\left(2\,a\cos\left(fx+e\right)^{4}-2\,(3\,a-4\,b)\cos\left(f\right.\right.}{12\,f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a - 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x +
 e)^2) - 2*(2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)^2 + 3*b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/6*(3*(3*a - 2*b
)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)/b)*cos(f*x + e) - (2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)
```

```
^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41367, size = 219, normalized size = 1.35

$$\frac{\left(2\left(a\cos\left(fx+e\right)^{2}+b\right)^{\frac{3}{2}}af^{4}-6\sqrt{a\cos\left(fx+e\right)^{2}+b}a^{2}f^{4}+6\sqrt{a\cos\left(fx+e\right)^{2}+b}abf^{4}+\frac{3\sqrt{a\cos\left(fx+e\right)^{2}+b}abf^{4}}{\cos\left(fx+e\right)^{2}}-\frac{3\left(3a^{2}bf\right.}{}\right.}{6af^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(a*cos(f*x + e)^2 + b)^(3/2)*a*f^4 - 6*sqrt(a*cos(f*x + e)^2 + b)*a^
2*f^4 + 6*sqrt(a*cos(f*x + e)^2 + b)*a*b*f^4 + 3*sqrt(a*cos(f*x + e)^2 + b)
*a*b*f^4/cos(f*x + e)^2 - 3*(3*a^2*b*f^4 - 2*a*b^2*f^4)*arctan(sqrt(a*cos(f
*x + e)^2 + b)/sqrt(-b))/sqrt(-b))*sgn(cos(f*x + e))/(a*f^5)
```

3.82 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*f) + (3*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f) - (Cos[e + f*x]*(
a + b*Sec[e + f*x]^2)^(3/2))/f
```

Rubi [A] time = 0.0702479, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4134, 277, 195, 217, 206}

$$\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

```
[Out] (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2
*f) + (3*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f) - (Cos[e + f*x]*(
a + b*Sec[e + f*x]^2)^(3/2))/f
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{(3b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,\sec(e+ \ldots\right.}{f}$$

$$=\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{(3 \ldots}{\ldots}$$

$$=\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}+\frac{(3 \ldots}{\ldots}$$

$$=\frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}+\frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\cos(e+ \ldots}{\ldots}$$

Mathematica [C] time = 0.664805, size = 73, normalized size = 0.73

$$-\frac{a\cos(e+fx)(a\cos(2(e+fx))+a+2b)^2\sqrt{a+b\sec^2(e+fx)}\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\frac{a\cos^2(e+fx)}{b}+1\right)}{20b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

```
[Out] -(a*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Hypergeometric2F1[2, 5/2,
 7/2, 1 + (a*Cos[e + f*x]^2)/b]*Sqrt[a + b*Sec[e + f*x]^2])/(20*b^2*f)
```

Maple [A] time = 0.059, size = 121, normalized size = 1.2

$$-\frac{1}{fa\sec\left(fx+e\right)}\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^{\frac{5}{2}}+\frac{b\sec\left(fx+e\right)}{fa}\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^{\frac{3}{2}}+\frac{3b\sec\left(fx+e\right)}{2f}\sqrt{a+b\left(\sec\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(5/2)+1/f*b/a*sec(f*x+e)*(a+b*sec(f*x+
e)^2)^(3/2)+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f+3/2/f*b^(1/2)*a*ln(
sec(f*x+e)*b^(1/2)+(a+b*sec(f*x+e)^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.18364, size = 591, normalized size = 5.91

$$\left[\frac{3\,a\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos\left(fx+e\right)^2+2\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+2\,b}{\cos\left(fx+e\right)^2}\right)-2\left(2\,a\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{4\,f\cos\left(fx+e\right)},-\frac{3\,a\sqrt{-b}\,\mathrm{arc}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(
2*a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos
(f*x + e)), -1/2*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + (2*a*cos(f*x + e)^2 - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30538, size = 124, normalized size = 1.24

$$-\frac{\left(\frac{3\,b\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\sqrt{a\cos\left(fx+e\right)^2+b}-\frac{\sqrt{a\cos\left(fx+e\right)^2+b}b}{a\cos\left(fx+e\right)^2}\right)a\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*c
os(f*x + e)^2 + b) - sqrt(a*cos(f*x + e)^2 + b)*b/(a*cos(f*x + e)^2))*a*sgn
(cos(f*x + e))/f
```

3.83 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]])/f + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.13605, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4134, 416, 523, 217, 206, 377, 207}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]])/f + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{-1+x^2}dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{a(2a+b)+b(3a+2b)x^2}{(-1+x^2)\sqrt{a+bx^2}}dx, x, \sec(e+\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}dx, x, \sec\right.}{f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}dx, x, \frac{\sec}{\sqrt{a+b\,\text{s}}}\right.}{f}$$

$$= \frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{b}{}$$

Mathematica [A] time = 0.589641, size = 171, normalized size = 1.4

$$\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}b\sqrt{a\cos(2(e+fx))+a+2b}+2\sqrt{b}(3a+2b)\cos^2(e+fx)\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{b}}\right.\right.}{2\sqrt{2}f\sqrt{a\cos(2(e+fx))+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[b]*(3*a + 2*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos
[e + f*x]^2 - 4*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a
 + b]]*Cos[e + f*x]^2 + Sqrt[2]*b*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])*Sec[e
 + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e
+ f*x)]])
```

Maple [B] time = 0.329, size = 2563, normalized size = 21.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f/(a+b)^(9/2)/b^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*4^(1/2)*
cos(f*x+e)*(-1+cos(f*x+e))^3*(-2*cos(f*x+e)^2*b^(13/2)*ln(-4/(a+b)^(1/2)*(-
1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
+b)/sin(f*x+e)^2)-cos(f*x+e)^2*b^(13/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))-15*cos(f*x+e)^2*b^(9/2
)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-
1+cos(f*x+e)))*a^2-20*cos(f*x+e)^2*b^(7/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3-12*cos(f*x+e)^2
*b^(11/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a+6*cos(f*x+e)^2*b^(11/2)*l
n(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a-24*cos(f*x+e)^2*b^(9/2)*ln(-4/(a+b)^
(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+b)/sin(f*x+e)^2)*a^2+9*cos(f*x+e)^2*b^(9/2)*ln(-2/(a+b)^(1/2)*(-1+
cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b
)/sin(f*x+e)^2)*a^2-20*cos(f*x+e)^2*b^(7/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e
))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*co
s(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x
+e)^2)*a^3+2*cos(f*x+e)^2*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(
f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2))*(a+b)^(7/2)*b^3+cos(f*x+e)*b^(5/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^(3/2)*(a+b)^(7/2)*a-6*cos(f*x+e)^2*b^(11/2)*ln(-4*(cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a-15*cos(
f*x+e)^2*b^(5/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-9*cos(f*x+e)^2*b^(5/2)*ln(-2/(a+b)^(1/2)*
(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+b)/sin(f*x+e)^2)*a^4-6*cos(f*x+e)^2*b^(5/2)*ln(-4/(a+b)^(1/2)*(-1+cos(f*
x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a
*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(
f*x+e)^2)*a^4-6*cos(f*x+e)^2*b^(3/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/
```

```
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^5-6*cos(f*x+e)^2*b^(3/2
)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^5-cos(f*x+e)^2*b^(1/2)*ln(-4*(cos
(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)
))*a^6-cos(f*x+e)^2*b^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^6+cos(f
*x+e)*b^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a+3*c
os(f*x+e)^2*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)
-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2))*(a+b)^(7/2)*a^2*b+5*cos(f*x+e)^2*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+co
s(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(7/2)*a*b^2+cos(f*x+e)^2*b^(13/2
)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(5/2)*(a+b)^(7/2))/sin(f*x+e)^6/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

Fricas [A] time = 1.09593, size = 1867, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*
x + e)^2 - 1)) + (3*a + 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/co
s(f*x + e)^2) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
 + e)), 1/4*(4*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) + (3*a + 2*b)*s
qrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*b*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*((3*a + 2*b)*sqrt
(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/b)*cos(f*x + e) - (a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2
*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
 2*b)/(cos(f*x + e)^2 - 1)) - b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
)/(f*cos(f*x + e)), 1/2*(2*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) - (
3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)/b)*cos(f*x + e) + b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

3.84 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=161

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\text{co}$$

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (Sqrt[a + b]*(a + 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqr
t[a + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2
])/f - (Cot[e + f*x]*Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(2*f)
```

Rubi [A] time = 0.203121, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4134, 467, 528, 523, 217, 206, 377, 207}

$$\frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\frac{\sqrt{a+b}(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}-\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (Sqrt[a + b]*(a + 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqr
t[a + b*Sec[e + f*x]^2]])/(2*f) + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2
])/f - (Cot[e + f*x]*Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(2*f)
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\left(a+4bx^2\right)}{-1+x^2}\, a\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+\right.}{2f}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+\right.}{2f}$$

$$= \frac{\sqrt{b}(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{\sqrt{a+b}(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx}{\sqrt{a+b\sec^2(e+fx}}\right.}{2f}$$

Mathematica [A] time = 1.4986, size = 202, normalized size = 1.25

$$-\frac{\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{a\cos(2(e+fx))+a+2b}((a+2b)\cos(2(e+fx))+a)-\sqrt{b}(3a+4b)\right.}{4\sqrt{2}f\sqrt{a\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Csc[e + f*x]^2*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*Sqrt[a +
2*b + a*Cos[2*(e + f*x)]]*(a + (a + 2*b)*Cos[2*(e + f*x)]) - Sqrt[b]*(3*a +
 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Sin[2*(e + f*x)]^2 +
Sqrt[a + b]*(a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*S
in[2*(e + f*x)]^2))/(4*Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

Maple [B] time = 0.378, size = 5178, normalized size = 32.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Fricas [A] time = 1.12478, size = 2477, normalized size = 15.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + 4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2
*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + ((3*a + 4*b)*cos(f*x +
e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e
)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/4*(2*((a + 4*b)*cos(f*x +
e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + ((3*a + 4*b)*cos(f
*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqr
```

```
t(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*
x + e)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((3*a + 4*b)*co
s(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - ((a + 4*b)*cos(f*x + e
)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(
a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/
(cos(f*x + e)^2 - 1)) - 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/2*(((a +
4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a -
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - ((3
*a + 4*b)*cos(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt(-
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + ((a + 2*b)
*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*
x + e)^3 - f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

3.85 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=218

$$-\frac{3\left(a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}-\frac{3(a+2b)\csc^2(e+fx)\sec(e}{8f}$$

```
[Out] (3*Sqrt[b]*(a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (3*(a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/S
qrt[a + b*Sec[e + f*x]^2]])/(8*Sqrt[a + b]*f) + (3*(a + 4*b)*Sec[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*f) - (3*(a + 2*b)*Csc[e + f*x]^2*Sec[e + f*x]
*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a + b*Se
c[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.336147, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4134, 467, 577, 582, 523, 217, 206, 377, 207}

$$-\frac{3\left(a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}-\frac{3(a+2b)\csc^2(e+fx)\sec(e}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] (3*Sqrt[b]*(a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]
^2]])/(2*f) - (3*(a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/S
qrt[a + b*Sec[e + f*x]^2]])/(8*Sqrt[a + b]*f) + (3*(a + 4*b)*Sec[e + f*x]*S
qrt[a + b*Sec[e + f*x]^2])/(8*f) - (3*(a + 2*b)*Csc[e + f*x]^2*Sec[e + f*x]
*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - (Cot[e + f*x]*Csc[e + f*x]^3*(a + b*Se
c[e + f*x]^2)^(3/2))/(4*f)
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}(3a+6}{\left(-1+x^2\right)^2}\right.}{4}$$

$$= -\frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{\cot(e+fx)\csc^3}{}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+}{8f}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+}{8f}$$

$$= \frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{3(a+2b)\csc^2(e+fx)\sec(e+}{8f}$$

$$= \frac{3\sqrt{b}(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{3\left(a^2+8ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec}{\sqrt{a+b\sec}}\right.}{8\sqrt{a+b}f}$$

Mathematica [A] time = 3.57315, size = 262, normalized size = 1.2

$$-\frac{\sec(e+fx)\left(a\cos^2(e+fx)+b\right)\sqrt{a+b\sec^2(e+fx)}\left(-12b^{3/2}\left(a^2+3ab+2b^2\right)\cos^2(e+fx)\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{b}}\right)\right.}{2\sqrt{2}bf(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b + a*Cos[e + f*x]^2)*(-12*b^(3/2)*(a^2 + 3*a*b + 2*b^2)*ArcTanh[Sqrt[a
+ b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos[e + f*x]^2 + 3*b*Sqrt[a + b]*(a^2 + 8*
a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Cos[e + f*
x]^2 + (b*(a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(11*a + 4*b + 8*(a + 3
*b)*Cos[2*(e + f*x)] - 3*(a + 4*b)*Cos[4*(e + f*x)])*Csc[e + f*x]^4)/(8*Sqr
t[2]))*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*Sqrt[2]*b*(a + b)*f*(a +
 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.428, size = 10199, normalized size = 46.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Fricas [A] time = 1.32259, size = 3794, normalized size = 17.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos
(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + 3*a*b + 2*b^2)*cos(
f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)
*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3*(a^2 +
 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 +
 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos
(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), 1/8*(3*
((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e
)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + 6*((a^2
 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 +
 (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x
 + e)^2) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b
^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*co
s(f*x + e)), -1/16*(24*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a
*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(-b)*a
rctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)
- 3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x
+ e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a^2 + 5*a*b + 4*b^2)*cos(f*x
+ e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*
f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)), 1/8*(3*((a^2 + 8*a*b + 8*b^2)*c
os(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^3 + (a^2 + 8*a*b + 8*b
^2)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) - 12*((a^2 + 3*a*b + 2*b^2)*cos(f
*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*
cos(f*x + e))*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*cos(f*x + e)/b) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2
```

```
+ 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3
 + (a + b)*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

3.86 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx$

Optimal. Leaf size=298

$$\frac{\left(-45a^2b+5a^3+15ab^2+b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}-\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16af}+\frac{(5a^2$$

```
[Out] ((5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/(16*a^(3/2)*f) + ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - ((5*a^2 - 26*a*
b + b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a^2 -
40*a*b + 3*b^2)*Sin[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(48*a*f) + ((5*a - 3*b)*Sin[e + f*x]^4*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(24*f) - (Cos[e + f*x]*Sin[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(6*f)
```

Rubi [A] time = 0.473906, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4132, 467, 577, 578, 582, 523, 217, 206, 377, 203}

$$\frac{\left(-45a^2b+5a^3+15ab^2+b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}-\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16af}+\frac{(5a^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```


```
[Out] ((5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a +
 b + b*Tan[e + f*x]^2]])/(16*a^(3/2)*f) + ((3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - ((5*a^2 - 26*a*
b + b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a^2 -
40*a*b + 3*b^2)*Sin[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(48*a*f) + ((5*a - 3*b)*Sin[e + f*x]^4*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(24*f) - (Cos[e + f*x]*Sin[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(6*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^4}dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f}+\frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{(1+}\right.}{}$$

$$=\frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f}-\frac{\cos(e+fx)\sin}{}$$

$$=\frac{\left(5a^2-40ab+3b^2\right)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af}+\frac{(5a-}{}$$

$$=-\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af}+\frac{\left(5a^2-40ab+3b\right.}{}$$

$$=-\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af}+\frac{\left(5a^2-40ab+3b\right.}{}$$

$$=-\frac{\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af}+\frac{\left(5a^2-40ab+3b\right.}{}$$

$$=\frac{\left(5a^3-45a^2b+15ab^2+b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f}+\frac{(3a-5b)\sqrt{b}\tanh^{-1}\left(\right.}{2f}$$

Mathematica [F] time = 10.5733, size = 0, normalized size = 0.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^6(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6, x]
```

Maple [C] time = 0.834, size = 3067, normalized size = 10.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x)
```

```
[Out] -1/48/f/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(-6*cos(f*x+e)^2*sin(f*x+
e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*b^3+15*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-144*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2
*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
))*a^2*b+240*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2-63*sin(f*x+e)*
cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-75*sin
(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^
2+22*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+17*cos(f*x+
e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+8*cos(f*x+e)^9*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b
```

```
)/(a+b))^(1/2)*a^3+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b
^3-17*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-22*cos(f*x
+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+270*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*a^2*b-90*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*a*b^2-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-30*cos(f*
x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3-26*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^3+26*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a^3+33*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-33*c
os(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+3*sin(f*x+e)*cos(f*
x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3+24*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-94*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a^2*b+94*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a^2*b+9*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-68*co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-9*cos(f*x+e)^2*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+68*cos(f*x+e)^2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b^2-24*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*sin(f*x+
e)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

Fricas [A] time = 34.2621, size = 4521, normalized size = 15.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(-a)*cos(f*x + e)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 48*(3*a^3 - 5*
a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*
b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqr
t(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(
f*x + e)^4) + 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x + e)^4
 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)), 1/384*(96*
(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(5*a^3 - 45*a^2*b + 15*a*b^2
+ b^3)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)
*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
 e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*
a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x
 + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)), -1/
192*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*
x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))
*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
```

```
 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
 e) + 24*(3*a^3 - 5*a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*co
s(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a
^2*b)*cos(f*x + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f
*x + e)), -1/192*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
)))*cos(f*x + e) - 48*(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 4*(8*a^3*
cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos(f*x + e)^4 - 24*a^2*b + (33*a^3 -
 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

3.87 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=217

$$\frac{3\left(a^2-6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}-\frac{3(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{3(a-b)\sin^2(e+fx)\tan}{}$$

```
[Out] (3*(a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 3*b)*Tan[e + f*x]*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(8*f) + (3*(a - b)*Sin[e + f*x]^2*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b +
 b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.340862, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4132, 467, 577, 582, 523, 217, 206, 377, 203}

$$\frac{3\left(a^2-6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}-\frac{3(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{3(a-b)\sin^2(e+fx)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*f) - (3*(a - 3*b)*Tan[e + f*x]*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(8*f) + (3*(a - b)*Sin[e + f*x]^2*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(8*f) - (Cos[e + f*x]*Sin[e + f*x]^3*(a + b +
 b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{(1+\cdots}\right.}{\cdots}$$

$$= \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin^3 \cdots}{\cdots}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(\cdots}{\cdots}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(\cdots}{\cdots}$$

$$= -\frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(\cdots}{\cdots}$$

$$= \frac{3\left(a^2-6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+\cdots}{\sqrt{a+b+b\tan^2(\cdots}}\right.}{2f}$$

Mathematica [A] time = 5.64237, size = 211, normalized size = 0.97

$$\frac{3\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(\frac{\left(a^2-6ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{a}} + 4\sqrt{b}(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)\right)}{2\sqrt{2}f(a\cos(2e+2fx)+a+2b)^{3/2}} + \frac{\tan(e\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (3*(((a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
 + f*x]^2]])/Sqrt[a] + 4*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqr
t[a + b - a*Sin[e + f*x]^2]])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/
(2*Sqrt[2]*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((-7*a + 26*b + (-6*a
```

```
+ 10*b)*Cos[2*(e + f*x)] + a*Cos[4*(e + f*x)])*Sqrt[a + b*Sec[e + f*x]^2]*T
an[e + f*x])/(32*f)
```

Maple [C] time = 0.541, size = 2309, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x)
```

```
[Out] 1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(2*cos(f*x+e)^7*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+24*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*a*b-24*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*cos(f*x+e)^2*sin(f*
x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+6*sin(f*x+e)*cos
(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+9*sin(f*x+e
)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+6*cos(
f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipt
```

```
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-36*cos(f*x+e)^2*sin(f*x+e)*2^(1/
2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2))*a*b+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-2*cos(f*
x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-5*cos(f*x+e)^5*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+7*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^2-7*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^3*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^3*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*b^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a*b-5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+4*cos(
f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-4*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*s
in(f*x+e)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Fricas [A] time = 11.4292, size = 4078, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")

[Out] [-1/64*(3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*
cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*
(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 24*(a^2 - a*b)*sqrt(b)*cos(f*x
+ e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 8*(2*a^2*
cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), 1/64*(48*(a^2 - a
*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*
sin(f*x + e)))*cos(f*x + e) - 3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*l
og(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b
^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^
3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqr
t(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*
cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), -1/32*(3*(a^2 - 6
*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2
*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + 12*(a^2 - a*b)*sqrt(b)*co
s(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x
+ e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(
2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), -1/32*(3*(a
^2 - 6*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - 24*(a^2 - a*b)*sqrt
(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x
+ e)))*cos(f*x + e) - 4*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^
2 + 4*a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*c
os(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

3.88 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=161

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}+\frac{\sqrt{a}(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{\sqrt{b}(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^
2])/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```

Rubi [A] time = 0.197733, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4132, 467, 528, 523, 217, 206, 377, 203}

$$\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}+\frac{\sqrt{a}(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{\sqrt{b}(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^
2])/f - (Cos[e + f*x]*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(2*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{x^2\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f}+\frac{\text{Subst}\left(\int\frac{\sqrt{a+b+bx^2}(a}{1+x}\right.}{}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\right.}{2f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\right.}{2f}$$

$$=\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\right.}{2f}$$

$$=\frac{\sqrt{a}(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{(3a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx}}\right.}{2f}$$

Mathematica [C] time = 5.88908, size = 493, normalized size = 3.06

$$\frac{e^{-i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{i\left(-1+e^{2i(e+fx)}\right)\left(a\left(1+e^{2i(e+fx)}\right)^2-4be^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2e^{2i(e+fx)}\left(-i\sqrt{a}(a-3b)\log\left(\sqrt{a}\sqrt{a\left(1+\right.}\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]^3*((I*(-1 + E^((2*I)*(e + f*x)))*(-4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2))/(1 + E^((2*I)*(e + f*x)))^2 + (2*E^((2*I)*(e + f*x))*(
2*Sqrt[a]*(a - 3*b)*f*x - I*Sqrt[a]*(a - 3*b)*Log[a + 2*b + a*E^((2*I)*(e +
 f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2]] + I*Sqrt[a]*(a - 3*b)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
+ f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + 2*Sqrt[b]*(-3*a + b)*Log[((Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*S
qrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/(b*(-3*a +
 b)*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*E^(I*(e + f*
x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.362, size = 1583, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(-2*cos(f*x+e)^2*sin(f*x+e)*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)
+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2))*a^2+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-6*s
in(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+2*sin(f*x+e)*cos(f*x+e)^2*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
```

```
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2))*b^2+cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^2-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*b^2+cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^2-cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/
(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Fricas [B] time = 3.92184, size = 3771, normalized size = 23.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(sqrt(-a)*(a - 3*b)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)
^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
```

```
^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b
)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7
*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e)) + 2*(3*a - b)*sqrt(b)*cos(f*x + e)*log(((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*c
os(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*(a*cos(f*x + e)^2 - b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e))
, 1/16*(4*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(
f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*
x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - sqrt(-a)*(a - 3*b)*cos(f*x +
e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(a*
cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(f*cos(f*x + e)), -1/8*((a - 3*b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)
^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2
*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) +
(3*a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*
(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4) + 4*(a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), -1/8*((a - 3*b)*sqrt(a)*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e) - 2*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 4*(a*cos(f*x +
e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(
f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

3.89 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0961235, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4128, 416, 523, 217, 206, 377, 203}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{1+x^2} dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}} dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}} dx, x, \tan(e+fx)\right)}{f} +$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1+ax^2} dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(b(}{}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{}}{}$$

Mathematica [C] time = 5.54206, size = 527, normalized size = 4.47

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-ia^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+ia^{3/2}\log\left(\sqrt{a}\sqrt{a(1+e^{2}}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
```

```
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.309, size = 1556, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(2*cos(f*x+e)^2*sin(f*x+e)*2
^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+3*sin(f*x+e)*cos(f*x+e
)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+sin(f*x+e)*cos(f*
x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-6*sin(f*x+e)*c
os(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2
*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
))*b^2-4*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
```

```
1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))
^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-cos(f*x+e)^3*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a*b-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)
/cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 1.76699, size = 3602, normalized size = 30.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
```

```
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx + e\right)^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.90 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=105

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{3\sqrt{b}(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{f}$$

```
[Out] (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*
x]^2]])/(2*f) + (3*b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f) - (
Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/f
```

Rubi [A] time = 0.104994, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4132, 277, 195, 217, 206}

$$\frac{3b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{3\sqrt{b}(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] (3*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*
x]^2]])/(2*f) + (3*b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f) - (
Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/f
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f}+\frac{(3b)\,\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx,x\right.}{f}$$

$$=\frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\left(a+b+b\tan^2(e+f\right.}{f}$$

$$=\frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}-\frac{\cot(e+fx)\left(a+b+b\tan^2(e+f\right.}{f}$$

$$=\frac{3\sqrt{b}(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+}}{2f}$$

Mathematica [C] time = 0.221493, size = 64, normalized size = 0.61

$$-\frac{(a+b)\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{b\sin^2(e+fx)}{-a\sin^2(e+fx)+a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + b)*Cot[e + f*x]*Hypergeometric2F1[-1/2, 2, 1/2, (b*Sin[e + f*x]^2)/
(a + b - a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

Maple [C] time = 0.345, size = 2032, normalized size = 19.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+
e)^2)^(3/2)*cos(f*x+e)*(6*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+6*s
in(f*x+e)*cos(f*x+e)^3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*sin(f*x+e)*cos(f*x+e)^3*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-3*sin(f*x+e)*cos(f*x+e)^3
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))
```

```
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+6*sin(f*x+e)*cos(f*x
+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*a*b+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2
-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a*b-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*b^2+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos
(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+cos(f*x+e)^2*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/sin(f*x+e)
/(b+a*cos(f*x+e)^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30849, size = 932, normalized size = 8.88

$$\left[\frac{3\,(a+b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^4}\right)}{8\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^
4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*
b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e)^2 - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x + e)), 1/4
*(3*(a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - 2*((2*a + 3*b)*cos(f*x +
 e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin
(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

3.91 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e\right.}{3f(a+b)}$$

```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 5*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*(a
 + b)*f)
```

Rubi [A] time = 0.152781, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4132, 453, 277, 195, 217, 206}

$$\frac{b(3a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^3(e+fx)\left(a+b\tan^2(e\right.}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```



```
[Out] (Sqrt[b]*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 5*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*(a
 + b)*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^{3/2}}{x^4}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} + \frac{(3a+5b)\text{Subst}\left(\int \frac{(a+b+bx^2)^{3/2}}{x^2}\right.}{3(a+b)f}$$

$$= -\frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{\cot^3(e+fx)\left(a+b+\right.}{3(a+}$$

$$= \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)\cot(e+fx)\left(a\right.}{3(a-}$$

$$= \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)\cot(e+fx)\left(a\right.}{3(a-}$$

$$= \frac{\sqrt{b}(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+}}{2(a+b)f}$$

Mathematica [C] time = 8.62628, size = 369, normalized size = 2.15

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\frac{i\left(4a\left(-4e^{2i(e+fx)}+e^{4i(e+fx)}+1\right)\left(1+e^{2i(e+fx)}\right)^2+b\left(-20e^{2i(e+fx)}-22e^{4i(e+fx)}-20e^{6i(e+f}\right.\right.}{\left(-1+e^{2i(e+fx)}\right)^3\left(1+e^{2i(e+fx)}\right)^2}\right.}{3f(a\cos(2e+2fx)+a+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(4*a*(1 + E^((2*I)*(e + f*x)))^2*(1 - 4
*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x))) + b*(15 - 20*E^((2*I)*(e + f*x)
) - 22*E^((4*I)*(e + f*x)) - 20*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)
))))/((-1 + E^((2*I)*(e + f*x)))^3*(1 + E^((2*I)*(e + f*x)))^2) - (3*Sqrt[b
]*(3*a + 5*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b
```

```
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e +
 f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a
 + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.396, size = 3925, normalized size = 22.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/6/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(18*cos(f*x+e)^5*sin(f*x+e)*2
^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*a*b+30*cos(f*x+e)^5*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-9*cos
(f*x+e)^5*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-
15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^5*sin(f*x+e
)*b^2+18*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))
^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+30*2^(1/2)*(1/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(
```

f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*cos(f*x+e)^4*sin(f*x+e)*b^2-9*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*cos(f*x+e)^4*sin(f*x+e)*b^2-18*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-30*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+9*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+15*sin(f*x+e)*cos(f*x+e)^3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-18*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-30*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+

```
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*b^2+9*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a*b+15*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+4*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a^2+15*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*
b-6*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-16*cos(f*x+e)^
4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+15*cos(f*x+e)^4*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b-20*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+
3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)^2/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.95812, size = 1180, normalized size = 6.86

$$\left[\frac{3\left((3a+5b)\cos\left(fx+e\right)^3-(3a+5b)\cos\left(fx+e\right)\right)\sqrt{b}\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\cos\left(fx+e\right)^3+2b\cos\right.}{\cos\left(fx+e\right)^4}\right.}{24\left(f\cos\left(fx+e\right)^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e))*sqrt(b)*lo
g(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*(
(4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b)*cos(f*x + e)^2 + 3*b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*s
in(f*x + e)), 1/12*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e
))*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*s
in(f*x + e)))*sin(f*x + e) - 2*((4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b
)*cos(f*x + e)^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*co
s(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

3.92 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=209

$$\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+\right.}{5f(a+b)}$$

```
[Out] (Sqrt[b]*(3*a + 7*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 7*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (2*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*
(a + b)*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(5/2))/(5*(a + b)*f
)
```

Rubi [A] time = 0.20454, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4132, 462, 453, 277, 195, 217, 206}

$$\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f(a+b)}+\frac{\sqrt{b}(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}-\frac{\cot^5(e+fx)\left(a+b\tan^2(e+\right.}{5f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(3*a + 7*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/(2*f) + (b*(3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*(a + b)*f) - ((3*a + 7*b)*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/
2))/(3*(a + b)*f) - (2*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*
(a + b)*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(5/2))/(5*(a + b)*f
)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}\left(10(a+b)+5(a+\right.}{x^4}\right.}{5(a+b)}$$

$$= -\frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f} - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e\right.}{5(a+b)f}$$

$$= -\frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{2\cot^3(e+fx)\left(a+b+\right.}{3(a+b}$$

$$= \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+7b)\cot(e+fx)\left(a+\right.}{3(a+}$$

$$= \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+7b)\cot(e+fx)\left(a+\right.}{3(a+}$$

$$= \frac{\sqrt{b}(3a+7b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b}}{2(a+b)f}$$

Mathematica [C] time = 10.4456, size = 512, normalized size = 2.45

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\frac{i\left(16a^2\left(1+e^{2i(e+fx)}\right)^2\left(-6e^{2i(e+fx)}+16e^{4i(e+fx)}-6e^{6i(e+fx)}+e^{8i(e+fx)}+1\right)+ab\left(-402e^{2i(e\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(16*a^2*(1 + E^((2*I)*(e + f*x)))^2*(1
- 6*E^((2*I)*(e + f*x)) + 16*E^((4*I)*(e + f*x)) - 6*E^((6*I)*(e + f*x)) +
E^((8*I)*(e + f*x))) + b^2*(105 - 350*E^((2*I)*(e + f*x)) + 231*E^((4*I)*(e
 + f*x)) + 412*E^((6*I)*(e + f*x)) + 231*E^((8*I)*(e + f*x)) - 350*E^((10*I
)*(e + f*x)) + 105*E^((12*I)*(e + f*x))) + a*b*(115 - 402*E^((2*I)*(e + f*x
)) + 317*E^((4*I)*(e + f*x)) + 708*E^((6*I)*(e + f*x)) + 317*E^((8*I)*(e +
f*x)) - 402*E^((10*I)*(e + f*x)) + 115*E^((12*I)*(e + f*x)))))/((a + b)*(-1
 + E^((2*I)*(e + f*x)))^5*(1 + E^((2*I)*(e + f*x)))^2) - (15*Sqrt[b]*(3*a +
 7*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))]
)/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec
[e + f*x]^2)^(3/2))/(15*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [C] time = 0.951, size = 8726, normalized size = 41.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 20.3657, size = 1732, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b + 7
*b^2)*cos(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(b)*log((
(a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((16
*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 245*b^2)*cos
(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a*b - 15*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^5 -
 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x + e)), 1/60*(
15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b + 7*b^2)*co
s(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(-b)*arctan(-1/2*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x +
e) - 2*((16*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 2
45*b^2)*cos(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a
*b - 15*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f
*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x +
e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \csc\left(fx+e\right)^6\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

3.93 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{\left(15a^2+20ab+8b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f}+\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f}-\frac{\cos^5(e+fx)\sqrt{a+\ldots}}{5a\ldots}$$

```
[Out] -((15*a^2 + 20*a*b + 8*b^2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(15*a^
3*f) + (2*(5*a + 2*b)*Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(15*a^2*f)
 - (Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2])/(5*a*f)
```

Rubi [A] time = 0.138716, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4134, 462, 453, 264}

$$-\frac{\left(15a^2+20ab+8b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f}+\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f}-\frac{\cos^5(e+fx)\sqrt{a+\ldots}}{5a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((15*a^2 + 20*a*b + 8*b^2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(15*a^
3*f) + (2*(5*a + 2*b)*Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(15*a^2*f)
 - (Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2])/(5*a*f)
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
```

```
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af} + \frac{\text{Subst}\left(\int \frac{-2(5a+2b)+5ax^2}{x^4\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af} + \frac{\left(15a^2+\right.}{}$$

$$= -\frac{\left(15a^2+20ab+8b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} + \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\,\text{se}}}{15a^2f}$$

Mathematica [A] time = 0.980078, size = 93, normalized size = 0.76

$$-\frac{\sec(e+fx)(a\cos(2(e+fx))+a+2b)\left(3a^2\cos(4(e+fx))+89a^2-4a(7a+4b)\cos(2(e+fx))+144ab+64b^2\right)}{240a^3f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(89*a^2 + 144*a*b + 64*b^2 - 4*a*(7*a + 4*
b)*Cos[2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x])/(240*a^3*f*Sqrt
[a + b*Sec[e + f*x]^2])
```

Maple [A] time = 0.427, size = 105, normalized size = 0.9

$$-\frac{\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)\left(3\left(\cos\left(fx+e\right)\right)^{4}a^{2}-10\left(\cos\left(fx+e\right)\right)^{2}a^{2}-4\left(\cos\left(fx+e\right)\right)^{2}ab+15a^{2}+20ab+8b^{2}\right)}{15fa^{3}\cos\left(fx+e\right)} \frac{1}{\sqrt{\frac{b+\cdots}{\cdots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f/a^3*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^4*a^2-10*cos(f*x+e)^2*a^2-4*co
s(f*x+e)^2*a*b+15*a^2+20*a*b+8*b^2)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)
/cos(f*x+e)
```

Maxima [A] time = 1.21052, size = 219, normalized size = 1.78

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a}-\frac{10\left(\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{3}-3\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\,b\cos\left(fx+e\right)\right)}{a^{2}}+\frac{3\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^{5}-10\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}b\cdots}{a^{3}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - 10*((a + b/cos(f*x +
e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a
^2 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a + b/cos(f*x + e
)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e
))/a^3)/f
```

Fricas [A] time = 0.573482, size = 213, normalized size = 1.73

$$-\frac{\left(3\,a^{2}\cos\left(fx+e\right)^{5}-2\left(5\,a^{2}+2\,ab\right)\cos\left(fx+e\right)^{3}+\left(15\,a^{2}+20\,ab+8\,b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{15\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(f*x + e)^5 - 2*(5*a^2 + 2*a*b)*cos(f*x + e)^3 + (15*a^2 +
20*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
a^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{5}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.94 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}-\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}$$

```
[Out] -((3*a + 2*b)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(3*a^2*f) + (Cos[e +
 f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a*f)
```

Rubi [A] time = 0.0910306, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4134, 453, 264}

$$\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}-\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] -((3*a + 2*b)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(3*a^2*f) + (Cos[e +
 f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a*f)
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af} + \frac{(3a+2b)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{3af}$$

$$= -\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2f} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}$$

Mathematica [A] time = 0.294836, size = 64, normalized size = 0.86

$$\frac{\sec(e+fx)(a\cos(2(e+fx))-5a-4b)(a\cos(2(e+fx))+a+2b)}{12a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-5*a - 4*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f
*x])/(12*a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A] time = 0.344, size = 69, normalized size = 0.9

$$\frac{\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)\left(a\left(\cos\left(fx+e\right)\right)^2-3\,a-2\,b\right)}{3\,fa^2\cos\left(fx+e\right)}\frac{1}{\sqrt{\frac{b+a\left(\cos\left(fx+e\right)\right)^2}{\left(\cos\left(fx+e\right)\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f/a^2*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-3*a-2*b)/((b+a*cos(f*x+e)^2)/c
os(f*x+e)^2)^(1/2)/cos(f*x+e)
```

Maxima [A] time = 1.02358, size = 112, normalized size = 1.51

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^2}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - ((a + b/cos(f*x + e)^2)
^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^2)/f
```

Fricas [A] time = 0.550606, size = 139, normalized size = 1.88

$$\frac{\left(a\cos\left(fx+e\right)^3-(3\,a+2\,b)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/(a^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.95 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=30

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

```
[Out] -((Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a*f))
```

Rubi [A] time = 0.0420239, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4134, 264}

$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a*f))
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} = -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

Mathematica [A] time = 0.114465, size = 48, normalized size = 1.6

$$-\frac{\sec(e+fx)(a\cos(2e+2fx)+a+2b)}{2af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x])/(2*a*f*Sqrt[a + b*Sec[e + f*
x]^2])
```

Maple [A] time = 0.076, size = 31, normalized size = 1.

$$-\frac{1}{fa\sec\left(fx+e\right)}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)
```

Maxima [A] time = 0.995049, size = 38, normalized size = 1.27

$$-\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

Fricas [A] time = 0.511495, size = 88, normalized size = 2.93

$$-\frac{\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [B] time = 1.66257, size = 81, normalized size = 2.7

$$\frac{\sqrt{b}\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{a|f|}-\frac{\sqrt{a\cos\left(fx+e\right)^{2}+b}}{a|f|\mathrm{sgn}\left(f\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b)*sgn(f)*sgn(cos(f*x + e))/(a*abs(f)) - sqrt(a*cos(f*x + e)^2 + b)/(a
*abs(f)*sgn(f)*sgn(cos(f*x + e)))
```

3.96 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(Sqrt[a +
b]*f))
```

Rubi [A] time = 0.0677984, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4134, 377, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(Sqrt[a +
b]*f))
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}f}\end{aligned}$$

Mathematica [A] time = 0.106268, size = 86, normalized size = 2.

$$-\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right)}{\sqrt{2}f\sqrt{a+b}\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + 2*b + a*Cos
[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a + b]*f*Sqrt[a + b*Sec[e + f*x]
^2]))
```

Maple [B] time = 0.379, size = 280, normalized size = 6.5

$$\frac{\left(\sin\left(fx+e\right)\right)^2}{2f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{b+a\left(\cos\left(fx+e\right)\right)^2}{\left(1+\cos\left(fx+e\right)\right)^2}}\left(\ln\left(-2\frac{-1+\cos\left(fx+e\right)}{\sqrt{a+b}\left(\sin\left(fx+e\right)\right)^2}\left(\cos\left(fx+e\right)\sqrt{\frac{b+a\left(\cos\right.}{\left(1+\cos\left(\right.\right.}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(ln(-2/(a+b)^
(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+b)/sin(f*x+e)^2)+ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e))))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/co
s(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A] time = 0.698004, size = 360, normalized size = 8.37

$$\left[\frac{\log\left(\frac{2\left(a\cos\left(fx+e\right)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+a+2\,b\right)}{\cos\left(fx+e\right)^2-1}\right)}{2\sqrt{a+b}f},\ \frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a+b}\right)}{(a+b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))/(sqrt(a + b)*f)
```

```
, sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*cos(f*x + e)/(a + b))/((a + b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.97 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=87

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{3/2}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}$$

```
[Out] -(a*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2*(a +
 b)^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a
 + b)*f)
```

Rubi [A] time = 0.109579, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4134, 471, 12, 377, 207}

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{3/2}}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] -(a*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2*(a +
 b)^(3/2)*f) - (Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a
 + b)*f)
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}+\frac{\text{Subst}\left(\int \frac{a}{\left(-1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}\\
&= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}+\frac{a\,\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}\\
&= -\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}+\frac{a\,\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)f}\\
&= -\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}
\end{aligned}
$$

Mathematica [A] time = 1.03311, size = 140, normalized size = 1.61

$$-\frac{a\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos(2e+2fx)+a+2b}\left(\frac{(a+b)\csc^2(e+fx)}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right)}{2\sqrt{2}f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(a*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]*Sqrt[a + b - a*Sin[e +
f*x]^2]*(((a + b)*Csc[e + f*x]^2)/a + ArcTanh[Sqrt[1 - (a*Sin[e + f*x]^2)/(
a + b)]]/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))/(2*Sqrt[2]*(a + b)^2*f*Sqrt
[a + b*Sec[e + f*x]^2])
```

Maple [B] time = 0.414, size = 2199, normalized size = 25.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4/f/(a+b)^(5/2)*(cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2+cos(f*x+e)^3*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b+cos(f*x+
e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2+cos(f*x+e
)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b+cos(f*x+e)
^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*
x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a
*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(
```

```
f*x+e)^2)*a^2+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-
2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2+cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b-2*cos(f*x+e)^2*(a+b)^(3/2)*a-cos(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+
e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*c
os(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*
x+e)^2)*a^2-cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a
+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b-cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/(-1+cos(f*x+e)))*a^2-cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)+b)/(-1+cos(f*x+e)))*a*b-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b-((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)
/(-1+cos(f*x+e)))*a*b-2*(a+b)^(3/2)*b)*sin(f*x+e)^2/(-1+cos(f*x+e))^2/cos(f
*x+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(1+cos(f*x+e))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A] time = 0.794466, size = 765, normalized size = 8.79

$$\left[\frac{2\,(a+b)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+a+2\,b\right)}{\cos\left(fx+e\right)^2-1}\right)}{4\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2-\left(a^2+2\,ab+b^2\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
(a*cos(f*x + e)^2 - a)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*
x + e)^2 - 1)))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)
*f), 1/2*((a*cos(f*x + e)^2 - a)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a + b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/((a^2 + 2*a*b + b^2)*f*co
s(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.98 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=138

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}-\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)^2}$$

```
[Out] (-3*a^2*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(8*
(a + b)^(5/2)*f) - ((5*a + 2*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(8*(a + b)^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(4*(a + b)*f)
```

Rubi [A] time = 0.165052, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4134, 470, 527, 12, 377, 207}

$$-\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}-\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (-3*a^2*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(8*
(a + b)^(5/2)*f) - ((5*a + 2*b)*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(8*(a + b)^2*f) - (Cot[e + f*x]^3*Csc[e + f*x]*Sqrt[a + b*Sec[e
+ f*x]^2])/(4*(a + b)*f)
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{-a-2(2a+b)x^2}{\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\se}}{4(a+b)f}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\se}}{4(a+b)f}$$

$$= -\frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\se}}{4(a+b)f}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{5/2}f} - \frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \text{cot}$$

Mathematica [C] time = 0.18188, size = 78, normalized size = 0.57

$$-\frac{a^2\sec(e+fx)(a\cos(2(e+fx))+a+2b)\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right)}{2f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(a^2*(a + 2*b + a*Cos[2*(e + f*x)])*Hypergeometric2F1[1/2, 3, 3/2, 1 - (a*
Sin[e + f*x]^2)/(a + b)]*Sec[e + f*x])/(2*(a + b)^3*f*Sqrt[a + b*Sec[e + f*
x]^2])
```

Maple [B] time = 0.412, size = 4983, normalized size = 36.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/16/f/(a+b)^(9/2)*(6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a
+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^2+6*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2-2*cos(f*x+e)^2*(a+b)^(5/2)*a*b+3*cos(f*x
+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos
(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/s
in(f*x+e)^2)*a^4+3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1
+cos(f*x+e)))*a^4+3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4+3*cos(f*x+e)^4*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-6*cos(f*x+e)^3*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4-6*co
s(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-6*c
os(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(
-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+b)/sin(f*x+e)^2)*a^4-6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
b)/(-1+cos(f*x+e)))*a^4+3*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4+3*cos(f*x+e)*((b+a*cos(f*x+
```

```
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4+3*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4+10*(a+b)^(5/2)*a*b+3*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4-6*cos(f*
x+e)^4*(a+b)^(5/2)*a^2+10*cos(f*x+e)^2*(a+b)^(5/2)*a^2+6*cos(f*x+e)^5*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(c
os(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x
+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2
)*a^3*b+3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a
+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^2+6*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b+3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2+6*cos(f*x+e)^4*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b+4
*(a+b)^(5/2)*b^2+3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^2+6*cos(f*x+e)^4*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b+3*cos(f*x+e)^4*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2-12*cos(f*x+e)^3*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(
cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^
2)*a^3*b-6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(
a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^2-12*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
```

```
1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b-6*cos(f*x+e)^3*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2-12*cos(f*x+e)^2*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*
b-6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1
/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/sin(f*x+e)^2)*a^2*b^2-12*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b-6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2+6*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b+3*cos(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(
f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/si
n(f*x+e)^2)*a^2*b^2+6*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-
1+cos(f*x+e)))*a^3*b+3*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(
-1+cos(f*x+e)))*a^2*b^2)/(-1+cos(f*x+e))^2/cos(f*x+e)/((b+a*cos(f*x+e)^2)/c
os(f*x+e)^2)^(1/2)/(1+cos(f*x+e))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.919154, size = 1185, normalized size = 8.59

$$\left[\frac{3\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^2-2\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)+a+2\,b\right)}{\cos\left(fx+e\right)^2-1}\right)+2\left(3\left(a^2+ab\right.\right.}{16\left(\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right)^4-2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(a + b)*log(
2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 + a*b)*cos(f
*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*f), 1/8*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(-a
 - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/(a + b)) + (3*(a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos
(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.99 $\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=193

$$-\frac{(33a^2+40ab+15b^2)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^3f}+\frac{5(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{(9a+5b}{}$$

```
[Out] (5*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(16*a^(7/2)*f) - ((33*a^2 + 40*a*b + 15*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqr
t[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + ((9*a + 5*b)*Cos[e + f*x]^3*Sin[e
 + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos[e + f*x]^3*Sin[e
+ f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```

Rubi [A] time = 0.278349, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4132, 470, 578, 527, 12, 377, 203}

$$-\frac{(33a^2+40ab+15b^2)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^3f}+\frac{5(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{(9a+5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```



```
[Out] (5*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(16*a^(7/2)*f) - ((33*a^2 + 40*a*b + 15*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqr
t[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + ((9*a + 5*b)*Cos[e + f*x]^3*Sin[e
 + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos[e + f*x]^3*Sin[e
+ f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)-2(3a+b)x^2\right)}{\left(1+x^2\right)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(\right.}{6af}$$

$$= \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a}}{6af}$$

$$= -\frac{\left(33a^2+40ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e}{}$$

$$= -\frac{\left(33a^2+40ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e}{}$$

$$= -\frac{\left(33a^2+40ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{(9a+5b)\cos^3(e}{}$$

$$= \frac{5(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} - \frac{\left(33a^2+40ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b}}{48a^3f}$$

Mathematica [A] time = 1.6466, size = 163, normalized size = 0.84

$$\frac{\sec(e+fx)\sqrt{a\cos(2(e+fx))+a+2b}\left(15(a+b)^3\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)-\sqrt{a}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,\right(}{48\sqrt{2}a^{7/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(15*(a + b)^3*ArcTan[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sq
```

```
rt[a + b - a*Sin[e + f*x]^2]*(15*(a + b)^2 + 10*a*(a + b)*Sin[e + f*x]^2 +
8*a^2*Sin[e + f*x]^4)))/(48*Sqrt[2]*a^(7/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.62, size = 2425, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/48/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^3*sin(f*x+e)*(15*2^(1/2)*
(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-30*2^(1/2)*(1/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*a^3*sin(f*x+e)+45*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2
*b*sin(f*x+e)+45*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*si
n(f*x+e)-90*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(
1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-90*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
```

```
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2
*sin(f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-30*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*b^3*sin(f*x+e)+15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
^3*sin(f*x+e)+8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*
cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-26*cos(f*x+e)^5*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+26*cos(f*x+e)^4*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^3+33*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^3-33*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+15*
cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-33*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^2*b-40*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b
^2-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*cos(f*x+e
)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+14*cos(f*x+e)^3*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a*b^2-14*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a^2*b-5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+33*c
os(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+40*cos(f*x+e)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2)/(-1+cos(f*x+e))/((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A] time = 4.38913, size = 1543, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)
^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 - 2*(1
3*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 15*a*b^2)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f), -1/1
92*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^3*co
s(f*x + e)^5 - 2*(13*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 1
5*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^4*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.100 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=135

$$\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}-\frac{(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+}{}$$

```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*a^(5/2)*f) - ((5*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(4*a*f)
```

Rubi [A] time = 0.15224, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4132, 470, 527, 12, 377, 203}

$$\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}-\frac{(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*a^(5/2)*f) - ((5*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(4*a*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{a+b-2(2a+b)x^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{4af}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a}}{4af}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a}}{4af}$$

$$= -\frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a}}{4af}$$

$$= \frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e}}{8a^2f}$$

Mathematica [A] time = 0.605738, size = 145, normalized size = 1.07

$$\frac{\sec(e+fx)\sqrt{a\cos(2(e+fx))+a+2b}\left(3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)-\sqrt{a}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\,(\right.}{8\sqrt{2}a^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(3*(a + b)^2*ArcTan[(Sqrt[
a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqr
t[a + b - a*Sin[e + f*x]^2]*(3*(a + b) + 2*a*Sin[e + f*x]^2)))/(8*Sqrt[2]*a
^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.425, size = 1701, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2*sin(f*x+e)*(-2*cos(f*x+e
)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-12*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*a*b*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+2*cos(f*x+e)^4*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a*b-5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+
e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*cos(f*x+e)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*b^2-5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+
```

```
e)^2)^(1/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A] time = 1.61722, size = 1362, normalized size = 10.09

$$\left[-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 + 3*a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*
f), -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*s
```

```
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
 a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x
+ e)^3 - (5*a^2 + 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.101 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=85

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rubi [A] time = 0.109313, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4132, 471, 12, 377, 203}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) - (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{\text{Subst}\left(\int \frac{a+b}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x,\right.}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e}{\sqrt{a+b+b\,\text{ta}}}\right.}{2af}$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

Mathematica [A] time = 0.274719, size = 125, normalized size = 1.47

$$\frac{\sec(e+fx)\sqrt{a\cos(2(e+fx))+a+2b}\left((a+b)\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)-\sqrt{a}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\right)}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]*S
in[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqrt[a
+ b - a*Sin[e + f*x]^2]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.355, size = 1055, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a*sin(f*x+e)*(2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)-2*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*b)/(-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+
e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 0.931211, size = 1218, normalized size = 14.33

$$\left[-\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-a}(a+b)\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) + sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)))/(a^2*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)*sin(f*x + e) + (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.102 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rubi [A] time = 0.0288085, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4128, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [B] time = 0.0668762, size = 87, normalized size = 2.23

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [C] time = 0.409, size = 380, normalized size = 9.7

$$-\frac{\sqrt{2}\left(\sin\left(fx+e\right)\right)^2}{f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{1}{(a+b)\left(1+\cos\left(fx+e\right)\right)}}\left(i\cos\left(fx+e\right)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+a\cos\left(fx+e\right)+b\right)\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*(EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*
x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.762084, size = 976, normalized size = 25.03

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+a^4-28\,a^3b\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
```

```
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.103 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

```
[Out] -((Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f))
```

Rubi [A] time = 0.0710493, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {4132, 264}

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f))
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

Mathematica [A] time = 0.112694, size = 55, normalized size = 1.67

$$-\frac{\csc(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+2b)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/(2*(a + b)*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Maple [A] time = 0.322, size = 48, normalized size = 1.5

$$-\frac{\cos(fx+e)}{f(a+b)\sin(fx+e)}\sqrt{\frac{b+a(\cos(fx+e))^2}{(\cos(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/(a+b)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.529402, size = 113, normalized size = 3.42

$$-\frac{\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{(a+b)f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a + b)*f*sin(f*
x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.104 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}-\frac{(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

```
[Out] -((3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)^2*f) -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rubi [A] time = 0.0970371, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4132, 453, 264}

$$-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}-\frac{(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] -((3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)^2*f) -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{(3a+b)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}\\ &= -\frac{(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}\end{aligned}$$

Mathematica [A] time = 0.209077, size = 74, normalized size = 0.95

$$\frac{\csc^3(e+fx)\sec(e+fx)(a\cos(2(e+fx))-2a-b)(a\cos(2(e+fx))+a+2b)}{6f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-2*a - b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x
]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A] time = 0.343, size = 66, normalized size = 0.9

$$\frac{\left(2\,a\left(\cos\left(fx+e\right)\right)^2-3\,a-b\right)\cos\left(fx+e\right)}{3\,f\,(a+b)^2\left(\sin\left(fx+e\right)\right)^3}\sqrt{\frac{b+a\left(\cos\left(fx+e\right)\right)^2}{\left(\cos\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f/(a+b)^2*(2*a*cos(f*x+e)^2-3*a-b)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1
/2)*cos(f*x+e)/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.729017, size = 235, normalized size = 3.01

$$-\frac{\left(2\,a\cos\left(fx+e\right)^{3}-\left(3\,a+b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\left(\left(a^{2}+2\,ab+b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{2}+2\,ab+b^{2}\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b +
 b^2)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.105 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=132

$$-\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f(a+b)}-\frac{2(5a+3b)\cot^3(e}{}$$

```
[Out] -((15*a^2 + 10*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(1
5*(a + b)^3*f) - (2*(5*a + 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^
2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5*(
a + b)*f)
```

Rubi [A] time = 0.139957, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4132, 462, 453, 264}

$$-\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5f(a+b)}-\frac{2(5a+3b)\cot^3(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((15*a^2 + 10*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(1
5*(a + b)^3*f) - (2*(5*a + 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^
2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5*(
a + b)*f)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{2(5a+3b)+5(a+b)x^2}{x^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} -$$

$$= -\frac{\left(15a^2+10ab+3b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} - \frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+}}{15(a+b)^2f}$$

Mathematica [A] time = 0.350556, size = 100, normalized size = 0.76

$$-\frac{\csc^5(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+2b)\left(a^2\cos(4(e+fx))+8a^2-2a(3a+b)\cos(2(e+fx))+8ab+3b^2\right)}{30f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(8*a^2 + 8*a*b + 3*b^2 - 2*a*(3*a + b)*Cos
[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/(30*(a +
 b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [A] time = 0.383, size = 101, normalized size = 0.8

$$-\frac{\left(8\left(\cos\left(fx+e\right)\right)^{4}a^{2}-20\left(\cos\left(fx+e\right)\right)^{2}a^{2}-4\left(\cos\left(fx+e\right)\right)^{2}ab+15a^{2}+10ab+3b^{2}\right)\cos\left(fx+e\right)}{15f\left(a+b\right)^{3}\left(\sin\left(fx+e\right)\right)^{5}}\sqrt{\frac{b+a\left(\cos\right.}{\left(\cos\left(f\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15/f/(a+b)^3*(8*cos(f*x+e)^4*a^2-20*cos(f*x+e)^2*a^2-4*cos(f*x+e)^2*a*b+
15*a^2+10*a*b+3*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*cos(f*x+e)/sin
(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81973, size = 409, normalized size = 3.1

$$-\frac{\left(8a^{2}\cos\left(fx+e\right)^{5}-4\left(5a^{2}+ab\right)\cos\left(fx+e\right)^{3}+\left(15a^{2}+10ab+3b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{15\left(\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*a^2*cos(f*x + e)^5 - 4*(5*a^2 + a*b)*cos(f*x + e)^3 + (15*a^2 + 10
*a*b + 3*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.106 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{2b\left(15a^2+40ab+24b^2\right)\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(15a^2+40ab+24b^2\right)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}}+\frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos^5(e}{5af\sqrt{a+b\,\mathrm{s}}}$$

```
[Out] -((15*a^2 + 40*a*b + 24*b^2)*Cos[e + f*x])/(15*a^3*f*Sqrt[a + b*Sec[e + f*x
]^2]) + (2*(5*a + 3*b)*Cos[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Sec[e + f*x]^2]
) - Cos[e + f*x]^5/(5*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(15*a^2 + 40*a
*b + 24*b^2)*Sec[e + f*x])/(15*a^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.187001, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4134, 462, 453, 271, 191}

$$-\frac{2b\left(15a^2+40ab+24b^2\right)\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}}+\frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(\frac{8b(5a+3b)}{a^2}+15\right)\cos(e+fx)}{15af\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((15 + (8*b*(5*a + 3*b))/a^2)*Cos[e + f*x])/(15*a*f*Sqrt[a + b*Sec[e + f*x
]^2]) + (2*(5*a + 3*b)*Cos[e + f*x]^3)/(15*a^2*f*Sqrt[a + b*Sec[e + f*x]^2]
) - Cos[e + f*x]^5/(5*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(15*a^2 + 40*a
*b + 24*b^2)*Sec[e + f*x])/(15*a^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-2(5a+3b)+5ax^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} - \frac{\left(-15a^2-8b(5a+3b)\right)\text{Subst}\left(\int\right.}{15a}$$

$$= -\frac{\left(15a^2+8b(5a+3b)\right)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+}}$$

$$= -\frac{\left(15a^2+8b(5a+3b)\right)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+}}$$

Mathematica [B] time = 7.61137, size = 432, normalized size = 2.53

$$-\frac{\sec^3(e+fx)\left(-2a^2\cos(4(e+fx))+27a^2+16a(a+2b)\cos(2(e+fx))+128ab+128b^2\right)(a\cos(2e+2fx)+a+2b)^{3/2}}{192a^3f\sqrt{a\cos(2(e+fx))+a+2b}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(64*a*f*Sqrt[a + 2*
b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) + ((2*a + 4*b + a*Cos
[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(32*a^2
*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) - ((27*
a^2 + 128*a*b + 128*b^2 + 16*a*(a + 2*b)*Cos[2*(e + f*x)] - 2*a^2*Cos[4*(e
+ f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(192*a^3*f*Sq
rt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2)) - ((40*a^3 +
 336*a^2*b + 768*a*b^2 + 512*b^3 + a*(25*a^2 + 128*a*b + 128*b^2)*Cos[2*(e
+ f*x)] - 4*a^2*(a + 2*b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])*(a + 2*b
 + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(320*a^4*f*Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] time = 2.064, size = 35190, normalized size = 205.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.04516, size = 338, normalized size = 1.98

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}-\frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^3}+\frac{15\,b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)}+\frac{30\,b^2}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^3\,\mathrm{c}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - 10*((a + b/cos(f*x
+ e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^3 + 15*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 30*b^2/(sqrt(a
+ b/cos(f*x + e)^2)*a^3*cos(f*x + e)) + 15*b^3/(sqrt(a + b/cos(f*x + e)^2)*
a^4*cos(f*x + e)) + 3*((a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a +
 b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b
^2*cos(f*x + e))/a^4)/f
```

Fricas [A] time = 0.863724, size = 324, normalized size = 1.89

$$-\frac{\left(3\,a^3\cos\left(fx+e\right)^7-2\left(5\,a^3+3\,a^2b\right)\cos\left(fx+e\right)^5+\left(15\,a^3+40\,a^2b+24\,ab^2\right)\cos\left(fx+e\right)^3+2\left(15\,a^2b+40\,ab^2+24\,b\right.}{15\left(a^5f\cos\left(fx+e\right)^2+a^4bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^3*cos(f*x + e)^7 - 2*(5*a^3 + 3*a^2*b)*cos(f*x + e)^5 + (15*a^3
+ 40*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 2*(15*a^2*b + 40*a*b^2 + 24*b^3)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)
^2 + a^4*b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.107 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -((3*a + 4*b)*Cos[e + f*x])/(3*a^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Cos[e +
f*x]^3/(3*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(3*a + 4*b)*Sec[e + f*x])/
(3*a^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.116966, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4134, 453, 271, 191}

$$-\frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a + 4*b)*Cos[e + f*x])/(3*a^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Cos[e +
f*x]^3/(3*a*f*Sqrt[a + b*Sec[e + f*x]^2]) - (2*b*(3*a + 4*b)*Sec[e + f*x])/
(3*a^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{(3a+4b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx,x,\sec(e+fx)\right)}{3af}$$

$$= -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{(2b(3a+4b))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3}}\right.}{3a^2f}$$

$$= -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A]   time = 3.68715, size = 93, normalized size = 0.82

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(a^2(-\cos(4(e+fx)))+9a^2+8a(a+2b)\cos(2(e+fx))+64ab+64b^2\right)}{48a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(9*a^2 + 64*a*b + 64*b^2 + 8*a*(a + 2*b)*C
os[2*(e + f*x)] - a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^3)/(48*a^3*f*(a + b*Se
c[e + f*x]^2)^(3/2))
```

Maple [B] time = 0.89, size = 12782, normalized size = 112.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.01991, size = 190, normalized size = 1.67

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}-\frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^3}+\frac{3b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^2\cos(fx+e)}+\frac{3b^2}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,a^3\cos(fx+e)}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - ((a + b/cos(f*x + e)^
2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^3
+ 3*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 3*b^2/(sqrt(a + b/cos
(f*x + e)^2)*a^3*cos(f*x + e)))/f
```

Fricas [A] time = 0.722797, size = 228, normalized size = 2.

$$\frac{\left(a^2\cos\left(fx+e\right)^5-\left(3a^2+4ab\right)\cos\left(fx+e\right)^3-2\left(3ab+4b^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(a^4f\cos\left(fx+e\right)^2+a^3bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*cos(f*x + e)^5 - (3*a^2 + 4*a*b)*cos(f*x + e)^3 - 2*(3*a*b + 4*b^2
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^4*f*cos(f*x
+ e)^2 + a^3*b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.108 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -(Cos[e + f*x]/(a*f*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*
f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.0561524, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4134, 271, 191}

$$-\frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/(a*f*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*
f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A] time = 1.42073, size = 64, normalized size = 1.03

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)(a\cos(2(e+fx))+a+4b)}{4a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(a + 4*b + a*Cos[2*(e + f*x)])*Sec[e + f*x
]^3)/(4*a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.052, size = 59, normalized size = 1.

$$\frac{1}{f}\left(-\frac{1}{a\sec\left(fx+e\right)}\frac{1}{\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}}-2\frac{b\sec\left(fx+e\right)}{a^2\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)-2*b/a^2*sec(f*x+e)/(a+b*sec(f
*x+e)^2)^(1/2))
```

Maxima [A] time = 1.00267, size = 77, normalized size = 1.24

$$-\frac{\frac{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^2}+\frac{b}{\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}a^2\cos\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 + b/(sqrt(a + b/cos(f*x + e)^
2)*a^2*cos(f*x + e)))/f
```

Fricas [A] time = 0.607297, size = 158, normalized size = 2.55

$$-\frac{\left(a\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}}{a^3f\cos\left(fx+e\right)^2+a^2bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(a*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(a^3*f*cos(f*x + e)^2 + a^2*b*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.109 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{b\sec(e+fx)}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
3/2)*f)) - (b*Sec[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.0921376, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4134, 382, 377, 207}

$$-\frac{b\sec(e+fx)}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
3/2)*f)) - (b*Sec[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= -\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{(a+b)f}\\
&= -\frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-1-(-a-b)x^2}\,dx, x, \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)f}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}f} - \frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.837625, size = 113, normalized size = 1.41

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(a\sqrt{-a\sin^2(e+fx)+a+b}\tanh^{-1}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{a+b}}\right)+b\sqrt{a+b}\right)}{2af(a+b)^{3/2}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(b*Sqrt[a + b] + a*ArcTanh[
Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + b - a*Sin[e + f*x]^2])
)/(2*a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] time = 0.355, size = 1094, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/(a+b)^(5/2)/a*(b+a*cos(f*x+e)^2)*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2+cos(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e)
)*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+
e)^2)*a*b+cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos
(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)
))*a^2+cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*
a*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f
*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-
a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin
(f*x+e)^2)*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2
)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)+b)/sin(f*x+e)^2)*a*b+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4
*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f
*x+e)))*a^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b+2*
(a+b)^(3/2)*b)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 0.847261, size = 821, normalized size = 10.26

$$\left[-\frac{2\left(ab+b^2\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)-\left(a^2\cos(fx+e)^2+ab\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\right.}{\cos(fx+e)^2-1}\right.}{2\left(\left(a^4+2a^3b+a^2b^2\right)f\cos(fx+e)^2+\left(a^3b+2a^2b^2+ab^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e) - (a^2*cos(f*x + e)^2 + a*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqr
t(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b
)/(cos(f*x + e)^2 - 1)))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3
*b + 2*a^2*b^2 + a*b^3)*f), ((a^2*cos(f*x + e)^2 + a*b)*sqrt(-a - b)*arctan
(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
 b)) - (a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)
*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.110 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{3b\sec(e+fx)}{2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{5/2}}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] -((a - 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])
/(2*(a + b)^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*Sqrt[a + b*
Sec[e + f*x]^2]) - (3*b*Sec[e + f*x])/(2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x
]^2])
```

Rubi [A] time = 0.153105, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4134, 471, 527, 12, 377, 207}

$$-\frac{3b\sec(e+fx)}{2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{5/2}}-\frac{\cot(e+fx)\csc(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])
/(2*(a + b)^(5/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*Sqrt[a + b*
Sec[e + f*x]^2]) - (3*b*Sec[e + f*x])/(2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x
]^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-2bx^2}{\left(-1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a(a-2b)}{\left(-1+x^2\right)\sqrt{a-}}\right.}{2a(a}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a-2b)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{(a-2b)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+}{2(a+b)^2f\sqrt{a+b\,s}}$$

Mathematica [C] time = 0.352677, size = 97, normalized size = 0.77

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left((a+b)\csc^2(e+fx)-(a-2b)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right.\right.}{4f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Csc[e + f*x]^2 - (a - 2*b)*Hyperg
eometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]^3)/
(4*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] time = 0.391, size = 3289, normalized size = 26.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/f/(a+b)^(9/2)/(b+a*cos(f*x+e)^2)^2*(cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3-2*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^3-cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3+2*cos(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e
))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*co
s(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x
+e)^2)*b^3-cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(co
s(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+
e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e
)))*a^3+2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos
(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)
))*b^3+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+
cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b
)/sin(f*x+e)^2)*a*b^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(
cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x
+e)))*a*b^2+cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/
(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3-2*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^3+cos(f*x+e)^3*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3-2*cos(f*x+e)^3
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^3+cos(f*x+e)^2*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e
))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*co
```

```
s(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x
+e)^2)*a^3-2*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2
/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^3-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3+4*cos(f*x+e)^2*(a+b)^(5/2)*
b+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f
*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-
a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin
(f*x+e)^2)*b^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b
^3-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3-2*cos(f*x+e
)^2*(a+b)^(5/2)*a-3*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b^2-3*cos(f*x+e)^3*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b^2-3*cos(f*x+e)^2*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b^
2-3*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a
*b^2+3*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(
1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)+b)/sin(f*x+e)^2)*a*b^2+3*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+b)/(-1+cos(f*x+e)))*a*b^2-6*(a+b)^(5/2)*b)*cos(f*x+e)^3*((b+a*cos(f*x
+e)^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.03824, size = 1277, normalized size = 10.13

$$\left[-\frac{\left(\left(a^{2}-2\,ab\right)\cos\left(fx+e\right)^{4}-\left(a^{2}-3\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{2}-ab+2\,b^{2}\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos\left(fx+e\right)^{2}+2\,\sqrt{a+b}\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\,\mathrm{co}\right.}{\cos\left(fx+e\right)^{2}-1}\right.}{4\left(\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{4}-\left(a^{4}+2\,a^{3}b-2\,ab^{3}-b^{4}\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a^2 - 2*a*b)*cos(f*x + e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2
 - a*b + 2*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^
2 - 1)) - 2*((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)
^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f), 1/2*(((a^2 - 2*a*b)*cos(f*x +
e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-a - b)*arc
tan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(
a + b)) + ((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 +
 a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2
 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{3}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.111 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{b(13a-2b)\sec(e+fx)}{8f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{7/2}}-\frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{5a\cot(e+fx}{8f(a+b)^2\sqrt{a+}}$$

```
[Out] (-3*a*(a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/(8*(a + b)^(7/2)*f) - (5*a*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*S
qrt[a + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*Sqr
t[a + b*Sec[e + f*x]^2]) - ((13*a - 2*b)*b*Sec[e + f*x])/(8*(a + b)^3*f*Sqr
t[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.216474, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4134, 470, 527, 12, 377, 207}

$$-\frac{b(13a-2b)\sec(e+fx)}{8f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{7/2}}-\frac{\cot^3(e+fx)\csc(e+fx)}{4f(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{5a\cot(e+fx}{8f(a+b)^2\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```



```
[Out] (-3*a*(a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]])/(8*(a + b)^(7/2)*f) - (5*a*Cot[e + f*x]*Csc[e + f*x])/(8*(a + b)^2*f*S
qrt[a + b*Sec[e + f*x]^2]) - (Cot[e + f*x]^3*Csc[e + f*x])/(4*(a + b)*f*Sqr
t[a + b*Sec[e + f*x]^2]) - ((13*a - 2*b)*b*Sec[e + f*x])/(8*(a + b)^3*f*Sqr
t[a + b*Sec[e + f*x]^2])
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a-4ax^2}{\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a(3a-2b)+10}{\left(-1+x^2\right)\left(a+bx\right.}\right.}{8(a+}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e}{8(a+b)^3f\sqrt{a+b\sec}}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e}{8(a+b)^3f\sqrt{a+b\sec}}$$

$$= -\frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e}{8(a+b)^3f\sqrt{a+b\sec}}$$

$$= -\frac{3a(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{7/2}f} - \frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\,c}{4(a+b)f\sqrt{a+b}}$$

Mathematica [C] time = 0.504585, size = 100, normalized size = 0.56

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left((a+b)^2\csc^4(e+fx)-a(a-4b)\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right.\right.}{8f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)^2*Csc[e + f*x]^4 - a*(a - 4*b)*Hy
pergeometric2F1[-1/2, 2, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]
^3)/(8*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [B] time = 0.446, size = 8268, normalized size = 46.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.24107, size = 1989, normalized size = 11.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^2)*cos
(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*s
qrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(a
^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5 - (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b
^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
+ a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*
b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*
b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 +
b^5)*f), 1/8*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^
```

```
2)*cos(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e
)^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e)/(a + b)) + (3*(a^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5
- (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b
^2 - 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^
5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*
a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*
a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b
+ 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.112 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=242

$$-\frac{b\left(81a^2+190ab+105b^2\right)\tan(e+fx)}{48a^4f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{5(a+b)^2(a+7b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}+\frac{(9a+7b)\sin(e+fx)\cos^3(e+}{24a^2f\sqrt{a+b\tan^2(e+fx)+}}$$

```
[Out] (5*(a + b)^2*(a + 7*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(16*a^(9/2)*f) - ((a + b)*(33*a + 35*b)*Cos[e + f*x]*Sin[e + f*x
])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((9*a + 7*b)*Cos[e + f*x]^3*
Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*S
in[e + f*x]^3)/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (b*(81*a^2 + 190*a*
b + 105*b^2)*Tan[e + f*x])/(48*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.373629, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4132, 470, 578, 527, 12, 377, 203}

$$-\frac{b\left(81a^2+190ab+105b^2\right)\tan(e+fx)}{48a^4f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{5(a+b)^2(a+7b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}+\frac{(9a+7b)\sin(e+fx)\cos^3(e+}{24a^2f\sqrt{a+b\tan^2(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (5*(a + b)^2*(a + 7*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(16*a^(9/2)*f) - ((a + b)*(33*a + 35*b)*Cos[e + f*x]*Sin[e + f*x
])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((9*a + 7*b)*Cos[e + f*x]^3*
Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*S
in[e + f*x]^3)/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (b*(81*a^2 + 190*a*
b + 105*b^2)*Tan[e + f*x])/(48*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}}-\frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+2(b-3(a+b))x^2\right)}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^{3/2}}\,dx,x,\tan(e+fx)\right)}{6af}$$

$$=\frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}}-\frac{\text{Subst}\left(\int \frac{(a+b)(9a+7\cdots}{(1+x^2\cdots}\right.}{}$$

$$=-\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{c}\cdots}{6a\cdots}$$

$$=-\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{c}\cdots}{6a\cdots}$$

$$=-\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{c}\cdots}{6a\cdots}$$

$$=-\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{c}\cdots}{6a\cdots}$$

$$=\frac{5(a+b)^2(a+7b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f}-\frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx\cdots}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] time = 8.64539, size = 256, normalized size = 1.06

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(120(a+b)^2(a+7b)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-2\sqrt{2}\sqrt{a}\sqrt{a}\right.}{1536a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(120*(a + b)^2*(a + 7*b)*Arc
Sin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*
Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(3
7*a^3 + 439*a^2*b + 830*a*b^2 + 420*b^3 + a*(29*a^2 + 108*a*b + 70*b^2)*Cos
[2*(e + f*x)] - 7*a^2*(a + b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])*Sin[
e + f*x]))/(1536*a^(9/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a
 + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [C] time = 0.695, size = 2437, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/48/f/a^4/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(b+a*cos(f*x+e)^2)*(105
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-30*2^(1/2
)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*
(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2))*a^3*sin(f*x+e)+135*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
```

```
(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*a^2*b*sin(f*x+e)+225*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*a*b^2*sin(f*x+e)-270*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-450*2^(1/2)*(1/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*a*b^2*sin(f*x+e)-105*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-21
0*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a^3*sin(f*x+e)+8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^3-8*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-26*
cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+26*cos(f*x+e)^4*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+33*cos(f*x+e)^3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^3-33*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^3+105*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-81*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-190*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b^2-14*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^2*b+14*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+68*cos
(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+35*cos(f*x+e)^3*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-68*cos(f*x+e)^2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^2*b-35*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b^2+81*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*
b+190*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2)*sin(f*x+e)/
(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A] time = 20.1154, size = 1932, normalized size = 7.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^2
*b^2 + 7*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^4*cos(f*x + e)^7 - 2*(13*a^4 + 7*a^
3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x + e)^3 + (81
*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), -1/192*(15
*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^2*b^2 + 7*a*
b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*
b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^
3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^4*cos(f*x + e)^7 - 2*(
13*a^4 + 7*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x
 + e)^3 + (81*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f
```

```
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.113 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{3(a+b)(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5(a+b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+}{4af\sqrt{a+}}$$

```
[Out] (3*(a + b)*(a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(8*a^(7/2)*f) - (5*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*Sqr
t[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*Sqrt[a
+ b + b*Tan[e + f*x]^2]) - (b*(13*a + 15*b)*Tan[e + f*x])/(8*a^3*f*Sqrt[a +
 b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.217153, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4132, 470, 527, 12, 377, 203}

$$\frac{3(a+b)(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5(a+b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+}{4af\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*(a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(8*a^(7/2)*f) - (5*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*Sqr
t[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*Sqrt[a
+ b + b*Tan[e + f*x]^2]) - (b*(13*a + 15*b)*Tan[e + f*x])/(8*a^3*f*Sqrt[a +
 b + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a+b-4(a+b)x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+5b)\cdot}{\left(1+x^2\right)\left(a+\right.}\right.}{}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e}{8a^3f\sqrt{a+b+b\tan^2}}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e}{8a^3f\sqrt{a+b+b\tan^2}}$$

$$= -\frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e}{8a^3f\sqrt{a+b+b\tan^2}}$$

$$= \frac{3(a+b)(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e}{4af\sqrt{a+}}$$

Mathematica [A] time = 3.71712, size = 229, normalized size = 1.31

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(24\left(a^2+6ab+5b^2\right)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-2\sqrt{2}\sqrt{a}\sqrt{a}\right.}{256a^{7/2}f\sqrt{a+b}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(24*(a^2 + 6*a*b + 5*b^2)*Ar
cSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2
*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(
7*a^2 + 62*a*b + 60*b^2 + 2*a*(3*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e +
 f*x)])*Sin[e + f*x]))/(256*a^(7/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3
/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [C] time = 0.424, size = 1714, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^3*(b+a*cos(f*x+e)^2)*(2*cos
(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-2*cos(f*x+e)^4*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a^2*sin(f*x+e)-18*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))
^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*a*b*sin(f*x+e)-15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*b^2*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+36*2^(1/2)*(1/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
```

```
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a
*b*sin(f*x+e)+30*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticP
i((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-5*cos(f*x+e)^3*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^2+5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-13*cos(f*x+e)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-15*cos(f*x+e)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*b^2+13*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+
15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*sin(f*x+e)/(-1+cos(f*x+e))/
cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A] time = 6.32618, size = 1667, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2*b + 6*a*b^2 + 5*b^3 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)
^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
+ 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
```

```
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
)) - 8*(2*a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^2*b +
 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/32*(3*(a^2*b + 6*a*b^2 + 5*b^3
 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*
a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^2*b + 15*a*b^2)
*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a
^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.114 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - (3*b*Tan[e + f*x])/(2*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.149528, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4132, 471, 527, 12, 377, 203}

$$\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - (3*b*Tan[e + f*x])/(2*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b-2bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a+b)(a+3b)}{\left(1+x^2\right)\sqrt{a+b+bx^2}} \cdots}{2a^2(a+ \cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+3b)\,\text{Subst}\left(\int \frac{\cdots}{(1+x^2 \cdots} \cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{1+ax^2} \cdots}{2a \cdots}$$

$$= \frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx \cdots}{2a^2f\sqrt{a+b+b\tan^2 \cdots}}$$

Mathematica [A] time = 1.32962, size = 190, normalized size = 1.57

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(4(a+3b)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b) - 2\sqrt{2}\sqrt{a}\sqrt{a+b}\sin(e+ \cdots}{32a^{5/2}f\sqrt{a+b}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(4*(a + 3*b)*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*Sqrt[2]*Sqrt
[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(a + 6*b + a*C
os[2*(e + f*x)])*Sin[e + f*x]))/(32*a^(5/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*
```

```
x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [C] time = 0.314, size = 1069, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2*(b+a*cos(f*x+e)^2)*(-2*2
^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*a*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))
^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-3*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*c
os(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.10061, size = 1458, normalized size = 12.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((a^2 + 3*a*b)*cos(f*x + e)^2 + a*b + 3*b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3
2*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos
(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(a^2*cos(f*x + e)^3
 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), -1/8*(((a^2 + 3*a*b)*cos(f*x + e)^2
 + a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3
*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(a^2*cos(f*x + e)^3 + 3*a*b*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f*
cos(f*x + e)^2 + a^3*b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.115 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0501041, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4128, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\ &= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}\\ &= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}\end{aligned}$$

Mathematica [B] time = 1.38777, size = 168, normalized size = 2.18

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\sqrt{2}\sqrt{a}b\sin(e+fx)\sqrt{\frac{a}{\cdots}}\right.}{4a^{3/2}f(a+b)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

Maple [C] time = 0.408, size = 1007, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a*(b+a*cos(f*x+e)^2)*(2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin
(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*sin(f*x+e)/(
```

```
-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.04019, size = 1438, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

3.116 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2b\tan(e+fx)}{f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] -(Cot[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f
*x])/((a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0914931, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4132, 271, 191}

$$-\frac{2b\tan(e+fx)}{f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f
*x])/((a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$
$$= -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] time = 1.77019, size = 76, normalized size = 1.12

$$-\frac{\csc(e+fx)\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)((a-b)\cos(2(e+fx))+a+3b)}{4f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(a + 3*b + (a - b)*Cos[2*(e + f*x)])*Csc[e
 + f*x]*Sec[e + f*x]^3)/(4*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.282, size = 89, normalized size = 1.3

$$-\frac{\left(a\left(\cos\left(fx+e\right)\right)^2-b\left(\cos\left(fx+e\right)\right)^2+2\,b\right)\left(\cos\left(fx+e\right)\right)^3}{f\left(a+b\right)^2\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)^2\sin\left(fx+e\right)}\left(\frac{b+a\left(\cos\left(fx+e\right)\right)^2}{\left(\cos\left(fx+e\right)\right)^2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/(a+b)^2/(b+a*cos(f*x+e)^2)^2*(a*cos(f*x+e)^2-b*cos(f*x+e)^2+2*b)*cos(f
*x+e)^3*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.85312, size = 238, normalized size = 3.5

$$-\frac{\left((a-b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{\left(\left(a^{3}+2\,a^{2}b+ab^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b+2\,ab^{2}+b^{3}\right)f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 + (a^2*b + 2*a*b^2
 + b^3)*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.117 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{2b(3a-b)\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^3(e+fx)}{3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(3a-b)\cot(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] -((3*a - b)*Cot[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) -
Cot[e + f*x]^3/(3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (2*(3*a - b)*
b*Tan[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.127824, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4132, 453, 271, 191}

$$-\frac{2b(3a-b)\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^3(e+fx)}{3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(3a-b)\cot(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a - b)*Cot[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) -
Cot[e + f*x]^3/(3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - (2*(3*a - b)*
b*Tan[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(2(3a-b)b)}{}$$

$$= -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(3a-}{3(a+b)^3f\sqrt{}}$$

Mathematica [A] time = 0.668803, size = 102, normalized size = 0.83

$$-\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\left(a^2-2ab-3b^2\right)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)-2a(a-3b)\right)}{6f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(-2*a*(a - 3*b) + (a^2 - 2*a*b - 3*b^2)*Cs
c[e + f*x]^2 + (a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x])/(6*(a
 + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.316, size = 137, normalized size = 1.1

$$\frac{\left(2\left(\cos\left(fx+e\right)\right)^{4}a^{2}-6\left(\cos\left(fx+e\right)\right)^{4}ab-3\left(\cos\left(fx+e\right)\right)^{2}a^{2}+10\left(\cos\left(fx+e\right)\right)^{2}ab-3\left(\cos\left(fx+e\right)\right)^{2}b^{2}-6a\cdots}{3f\left(a+b\right)^{3}\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)^{2}\left(\sin\left(fx+e\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f/(a+b)^3/(b+a*cos(f*x+e)^2)^2*(2*cos(f*x+e)^4*a^2-6*cos(f*x+e)^4*a*b-3
*cos(f*x+e)^2*a^2+10*cos(f*x+e)^2*a*b-3*cos(f*x+e)^2*b^2-6*a*b+2*b^2)*((b+a
*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21597, size = 428, normalized size = 3.48

$$-\frac{\left(2\left(a^{2}-3ab\right)\cos\left(fx+e\right)^{5}-\left(3a^{2}-10ab+3b^{2}\right)\cos\left(fx+e\right)^{3}-2\left(3ab-b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}\cdots}{\cos\left(fx+e\right)^{2}}}}{3\left(\left(a^{4}+3a^{3}b+3a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{4}-\left(a^{4}+2a^{3}b-2ab^{3}-b^{4}\right)f\cos\left(fx+e\right)^{2}-\left(a^{3}b+3a^{2}b^{2}+3ab^{3}+b^{4}\right)f\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2 - 3*a*b)*cos(f*x + e)^5 - (3*a^2 - 10*a*b + 3*b^2)*cos(f*x + e
)^3 - 2*(3*a*b - b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*
b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f
)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.118 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2b\left(15a^2-10ab-b^2\right)\tan(e+fx)}{15f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\left(15a^2-10ab-b^2\right)\cot(e+fx)}{15f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^5(e+fx)}{5f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{}{15}$$

```
[Out] -((15*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(15*(a + b)^3*f*Sqrt[a + b + b*Tan[
e + f*x]^2]) - (2*(5*a + 2*b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*Sqrt[a + b +
b*Tan[e + f*x]^2]) - Cot[e + f*x]^5/(5*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x
]^2]) - (2*b*(15*a^2 - 10*a*b - b^2)*Tan[e + f*x])/(15*(a + b)^4*f*Sqrt[a +
 b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.181575, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4132, 462, 453, 271, 191}

$$-\frac{2b\left(15a^2-10ab-b^2\right)\tan(e+fx)}{15f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\left(15a^2-10ab-b^2\right)\cot(e+fx)}{15f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^5(e+fx)}{5f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{}{15}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((15*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(15*(a + b)^3*f*Sqrt[a + b + b*Tan[
e + f*x]^2]) - (2*(5*a + 2*b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*Sqrt[a + b +
b*Tan[e + f*x]^2]) - Cot[e + f*x]^5/(5*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x
]^2]) - (2*b*(15*a^2 - 10*a*b - b^2)*Tan[e + f*x])/(15*(a + b)^4*f*Sqrt[a +
 b + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^6(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(5a+2b)+5(a+b)x^2}{x^4(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(15a^2-1}{}$$

$$= -\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{}{5(a+b}$$

$$= -\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{}{5(a+b}$$

Mathematica [A] time = 0.986108, size = 126, normalized size = 0.69

$$-\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(4a\left(a^2-4ab-5b^2\right)\csc^2(e+fx)-8a^2(a-5b)+3(a+b)^3\csc^6(e}{30f(a+b)^4\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(-8*a^2*(a - 5*b) + 4*a*(a^2 - 4*a*b - 5*b
^2)*Csc[e + f*x]^2 + (a - 5*b)*(a + b)^2*Csc[e + f*x]^4 + 3*(a + b)^3*Csc[e
 + f*x]^6)*Sec[e + f*x]^2*Tan[e + f*x])/(30*(a + b)^4*f*(a + b*Sec[e + f*x]
^2)^(3/2))
```

Maple [A] time = 0.413, size = 204, normalized size = 1.1

$$-\frac{\left(8\left(\cos\left(fx+e\right)\right)^6a^3-40\left(\cos\left(fx+e\right)\right)^6a^2b-20\left(\cos\left(fx+e\right)\right)^4a^3+104\left(\cos\left(fx+e\right)\right)^4a^2b-20\left(\cos\left(fx+e\right)\right)^4}{15\,f\,(a+b)^4\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15/f/(a+b)^4/(b+a*cos(f*x+e)^2)^2*(8*cos(f*x+e)^6*a^3-40*cos(f*x+e)^6*a^
2*b-20*cos(f*x+e)^4*a^3+104*cos(f*x+e)^4*a^2*b-20*cos(f*x+e)^4*a*b^2+15*cos
(f*x+e)^2*a^3-85*cos(f*x+e)^2*a^2*b+49*cos(f*x+e)^2*a*b^2+5*cos(f*x+e)^2*b^
3+30*a^2*b-20*a*b^2-2*b^3)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(
3/2)/sin(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.9212, size = 706, normalized size = 3.86

$$-\frac{\left(8\left(a^3-5a^2b\right)\cos\left(fx+e\right)^7-4\left(5a^3-26a^2b+5ab^2\right)\cos\left(fx+e\right)^5+\left(15a^3-85a^2b+\right.}{15\left(\left(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4\right)f\cos\left(fx+e\right)^6-\left(2a^5+7a^4b+8a^3b^2+2a^2b^3-2ab^4-b^5\right)f\cos\left(fx+e\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^3 - 5*a^2*b)*cos(f*x + e)^7 - 4*(5*a^3 - 26*a^2*b + 5*a*b^2)*co
s(f*x + e)^5 + (15*a^3 - 85*a^2*b + 49*a*b^2 + 5*b^3)*cos(f*x + e)^3 + 2*(1
5*a^2*b - 10*a*b^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6
 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x + e)
^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x +
e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.119 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{8b\left(5a^2+20ab+16b^2\right)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\left(5a^2+20ab+16b^2\right)\sec(e+fx)}{15a^4f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\left(5a^2+20ab+16b^2\right)\cos(e+fx)}{5a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{2(5a}{15a^2f\,(}$$

```
[Out] -((5*a^2 + 20*a*b + 16*b^2)*Cos[e + f*x])/(5*a^3*f*(a + b*Sec[e + f*x]^2)^(
3/2)) + (2*(5*a + 4*b)*Cos[e + f*x]^3)/(15*a^2*f*(a + b*Sec[e + f*x]^2)^(3/
2)) - Cos[e + f*x]^5/(5*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 + 2
0*a*b + 16*b^2)*Sec[e + f*x])/(15*a^4*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*
b*(5*a^2 + 20*a*b + 16*b^2)*Sec[e + f*x])/(15*a^5*f*Sqrt[a + b*Sec[e + f*x]
^2])
```

Rubi [A] time = 0.215292, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4134, 462, 453, 271, 192, 191}

$$-\frac{8b\left(5a^2+20ab+16b^2\right)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\left(5a^2+20ab+16b^2\right)\sec(e+fx)}{15a^4f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\left(\frac{4b(5a+4b)}{a^2}\right.}{5af\,\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((5 + (4*b*(5*a + 4*b))/a^2)*Cos[e + f*x])/(5*a*f*(a + b*Sec[e + f*x]^2)^(
3/2)) + (2*(5*a + 4*b)*Cos[e + f*x]^3)/(15*a^2*f*(a + b*Sec[e + f*x]^2)^(3/
2)) - Cos[e + f*x]^5/(5*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(5*a^2 + 2
0*a*b + 16*b^2)*Sec[e + f*x])/(15*a^4*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*
b*(5*a^2 + 20*a*b + 16*b^2)*Sec[e + f*x])/(15*a^5*f*Sqrt[a + b*Sec[e + f*x]
^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
```

```
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)}{5af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-2(5a+4b)+5ax^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5af}$$

$$= \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cos^5(e+fx)}{5af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\left(5a^2+20ab+16b^2\right)\text{Subst}}{}$$

$$= -\frac{\left(5a^2+20ab+16b^2\right)\cos(e+fx)}{5a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cos^5(e+fx)}{5af\left(a+b\sec^2(e+\right.}$$

$$= -\frac{\left(5a^2+20ab+16b^2\right)\cos(e+fx)}{5a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cos^5(e+fx)}{5af\left(a+b\sec^2(e+\right.}$$

$$= -\frac{\left(5a^2+20ab+16b^2\right)\cos(e+fx)}{5a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cos^5(e+fx)}{5af\left(a+b\sec^2(e+\right.}$$

Mathematica [A] time = 3.41962, size = 182, normalized size = 0.83

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(12a^2\left(7a^2+64ab+64b^2\right)\cos(4(e+fx))+48a\left(150a^2b+11a^3+384ab^2+256\right.}{384}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(425*a^4 + 6400*a^3*b + 22784*a^2*b^2 + 32
768*a*b^3 + 16384*b^4 + 48*a*(11*a^3 + 150*a^2*b + 384*a*b^2 + 256*b^3)*Cos
[2*(e + f*x)] + 12*a^2*(7*a^2 + 64*a*b + 64*b^2)*Cos[4*(e + f*x)] - 16*a^4*
Cos[6*(e + f*x)] - 32*a^3*b*Cos[6*(e + f*x)] + 3*a^4*Cos[8*(e + f*x)])*Sec[
e + f*x]^5)/(3840*a^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] time = 2.01, size = 229, normalized size = 1.1

$$\frac{a^2\sqrt{4}\,(a+b)^7\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)\left(3\left(\cos\left(fx+e\right)\right)^8a^4-10\left(\cos\left(fx+e\right)\right)^6a^4-8\left(\cos\left(fx+e\right)\right)^6a^3b+15\left(\cos\left(f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/30/f*a^2/((-a*b)^(1/2)-a)^7/((-a*b)^(1/2)+a)^7*4^(1/2)*(a+b)^7*(b+a*cos(f
*x+e)^2)*(3*cos(f*x+e)^8*a^4-10*cos(f*x+e)^6*a^4-8*cos(f*x+e)^6*a^3*b+15*co
s(f*x+e)^4*a^4+60*cos(f*x+e)^4*a^3*b+48*cos(f*x+e)^4*a^2*b^2+60*cos(f*x+e)^
2*a^3*b+240*cos(f*x+e)^2*a^2*b^2+192*cos(f*x+e)^2*a*b^3+40*a^2*b^2+160*a*b^
3+128*b^4)/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)
```

Maxima [A] time = 1.01714, size = 451, normalized size = 2.06

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^3}-\frac{10\left(\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-9\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\,b\cos\left(fx+e\right)\right)}{a^4}+\frac{3\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{5}{2}}\cos\left(fx+e\right)^5-20\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)\cdots}{a^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - 10*((a + b/cos(f*x
+ e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^4 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a + b/cos(f*x +
 e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x +
 e))/a^5 + 5*(6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(
f*x + e)^2)^(3/2)*a^3*cos(f*x + e)^3) + 10*(9*(a + b/cos(f*x + e)^2)*b^2*co
s(f*x + e)^2 - b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3) + 5*(
12*(a + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a + b/cos(f*x + e)^2)
^(3/2)*a^5*cos(f*x + e)^3))/f
```

Fricas [A] time = 1.43799, size = 441, normalized size = 2.01

$$-\frac{\left(3\,a^4\cos\left(fx+e\right)^9-2\left(5\,a^4+4\,a^3b\right)\cos\left(fx+e\right)^7+3\left(5\,a^4+20\,a^3b+16\,a^2b^2\right)\cos\left(fx+e\right)^5+12\left(5\,a^3b+20\,a^2b^2+1\right.}{15\left(a^7f\cos\left(fx+e\right)^4+2\,a^6bf\cos\left(fx+e\right)^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^4*cos(f*x + e)^9 - 2*(5*a^4 + 4*a^3*b)*cos(f*x + e)^7 + 3*(5*a^4
 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^5 + 12*(5*a^3*b + 20*a^2*b^2 + 16*a*
b^3)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*co
s(f*x + e)^2 + a^5*b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.120 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{8b(a+2b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] -(((a + 2*b)*Cos[e + f*x])/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))) + Cos[e +
f*x]^3/(3*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + 2*b)*Sec[e + f*x])/
(3*a^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a + 2*b)*Sec[e + f*x])/(3*a^
4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.138049, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4134, 453, 271, 192, 191}

$$-\frac{8b(a+2b)\sec(e+fx)}{3a^4f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] -(((a + 2*b)*Cos[e + f*x])/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))) + Cos[e +
f*x]^3/(3*a*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (4*b*(a + 2*b)*Sec[e + f*x])/
(3*a^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*(a + 2*b)*Sec[e + f*x])/(3*a^
4*f*Sqrt[a + b*Sec[e + f*x]^2])
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b(a+2b))\,\text{Subst}\left(\int \frac{}{(a+b}\right.}{a^2f}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^3}$$

$$= -\frac{(a+2b)\cos(e+fx)}{a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(a+2b)\sec(e+fx)}{3a^3f\left(a+b\sec^2(e+fx)\right)^3}$$

Mathematica [A] time = 2.52247, size = 129, normalized size = 0.88

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(3a\left(11a^2+96ab+128b^2\right)\cos(2(e+fx))+6a^2(a+4b)\cos(4(e+fx))+264a\right.}{192a^4f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(26*a^3 + 264*a^2*b + 640*a*b^2 + 512*b^3
+ 3*a*(11*a^2 + 96*a*b + 128*b^2)*Cos[2*(e + f*x)] + 6*a^2*(a + 4*b)*Cos[4*
(e + f*x)] - a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^5)/(192*a^4*f*(a + b*Sec[e
+ f*x]^2)^(5/2))
```

Maple [A] time = 1.063, size = 159, normalized size = 1.1

$$-\frac{a\sqrt{4}\,(a+b)^5\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)\left(\left(\cos\left(fx+e\right)\right)^6a^3-3\left(\cos\left(fx+e\right)\right)^4a^3-6\left(\cos\left(fx+e\right)\right)^4a^2b-12\left(\cos\left(fx-\right.\right.\right.}{6f\left(\cos\left(fx+e\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/6/f*a/((-a*b)^(1/2)+a)^5/((-a*b)^(1/2)-a)^5*4^(1/2)*(a+b)^5*(b+a*cos(f*x
+e)^2)*(cos(f*x+e)^6*a^3-3*cos(f*x+e)^4*a^3-6*cos(f*x+e)^4*a^2*b-12*cos(f*x
+e)^2*a^2*b-24*cos(f*x+e)^2*a*b^2-8*a*b^2-16*b^3)/cos(f*x+e)^5/((b+a*cos(f*
x+e)^2)/cos(f*x+e)^2)^(5/2)
```

<u>Maxima</u> [A] time = 0.989938, size = 263, normalized size = 1.8

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)}{a^3}-\frac{\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3-9\sqrt{a+\frac{b}{\cos\left(fx+e\right)^2}}b\cos\left(fx+e\right)}{a^4}+\frac{6\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)b\cos\left(fx+e\right)^2-b^2}{\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}a^3\cos\left(fx+e\right)^3}+\frac{9\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)b^2 \mathrm{c}}{\left(a+\frac{b}{\cos\left(fx+e\right)^2}\right)^{\frac{3}{2}}a^4}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - ((a + b/cos(f*x + e)^
2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^4
+ (6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)
^(3/2)*a^3*cos(f*x + e)^3) + (9*(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e)^2 -
 b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3))/f
```

<u>Fricas</u> [A] time = 1.10187, size = 321, normalized size = 2.2

$$\frac{\left(a^3\cos\left(fx+e\right)^7-3\left(a^3+2\,a^2b\right)\cos\left(fx+e\right)^5-12\left(a^2b+2\,ab^2\right)\cos\left(fx+e\right)^3-8\left(ab^2+2\,b^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right.}{\cos\left(fx+e\right.}}}{3\left(a^6f\cos\left(fx+e\right)^4+2\,a^5bf\cos\left(fx+e\right)^2+a^4b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*cos(f*x + e)^7 - 3*(a^3 + 2*a^2*b)*cos(f*x + e)^5 - 12*(a^2*b + 2*
a*b^2)*cos(f*x + e)^3 - 8*(a*b^2 + 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 +
 a^4*b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.121 $\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] -(Cos[e + f*x]/(a*f*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e + f*x])/(3*
a^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x])/(3*a^3*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.0684935, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4134, 271, 192, 191}

$$-\frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] -(Cos[e + f*x]/(a*f*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*Sec[e + f*x])/(3*
a^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (8*b*Sec[e + f*x])/(3*a^3*f*Sqrt[a +
b*Sec[e + f*x]^2])
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{af}$$

$$= -\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(8b)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx,\right.}{3a^2f}$$

$$= -\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [A] time = 1.45492, size = 88, normalized size = 0.91

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(3a^2\cos(4(e+fx))+12a(a+4b)\cos(2(e+fx))+(3a+8b)^2\right)}{48a^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*((3*a + 8*b)^2 + 12*a*(a + 4*b)*Cos[2*(e +
 f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^5)/(48*a^3*f*(a + b*Sec[e + f
*x]^2)^(5/2))
```

Maple [A] time = 0.052, size = 90, normalized size = 0.9

$$\frac{1}{f}\left(-\frac{1}{a\sec\left(fx+e\right)}\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{-\frac{3}{2}}-4\,\frac{b}{a}\left(1/3\,\frac{\sec\left(fx+e\right)}{a\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{3/2}}+2/3\,\frac{\sec\left(fx+e\right)}{a^{2}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^{2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2)-4*b/a*(1/3*sec(f*x+e)/a/(a+b*
sec(f*x+e)^2)^(3/2)+2/3/a^2*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)))
```

Maxima [A] time = 1.00032, size = 116, normalized size = 1.2

$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)}{a^{3}}+\frac{6\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)b\cos\left(fx+e\right)^{2}-b^{2}}{\left(a+\frac{b}{\cos\left(fx+e\right)^{2}}\right)^{\frac{3}{2}}a^{3}\cos\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 + (6*(a + b/cos(f*x + e
)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)^(3/2)*a^3*cos(f*x + e)
^3))/f
```

Fricas [A] time = 0.812738, size = 243, normalized size = 2.51

$$-\frac{\left(3 a^{2} \cos \left(f x+e\right)^{5}+12 a b \cos \left(f x+e\right)^{3}+8 b^{2} \cos \left(f x+e\right)\right) \sqrt{\frac{a \cos \left(f x+e\right)^{2}+b}{\cos \left(f x+e\right)^{2}}}}{3 \left(a^{5} f \cos \left(f x+e\right)^{4}+2 a^{4} b f \cos \left(f x+e\right)^{2}+a^{3} b^{2} f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(f*x + e)^5 + 12*a*b*cos(f*x + e)^3 + 8*b^2*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f
*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin \left(f x+e\right)}{\left(b \sec \left(f x+e\right)^{2}+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.122 $\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{b(5a+2b)\sec(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b\sec(e+fx)}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
5/2)*f)) - (b*Sec[e + f*x])/(3*a*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) -
(b*(5*a + 2*b)*Sec[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.141761, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4134, 414, 527, 12, 377, 207}

$$-\frac{b(5a+2b)\sec(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b\sec(e+fx)}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/((a + b)^(
5/2)*f)) - (b*Sec[e + f*x])/(3*a*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) -
(b*(5*a + 2*b)*Sec[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(-1+x^2)(a+bx^2)^{5/2}}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+2b-2bx^2}{(-1+x^2)(a+bx^2)^{3/2}}\,dx,x,\sec(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(-1+x}\right.}{3}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(-1+x}\right.}{}$$

$$= -\frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{-1-(-}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{5/2}f} - \frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+}{3a^2(a+b)^2f\sqrt{a+b\sec^2}}$$

Mathematica [C] time = 5.33787, size = 108, normalized size = 0.85

$$\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(a^2\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)\left(3a\sin^2(e+fx)-\right.\right.}{6a^2f(a+b)\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(a^2*Hypergeometric2F1[-3/2,
 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*(-2*(2*a + b) + 3*a*Sin
[e + f*x]^2)))/(6*a^2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] time = 0.68, size = 5056, normalized size = 39.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 0.987878, size = 1341, normalized size = 10.56

$$\left[\frac{3\left(a^4\cos(fx+e)^4+2\,a^3b\cos(fx+e)^2+a^2b^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2\,b\right)}{\cos(fx+e)^2-1}\right)-2\left(3\left(2\right.\right.}{6\left(\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)f\cos(fx+e)^4+2\left(a^6b+3\,a^5b^2+3\,a^4b^3+a^3b^4\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(a + b)
*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(2*a^3*b + 3
*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 3*a^6*b + 3*a^5*b^
```

```
2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)
*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f), 1/3*(3*
(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(-a - b)*arctan
(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
 b)) - (3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b
^3 + 2*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^
7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
+ a^2*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.123 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{b(13a-2b)\sec(e+fx)}{6af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{5b\sec(e+fx)}{6f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{7/2}}-\frac{\cot(e+}{2f(a+b)\left(a\right.}$$

```
[Out] -((a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])
/(2*(a + b)^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*(a + b*Sec[
e + f*x]^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^
2)^(3/2)) - ((13*a - 2*b)*b*Sec[e + f*x])/(6*a*(a + b)^3*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

Rubi [A] time = 0.206449, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4134, 471, 527, 12, 377, 207}

$$-\frac{b(13a-2b)\sec(e+fx)}{6af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{5b\sec(e+fx)}{6f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f(a+b)^{7/2}}-\frac{\cot(e+}{2f(a+b)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])
/(2*(a + b)^(7/2)*f) - (Cot[e + f*x]*Csc[e + f*x])/(2*(a + b)*f*(a + b*Sec[
e + f*x]^2)^(3/2)) - (5*b*Sec[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^
2)^(3/2)) - ((13*a - 2*b)*b*Sec[e + f*x])/(6*a*(a + b)^3*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(-1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a-4bx^2}{\left(-1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a(3a}{(-1+}\right.}{}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-2b)}{6a(a+b)^3f\sqrt{}}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-2b)}{6a(a+b)^3f\sqrt{}}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-2b)}{6a(a+b)^3f\sqrt{}}$$

$$= -\frac{(a-4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec}{6(a+b)^2f\left(a+\right.}$$

Mathematica [C] time = 1.54846, size = 151, normalized size = 0.88

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left((a+b)\csc^2(e+fx)\left(\left(3a^2+2b^2\right)\cos(2(e+fx))+3a^2+6ab-2b^2\right)-3a(a-\right.}{24af(a+b)^3\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*(3*a^2 + 6*a*b - 2*b^2 + (3*a^2 +
 2*b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - 3*a*(a - 4*b)*(a + 2*b + a*Cos[2
*(e + f*x)])*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)
```

```
])*Sec[e + f*x]^5)/(24*a*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] time = 1.021, size = 11110, normalized size = 65.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.31295, size = 2086, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((a^4 - 4*a^3*b)*cos(f*x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos
(f*x + e)^4 - a^2*b^2 + 4*a*b^3 - (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x +
 e)^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) -
 2*(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*b^2 -
 4*a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^
```

```
2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*
a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a^5*b^2 +
8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*
a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/6*(3*((a^4 - 4*a^3*b)*cos(f*
x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 4*a*b^3 -
 (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) +
(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*b^2 - 4*
a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4
*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a
^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4
*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.124 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(3a^2-24ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{9/2}}-\frac{5b(11a-10b)\sec(e+fx)}{24f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}-\frac{b(23a-12b)\sec(e+fx)}{24f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}-$$

```
[Out] -((3*a^2 - 24*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Se
c[e + f*x]^2]])/(8*(a + b)^(9/2)*f) - ((5*a - 2*b)*Cot[e + f*x]*Csc[e + f*x
])/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e + f
*x])/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 12*b)*b*Sec[e +
f*x])/(24*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 10*b)*b*Se
c[e + f*x])/(24*(a + b)^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.323945, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4134, 470, 527, 12, 377, 207}

$$-\frac{\left(3a^2-24ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8f(a+b)^{9/2}}-\frac{5b(11a-10b)\sec(e+fx)}{24f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}-\frac{b(23a-12b)\sec(e+fx)}{24f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((3*a^2 - 24*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Se
c[e + f*x]^2]])/(8*(a + b)^(9/2)*f) - ((5*a - 2*b)*Cot[e + f*x]*Csc[e + f*x
])/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (Cot[e + f*x]^3*Csc[e + f
*x])/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - ((23*a - 12*b)*b*Sec[e +
f*x])/(24*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (5*(11*a - 10*b)*b*Se
c[e + f*x])/(24*(a + b)^4*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
```

```
&& IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^3\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a-2(2a-b)x^2}{\left(-1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a(3a-}{(-1-}\right.}{}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b}{24(a+b)^3f\left(a-\right.}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b}{24(a+b)^3f\left(a-\right.}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b}{24(a+b)^3f\left(a-\right.}$$

$$= -\frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b}{24(a+b)^3f\left(a-\right.}$$

$$= -\frac{\left(3a^2-24ab+8b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{9/2}f} - \frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{c}{4(a+}$$

Mathematica [C] time = 1.92286, size = 129, normalized size = 0.55

$$-\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(3(a+b)\csc^4(e+fx)((a+8b)\cos(2(e+fx))+3a-4b)-2\left(3a^2-24ab+8b^2\right.\right.}{96f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a + b)*(3*a - 4*b + (a + 8*b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^4 - 2*(3*a^2 - 24*a*b + 8*b^2)*Hypergeometric2F1[-3/
2, 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]^5)/(96*(a + b)^3*
f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] time = 1.572, size = 15551, normalized size = 66.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.80722, size = 2969, normalized size = 12.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^3
*b + 32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b^2
 - 56*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(3*a
^3*b - 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2*(a*
```

```
cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(3*a^4 - 21*a^3*b - 16
*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117*a^3*b + 4*a^2*b^2 + 104*
a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^2*b^2 - 61*a*b^3 + 88*b^4
)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^
3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 -
5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2
- 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4 + 2*(
a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x + e)^2
 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f), 1/24
*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^3*b +
32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b^2 - 56
*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(3*a^3*b
- 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) +
(3*(3*a^4 - 21*a^3*b - 16*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117
*a^3*b + 4*a^2*b^2 + 104*a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^
2*b^2 - 61*a*b^3 + 88*b^4)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)
*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b
 + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7
 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 +
(a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^
7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^
6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5
+ 5*a*b^6 + b^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.125 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=288

$$\frac{5(a+b)\left(a^2+14ab+21b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}-\frac{b\left(113a^2+420ab+315b^2\right)\tan(e+fx)}{48a^5f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{7b(a+b)(7a+15b)\,\mathrm{ta}}{48a^4f\left(a+b\tan^2(e+\right.}$$

```
[Out] (5*(a + b)*(a^2 + 14*a*b + 21*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(16*a^(11/2)*f) - ((a + b)*(11*a + 21*b)*Cos[e + f*x
]*Sin[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (3*(a + b)*Co
s[e + f*x]^3*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Co
s[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (7*
b*(a + b)*(7*a + 15*b)*Tan[e + f*x])/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^(
3/2)) - (b*(113*a^2 + 420*a*b + 315*b^2)*Tan[e + f*x])/(48*a^5*f*Sqrt[a + b
 + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.441857, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4132, 470, 578, 527, 12, 377, 203}

$$\frac{5(a+b)\left(a^2+14ab+21b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}-\frac{b\left(113a^2+420ab+315b^2\right)\tan(e+fx)}{48a^5f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{7b(a+b)(7a+15b)\,\mathrm{ta}}{48a^4f\left(a+b\tan^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (5*(a + b)*(a^2 + 14*a*b + 21*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/(16*a^(11/2)*f) - ((a + b)*(11*a + 21*b)*Cos[e + f*x
]*Sin[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (3*(a + b)*Co
s[e + f*x]^3*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Co
s[e + f*x]^3*Sin[e + f*x]^3)/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (7*
b*(a + b)*(7*a + 15*b)*Tan[e + f*x])/(48*a^4*f*(a + b + b*Tan[e + f*x]^2)^(
3/2)) - (b*(113*a^2 + 420*a*b + 315*b^2)*Tan[e + f*x])/(48*a^5*f*Sqrt[a + b
 + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^4\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)-6(a+b)x^2\right)}{(1+x^2)^3\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{9(a+b)^2-}{(1+x^2)}\right.}{}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{co}}{6af}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{co}}{6af}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{co}}{6af}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{co}}{6af}$$

$$= -\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{co}}{6af}$$

$$= \frac{5(a+b)\left(a^2+14ab+21b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} - \frac{(a+b)(11a+21b)\cos(e+fx)\,\text{s}}{16a^3f\left(a+b+b\tan^2(e+f\right.}$$

Mathematica [B] time = 19.5767, size = 1705, normalized size = 5.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]

[Out] -(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*e + 2*f
*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a*b^2 + 1
4*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f
*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]*(3*
(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4 + 1120*b^5
) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 2240*b^4)*Sin[e
 + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a + b)^2*Sin[e +
f*x]^6)))/(3072*Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a +
 b*Sec[e + f*x]^2)^(5/2)) + (((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)
*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(420*Sqrt[a + b]*(a^4
+ 9*a^3*b + 26*a^2*b^2 + 30*a*b^3 + 12*b^4)*ArcSin[(Sqrt[a]*Sin[e + f*x])/S
qrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - Sqrt[a]*Sin[e + f*x]*Sqrt[(a
 + b - a*Sin[e + f*x]^2)/(a + b)]*(3*(561*a^6 + 6161*a^5*b + 25200*a^4*b^2
+ 50960*a^3*b^3 + 54880*a^2*b^4 + 30240*a*b^5 + 6720*b^6) - 2*a*(1151*a^5 +
 11230*a^4*b + 39200*a^3*b^2 + 62720*a^2*b^3 + 47040*a*b^4 + 13440*b^5)*Sin
[e + f*x]^2 + 672*a^2*(a + b)^2*(a^2 + 3*a*b + 6*b^2)*Sin[e + f*x]^4 - 576*
a^3*(a - 2*b)*(a + b)^2*Sin[e + f*x]^6 + 512*a^4*(a + b)^2*Sin[e + f*x]^8))
)/(3072*Sqrt[2]*a^(11/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a + b*Sec[
e + f*x]^2)^(5/2)) - (5*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*S
ec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin
[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e
 + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/
(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Si
n[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt
[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(12
288*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)
) + (5*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Si
n[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a +
 b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b -
a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f
*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b
 - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x
])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^
3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a
*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a +
 b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a
+ b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*
x]^2)/(a + b))^2))/a^4))/(12288*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a +
 b - a*Sin[e + f*x]^2)^(3/2)) + (5*(2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*
b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(3072*(a + b)^2*
f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (5*(
b + (3*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[
```

```
e + f*x]^4*Tan[e + f*x])/(3072*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
3/2)*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] time = 1.371, size = 4477, normalized size = 15.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/48/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^5*sin(f*x+e)*(b+a*cos(f*x
+e)^2)*(-630*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3+315*sin(f*x+e
)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)
-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^4-630*sin(f*x+e)*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*b^4-113*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-420*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+33*cos(f*x+e)^5*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+8*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^4+315*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-3
3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-8*cos(f*x+e)^8*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+26*cos(f*x+e)^6*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^4-26*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^4-315*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+420*cos(f*x+e)
^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-162*cos(f*x+e)^2*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-574*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a^2*b^2-420*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b^3+113*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b
^2+420*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-30*sin(f*x+
e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
```

```
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*a^3*b-450*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b^2-1050*
sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3+15*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*a^3*b+225*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*a^2*b^2+525*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*a*b^3-30*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(
1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^4+
15*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a^4+225*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
```

```
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a^3*b+525*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a^2*b^2+315*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^3-450*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*a^3*b-1050*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2
)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b^2-18*cos
(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+18*cos(f*x+e)^6*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+96*cos(f*x+e)^5*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^3*b+63*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^2*b^2-96*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^3*b-63*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+162*c
os(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+574*cos(f*x+e)^3*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2)/(-1+cos(f*x+e))/cos(f*x+e)
^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [A] time = 72.5447, size = 2375, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b + 3
5*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*b^3
 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 9*a^4*b
)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3*b^2)*cos(f*x + e)^5 + 2*(8
1*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e)^3 + (113*a^3*b^2 + 420*a^
2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*
f), -1/192*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b
+ 35*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*
b^3 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^5*cos(f*x +
 e)^9 - 2*(13*a^5 + 9*a^4*b)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3
*b^2)*cos(f*x + e)^5 + 2*(81*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e
)^3 + (113*a^3*b^2 + 420*a^2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*
f*cos(f*x + e)^2 + a^6*b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.126 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=227

$$\frac{\left(3a^2+30ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}-\frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(5}{8}$$

```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) - ((5*a + 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 35*b)*Tan[e + f*x])/(24*a
^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 21*b)*Tan[e + f*x])/(
24*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.302317, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4132, 470, 527, 12, 377, 203}

$$\frac{\left(3a^2+30ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}-\frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{(5}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] ((3*a^2 + 30*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) - ((5*a + 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(23*a + 35*b)*Tan[e + f*x])/(24*a
^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (5*b*(11*a + 21*b)*Tan[e + f*x])/(
24*a^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
```

```
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b-2(2a+3b)x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a}{(1+x}\right.}{}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)}{24a^3f\left(a+b+b\right.}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)}{24a^3f\left(a+b+b\right.}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)}{24a^3f\left(a+b+b\right.}$$

$$= -\frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)}{24a^3f\left(a+b+b\right.}$$

$$= \frac{\left(3a^2+30ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{c}{4af}$$

Mathematica [B] time = 14.3601, size = 1315, normalized size = 5.79

$$-\frac{\left(\frac{\cos(2(e+fx))a+a+2b}{a+b}\right)^{3/2}(\cos(2e+2fx)a+a+2b)^{5/2}\left(\sqrt{a}\sin(e+fx)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(192a^3(a+b)^2\sin^6(e+fx)+672\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*e + 2*f
*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a*b^2 + 1
4*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f
*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]*(3*
(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4 + 1120*b^5
) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 2240*b^4)*Sin[e
 + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a + b)^2*Sin[e +
f*x]^6)))/(768*Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(7/2)*(a +
b*Sec[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*
x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])
*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*S
in[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]
^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b -
a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/
Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))
/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/
2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Si
n[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a +
 b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b -
a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f
*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b
 - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x
])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^
3 - (160*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a
*(a + b)^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a +
 b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a
+ b - a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*
x]^2)/(a + b))^2))/a^4))/(3072*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a +
b - a*Sin[e + f*x]^2)^(3/2)) + ((2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b +
```

```
 a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(256*(a + b)^2*f*(a
 + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((b + (3
*a + 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*
x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a
 + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] time = 0.744, size = 3223, normalized size = 14.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/24/f/a^4/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(b+a*cos(f*x+
e)^2)*(-105*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e
)-90*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a^2*b-105*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a*b^2-9*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellip
ticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*
b*sin(f*x+e)-90*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin
(f*x+e)+18*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
```

/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+180*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2
*sin(f*x+e)-9*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a^3+18*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+105*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+210*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+
e)+180*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+210*cos(f*x+e)^2*si
n(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2
)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+6*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^3-6*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^
3-15*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+15*cos(f*x+e)
^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-105*cos(f*x+e)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+55*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
*b^2-21*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+21*cos(f
x+e)^4((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-78*cos(f*x+e)^3*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-140*cos(f*x+e)^3*((2*I*a^(1/2)*b^(

```
1/2)+a-b)/(a+b))^(1/2)*a*b^2+78*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2*b+140*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*
b^2-55*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2)/(-1+cos(f*
x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [A] time = 24.5783, size = 2060, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((3*a^4 + 30*a^3*b + 35*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30
*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(
-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(6
*a^4*cos(f*x + e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b + 70
*a^2*b^2)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2
*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f), -1/96*(3*((3*a^4 + 30*a^3*b + 35*a^2*
b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b
^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
```

```
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(6*a^4*cos(f*x +
e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b + 70*a^2*b^2)*cos(f
*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*
x + e)^2 + a^5*b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.127 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=167

$$\frac{(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{6a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5b\tan(e+fx)}{6a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\sin(e}{2af\left(a+b\right.}$$

```
[Out] ((a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) - (5*b*Tan[e + f*x])/(6*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) -
(b*(13*a + 15*b)*Tan[e + f*x])/(6*a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rubi [A] time = 0.201752, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4132, 471, 527, 12, 377, 203}

$$\frac{(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}-\frac{b(13a+15b)\tan(e+fx)}{6a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{5b\tan(e+fx)}{6a^2f\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\sin(e}{2af\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) - (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) - (5*b*Tan[e + f*x])/(6*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) -
(b*(13*a + 15*b)*Tan[e + f*x])/(6*a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-4bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+}{(1+x^2)}\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)}{6a^3(a+b)f\sqrt{a+}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)}{6a^3(a+b)f\sqrt{a+}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)}{6a^3(a+b)f\sqrt{a+}}$$

$$= \frac{(a+5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e-}{6a^2f\left(a+b+b\tan\right.}$$

Mathematica [B] time = 10.7456, size = 983, normalized size = 5.89

$$-\frac{(\cos(2e+2fx)a+a+2b)^{5/2}\csc(e+fx)\left(-\frac{12\sin^4(e+fx)}{a+b} + \frac{(\cos(2(e+fx))a+a+2b)\sin^2(e+fx)}{(a+b)^2} + \frac{\sin^2(e+fx)}{a+b} + \frac{16\left(-a\sin^2(e+fx)+a+b\right)\left(1-\right.}{}\right.}{256\sqrt{2}f\left(b\sec^2(e+fx)+a\right)^{5/2}\left(-a\sin^2(e+fx\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e +
 f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2
 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin
[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e
+ f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (3*S
qrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])
/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(256*Sqrt[2]*f*(a + b*Sec
[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[2*
e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a
 + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (24*Sin[e + f*x]^4
)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b - a*Sin[e + f*x]^2)*(1 - (a*
Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*
(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (
3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*
x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3 - (160*(a + b - a*Sin[e
+ f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)^2*Sin[e + f*x]^2)
/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a + b)^(3/2)*ArcSin[(Sqrt[a]*
Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a
 + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*x]^2)/(a + b))^2))/a^4))/(
768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)
) + (5*(2*a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2
)*Sec[e + f*x]^4*Tan[e + f*x])/(384*(a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x
)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((b + (3*a + 2*b)*Cos[2*(e + f*x)
])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e + f*x])/(384*(
a + b)^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2
))
```

Maple [C] time = 0.582, size = 3158, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/6/f/a^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(b+a*cos(f*x+
e)^2)*(3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))
^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
```

```
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*a^3+18*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*a^2*b+15*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a*b^2-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^
3-36*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-30*cos(f*x+e)^2*sin(f
*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^3+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b
+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+18*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+15*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-6*2^(1/2)*(1/(a+b
```

```
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*a^2*b*sin(f*x+e)-36*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)-30*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*b^3*sin(f*x+e)-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^3-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+18*cos(f
*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+20*cos(f*x+e)^3*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-18*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2*b-20*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b^2+13*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+
15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-13*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^3)/(a+b)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 7.39595, size = 2037, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 6*a*b^3 +
 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a
^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b
 + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4
 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*
x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(3*(a^4 + a^3*b
)*cos(f*x + e)^5 + 2*(9*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 +
15*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)
^2 + (a^5*b^2 + a^4*b^3)*f), -1/24*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^2*b^2 + 6*a*b^3 + 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*
x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*(a^4 + a^3*b)*cos(f*x + e)^5 + 2*(9
*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 + 15*a*b^3)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f
*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^2 + a^4*b^3
)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.128 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0990468, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan}{3a^2(a+b)^2f\sqrt{a+b}}$$

Mathematica [C] time = 17.2824, size = 1927, normalized size = 15.42

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
```

```
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [C] time = 0.423, size = 3024, normalized size = 24.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/(a^2+2*a*b+b^2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2*sin(f*x+
e)*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(
a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2))*a^3-12*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-6*cos(f
*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
```

```
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^3*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-12*2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*a*b^2*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2
)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+6*cos(f*x+e)^
3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^3*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-6*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a*b^2+5*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*cos(f*x+
e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-5*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(
f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31132, size = 2021, normalized size = 16.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

3.129 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{8b\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\tan(e+fx)}{3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -(Cot[e + f*x]/((a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e +
 f*x])/(3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x]
)/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.108249, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4132, 271, 192, 191}

$$-\frac{8b\tan(e+fx)}{3f(a+b)^3\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\tan(e+fx)}{3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cot[e + f*x]/((a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*Tan[e +
 f*x])/(3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*b*Tan[e + f*x]
)/(3*(a + b)^3*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4b)\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(8b)\,\text{Su}}{}$$

$$= -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{3(a+}$$

Mathematica [A] time = 2.09656, size = 108, normalized size = 1.02

$$-\frac{\tan^3(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-6\left(a^2-b^2\right)\csc^2(e+fx)+3a^2+3(a+b)^2\csc^4(e+fx)-6ab-}{6f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(3*a^2 - 6*a*b - b^2 - 6*(a^2 - b^2)*Csc[e
 + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x]^3)/(6*(
a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] time = 0.403, size = 146, normalized size = 1.4

$$-\frac{\left(3\left(\cos\left(fx+e\right)\right)^{4}a^{2}-6\left(\cos\left(fx+e\right)\right)^{4}ab-\left(\cos\left(fx+e\right)\right)^{4}b^{2}+12\left(\cos\left(fx+e\right)\right)^{2}ab-4\left(\cos\left(fx+e\right)\right)^{2}b^{2}+8b^{2}\right)}{3f\left(a^{2}+2ab+b^{2}\right)\left(a+b\right)\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)^{4}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/(a^2+2*a*b+b^2)/(a+b)/(b+a*cos(f*x+e)^2)^4*(3*cos(f*x+e)^4*a^2-6*cos
(f*x+e)^4*a*b-cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-4*cos(f*x+e)^2*b^2+8*b^2
)*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/sin(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40064, size = 429, normalized size = 4.05

$$-\frac{\left(\left(3a^{2}-6ab-b^{2}\right)\cos\left(fx+e\right)^{5}+4\left(3ab-b^{2}\right)\cos\left(fx+e\right)^{3}+8b^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}}{3\left(\left(a^{5}+3a^{4}b+3a^{3}b^{2}+a^{2}b^{3}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{4}b+3a^{3}b^{2}+3a^{2}b^{3}+ab^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{3}b^{2}+3a^{2}b^{3}+3ab^{4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^5 + 4*(3*a*b - b^2)*cos(f*x + e)^3
 + 8*b^2*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3
*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*
f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.130 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{8b(a-b)\tan(e+fx)}{3f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b(a-b)\tan(e+fx)}{3f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot^3(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-$$

```
[Out] -(((a - b)*Cot[e + f*x])/((a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) -
Cot[e + f*x]^3/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - b)*
b*Tan[e + f*x])/(3*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*(a -
b)*b*Tan[e + f*x])/(3*(a + b)^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.160717, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4132, 453, 271, 192, 191}

$$-\frac{8b(a-b)\tan(e+fx)}{3f(a+b)^4\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b(a-b)\tan(e+fx)}{3f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\cot^3(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((a - b)*Cot[e + f*x])/((a + b)^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2))) -
Cot[e + f*x]^3/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (4*(a - b)*
b*Tan[e + f*x])/(3*(a + b)^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (8*(a -
b)*b*Tan[e + f*x])/(3*(a + b)^4*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^4\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(4(a-b)}{}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{3(a+b)}$$

$$= -\frac{(a-b)\cot(e+fx)}{(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{3(a+b)}$$

Mathematica [A] time = 4.64024, size = 138, normalized size = 0.87

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\frac{4b^2(a+b)}{(a\cos(2(e+fx))+a+2b)^2}+\frac{4b(b-3a)}{a\cos(2(e+fx))+a+2b}-(a+b)\csc^4(e+fx)-2(a\right.}{24f(a+b)^4\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*((4*b^2*(a + b))/(a + 2*b + a*Cos[2*(e +
f*x)])^2 + (4*b*(-3*a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a - 3*b)*Cs
c[e + f*x]^2 - (a + b)*Csc[e + f*x]^4)*Sec[e + f*x]^4*Tan[e + f*x])/(24*(a
+ b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] time = 0.411, size = 225, normalized size = 1.4

$$\frac{\left(2\left(\cos\left(fx+e\right)\right)^6a^3-12\left(\cos\left(fx+e\right)\right)^6a^2b+2\left(\cos\left(fx+e\right)\right)^6ab^2-3\left(\cos\left(fx+e\right)\right)^4a^3+21\left(\cos\left(fx+e\right)\right)^4a^2b-\right.}{3f\left(a^2+2ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/(a^2+2*a*b+b^2)/(a+b)^2/(b+a*cos(f*x+e)^2)^4*(2*cos(f*x+e)^6*a^3-12*c
os(f*x+e)^6*a^2*b+2*cos(f*x+e)^6*a*b^2-3*cos(f*x+e)^4*a^3+21*cos(f*x+e)^4*a
^2*b-21*cos(f*x+e)^4*a*b^2+3*cos(f*x+e)^4*b^3-12*cos(f*x+e)^2*a^2*b+24*cos(
f*x+e)^2*a*b^2-12*cos(f*x+e)^2*b^3-8*a*b^2+8*b^3)*cos(f*x+e)^5*((b+a*cos(f*
x+e)^2)/cos(f*x+e)^2)^(5/2)/sin(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.20669, size = 701, normalized size = 4.44

$$-\frac{\left(2\left(a^{3}-6\,a^{2}b+ab^{2}\right)\cos\left(fx+e\right)^{7}-3\left(a^{3}-7\,a^{2}b+7\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{5}-\cdots\right.}{3\left(\left(a^{6}+4\,a^{5}b+6\,a^{4}b^{2}+4\,a^{3}b^{3}+a^{2}b^{4}\right)f\cos\left(fx+e\right)^{6}-\left(a^{6}+2\,a^{5}b-2\,a^{4}b^{2}-8\,a^{3}b^{3}-7\,a^{2}b^{4}-2\,ab^{5}\right)f\cos\left(fx+e\right)^{4}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^7 - 3*(a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e)^5 - 12*(a^2*b - 2*a*b^2 + b^3)*cos(f*x + e)^3 - 8*(a*b
^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^6
+ 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^
5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^
5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 -
 (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.131 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=226

$$-\frac{8b\left(5a^2-10ab+b^2\right)\tan(e+fx)}{15f(a+b)^5\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\left(5a^2-10ab+b^2\right)\tan(e+fx)}{15f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx)}{5f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -((5*a^2 - 10*a*b + b^2)*Cot[e + f*x])/(5*(a + b)^3*f*(a + b + b*Tan[e + f*
x]^2)^(3/2)) - (2*(5*a + b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*(a + b + b*Tan[
e + f*x]^2)^(3/2)) - Cot[e + f*x]^5/(5*(a + b)*f*(a + b + b*Tan[e + f*x]^2)
^(3/2)) - (4*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(15*(a + b)^4*f*(a + b
+ b*Tan[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(15*
(a + b)^5*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.23976, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4132, 462, 453, 271, 192, 191}

$$-\frac{8b\left(5a^2-10ab+b^2\right)\tan(e+fx)}{15f(a+b)^5\sqrt{a+b\tan^2(e+fx)+b}}-\frac{4b\left(5a^2-10ab+b^2\right)\tan(e+fx)}{15f(a+b)^4\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx)}{5f(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] -((5*a^2 - 10*a*b + b^2)*Cot[e + f*x])/(5*(a + b)^3*f*(a + b + b*Tan[e + f*
x]^2)^(3/2)) - (2*(5*a + b)*Cot[e + f*x]^3)/(15*(a + b)^2*f*(a + b + b*Tan[
e + f*x]^2)^(3/2)) - Cot[e + f*x]^5/(5*(a + b)*f*(a + b + b*Tan[e + f*x]^2)
^(3/2)) - (4*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(15*(a + b)^4*f*(a + b
+ b*Tan[e + f*x]^2)^(3/2)) - (8*b*(5*a^2 - 10*a*b + b^2)*Tan[e + f*x])/(15*
(a + b)^5*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
```

```
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^6\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{2(5a+b)+5(a+b)x^2}{x^4\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= -\frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\left(5a^2\right.}{}$$

$$= -\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{5(a}$$

$$= -\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{5(a}$$

$$= -\frac{\left(5a^2-10ab+b^2\right)\cot(e+fx)}{5(a+b)^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{}{5(a}$$

Mathematica [A] time = 7.30988, size = 173, normalized size = 0.77

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\left(-8a^2+50ab-15b^2\right)\csc^2(e+fx)+\frac{20ab^2(a+b)}{(a\cos(2(e+fx))+a+2b)^2}+\frac{10a}{a\cos(2(}\right.}{120f(a+b)^5\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*((20*a*b^2*(a + b))/(a + 2*b + a*Cos[2*(e
 + f*x)])^2 + (10*a*b*(-6*a + 5*b))/(a + 2*b + a*Cos[2*(e + f*x)]) + (-8*a^
2 + 50*a*b - 15*b^2)*Csc[e + f*x]^2 + 2*(a + b)*(-2*a + 5*b)*Csc[e + f*x]^4
 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x]^4*Tan[e + f*x])/(120*(a + b)^5*
f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] time = 0.409, size = 324, normalized size = 1.4

$$-\frac{\left(8\left(\cos\left(fx+e\right)\right)^{8}a^{4}-80\left(\cos\left(fx+e\right)\right)^{8}a^{3}b+40\left(\cos\left(fx+e\right)\right)^{8}a^{2}b^{2}-20\left(\cos\left(fx+e\right)\right)^{6}a^{4}+212\left(\cos\left(fx+e\right)\right)^{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/15/f/(a^2+2*a*b+b^2)/(a+b)^3/(b+a*cos(f*x+e)^2)^4*(8*cos(f*x+e)^8*a^4-80
*cos(f*x+e)^8*a^3*b+40*cos(f*x+e)^8*a^2*b^2-20*cos(f*x+e)^6*a^4+212*cos(f*x
+e)^6*a^3*b-220*cos(f*x+e)^6*a^2*b^2+60*cos(f*x+e)^6*a*b^3+15*cos(f*x+e)^4*
a^4-180*cos(f*x+e)^4*a^3*b+378*cos(f*x+e)^4*a^2*b^2-180*cos(f*x+e)^4*a*b^3+
15*cos(f*x+e)^4*b^4+60*cos(f*x+e)^2*a^3*b-220*cos(f*x+e)^2*a^2*b^2+212*cos(
f*x+e)^2*a*b^3-20*cos(f*x+e)^2*b^4+40*a^2*b^2-80*a*b^3+8*b^4)*cos(f*x+e)^5*
((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/sin(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 29.0847, size = 1033, normalized size = 4.57

$$-\frac{\left(8\left(a^{4}-10\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{9}-4\left(5\,a^{4}-53\,a^{3}b+55\,a^{2}b^{2}-15\,ab^{3}\right)\cos\right.}{15\left(\left(a^{7}+5\,a^{6}b+10\,a^{5}b^{2}+10\,a^{4}b^{3}+5\,a^{3}b^{4}+a^{2}b^{5}\right)f\cos\left(fx+e\right)^{8}-2\left(a^{7}+4\,a^{6}b+5\,a^{5}b^{2}-5\,a^{3}b^{4}-4\,a^{2}b^{5}-ab^{6}\right)f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f*x + e)^9 - 4*(5*a^4 - 53*a^3*b
+ 55*a^2*b^2 - 15*a*b^3)*cos(f*x + e)^7 + 3*(5*a^4 - 60*a^3*b + 126*a^2*b^2
 - 60*a*b^3 + 5*b^4)*cos(f*x + e)^5 + 4*(15*a^3*b - 55*a^2*b^2 + 53*a*b^3 -
 5*b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 - 10*a*b^3 + b^4)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a
^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b
^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5
*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x + e)^4
+ 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*cos(f*x +
 e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f)*
sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.132 $\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx$

Optimal. Leaf size=123

$$\frac{\tan(e+fx)\cos^2(e+fx)^{p+\frac{1}{2}}(d\sin(e+fx))^m\left(\frac{-a\sin^2(e+fx)+a+b}{a+b}\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p F_1\left(\frac{m+1}{2};p+\frac{1}{2},-p;\frac{m+3}{2};\sin^2(e+f\right.}{f(m+1)}$$

```
[Out] (AppellF1[(1 + m)/2, 1/2 + p, -p, (3 + m)/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)]*(Cos[e + f*x]^2)^(1/2 + p)*(a + b*Sec[e + f*x]^2)^p*(d*Sin[e
+ f*x])^m*Tan[e + f*x])/(f*(1 + m)*((a + b - a*Sin[e + f*x]^2)/(a + b))^p)
```

Rubi [F] time = 0.0454144, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m, x]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx=\int \left(a+b\sec^2(e+fx)\right)^p (d\sin(e+fx))^m\,dx$$

Mathematica [B] time = 4.05454, size = 286, normalized size = 2.33

$$\frac{\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^m\left(a+b\sec^2(e+fx)\right)^p F_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-\tan^2(e\right.}{f(m+1)\left(F_1\left(\frac{m+1}{2};\frac{m+2}{2},-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)-\frac{\tan^2(e+fx)\left((m+2)(a+b)F_1\left(\frac{m+3}{2};\frac{m+4}{2},-p;\frac{m+5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx}{a+b}\right.\right.}{(m+3)(a+b)}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*S
in[e + f*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - ((-2*b*p*AppellF1[(3 + m)/
2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))] + (a + b)*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)/((a + b)*(3 + m)
)))
```

Maple [F] time = 1.088, size = 0, normalized size = 0.

$$\int \left(a + b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p} \left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2} + a\right)^{p} \left(d\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^{2} + a\right)^{p} \left(d\sin\left(fx+e\right)\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*(d*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

3.133 $\int \left(a+b\sec^2(e+fx)\right)^p \sin^5(e+fx)\,dx$

Optimal. Leaf size=182

$$-\frac{\left(15a^2+10ab(1-2p)+b^2\left(4p^2-8p+3\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}}{15a^2f}$$

```
[Out] ((10*a + b*(3 - 2*p))*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(15*a^
2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(1 + p))/(5*a*f) - ((15*a^2 +
 10*a*b*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Cos[e + f*x]*Hypergeometric2F1[-
1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a + b*Sec[e + f*x]^2)^p)/(15*a^2*f*
(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.192125, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4134, 462, 453, 365, 364}

$$-\frac{\left(15a^2+10ab(1-2p)+b^2\left(4p^2-8p+3\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e}{a}\right.}{15a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```


```
[Out] ((10*a + b*(3 - 2*p))*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(15*a^
2*f) - (Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(1 + p))/(5*a*f) - ((15*a^2 +
 10*a*b*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Cos[e + f*x]*Hypergeometric2F1[-
1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a + b*Sec[e + f*x]^2)^p)/(15*a^2*f*
(1 + (b*Sec[e + f*x]^2)/a)^p)
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^p}{x^6}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= -\frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{5af} + \frac{\text{Subst}\left(\int \frac{\left(-10a-b(3-2p)+5ax^2\right)\left(a+bx^2\right)^p}{x^4}\,d\right.}{5af}$$

$$= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\right.}{5af}$$

$$= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\right.}{5af}$$

$$= \frac{(10a+b(3-2p))\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\right.}{5af}$$

Mathematica [A] time = 7.7896, size = 253, normalized size = 1.39

$$\frac{2\sin^4(e+fx)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(4\left(15a^2+10ab(1-2p)+b^2\left(4p^2-8p+3\right)\right)\text{Hypergeometric2F1}\left(-\right.\right.}{15a^2f\left(4\cos(2(e+fx))\left(\frac{a+b\tan^2(e+fx)+b}{a}\right)^p-2^{-p}\left(2^p\cos(4(e+fx))\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

```
[Out] (2*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4*(4*(15*a^2 + 10*a*b
*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Hypergeometric2F1[-1/2, -p, 1/2, -((b*S
ec[e + f*x]^2)/a)] + (a + 2*b + a*Cos[2*(e + f*x)])*(-17*a - 6*b + 4*b*p +
3*a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(15*a^2*f*(4*Cos[2
*(e + f*x)]*((a + b + b*Tan[e + f*x]^2)/a)^p - (3*(((a + 2*b + a*Cos[2*(e +
 f*x)])*Sec[e + f*x]^2)/a)^p + 2^p*Cos[4*(e + f*x)]*((a + b + b*Tan[e + f*x
]^2)/a)^p)/2^p))
```

Maple [F] time = 1.09, size = 0, normalized size = 0.

$$\int \left(a + b\left(\sec\left(fx + e\right)\right)^{2}\right)^{p} \left(\sin\left(fx + e\right)\right)^{5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx + e\right)^{2} + a\right)^{p} \sin\left(fx + e\right)^{5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\cos\left(fx + e\right)^{4} - 2\cos\left(fx + e\right)^{2} + 1\right)\left(b\sec\left(fx + e\right)^{2} + a\right)^{p} \sin\left(fx + e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p*s
in(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

3.134 $\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx$

Optimal. Leaf size=117

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3af}-\frac{(3a-2bp+b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeome}}{3af}$$

```
[Out] (Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(3*a*f) - ((3*a + b - 2*b*p
)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a
 + b*Sec[e + f*x]^2)^p)/(3*a*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0953044, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4134, 453, 365, 364}

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3af}-\frac{(3a-2bp+b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2}\right.}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

```
[Out] (Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(3*a*f) - ((3*a + b - 2*b*p
)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(a
 + b*Sec[e + f*x]^2)^p)/(3*a*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)^p \sin^3(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^p}{x^4}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} + \frac{(3a+b-2bp)\,\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2}\,dx, x\right.}{3af}\\ &= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} + \frac{\left((3a+b-2bp)\left(a+b\sec^2(e+fx)\right)^p\right.}{}\\ &= \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{3af} - \frac{(3a+b-2bp)\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\right.}{}\end{aligned}$$

Mathematica [A] time = 3.84903, size = 178, normalized size = 1.52

$$-\frac{\sin^2(e+fx)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\left(a\cos(2(e+fx))+a+2b\right)\left(\frac{a+b\tan^2(e+fx)+b}{a}\right)^p-2(3a-2bp+b)\text{Hyper}\right.}{3af\left(\left(\frac{a+b\tan^2(e+fx)+b}{a}\right)^p-2\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p+\cos(2(e+fx))\left(\frac{a+b\tan^2(e+fx)+b}{a}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

```
[Out] -(Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*(-2*(3*a + b - 2*b*p
)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)] + (a + 2*b + a*
Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(3*a*f*(-2*(1 + (b*Sec
[e + f*x]^2)/a)^p + ((a + b + b*Tan[e + f*x]^2)/a)^p + Cos[2*(e + f*x)]*((a
 + b + b*Tan[e + f*x]^2)/a)^p))
```

Maple [F] time = 0.972, size = 0, normalized size = 0.

$$\int \left(a + b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p} \left(\sin\left(fx+e\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2} + a\right)^{p} \sin\left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2} - 1\right)\left(b\sec\left(fx+e\right)^{2} + a\right)^{p} \sin\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}\sin \left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

3.135 $\int \left(a+b\sec^2(e+fx)\right)^p \sin(e+fx)\,dx$

Optimal. Leaf size=68

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rubi [A] time = 0.0486125, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4134, 365, 364}

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sec^2(e+fx)\right)^p \left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{x^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^p \left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

Mathematica [A] time = 1.66542, size = 68, normalized size = 1.

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p \left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x + e\right)^2 + a\right)^p \sin \left(f x + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.136 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=77

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e
 + f*x]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

Rubi [A] time = 0.077101, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4134, 430, 429}

$$-\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```



```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e
 + f*x]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```


Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{-1+x^2}\,dx,x,\sec(e+fx\right.}{f}$$

$$=-\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\right.}{f}$$

Mathematica [B] time = 17.1162, size = 1532, normalized size = 19.9

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Csc[e + f*x]*(Sec[e + f*x]^2)^p*(a + b*Se
c[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2,
-(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a +
b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/((a + b + b*T
an[e + f*x]^2)/(a + b))^p))/(2*f*(-(a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1
+ p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((2*AppellF1[-1/2 - p, -1/2, -p, 1
/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2
])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (A
ppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[
e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p)) + p*(a + 2*b + a*Cos[2
*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((2*AppellF1[-1/2 - p, -1/2,
 -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e +
f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]
) - (AppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
```

```
]*Tan[e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p) + ((a + 2*b + a*C
os[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2
- p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Sqrt[Sec[
e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x
]^2]) + (4*(a + b)*p*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2,
 -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*(1 + ((a + b)*Cot[e + f*x]^2)/
b)^(-1 - p)*Sqrt[Csc[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])/(b*(1 + 2*p)) + (2*(
(-2*(a + b)*(-1/2 - p)*p*AppellF1[1/2 - p, -1/2, 1 - p, 3/2 - p, -Cot[e + f
*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(b*(1/2
- p)) - ((-1/2 - p)*AppellF1[1/2 - p, 1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(
((a + b)*Cot[e + f*x]^2)/b)]*Cot[e + f*x]*Csc[e + f*x]^2)/(1/2 - p))*Sqrt[S
ec[e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e +
f*x]^2]) + (2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a
+ b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2]*Tan[e + f*x])/((1 + 2*p)*(1 +
 ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) + (2*b*p*AppellF1[1, 1
/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*T
an[e + f*x]^3*((a + b + b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) - (2*A
ppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[
e + f*x]^2*Tan[e + f*x])/((a + b + b*Tan[e + f*x]^2)/(a + b))^p - (Tan[e +
f*x]^2*((b*p*AppellF1[2, 1/2, 1 - p, 3, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(a + b) - (AppellF1[2, 3/2, -p, 3
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/2))/((a + b + b*Tan[e + f*x]^2)/(a + b))^p))/2))
```

Maple [F] time = 0.383, size = 0, normalized size = 0.

$$\int \csc\left(fx+e\right)\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)

[Out] int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

3.137 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=81

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{3f}$$

```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e +
 f*x]^3*(a + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0914773, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4134, 511, 510}

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e +
 f*x]^3*(a + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rule 4134

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Di
st[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)
/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^p}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a}\right)^p}{\left(-1+x^2\right)^2}\,dx, x, \sec(e+\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p}{3f}$$

Mathematica [B] time = 4.47006, size = 266, normalized size = 3.28

$$\frac{\csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+f\right.}{f(2p-1)\left(\sec(e+fx)F_1\left(\frac{1}{2}-p;-\frac{1}{2},-p;\frac{3}{2}-p;-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)-\frac{\cot(e+fx)\csc(e+fx)\left(2p(a+b)F_1\left(\frac{3}{2}-p;-\frac{1}{2},1-p;\frac{5}{2}-p\right.\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f
*x]^2)/b)]*Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(f*(-1 + 2*p)*(-(((2*(a
 + b)*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p, -Cot[e + f*x]^2, -(((a + b)
*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5/2 - p, -Cot[e + f*x]^
2, -(((a + b)*Cot[e + f*x]^2)/b)])*Cot[e + f*x]*Csc[e + f*x])/(b*(-3 + 2*p)
)) + AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e
 + f*x]^2)/b)]*Sec[e + f*x]))
```

Maple [F] time = 0.385, size = 0, normalized size = 0.

$$\int \left(\csc \left(f x+e\right)\right)^{3} \left(a+b \left(\sec \left(f x+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)

[Out] int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x+e\right)^{2}+a\right)^{p} \csc \left(f x+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(f x+e\right)^{2}+a\right)^{p} \csc \left(f x+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^2+a\right)^p \csc \left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

3.138 $\int \left(a+b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};3,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

```
[Out] (AppellF1[5/2, 3, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rubi [A] time = 0.121844, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4132, 511, 510}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};3,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

```
[Out] (AppellF1[5/2, 3, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^p}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a+b}\right)^p}{\left(1+x^2\right)^3}\,dx, x, \tan\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2};3,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\tan^2(e+\right.}{5f}$$

Mathematica [B] time = 25.7928, size = 5878, normalized size = 66.8

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.049, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p \left(\sin\left(fx+e\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\left(b\sec\left(fx+e\right)^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

3.139 $\int \left(a+b\sec^2(e+fx)\right)^p \sin^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rubi [A] time = 0.0985019, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4132, 511, 510}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

```
[Out] (AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \sin^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^p}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a+b}\right)^p}{\left(1+x^2\right)^2}\,dx, x, \tan\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(a+b+b\tan^2(e+\right.}{3f}$$

Mathematica [B] time = 21.9097, size = 3781, normalized size = 42.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

```
[Out] (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*(a +
b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*Appel
lF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) +
2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2
```

, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*
AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*
Tan[e + f*x]^2)))/(f*(3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f
x]^2)^(-1 + p)(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[
e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 6*a*(a + b)*p
*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-2 + p)*Sin[2*(e
 + f*x)]*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)
+ (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 6*(a +
b)*(-2 + p)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[
e + f*x]^2*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f
x]^2)/(3(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + 3*(a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e + f*x]*(((2*b*p*Ap
pellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3
)/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (2*AppellF1[1/2, -
p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*T
an[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +

```
b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Sec[e + f*x]
^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p
, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Ta
n[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (AppellF1[
1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(4*(-(b*p*
AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3/2
, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*
x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[
e + f*x]^2*(-(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b))
 - (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5)) + 2*(a + b)*((6*b*p*AppellF1[5/2,
 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x
]^2*Tan[e + f*x])/(5*(a + b)) - (18*AppellF1[5/2, 4, -p, 7/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(-3*(
a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2 - (AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*(4*(b*p*
AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^
2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e +
 f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[
5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2
, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*
(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -
```

```
Tan[e + f*x]^2])*Tan[e + f*x]^2)^2)))
```

Maple [F] time = 0.872, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\left(\sin\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(b\sec\left(fx+e\right)^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

3.140 $\int \left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rubi [A] time = 0.0502151, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

Mathematica [B] time = 15.3127, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
```

```
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int \left(a + b \left(\sec \left(f x + e\right)\right)^{2}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x + e\right)^{2} + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(f x + e\right)^{2} + a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \sec^{2}\left(e + f x\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.141 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=73

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

Rubi [A] time = 0.0666715, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4132, 365, 364}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```


Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^2}\,dx, x, \tan(e\right.}{f}$$

$$= -\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2}{a}\right.}{f}$$

Mathematica [A] time = 1.09503, size = 72, normalized size = 0.99

$$-\frac{\cot(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
b))]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

Maple [F] time = 0.411, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^2\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^2+a\right)^p \csc \left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

3.142 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=128

$$-\frac{(3a+2b(p+1))\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+f}{a+b}\right.}{3f(a+b)}$$

```
[Out] -(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(3*(a + b)*f) - ((3*a
+ 2*b*(1 + p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f
*x]^2)/(a + b))]*(a + b + b*Tan[e + f*x]^2)^p)/(3*(a + b)*f*(1 + (b*Tan[e +
 f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.106278, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4132, 453, 365, 364}

$$-\frac{(3a+2b(p+1))\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f(a+b)}-\frac{\cot^3(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(3*(a + b)*f) - ((3*a
+ 2*b*(1 + p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f
*x]^2)/(a + b))]*(a + b + b*Tan[e + f*x]^2)^p)/(3*(a + b)*f*(1 + (b*Tan[e +
 f*x]^2)/(a + b))^p)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)(a+b+bx^2)^p}{x^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} + \frac{(3a+2b(1+p))\,\text{Subst}\left(\int \frac{(a+b+b}{x^2}\right.}{3(a+b)f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} + \frac{\left((3a+2b(1+p))\left(a+b+b\tan^2\right.\right.}{}$$

$$= -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f} - \frac{(3a+2b(1+p))\cot(e+fx)\,{}_2F_1}{}$$

Mathematica [A] time = 2.28075, size = 132, normalized size = 1.03

$$-\frac{\cot(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left((3a+2b(p+1))\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+\right.}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p*((3*a + 2*b*(1 + p))*Hypergeometric
2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))] + Cot[e + f*x]^2*(a + b +
 b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*(a + b)*f*(1 + (
b*Tan[e + f*x]^2)/(a + b))^p)
```

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^{4} \left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

3.143 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=192

$$-\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2}}{15f(a+b)^2}$$

```
[Out] -((10*a + b*(7 + 2*p))*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(
15*(a + b)^2*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(5*(a
 + b)*f) - ((15*a^2 + 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cot[e + f*x]*
Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b + b*
Tan[e + f*x]^2)^p)/(15*(a + b)^2*f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.173566, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4132, 462, 453, 365, 364}

$$-\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\,t}{}\right.}{15f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((10*a + b*(7 + 2*p))*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(
15*(a + b)^2*f) - (Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(5*(a
 + b)*f) - ((15*a^2 + 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cot[e + f*x]*
Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b + b*
Tan[e + f*x]^2)^p)/(15*(a + b)^2*f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 4132

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m
+ 1)/f, Subst[Int[(x^m*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p)/(1 + f
f^2*x^2)^(m/2 + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
x] && IntegerQ[m/2] && IntegerQ[n/2]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a+b+bx^2)^p}{x^6}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{(a+b+bx^2)^p(10a+b(7+2p)\cdots}{x^4}\right.}{5(a+\cdots}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\cdots}{}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\cdots}{}$$

$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\cdots}{}$$

Mathematica [A] time = 1.98683, size = 149, normalized size = 0.78

$$-\frac{\cot(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(15\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+3\cot^4(e+\cdots\right.}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -((b*Tan
[e + f*x]^2)/(a + b))] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -p, -1/2
, -((b*Tan[e + f*x]^2)/(a + b))] + 15*Hypergeometric2F1[-1/2, -p, 1/2, -((b
*Tan[e + f*x]^2)/(a + b))])*(a + b*Sec[e + f*x]^2)^p)/(15*f*(1 + (b*Tan[e +
 f*x]^2)/(a + b))^p)
```

Maple [F] time = 0.454, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^6\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

3.144 $\int \left(a-a\sec^2(c+dx)\right)^4\,dx$

Optimal. Leaf size=74

$$\frac{a^4\tan^7(c+dx)}{7d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan(c+dx)}{d}+a^4x$$

```
[Out] a^4*x - (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d) - (a^4*Tan[c + d*
x]^5)/(5*d) + (a^4*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.047288, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 3473, 8}

$$\frac{a^4\tan^7(c+dx)}{7d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan(c+dx)}{d}+a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*x - (a^4*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d) - (a^4*Tan[c + d*
x]^5)/(5*d) + (a^4*Tan[c + d*x]^7)/(7*d)
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sec^2(c+dx)\right)^4\,dx &= a^4\int \tan^8(c+dx)\,dx\\ &= \frac{a^4\tan^7(c+dx)}{7d}-a^4\int \tan^6(c+dx)\,dx\\ &= -\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}+a^4\int \tan^4(c+dx)\,dx\\ &= \frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}-a^4\int \tan^2(c+dx)\,dx\\ &= -\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}+a^4\int 1\,dx\\ &= a^4x-\frac{a^4\tan(c+dx)}{d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] time = 0.038957, size = 72, normalized size = 0.97

$$a^4\left(\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}+\frac{\tan^{-1}(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c
 + d*x]^5/(5*d) + Tan[c + d*x]^7/(7*d))
```

Maple [A] time = 0.03, size = 125, normalized size = 1.7

$$\frac{1}{d}\left(a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-2/3-1/3(\sec(dx+c))^2\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-1/5(\sec(dx+c))^4-\frac{4(\ldots}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^4,x)
```

```
[Out] 1/d*(a^4*(d*x+c)-4*a^4*tan(d*x+c)-6*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+
4*a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-a^4*(-16/35-1/7
*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.0147, size = 174, normalized size = 2.35

$$a^4x + \frac{\left(5\,\tan\left(dx+c\right)^7 + 21\,\tan\left(dx+c\right)^5 + 35\,\tan\left(dx+c\right)^3 + 35\,\tan\left(dx+c\right)\right)a^4}{35\,d} - \frac{4\left(3\,\tan\left(dx+c\right)^5 + 10\,\tan\left(dx+c\right.\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/35*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35
*tan(d*x + c))*a^4/d - 4/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(
d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - 4*a^4*tan(d*x
 + c)/d
```

Fricas [A] time = 0.499195, size = 207, normalized size = 2.8

$$\frac{105\,a^4dx\cos\left(dx+c\right)^7 - \left(176\,a^4\cos\left(dx+c\right)^6 - 122\,a^4\cos\left(dx+c\right)^4 + 66\,a^4\cos\left(dx+c\right)^2 - 15\,a^4\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^4*d*x*cos(d*x + c)^7 - (176*a^4*cos(d*x + c)^6 - 122*a^4*cos(d
*x + c)^4 + 66*a^4*cos(d*x + c)^2 - 15*a^4)*sin(d*x + c))/(d*cos(d*x + c)^7
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int 1\,dx + \int -4\sec^2\left(c+dx\right)dx + \int 6\sec^4\left(c+dx\right)dx + \int -4\sec^6\left(c+dx\right)dx + \int \sec^8\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**4,x)
```

```
[Out] a**4*(Integral(1, x) + Integral(-4*sec(c + d*x)**2, x) + Integral(6*sec(c +
 d*x)**4, x) + Integral(-4*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**8,
```

```
x))
```

Giac [A] time = 1.30071, size = 89, normalized size = 1.2

$$\frac{15\,a^4\tan\left(dx+c\right)^7-21\,a^4\tan\left(dx+c\right)^5+35\,a^4\tan\left(dx+c\right)^3+105\,(dx+c)a^4-105\,a^4\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^4*tan(d*x + c)^7 - 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)^
3 + 105*(d*x + c)*a^4 - 105*a^4*tan(d*x + c))/d
```

3.145 $\int \left(a-a\sec^2(c+dx)\right)^3 dx$

Optimal. Leaf size=56

$$-\frac{a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x$$

```
[Out] a^3*x - (a^3*Tan[c + d*x])/d + (a^3*Tan[c + d*x]^3)/(3*d) - (a^3*Tan[c + d*
x]^5)/(5*d)
```

Rubi [A] time = 0.0385128, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 3473, 8}

$$-\frac{a^3\tan^5(c+dx)}{5d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^3,x]
```

```
[Out] a^3*x - (a^3*Tan[c + d*x])/d + (a^3*Tan[c + d*x]^3)/(3*d) - (a^3*Tan[c + d*
x]^5)/(5*d)
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a - a\sec^2(c+dx)\right)^3 dx = -\left(a^3 \int \tan^6(c+dx)\,dx\right)$$
$$= -\frac{a^3\tan^5(c+dx)}{5d} + a^3 \int \tan^4(c+dx)\,dx$$
$$= \frac{a^3\tan^3(c+dx)}{3d} - \frac{a^3\tan^5(c+dx)}{5d} - a^3 \int \tan^2(c+dx)\,dx$$
$$= -\frac{a^3\tan(c+dx)}{d} + \frac{a^3\tan^3(c+dx)}{3d} - \frac{a^3\tan^5(c+dx)}{5d} + a^3 \int 1\,dx$$
$$= a^3x - \frac{a^3\tan(c+dx)}{d} + \frac{a^3\tan^3(c+dx)}{3d} - \frac{a^3\tan^5(c+dx)}{5d}$$

Mathematica [A] time = 0.03132, size = 58, normalized size = 1.04

$$-a^3\left(\frac{\tan^5(c+dx)}{5d} - \frac{\tan^3(c+dx)}{3d} - \frac{\tan^{-1}(\tan(c+dx))}{d} + \frac{\tan(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^3,x]
```

```
[Out] -(a^3*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) +
Tan[c + d*x]^5/(5*d)))
```

Maple [A] time = 0.026, size = 81, normalized size = 1.5

$$\frac{1}{d}\left(a^3(dx+c) - 3a^3\tan(dx+c) - 3a^3\left(-2/3 - 1/3\,(\sec(dx+c))^2\right)\tan(dx+c) + a^3\left(-\frac{8}{15} - \frac{(\sec(dx+c))^4}{5} - \frac{4\,(\sec(c}{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)-3*a^3*tan(d*x+c)-3*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+
a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.04516, size = 109, normalized size = 1.95

$$a^3x - \frac{\left(3 \tan\left(dx+c\right)^5 + 10 \tan\left(dx+c\right)^3 + 15 \tan\left(dx+c\right)\right)a^3}{15\,d} + \frac{\left(\tan\left(dx+c\right)^3 + 3 \tan\left(dx+c\right)\right)a^3}{d} - \frac{3\,a^3\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3/d
 + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3/d - 3*a^3*tan(d*x + c)/d
```

Fricas [A] time = 0.48466, size = 167, normalized size = 2.98

$$\frac{15\,a^3dx\cos\left(dx+c\right)^5 - \left(23\,a^3\cos\left(dx+c\right)^4 - 11\,a^3\cos\left(dx+c\right)^2 + 3\,a^3\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*d*x*cos(d*x + c)^5 - (23*a^3*cos(d*x + c)^4 - 11*a^3*cos(d*x +
 c)^2 + 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^3\left(\int(-1)\;dx + \int 3\sec^2\left(c+dx\right)dx + \int -3\sec^4\left(c+dx\right)dx + \int \sec^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**3,x)
```

```
[Out] -a**3*(Integral(-1, x) + Integral(3*sec(c + d*x)**2, x) + Integral(-3*sec(c
 + d*x)**4, x) + Integral(sec(c + d*x)**6, x))
```

Giac [A] time = 1.17522, size = 72, normalized size = 1.29

$$-\frac{3\,a^3\tan\left(dx+c\right)^5-5\,a^3\tan\left(dx+c\right)^3-15\,(dx+c)a^3+15\,a^3\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(3*a^3*tan(d*x + c)^5 - 5*a^3*tan(d*x + c)^3 - 15*(d*x + c)*a^3 + 15*
a^3*tan(d*x + c))/d
```

3.146 $\int \left(a-a\sec^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=38

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x$$

```
[Out] a^2*x - (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0300588, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 3473, 8}

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^2,x]
```

```
[Out] a^2*x - (a^2*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sec^2(c+dx)\right)^2\,dx &= a^2\int \tan^4(c+dx)\,dx\\ &= \frac{a^2\tan^3(c+dx)}{3d}-a^2\int \tan^2(c+dx)\,dx\\ &= -\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}+a^2\int 1\,dx\\ &= a^2x-\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0203196, size = 42, normalized size = 1.11

$$a^2\left(\frac{\tan^3(c+dx)}{3d}+\frac{\tan^{-1}(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^2,x]
```

```
[Out] a^2*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))
```

Maple [A] time = 0.024, size = 49, normalized size = 1.3

$$\frac{1}{d}\left(a^2\,(dx+c)-2\,a^2\tan\,(dx+c)-a^2\left(-\frac{2}{3}-\frac{(\sec\,(dx+c))^2}{3}\right)\tan\,(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c)-2*a^2*tan(d*x+c)-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.0022, size = 61, normalized size = 1.61

$$a^2x+\frac{\left(\tan\,(dx+c)^3+3\,\tan\,(dx+c)\right)a^2}{3\,d}-\frac{2\,a^2\tan\,(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2/d - 2*a^2*tan(d*x + c)/d
```

Fricas [A] time = 0.475513, size = 128, normalized size = 3.37

$$\frac{3\,a^2dx\cos\left(dx+c\right)^3-\left(4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x*cos(d*x + c)^3 - (4*a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))/
(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 1\,dx+\int -2\sec^2\left(c+dx\right)dx+\int \sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**2,x)
```

```
[Out] a**2*(Integral(1, x) + Integral(-2*sec(c + d*x)**2, x) + Integral(sec(c + d
*x)**4, x))
```

Giac [A] time = 1.27645, size = 53, normalized size = 1.39

$$\frac{a^2\tan\left(dx+c\right)^3+3\left(dx+c\right)a^2-3\,a^2\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + 3*(d*x + c)*a^2 - 3*a^2*tan(d*x + c))/d
```

3.147 $\int \left(a - a\sec^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$ax - \frac{a\tan(c+dx)}{d}$$

```
[Out] a*x - (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0126255, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3767, 8}

$$ax - \frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a - a*Sec[c + d*x]^2,x]
```

```
[Out] a*x - (a*Tan[c + d*x])/d
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a - a\sec^2(c+dx)\right)\,dx &= ax - a\int \sec^2(c+dx)\,dx\\ &= ax + \frac{a\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d}\\ &= ax - \frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0062625, size = 26, normalized size = 1.62

$$-a\left(\frac{\tan(c+dx)}{d}-\frac{\tan^{-1}(\tan(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - a*Sec[c + d*x]^2,x]
```

```
[Out] -(a*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d))
```

Maple [A] time = 0.014, size = 17, normalized size = 1.1

$$ax-\frac{a\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a-a*sec(d*x+c)^2,x)
```

```
[Out] a*x-a*tan(d*x+c)/d
```

Maxima [A] time = 1.05065, size = 22, normalized size = 1.38

$$ax-\frac{a\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x - a*tan(d*x + c)/d
```

Fricas [A] time = 0.465543, size = 76, normalized size = 4.75

$$\frac{adx\cos(dx+c)-a\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*d*x*cos(d*x + c) - a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a\left(\int(-1)\,dx+\int\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)**2,x)
```

```
[Out] -a*(Integral(-1, x) + Integral(sec(c + d*x)**2, x))
```

Giac [A] time = 1.31491, size = 22, normalized size = 1.38

$$ax-\frac{a\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x - a*tan(d*x + c)/d
```

3.148 $\int \frac{1}{a-a\sec^2(c+dx)}\,dx$

Optimal. Leaf size=19

$$\frac{\cot(c+dx)}{ad}+\frac{x}{a}$$

```
[Out] x/a + Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.0214506, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 3473, 8}

$$\frac{\cot(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-1),x]
```

```
[Out] x/a + Cot[c + d*x]/(a*d)
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = -\frac{\int \cot^2(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)}{ad} + \frac{\int 1\,dx}{a}$$
$$= \frac{x}{a} + \frac{\cot(c+dx)}{ad}$$

Mathematica [C] time = 0.0279704, size = 31, normalized size = 1.63

$$\frac{\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-1),x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/(a*d)
```

Maple [A] time = 0.047, size = 31, normalized size = 1.6

$$\frac{\arctan\left(\tan\left(dx+c\right)\right)}{ad} + \frac{1}{ad\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2),x)
```

```
[Out] 1/a/d*arctan(tan(d*x+c))+1/a/d/tan(d*x+c)
```

Maxima [A] time = 1.52573, size = 35, normalized size = 1.84

$$\frac{\frac{dx+c}{a} + \frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] ((d*x + c)/a + 1/(a*tan(d*x + c)))/d
```

Fricas [A] time = 0.463929, size = 73, normalized size = 3.84

$$\frac{dx\sin(dx+c)+\cos(dx+c)}{ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (d*x*sin(d*x + c) + cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{1}{\sec^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2),x)
```

```
[Out] -Integral(1/(sec(c + d*x)**2 - 1), x)/a
```

Giac [B] time = 1.29523, size = 61, normalized size = 3.21

$$\frac{\frac{2(dx+c)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a - tan(1/2*d*x + 1/2*c)/a + 1/(a*tan(1/2*d*x + 1/2*c)))/d
```

3.149 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.0299385, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 3473, 8}

$$-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-2),x]
```

```
[Out] x/a^2 + Cot[c + d*x]/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d)
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^2}\,dx = \frac{\int \cot^4(c+dx)\,dx}{a^2}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d} - \frac{\int \cot^2(c+dx)\,dx}{a^2}$$
$$= \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\int 1\,dx}{a^2}$$
$$= \frac{x}{a^2} + \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d}$$

Mathematica [C] time = 0.0267814, size = 36, normalized size = 0.97

$$-\frac{\cot^3(c+dx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-2),x]
```

```
[Out] -(Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*a^2*
d)
```

Maple [A] time = 0.05, size = 47, normalized size = 1.3

$$\frac{\arctan(\tan(dx+c))}{da^2} - \frac{1}{3\,da^2(\tan(dx+c))^3} + \frac{1}{da^2\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*arctan(tan(d*x+c))-1/3/d/a^2/tan(d*x+c)^3+1/d/a^2/tan(d*x+c)
```

Maxima [A] time = 1.5722, size = 54, normalized size = 1.46

$$\frac{\frac{3(dx+c)}{a^2} + \frac{3\tan(dx+c)^2-1}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^2 - 1)/(a^2*tan(d*x + c)^3))/d
```

Fricas [B] time = 0.472582, size = 177, normalized size = 4.78

$$\frac{4\cos(dx+c)^3+3\left(dx\cos(dx+c)^2-dx\right)\sin(dx+c)-3\cos(dx+c)}{3\left(a^2d\cos(dx+c)^2-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(d*x + c)^3 + 3*(d*x*cos(d*x + c)^2 - d*x)*sin(d*x + c) - 3*cos(d
*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^4(c+dx)-2\sec^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**4 - 2*sec(c + d*x)**2 + 1), x)/a**2
```

Giac [B] time = 1.20012, size = 108, normalized size = 2.92

$$\frac{\frac{24(dx+c)}{a^2}+\frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)/a^2 + (15*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/2*d*x +
 1/2*c)^3) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

3.150 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=55

$$\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{x}{a^3}$$

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d)
```

Rubi [A] time = 0.0387751, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 3473, 8}

$$\frac{\cot^5(c+dx)}{5a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-3),x]
```

```
[Out] x/a^3 + Cot[c + d*x]/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + Cot[c + d*x]^5/(5
*a^3*d)
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^3}\,dx = -\frac{\int \cot^6(c+dx)\,dx}{a^3}$$
$$= \frac{\cot^5(c+dx)}{5a^3d} + \frac{\int \cot^4(c+dx)\,dx}{a^3}$$
$$= -\frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} - \frac{\int \cot^2(c+dx)\,dx}{a^3}$$
$$= \frac{\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d} + \frac{\int 1\,dx}{a^3}$$
$$= \frac{x}{a^3} + \frac{\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{\cot^5(c+dx)}{5a^3d}$$

Mathematica [C] time = 0.0473571, size = 36, normalized size = 0.65

$$\frac{\cot^5(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-3),x]
```

```
[Out] (Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*a^3*d
)
```

Maple [A] time = 0.049, size = 63, normalized size = 1.2

$$\frac{\arctan(\tan(dx+c))}{da^3} - \frac{1}{3\,da^3\,(\tan(dx+c))^3} + \frac{1}{5\,da^3\,(\tan(dx+c))^5} + \frac{1}{da^3\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^3,x)
```

```
[Out] 1/d/a^3*arctan(tan(d*x+c))-1/3/d/a^3/tan(d*x+c)^3+1/5/d/a^3/tan(d*x+c)^5+1/
d/a^3/tan(d*x+c)
```

Maxima [A] time = 1.51894, size = 68, normalized size = 1.24

$$\frac{\frac{15\,(dx+c)}{a^3}+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{a^3\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(a^3*ta
n(d*x + c)^5))/d
```

Fricas [B] time = 0.484161, size = 274, normalized size = 4.98

$$\frac{23\,\cos(dx+c)^5-35\,\cos(dx+c)^3+15\left(dx\cos(dx+c)^4-2\,dx\cos(dx+c)^2+dx\right)\sin(dx+c)+15\,\cos(dx+c)}{15\left(a^3d\cos(dx+c)^4-2\,a^3d\cos(dx+c)^2+a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*(d*x*cos(d*x + c)^4 - 2*d*
x*cos(d*x + c)^2 + d*x)*sin(d*x + c) + 15*cos(d*x + c))/((a^3*d*cos(d*x + c
)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{1}{\sec^6(c+dx)-3\sec^4(c+dx)+3\sec^2(c+dx)-1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**3,x)
```

```
[Out] -Integral(1/(sec(c + d*x)**6 - 3*sec(c + d*x)**4 + 3*sec(c + d*x)**2 - 1),
x)/a**3
```

Giac [B] time = 1.30958, size = 150, normalized size = 2.73

$$\frac{\frac{480\,(dx+c)}{a^3}+\frac{330\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+330\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(480*(d*x + c)/a^3 + (330*tan(1/2*d*x + 1/2*c)^4 - 35*tan(1/2*d*x + 1
/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
- 35*a^12*tan(1/2*d*x + 1/2*c)^3 + 330*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.151 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=73

$$-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot(c+dx)}{a^4d}+\frac{x}{a^4}$$

```
[Out] x/a^4 + Cot[c + d*x]/(a^4*d) - Cot[c + d*x]^3/(3*a^4*d) + Cot[c + d*x]^5/(5
*a^4*d) - Cot[c + d*x]^7/(7*a^4*d)
```

Rubi [A] time = 0.0453561, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4120, 3473, 8}

$$-\frac{\cot^7(c+dx)}{7a^4d}+\frac{\cot^5(c+dx)}{5a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{\cot(c+dx)}{a^4d}+\frac{x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] x/a^4 + Cot[c + d*x]/(a^4*d) - Cot[c + d*x]^3/(3*a^4*d) + Cot[c + d*x]^5/(5
*a^4*d) - Cot[c + d*x]^7/(7*a^4*d)
```

Rule 4120

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx &= \frac{\int \cot^8(c+dx)\,dx}{a^4} \\
&= -\frac{\cot^7(c+dx)}{7a^4d} - \frac{\int \cot^6(c+dx)\,dx}{a^4} \\
&= \frac{\cot^5(c+dx)}{5a^4d} - \frac{\cot^7(c+dx)}{7a^4d} + \frac{\int \cot^4(c+dx)\,dx}{a^4} \\
&= -\frac{\cot^3(c+dx)}{3a^4d} + \frac{\cot^5(c+dx)}{5a^4d} - \frac{\cot^7(c+dx)}{7a^4d} - \frac{\int \cot^2(c+dx)\,dx}{a^4} \\
&= \frac{\cot(c+dx)}{a^4d} - \frac{\cot^3(c+dx)}{3a^4d} + \frac{\cot^5(c+dx)}{5a^4d} - \frac{\cot^7(c+dx)}{7a^4d} + \frac{\int 1\,dx}{a^4} \\
&= \frac{x}{a^4} + \frac{\cot(c+dx)}{a^4d} - \frac{\cot^3(c+dx)}{3a^4d} + \frac{\cot^5(c+dx)}{5a^4d} - \frac{\cot^7(c+dx)}{7a^4d}
\end{aligned}
$$

Mathematica [C] time = 0.0175616, size = 36, normalized size = 0.49

$$-\frac{\cot^7(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] -(Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(7*a^4*
d)
```

Maple [A] time = 0.05, size = 79, normalized size = 1.1

$$\frac{\arctan(\tan(dx+c))}{da^4} - \frac{1}{7\,da^4(\tan(dx+c))^7} - \frac{1}{3\,da^4(\tan(dx+c))^3} + \frac{1}{5\,da^4(\tan(dx+c))^5} + \frac{1}{da^4\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^4,x)
```

```
[Out] 1/d/a^4*arctan(tan(d*x+c))-1/7/d/a^4/tan(d*x+c)^7-1/3/d/a^4/tan(d*x+c)^3+1/
5/d/a^4/tan(d*x+c)^5+1/d/a^4/tan(d*x+c)
```

Maxima [A] time = 1.5127, size = 81, normalized size = 1.11

$$\frac{\frac{105\,(dx+c)}{a^4}+\frac{105\,\tan(dx+c)^6-35\,\tan(dx+c)^4+21\,\tan(dx+c)^2-15}{a^4\,\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/105*(105*(d*x + c)/a^4 + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*tan
(d*x + c)^2 - 15)/(a^4*tan(d*x + c)^7))/d
```

Fricas [B] time = 0.494052, size = 374, normalized size = 5.12

$$\frac{176\,\cos(dx+c)^7-406\,\cos(dx+c)^5+350\,\cos(dx+c)^3+105\left(dx\cos(dx+c)^6-3\,dx\cos(dx+c)^4+3\,dx\cos(dx+c\right.}{105\left(a^4d\cos(dx+c)^6-3\,a^4d\cos(dx+c)^4+3\,a^4d\cos(dx+c)^2-a^4d\right)\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(176*cos(d*x + c)^7 - 406*cos(d*x + c)^5 + 350*cos(d*x + c)^3 + 105*(
d*x*cos(d*x + c)^6 - 3*d*x*cos(d*x + c)^4 + 3*d*x*cos(d*x + c)^2 - d*x)*sin
(d*x + c) - 105*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)
^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^8(c+dx)-4\sec^6(c+dx)+6\sec^4(c+dx)-4\sec^2(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**4,x)
```

```
[Out] Integral(1/(sec(c + d*x)**8 - 4*sec(c + d*x)**6 + 6*sec(c + d*x)**4 - 4*sec
(c + d*x)**2 + 1), x)/a**4
```

Giac [B] time = 1.1497, size = 188, normalized size = 2.58

$$\frac{\frac{13440\,(dx+c)}{a^4} + \frac{9765\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 1295\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 189\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 15}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7} + \frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 189\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 1295\,a^{24}\tan\left(\frac{1}{2}\,d\right.}{a^{28}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/13440*(13440*(d*x + c)/a^4 + (9765*tan(1/2*d*x + 1/2*c)^6 - 1295*tan(1/2*
d*x + 1/2*c)^4 + 189*tan(1/2*d*x + 1/2*c)^2 - 15)/(a^4*tan(1/2*d*x + 1/2*c)
^7) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*a^24*tan(1/2*d*x + 1/2*c)^5 + 1
295*a^24*tan(1/2*d*x + 1/2*c)^3 - 9765*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.152 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=98

$$\frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(6a+5b)\tan(e+fx)\sec^3(e+fx)}{24f}+\frac{(6a+5b)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b\tan(e+f$$

```
[Out] ((6*a + 5*b)*ArcTanh[Sin[e + f*x]])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]*Tan[
e + f*x])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]^3*Tan[e + f*x])/(24*f) + (b*Se
c[e + f*x]^5*Tan[e + f*x])/(6*f)
```

Rubi [A] time = 0.0593756, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3768, 3770}

$$\frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(6a+5b)\tan(e+fx)\sec^3(e+fx)}{24f}+\frac{(6a+5b)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b\tan(e+f$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```



```
[Out] ((6*a + 5*b)*ArcTanh[Sin[e + f*x]])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]*Tan[
e + f*x])/(16*f) + ((6*a + 5*b)*Sec[e + f*x]^3*Tan[e + f*x])/(24*f) + (b*Se
c[e + f*x]^5*Tan[e + f*x])/(6*f)
```


Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\sec^5(e+fx)\tan(e+fx)}{6f}+\frac{1}{6}(6a+5b)\int \sec^5(e+fx)\,dx$$
$$=\frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}+\frac{1}{8}(6a+5b)$$
$$=\frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}+\frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{b\sec}{}$$
$$=\frac{(6a+5b)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}+\frac{(6a+5b}{}$$

Mathematica [A] time = 0.313931, size = 75, normalized size = 0.77

$$\frac{3(6a+5b)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\sec(e+fx)\left(2(6a+5b)\sec^2(e+fx)+3(6a+5b)+8b\sec^4(e+fx)\right)}{48f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (3*(6*a + 5*b)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(6*a + 5*b) + 2*(6*a
 + 5*b)*Sec[e + f*x]^2 + 8*b*Sec[e + f*x]^4)*Tan[e + f*x])/(48*f)
```

Maple [A] time = 0.031, size = 138, normalized size = 1.4

$$\frac{a\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^3}{4f}+\frac{3a\tan\left(fx+e\right)\sec\left(fx+e\right)}{8f}+\frac{3a\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}+\frac{b\left(\sec\left(fx+e\right)\right)^5}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/4/f*a*tan(f*x+e)*sec(f*x+e)^3+3/8/f*a*tan(f*x+e)*sec(f*x+e)+3/8/f*a*ln(se
c(f*x+e)+tan(f*x+e))+1/6*b*sec(f*x+e)^5*tan(f*x+e)/f+5/24*b*sec(f*x+e)^3*ta
```

```
n(f*x+e)/f+5/16*b*sec(f*x+e)*tan(f*x+e)/f+5/16/f*b*ln(sec(f*x+e)+tan(f*x+e)
)
```

Maxima [A] time = 0.99761, size = 170, normalized size = 1.73

$$\frac{3\,(6\,a+5\,b)\log\left(\sin\left(fx+e\right)+1\right)-3\,(6\,a+5\,b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\,(6\,a+5\,b)\sin\left(fx+e\right)^{5}-8\,(6\,a+5\,b)\sin\left(fx+e\right)^{3}+3\,(10\,a+11}{\sin\left(fx+e\right)^{6}-3\,\sin\left(fx+e\right)^{4}+3\,\sin\left(fx+e\right)^{2}-1}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/96*(3*(6*a + 5*b)*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*log(sin(f*x + e)
- 1) - 2*(3*(6*a + 5*b)*sin(f*x + e)^5 - 8*(6*a + 5*b)*sin(f*x + e)^3 + 3*(
10*a + 11*b)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x +
 e)^2 - 1))/f
```

Fricas [A] time = 0.517976, size = 293, normalized size = 2.99

$$\frac{3\,(6\,a+5\,b)\cos\left(fx+e\right)^{6}\log\left(\sin\left(fx+e\right)+1\right)-3\,(6\,a+5\,b)\cos\left(fx+e\right)^{6}\log\left(-\sin\left(fx+e\right)+1\right)+2\left(3\,(6\,a+5\,b)\,\mathrm{c}}{96\,f\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(6*a + 5*b)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*co
s(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(6*a + 5*b)*cos(f*x + e)^4 + 2*(
6*a + 5*b)*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

Giac [A] time = 1.32571, size = 176, normalized size = 1.8

$$\frac{3\,(6\,a+5\,b)\log\left(\sin\left(fx+e\right)+1\right)-3\,(6\,a+5\,b)\log\left(-\sin\left(fx+e\right)+1\right)-\frac{2\left(18\,a\sin\left(fx+e\right)^{5}+15\,b\sin\left(fx+e\right)^{5}-48\,a\sin\left(fx+e\right)^{3}-40\,b\sin\left(fx+e\right)^{3}+30\,a\sin\left(fx+e\right)+33\,b\sin\left(fx+e\right)\right)}{\left(\sin\left(fx+e\right)^{2}-1\right)^{3}}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/96*(3*(6*a + 5*b)*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*log(-sin(f*x + e)
 + 1) - 2*(18*a*sin(f*x + e)^5 + 15*b*sin(f*x + e)^5 - 48*a*sin(f*x + e)^3
- 40*b*sin(f*x + e)^3 + 30*a*sin(f*x + e) + 33*b*sin(f*x + e))/(sin(f*x + e
)^2 - 1)^3)/f
```

3.153 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4a+3b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

```
[Out] ((4*a + 3*b)*ArcTanh[Sin[e + f*x]])/(8*f) + ((4*a + 3*b)*Sec[e + f*x]*Tan[e
 + f*x])/(8*f) + (b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

Rubi [A] time = 0.0457003, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3768, 3770}

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4a+3b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```


```
[Out] ((4*a + 3*b)*ArcTanh[Sin[e + f*x]])/(8*f) + ((4*a + 3*b)*Sec[e + f*x]*Tan[e
 + f*x])/(8*f) + (b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```


Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)dx = \frac{b\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{1}{4}(4a+3b)\int \sec^3(e+fx)\,dx$$
$$= \frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{1}{8}(4a+3b)$$
$$= \frac{(4a+3b)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f}+\frac{b\sec^3(e}{}$$

Mathematica [A] time = 0.123698, size = 54, normalized size = 0.77

$$\frac{(4a+3b)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\sec(e+fx)\left(4a+2b\sec^2(e+fx)+3b\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((4*a + 3*b)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(4*a + 3*b + 2*b*Sec[e +
f*x]^2)*Tan[e + f*x])/(8*f)
```

Maple [A] time = 0.028, size = 98, normalized size = 1.4

$$\frac{a\tan(fx+e)\sec(fx+e)}{2f}+\frac{a\ln\left(\sec(fx+e)+\tan(fx+e)\right)}{2f}+\frac{b\left(\sec(fx+e)\right)^3\tan(fx+e)}{4f}+\frac{3b\sec(fx+e)\tan}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2/f*a*tan(f*x+e)*sec(f*x+e)+1/2/f*a*ln(sec(f*x+e)+tan(f*x+e))+1/4*b*sec(f
*x+e)^3*tan(f*x+e)/f+3/8*b*sec(f*x+e)*tan(f*x+e)/f+3/8/f*b*ln(sec(f*x+e)+ta
n(f*x+e))
```

Maxima [A] time = 0.994445, size = 131, normalized size = 1.87

$$\frac{(4\,a+3\,b)\log\left(\sin\left(fx+e\right)+1\right)-(4\,a+3\,b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left((4\,a+3\,b)\sin\left(fx+e\right)^{3}-(4\,a+5\,b)\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/16*((4*a + 3*b)*log(sin(f*x + e) + 1) - (4*a + 3*b)*log(sin(f*x + e) - 1)
 - 2*((4*a + 3*b)*sin(f*x + e)^3 - (4*a + 5*b)*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.507906, size = 243, normalized size = 3.47

$$\frac{(4\,a+3\,b)\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-(4\,a+3\,b)\cos\left(fx+e\right)^{4}\log\left(-\sin\left(fx+e\right)+1\right)+2\left((4\,a+3\,b)\cos\left(f\right.\right.}{16\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/16*((4*a + 3*b)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (4*a + 3*b)*cos(f*
x + e)^4*log(-sin(f*x + e) + 1) + 2*((4*a + 3*b)*cos(f*x + e)^2 + 2*b)*sin(
f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

Giac [A] time = 1.29215, size = 142, normalized size = 2.03

$$\frac{(4a+3b)\log\left(\sin\left(fx+e\right)+1\right)-(4a+3b)\log\left(-\sin\left(fx+e\right)+1\right)-\frac{2\left(4a\sin\left(fx+e\right)^{3}+3b\sin\left(fx+e\right)^{3}-4a\sin\left(fx+e\right)-5b\sin\left(fx+e\right)\right)}{\left(\sin\left(fx+e\right)^{2}-1\right)^{2}}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/16*((4*a + 3*b)*log(sin(f*x + e) + 1) - (4*a + 3*b)*log(-sin(f*x + e) + 1
) - 2*(4*a*sin(f*x + e)^3 + 3*b*sin(f*x + e)^3 - 4*a*sin(f*x + e) - 5*b*sin
(f*x + e))/(sin(f*x + e)^2 - 1)^2)/f
```

3.154 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] ((2*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e + f*x]*Tan[e + f*x])/(2*
f)
```

Rubi [A] time = 0.0248988, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4046, 3770}

$$\frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e + f*x]*Tan[e + f*x])/(2*
f)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}(2a+b)\int \sec(e+fx)\,dx$$
$$= \frac{(2a+b)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

Mathematica [A] time = 0.0206032, size = 48, normalized size = 1.2

$$\frac{a\tanh^{-1}(\sin(e+fx))}{f} + \frac{b\tanh^{-1}(\sin(e+fx))}{2f} + \frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*ArcTanh[Sin[e + f*x]])/f + (b*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e +
f*x]*Tan[e + f*x])/(2*f)
```

Maple [A] time = 0.027, size = 59, normalized size = 1.5

$$\frac{a\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f} + \frac{b\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f} + \frac{b\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*a*ln(sec(f*x+e)+tan(f*x+e))+1/2*b*sec(f*x+e)*tan(f*x+e)/f+1/2/f*b*ln(se
c(f*x+e)+tan(f*x+e))
```

Maxima [A] time = 0.986785, size = 78, normalized size = 1.95

$$\frac{(2a+b)\log\left(\sin\left(fx+e\right)+1\right) - (2a+b)\log\left(\sin\left(fx+e\right)-1\right) - \frac{2b\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*a + b)*log(sin(f*x + e) + 1) - (2*a + b)*log(sin(f*x + e) - 1) - 2*
b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Fricas [A] time = 0.494837, size = 192, normalized size = 4.8

$$\frac{(2a+b)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2a+b)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2b\sin\left(fx+e\right)}{4f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a + b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a + b)*cos(f*x + e
)^2*log(-sin(f*x + e) + 1) + 2*b*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

Giac [A] time = 1.29418, size = 86, normalized size = 2.15

$$\frac{(2a+b)\log\left(\sin\left(fx+e\right)+1\right)-(2a+b)\log\left(-\sin\left(fx+e\right)+1\right)-\frac{2b\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/4*((2*a + b)*log(sin(f*x + e) + 1) - (2*a + b)*log(-sin(f*x + e) + 1) - 2
*b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

3.155 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(e+fx)}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{f}$$

```
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Sin[e + f*x])/f
```

Rubi [A] time = 0.0275183, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4045, 3770}

$$\frac{a\sin(e+fx)}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Sin[e + f*x])/f
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{a\sin(e+fx)}{f}+b\int \sec(e+fx)\,dx\\ &= \frac{b\tanh^{-1}(\sin(e+fx))}{f}+\frac{a\sin(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0172255, size = 35, normalized size = 1.46

$$\frac{a\sin(e)\cos(fx)}{f}+\frac{a\cos(e)\sin(fx)}{f}+\frac{b\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/f + (a*Cos[f*x]*Sin[e])/f + (a*Cos[e]*Sin[f*x])/f
```

Maple [A] time = 0.046, size = 32, normalized size = 1.3

$$\frac{\sin\left(fx+e\right)a}{f}+\frac{b\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

```
[Out] a*sin(f*x+e)/f+1/f*b*ln(sec(f*x+e)+tan(f*x+e))
```

Maxima [A] time = 0.986977, size = 51, normalized size = 2.12

$$\frac{b\left(\log\left(\sin\left(fx+e\right)+1\right)-\log\left(\sin\left(fx+e\right)-1\right)\right)+2\,a\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)) + 2*a*sin(f*x + e))/
f
```

Fricas [A] time = 0.50083, size = 107, normalized size = 4.46

$$\frac{b\log\left(\sin\left(fx+e\right)+1\right)-b\log\left(-\sin\left(fx+e\right)+1\right)+2\,a\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(sin(f*x + e) + 1) - b*log(-sin(f*x + e) + 1) + 2*a*sin(f*x + e))
/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\cos\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

Giac [A] time = 1.28577, size = 58, normalized size = 2.42

$$\frac{b\log\left(\sin\left(fx + e\right) + 1\right) - b\log\left(-\sin\left(fx + e\right) + 1\right) + 2\,a\sin\left(fx + e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(sin(f*x + e) + 1) - b*log(-sin(f*x + e) + 1) + 2*a*sin(f*x + e))
/f
```

3.156 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

```
[Out] ((a + b)*Sin[e + f*x])/f - (a*Sin[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.045325, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4044, 3013}

$$\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + b)*Sin[e + f*x])/f - (a*Sin[e + f*x]^3)/(3*f)
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\int \cos(e+fx)\left(b+a\cos^2(e+fx)\right)\,dx$$
$$=-\frac{\text{Subst}\left(\int\left(a+b-ax^2\right)\,dx,x,-\sin(e+fx)\right)}{f}$$
$$=\frac{(a+b)\sin(e+fx)}{f}-\frac{a\sin^3(e+fx)}{3f}$$

Mathematica [A] time = 0.0226969, size = 50, normalized size = 1.67

$$-\frac{a\sin^3(e+fx)}{3f}+\frac{a\sin(e+fx)}{f}+\frac{b\sin(e)\cos(fx)}{f}+\frac{b\cos(e)\sin(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*Cos[f*x]*Sin[e])/f + (b*Cos[e]*Sin[f*x])/f + (a*Sin[e + f*x])/f - (a*Sin
[e + f*x]^3)/(3*f)
```

Maple [A] time = 0.05, size = 33, normalized size = 1.1

$$\frac{1}{f}\left(\frac{a\left(2+\left(\cos\left(fx+e\right)\right)^2\right)\sin\left(fx+e\right)}{3}+\sin\left(fx+e\right)b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(1/3*a*(2+cos(f*x+e)^2)*sin(f*x+e)+sin(f*x+e)*b)
```

Maxima [A] time = 0.991559, size = 36, normalized size = 1.2

$$-\frac{a\sin\left(fx+e\right)^3-3\,(a+b)\sin\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(a*sin(f*x + e)^3 - 3*(a + b)*sin(f*x + e))/f
```

Fricas [A] time = 0.47222, size = 69, normalized size = 2.3

$$\frac{\left(a\cos\left(fx+e\right)^{2}+2\,a+3\,b\right)\sin\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(f*x + e)^2 + 2*a + 3*b)*sin(f*x + e)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30849, size = 50, normalized size = 1.67

$$-\frac{a\sin\left(fx+e\right)^{3}-3\,a\sin\left(fx+e\right)-3\,b\sin\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(a*sin(f*x + e)^3 - 3*a*sin(f*x + e) - 3*b*sin(f*x + e))/f
```

3.157 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=50

$$-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)\sin(e+fx)}{f}+\frac{a\sin^5(e+fx)}{5f}$$

```
[Out] ((a + b)*Sin[e + f*x])/f - ((2*a + b)*Sin[e + f*x]^3)/(3*f) + (a*Sin[e + f*
x]^5)/(5*f)
```

Rubi [A] time = 0.0662125, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4044, 3013, 373}

$$-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)\sin(e+fx)}{f}+\frac{a\sin^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + b)*Sin[e + f*x])/f - ((2*a + b)*Sin[e + f*x]^3)/(3*f) + (a*Sin[e + f*
x]^5)/(5*f)
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \cos^3(e+fx)\left(b+a\cos^2(e+fx)\right)\,dx$$
$$= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-ax^2\right)\,dx, x, -\sin(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(a\left(1+\frac{b}{a}\right)-(2a+b)x^2+ax^4\right)\,dx, x, -\sin(e+fx)\right)}{f}$$
$$= \frac{(a+b)\sin(e+fx)}{f}-\frac{(2a+b)\sin^3(e+fx)}{3f}+\frac{a\sin^5(e+fx)}{5f}$$

Mathematica [A] time = 0.0238516, size = 71, normalized size = 1.42

$$\frac{a\sin^5(e+fx)}{5f}-\frac{2a\sin^3(e+fx)}{3f}+\frac{a\sin(e+fx)}{f}-\frac{b\sin^3(e+fx)}{3f}+\frac{b\sin(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*Sin[e + f*x])/f + (b*Sin[e + f*x])/f - (2*a*Sin[e + f*x]^3)/(3*f) - (b*S
in[e + f*x]^3)/(3*f) + (a*Sin[e + f*x]^5)/(5*f)
```

Maple [A] time = 0.055, size = 54, normalized size = 1.1

$$\frac{1}{f}\left(\frac{\sin\left(fx+e\right)a}{5}\left(\frac{8}{3}+\left(\cos\left(fx+e\right)\right)^4+\frac{4\left(\cos\left(fx+e\right)\right)^2}{3}\right)+\frac{b\left(2+\left(\cos\left(fx+e\right)\right)^2\right)\sin\left(fx+e\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(1/5*a*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+1/3*b*(2+cos(f*x+
e)^2)*sin(f*x+e))
```

Maxima [A] time = 1.00095, size = 58, normalized size = 1.16

$$\frac{3\,a\sin\left(fx+e\right)^{5}-5\,(2\,a+b)\sin\left(fx+e\right)^{3}+15\,(a+b)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*a*sin(f*x + e)^5 - 5*(2*a + b)*sin(f*x + e)^3 + 15*(a + b)*sin(f*x
+ e))/f
```

Fricas [A] time = 0.477763, size = 113, normalized size = 2.26

$$\frac{\left(3\,a\cos\left(fx+e\right)^{4}+(4\,a+5\,b)\cos\left(fx+e\right)^{2}+8\,a+10\,b\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*cos(f*x + e)^4 + (4*a + 5*b)*cos(f*x + e)^2 + 8*a + 10*b)*sin(f*x
 + e)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22154, size = 84, normalized size = 1.68

$$\frac{3\,a\sin\left(fx+e\right)^{5}-10\,a\sin\left(fx+e\right)^{3}-5\,b\sin\left(fx+e\right)^{3}+15\,a\sin\left(fx+e\right)+15\,b\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*a*sin(f*x + e)^5 - 10*a*sin(f*x + e)^3 - 5*b*sin(f*x + e)^3 + 15*a*
sin(f*x + e) + 15*b*sin(f*x + e))/f
```

3.158 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=87

$$\frac{(7a+6b)\tan^5(e+fx)}{35f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

```
[Out] ((7*a + 6*b)*Tan[e + f*x])/(7*f) + (b*Sec[e + f*x]^6*Tan[e + f*x])/(7*f) +
(2*(7*a + 6*b)*Tan[e + f*x]^3)/(21*f) + ((7*a + 6*b)*Tan[e + f*x]^5)/(35*f)
```

Rubi [A] time = 0.049371, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4046, 3767}

$$\frac{(7a+6b)\tan^5(e+fx)}{35f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}+\frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((7*a + 6*b)*Tan[e + f*x])/(7*f) + (b*Sec[e + f*x]^6*Tan[e + f*x])/(7*f) +
(2*(7*a + 6*b)*Tan[e + f*x]^3)/(21*f) + ((7*a + 6*b)*Tan[e + f*x]^5)/(35*f)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{1}{7}(7a+6b)\int \sec^6(e+fx)\,dx$$
$$= \frac{b\sec^6(e+fx)\tan(e+fx)}{7f}-\frac{(7a+6b)\,\text{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,-\tan(e\right.}{7f}$$
$$= \frac{(7a+6b)\tan(e+fx)}{7f}+\frac{b\sec^6(e+fx)\tan(e+fx)}{7f}+\frac{2(7a+6b)\tan^3(e+fx)}{21f}$$

Mathematica [A] time = 0.311456, size = 81, normalized size = 0.93

$$\frac{a\left(\frac{1}{5}\tan^5(e+fx)+\frac{2}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}+\frac{b\left(\frac{1}{7}\tan^7(e+fx)+\frac{3}{5}\tan^5(e+fx)+\tan^3(e+fx)+\tan(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5))/f + (b*(Tan[e
+ f*x] + Tan[e + f*x]^3 + (3*Tan[e + f*x]^5)/5 + Tan[e + f*x]^7/7))/f
```

Maple [A] time = 0.03, size = 78, normalized size = 0.9

$$\frac{1}{f}\left(-a\left(-\frac{8}{15}-\frac{\left(\sec\left(fx+e\right)\right)^4}{5}-\frac{4\left(\sec\left(fx+e\right)\right)^2}{15}\right)\tan\left(fx+e\right)-b\left(-\frac{16}{35}-\frac{\left(\sec\left(fx+e\right)\right)^6}{7}-\frac{6\left(\sec\left(fx+e\right)\right)^4}{35}-\frac{8\left(\text{se}\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-b*(-16/35-1/7
*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 0.983035, size = 81, normalized size = 0.93

$$\frac{15\,b\tan\left(fx+e\right)^7+21\,(a+3\,b)\tan\left(fx+e\right)^5+35\,(2\,a+3\,b)\tan\left(fx+e\right)^3+105\,(a+b)\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*(a + 3*b)*tan(f*x + e)^5 + 35*(2*a + 3*b)*t
an(f*x + e)^3 + 105*(a + b)*tan(f*x + e))/f
```

Fricas [A] time = 0.475483, size = 188, normalized size = 2.16

$$\frac{\left(8\,(7\,a+6\,b)\cos\left(fx+e\right)^{6}+4\,(7\,a+6\,b)\cos\left(fx+e\right)^{4}+3\,(7\,a+6\,b)\cos\left(fx+e\right)^{2}+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(7*a + 6*b)*cos(f*x + e)^6 + 4*(7*a + 6*b)*cos(f*x + e)^4 + 3*(7*a
 + 6*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

Giac [A] time = 1.32198, size = 116, normalized size = 1.33

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,a\tan\left(fx+e\right)^{5}+63\,b\tan\left(fx+e\right)^{5}+70\,a\tan\left(fx+e\right)^{3}+105\,b\tan\left(fx+e\right)^{3}+105\,a\tan\left(fx\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 + 63*b*tan(f*x + e)^5 + 70
*a*tan(f*x + e)^3 + 105*b*tan(f*x + e)^3 + 105*a*tan(f*x + e) + 105*b*tan(f
*x + e))/f
```

3.159 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=65

$$\frac{(5a+4b)\tan^3(e+fx)}{15f}+\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

```
[Out] ((5*a + 4*b)*Tan[e + f*x])/(5*f) + (b*Sec[e + f*x]^4*Tan[e + f*x])/(5*f) +
((5*a + 4*b)*Tan[e + f*x]^3)/(15*f)
```

Rubi [A] time = 0.0432743, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4046, 3767}

$$\frac{(5a+4b)\tan^3(e+fx)}{15f}+\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a + 4*b)*Tan[e + f*x])/(5*f) + (b*Sec[e + f*x]^4*Tan[e + f*x])/(5*f) +
((5*a + 4*b)*Tan[e + f*x]^3)/(15*f)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{1}{5}(5a+4b)\int \sec^4(e+fx)\,dx$$
$$=\frac{b\sec^4(e+fx)\tan(e+fx)}{5f}-\frac{(5a+4b)\,\text{Subst}\left(\int\left(1+x^2\right)dx,x,-\tan(e+fx)\right)}{5f}$$
$$=\frac{(5a+4b)\tan(e+fx)}{5f}+\frac{b\sec^4(e+fx)\tan(e+fx)}{5f}+\frac{(5a+4b)\tan^3(e+fx)}{15f}$$

Mathematica [A] time = 0.194999, size = 61, normalized size = 0.94

$$\frac{a\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}+\frac{b\left(\frac{1}{5}\tan^5(e+fx)+\frac{2}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f + (b*(Tan[e + f*x] + (2*Tan[e + f*x
]^3)/3 + Tan[e + f*x]^5/5))/f
```

Maple [A] time = 0.032, size = 58, normalized size = 0.9

$$\frac{1}{f}\left(-a\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)-b\left(-\frac{8}{15}-\frac{\left(\sec\left(fx+e\right)\right)^4}{5}-\frac{4\left(\sec\left(fx+e\right)\right)^2}{15}\right)\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b*(-8/15-1/5*sec(f*x+e)^4-4/15*s
ec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 1.01184, size = 58, normalized size = 0.89

$$\frac{3\,b\tan\left(fx+e\right)^5+5\,(a+2\,b)\tan\left(fx+e\right)^3+15\,(a+b)\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*(a + 2*b)*tan(f*x + e)^3 + 15*(a + b)*tan(f*x
+ e))/f
```

Fricas [A] time = 0.46819, size = 140, normalized size = 2.15

$$\frac{\left(2\,(5\,a+4\,b)\cos\left(fx+e\right)^{4}+(5\,a+4\,b)\cos\left(fx+e\right)^{2}+3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*a + 4*b)*cos(f*x + e)^4 + (5*a + 4*b)*cos(f*x + e)^2 + 3*b)*sin(
f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

Giac [A] time = 1.29146, size = 84, normalized size = 1.29

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,a\tan\left(fx+e\right)^{3}+10\,b\tan\left(fx+e\right)^{3}+15\,a\tan\left(fx+e\right)+15\,b\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 10*b*tan(f*x + e)^3 + 15*a*
tan(f*x + e) + 15*b*tan(f*x + e))/f
```

3.160 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx$

Optimal. Leaf size=43

$$\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

```
[Out] ((3*a + 2*b)*Tan[e + f*x])/(3*f) + (b*Sec[e + f*x]^2*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.0383772, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4046, 3767, 8}

$$\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a + 2*b)*Tan[e + f*x])/(3*f) + (b*Sec[e + f*x]^2*Tan[e + f*x])/(3*f)
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\sec^2(e+fx)\tan(e+fx)}{3f}+\frac{1}{3}(3a+2b)\int \sec^2(e+fx)\,dx$$
$$= \frac{b\sec^2(e+fx)\tan(e+fx)}{3f}-\frac{(3a+2b)\,\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{3f}$$
$$= \frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\sec^2(e+fx)\tan(e+fx)}{3f}$$

Mathematica [A] time = 0.0878204, size = 36, normalized size = 0.84

$$\frac{a\tan(e+fx)}{f}+\frac{b\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + (b*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

Maple [A] time = 0.028, size = 35, normalized size = 0.8

$$\frac{1}{f}\left(a\tan\left(fx+e\right)-b\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*tan(f*x+e)-b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 1.02257, size = 46, normalized size = 1.07

$$\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*b + 3*a*tan(f*x + e))/f
```

Fricas [A] time = 0.458157, size = 95, normalized size = 2.21

$$\frac{\left((3\,a+2\,b)\cos\left(fx+e\right)^{2}+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a + 2*b)*cos(f*x + e)^2 + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

Giac [A] time = 1.26565, size = 50, normalized size = 1.16

$$\frac{b\tan\left(fx+e\right)^{3}+3\,a\tan\left(fx+e\right)+3\,b\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 3*b*tan(f*x + e))/f
```

3.161 $\int \left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0125057, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767, 8}

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\,dx &= ax+b\int \sec^2(e+fx)\,dx\\ &= ax-\frac{b\,\text{Subst}\left(\int 1\,dx,x,-\tan(e+fx)\right)}{f}\\ &= ax+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0025554, size = 15, normalized size = 1.

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.013, size = 16, normalized size = 1.1

$$ax+\frac{b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

Maxima [A] time = 0.963718, size = 20, normalized size = 1.33

$$ax+\frac{b\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Fricas [B] time = 0.466816, size = 76, normalized size = 5.07

$$\frac{afx\cos\left(fx+e\right)+b\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Giac [A] time = 1.29631, size = 22, normalized size = 1.47

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

3.162 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}x(a+2b)+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] ((a + 2*b)*x)/2 + (a*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.0273794, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4045, 8}

$$\frac{1}{2}x(a+2b)+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*x)/2 + (a*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{a\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}(a+2b)\int 1\,dx\\ &= \frac{1}{2}(a+2b)x+\frac{a\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.0305511, size = 33, normalized size = 1.06

$$\frac{a(e+fx)}{2f}+\frac{a\sin(2(e+fx))}{4f}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] b*x + (a*(e + f*x))/(2*f) + (a*Sin[2*(e + f*x)])/(4*f)
```

Maple [A] time = 0.054, size = 37, normalized size = 1.2

$$\frac{1}{f}\left(a\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+\left(fx+e\right)b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+(f*x+e)*b)
```

Maxima [A] time = 1.5004, size = 50, normalized size = 1.61

$$\frac{\left(fx+e\right)(a+2b)+\frac{a\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.475012, size = 72, normalized size = 2.32

$$\frac{(a+2b)fx+a\cos\left(fx+e\right)\sin\left(fx+e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e))/f
```

Sympy [A] time = 21.2814, size = 51, normalized size = 1.65

$$a\left(\begin{cases} \frac{x\sin^2\left(e+fx\right)}{2} + \frac{x\cos^2\left(e+fx\right)}{2} + \frac{\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for } f \neq 0 \\ x\cos^2\left(e\right) & \text{otherwise} \end{cases}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] a*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 + sin(e + f*x)*cos(e
 + f*x)/(2*f), Ne(f, 0)), (x*cos(e)**2, True)) + b*x
```

Giac [A] time = 1.24232, size = 54, normalized size = 1.74

$$\frac{\left(fx+e\right)(a+2b) + \frac{a\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

3.163 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(3a+4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(3a+4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rubi [A] time = 0.041084, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4045, 2635, 8}

$$\frac{(3a+4b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(3a+4b)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```


```
[Out] ((3*a + 4*b)*x)/8 + ((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*Cos[
e + f*x]^3*Sin[e + f*x])/(4*f)
```


Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{4}(3a+4b)\int \cos^2(e+fx)\,dx$$
$$= \frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}+\frac{1}{8}(3a+4b)\int$$
$$= \frac{1}{8}(3a+4b)x+\frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f}+\frac{a\cos^3(e+fx)\sin(e+fx)}{4f}$$

Mathematica [A] time = 0.0884978, size = 45, normalized size = 0.74

$$\frac{4(3a+4b)(e+fx)+8(a+b)\sin(2(e+fx))+a\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (4*(3*a + 4*b)*(e + f*x) + 8*(a + b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)])
/(32*f)
```

Maple [A] time = 0.058, size = 65, normalized size = 1.1

$$\frac{1}{f}\left(a\left(\frac{\sin\left(fx+e\right)}{4}\left(\left(\cos\left(fx+e\right)\right)^3+\frac{3\cos\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+b*(1/2*
sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.48162, size = 99, normalized size = 1.62

$$\frac{\left(fx+e\right)(3a+4b)+\frac{(3a+4b)\tan\left(fx+e\right)^3+(5a+4b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((f*x + e)*(3*a + 4*b) + ((3*a + 4*b)*tan(f*x + e)^3 + (5*a + 4*b)*tan(
f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.483855, size = 119, normalized size = 1.95

$$\frac{(3\,a+4\,b)fx+\left(2\,a\cos\left(fx+e\right)^{3}+(3\,a+4\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((3*a + 4*b)*f*x + (2*a*cos(f*x + e)^3 + (3*a + 4*b)*cos(f*x + e))*sin(
f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34478, size = 107, normalized size = 1.75

$$\frac{\left(fx+e\right)(3\,a+4\,b)+\frac{3\,a\tan\left(fx+e\right)^{3}+4\,b\tan\left(fx+e\right)^{3}+5\,a\tan\left(fx+e\right)+4\,b\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*((f*x + e)*(3*a + 4*b) + (3*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 5*a
*tan(f*x + e) + 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)/f
```

3.164 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{(5a+6b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(5a+6b)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

```
[Out] ((5*a + 6*b)*x)/16 + ((5*a + 6*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((5*a
 + 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rubi [A] time = 0.052592, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4045, 2635, 8}

$$\frac{(5a+6b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(5a+6b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(5a+6b)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((5*a + 6*b)*x)/16 + ((5*a + 6*b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((5*a
 + 6*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (a*Cos[e + f*x]^5*Sin[e + f*x
])/(6*f)
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{6}(5a+6b)\int \cos^4(e+fx)\,dx$$
$$=\frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a\cos^5(e+fx)\sin(e+fx)}{6f}+\frac{1}{8}(5a+6b)$$
$$=\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a\cos}{}$$
$$=\frac{1}{16}(5a+6b)x+\frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f}+\frac{(5a+6b)\cos^3(e+fx)\sin}{24f}$$

Mathematica [A] time = 0.102777, size = 68, normalized size = 0.76

$$\frac{(45a+48b)\sin(2(e+fx))+(9a+6b)\sin(4(e+fx))+a\sin(6(e+fx))+60ae+60afx+72be+72bfx}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (60*a*e + 72*b*e + 60*a*f*x + 72*b*f*x + (45*a + 48*b)*Sin[2*(e + f*x)] + (
9*a + 6*b)*Sin[4*(e + f*x)] + a*Sin[6*(e + f*x)])/(192*f)
```

Maple [A] time = 0.053, size = 86, normalized size = 1.

$$\frac{1}{f}\left(a\left(\frac{\sin(fx+e)}{6}\left((\cos(fx+e))^5+\frac{5\,(\cos(fx+e))^3}{4}+\frac{15\cos(fx+e)}{8}\right)+\frac{5\,fx}{16}+\frac{5\,e}{16}\right)+b\left(\frac{\sin(fx+e)}{4}\left((\cos(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16
*f*x+5/16*e)+b*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)
```

)

Maxima [A] time = 1.4862, size = 139, normalized size = 1.56

$$\frac{3\left(fx+e\right)(5a+6b)+\frac{3(5a+6b)\tan\left(fx+e\right)^{5}+8(5a+6b)\tan\left(fx+e\right)^{3}+3(11a+10b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{6}+3\tan\left(fx+e\right)^{4}+3\tan\left(fx+e\right)^{2}+1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(f*x + e)*(5*a + 6*b) + (3*(5*a + 6*b)*tan(f*x + e)^5 + 8*(5*a + 6*
b)*tan(f*x + e)^3 + 3*(11*a + 10*b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f
*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.495716, size = 167, normalized size = 1.88

$$\frac{3(5a+6b)fx+\left(8a\cos\left(fx+e\right)^{5}+2(5a+6b)\cos\left(fx+e\right)^{3}+3(5a+6b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a + 6*b)*f*x + (8*a*cos(f*x + e)^5 + 2*(5*a + 6*b)*cos(f*x + e)^
3 + 3*(5*a + 6*b)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34248, size = 140, normalized size = 1.57

$$\frac{3\left(fx+e\right)(5\,a+6\,b)+\frac{15\,a\tan\left(fx+e\right)^{5}+18\,b\tan\left(fx+e\right)^{5}+40\,a\tan\left(fx+e\right)^{3}+48\,b\tan\left(fx+e\right)^{3}+33\,a\tan\left(fx+e\right)+30\,b\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^{2}+1\right)^{3}}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(f*x + e)*(5*a + 6*b) + (15*a*tan(f*x + e)^5 + 18*b*tan(f*x + e)^5
+ 40*a*tan(f*x + e)^3 + 48*b*tan(f*x + e)^3 + 33*a*tan(f*x + e) + 30*b*tan(
f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

3.165 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=165

$$\frac{\left(48a^2+80ab+35b^2\right)\tanh^{-1}(\sin(e+fx))}{128f}+\frac{\left(48a^2+80ab+35b^2\right)\tan(e+fx)\sec^3(e+fx)}{192f}+\frac{\left(48a^2+80ab+35b^2\right)\text{t}}{128}$$

```
[Out] ((48*a^2 + 80*a*b + 35*b^2)*ArcTanh[Sin[e + f*x]])/(128*f) + ((48*a^2 + 80*
a*b + 35*b^2)*Sec[e + f*x]*Tan[e + f*x])/(128*f) + ((48*a^2 + 80*a*b + 35*b
^2)*Sec[e + f*x]^3*Tan[e + f*x])/(192*f) + (b*(10*a + 7*b)*Sec[e + f*x]^5*T
an[e + f*x])/(48*f) + (b*Sec[e + f*x]^7*(a + b - a*Sin[e + f*x]^2)*Tan[e +
f*x])/(8*f)
```

Rubi [A] time = 0.141082, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 413, 385, 199, 206}

$$\frac{\left(48a^2+80ab+35b^2\right)\tanh^{-1}(\sin(e+fx))}{128f}+\frac{\left(48a^2+80ab+35b^2\right)\tan(e+fx)\sec^3(e+fx)}{192f}+\frac{\left(48a^2+80ab+35b^2\right)\text{t}}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] ((48*a^2 + 80*a*b + 35*b^2)*ArcTanh[Sin[e + f*x]])/(128*f) + ((48*a^2 + 80*
a*b + 35*b^2)*Sec[e + f*x]*Tan[e + f*x])/(128*f) + ((48*a^2 + 80*a*b + 35*b
^2)*Sec[e + f*x]^3*Tan[e + f*x])/(192*f) + (b*(10*a + 7*b)*Sec[e + f*x]^5*T
an[e + f*x])/(48*f) + (b*Sec[e + f*x]^7*(a + b - a*Sin[e + f*x]^2)*Tan[e +
f*x])/(8*f)
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^5}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f} - \frac{\text{Subst}\left(\int \frac{-(a+b)(8a+7b)+}{(1-x^2}\right.}{}$$

$$= \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} + \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx\right.}{8f}$$

$$= \frac{\left(48a^2+80ab+35b^2\right)\sec^3(e+fx)\tan(e+fx)}{192f} + \frac{b(10a+7b)\sec^5(e+fx)\,\mathrm{t}}{48f}$$

$$= \frac{\left(48a^2+80ab+35b^2\right)\sec(e+fx)\tan(e+fx)}{128f} + \frac{\left(48a^2+80ab+35b^2\right)\sec^3}{192f}$$

$$= \frac{\left(48a^2+80ab+35b^2\right)\tanh^{-1}(\sin(e+fx))}{128f} + \frac{\left(48a^2+80ab+35b^2\right)\sec(e+}{128f}$$

Mathematica [A] time = 0.535401, size = 119, normalized size = 0.72

$$\frac{3\left(48a^2+80ab+35b^2\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\sec(e+fx)\left(2\left(48a^2+80ab+35b^2\right)\sec^2(e+fx)+3\left(48a^2\right.\right.}{384f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(48*a
^2 + 80*a*b + 35*b^2) + 2*(48*a^2 + 80*a*b + 35*b^2)*Sec[e + f*x]^2 + 8*b*(
16*a + 7*b)*Sec[e + f*x]^4 + 48*b^2*Sec[e + f*x]^6)*Tan[e + f*x])/(384*f)
```

Maple [A] time = 0.041, size = 256, normalized size = 1.6

$$\frac{a^2\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^3}{4f} + \frac{3a^2\tan\left(fx+e\right)\sec\left(fx+e\right)}{8f} + \frac{3a^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f} + \frac{ab\tan\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/4/f*a^2*tan(f*x+e)*sec(f*x+e)^3+3/8/f*a^2*tan(f*x+e)*sec(f*x+e)+3/8/f*a^2
*ln(sec(f*x+e)+tan(f*x+e))+1/3/f*a*b*tan(f*x+e)*sec(f*x+e)^5+5/12/f*a*b*tan
(f*x+e)*sec(f*x+e)^3+5/8/f*a*b*tan(f*x+e)*sec(f*x+e)+5/8/f*a*b*ln(sec(f*x+e
)+tan(f*x+e))+1/8/f*b^2*tan(f*x+e)*sec(f*x+e)^7+7/48/f*b^2*tan(f*x+e)*sec(f
*x+e)^5+35/192/f*b^2*tan(f*x+e)*sec(f*x+e)^3+35/128*b^2*sec(f*x+e)*tan(f*x+
e)/f+35/128/f*b^2*ln(sec(f*x+e)+tan(f*x+e))
```

Maxima [A] time = 1.02563, size = 270, normalized size = 1.64

$$\frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\left(48\,a^2+80\,ab+35\,b^2\right)\sin(\cdots}{\cdots}}{768\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(sin(f*x + e) + 1) - 3*(48*a^2 + 80*
a*b + 35*b^2)*log(sin(f*x + e) - 1) - 2*(3*(48*a^2 + 80*a*b + 35*b^2)*sin(f
*x + e)^7 - 11*(48*a^2 + 80*a*b + 35*b^2)*sin(f*x + e)^5 + (624*a^2 + 1168*
a*b + 511*b^2)*sin(f*x + e)^3 - 3*(80*a^2 + 176*a*b + 93*b^2)*sin(f*x + e))
/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 +
 1))/f
```

Fricas [A] time = 0.548484, size = 428, normalized size = 2.59

$$\frac{3\left(48\,a^2+80\,ab+35\,b^2\right)\cos\left(fx+e\right)^8\log\left(\sin\left(fx+e\right)+1\right)-3\left(48\,a^2+80\,ab+35\,b^2\right)\cos\left(fx+e\right)^8\log\left(-\sin\left(fx+e\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(sin(f*x + e) + 1) -
3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(-sin(f*x + e) + 1) + 2*(3*(
48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^6 + 2*(48*a^2 + 80*a*b + 35*b^2)*cos
```

```
(f*x + e)^4 + 8*(16*a*b + 7*b^2)*cos(f*x + e)^2 + 48*b^2)*sin(f*x + e))/(f*
cos(f*x + e)^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^2 \sec^5\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**5, x)
```

Giac [A] time = 1.34345, size = 319, normalized size = 1.93

$$\frac{3\left(48\,a^2 + 80\,ab + 35\,b^2\right)\log\left(\sin\left(fx + e\right) + 1\right) - 3\left(48\,a^2 + 80\,ab + 35\,b^2\right)\log\left(-\sin\left(fx + e\right) + 1\right) - \frac{2\left(144\,a^2\sin\left(fx+e\right)^7 + 24}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(sin(f*x + e) + 1) - 3*(48*a^2 + 80*
a*b + 35*b^2)*log(-sin(f*x + e) + 1) - 2*(144*a^2*sin(f*x + e)^7 + 240*a*b*
sin(f*x + e)^7 + 105*b^2*sin(f*x + e)^7 - 528*a^2*sin(f*x + e)^5 - 880*a*b*
sin(f*x + e)^5 - 385*b^2*sin(f*x + e)^5 + 624*a^2*sin(f*x + e)^3 + 1168*a*b
*sin(f*x + e)^3 + 511*b^2*sin(f*x + e)^3 - 240*a^2*sin(f*x + e) - 528*a*b*s
in(f*x + e) - 279*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^4)/f
```

3.166 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$

Optimal. Leaf size=129

$$\frac{\left(8a^2+12ab+5b^2\right)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{\left(8a^2+12ab+5b^2\right)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b(8a+5b)\tan(e+fx)\sec^3(e+}{24f}$$

```
[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTanh[Sin[e + f*x]])/(16*f) + ((8*a^2 + 12*a*b
+ 5*b^2)*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (b*(8*a + 5*b)*Sec[e + f*x]^3*
Tan[e + f*x])/(24*f) + (b*Sec[e + f*x]^5*(a + b - a*Sin[e + f*x]^2)*Tan[e +
 f*x])/(6*f)
```

Rubi [A] time = 0.134058, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 413, 385, 199, 206}

$$\frac{\left(8a^2+12ab+5b^2\right)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{\left(8a^2+12ab+5b^2\right)\tan(e+fx)\sec(e+fx)}{16f}+\frac{b(8a+5b)\tan(e+fx)\sec^3(e+}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] ((8*a^2 + 12*a*b + 5*b^2)*ArcTanh[Sin[e + f*x]])/(16*f) + ((8*a^2 + 12*a*b
+ 5*b^2)*Sec[e + f*x]*Tan[e + f*x])/(16*f) + (b*(8*a + 5*b)*Sec[e + f*x]^3*
Tan[e + f*x])/(24*f) + (b*Sec[e + f*x]^5*(a + b - a*Sin[e + f*x]^2)*Tan[e +
 f*x])/(6*f)
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
```

```
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^4}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{6f} - \frac{\text{Subst}\left(\int \frac{-(a+b)(6a+5b)+3a}{\left(1-x^2\right)^3}\right.}{6}$$

$$= \frac{b(8a+5b)\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{6f}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\sec(e+fx)\tan(e+fx)}{16f} + \frac{b(8a+5b)\sec^3(e+fx)\tan(e+}{24f}$$

$$= \frac{\left(8a^2+12ab+5b^2\right)\tanh^{-1}(\sin(e+fx))}{16f} + \frac{\left(8a^2+12ab+5b^2\right)\sec(e+fx)\tan}{16f}$$

Mathematica [A] time = 0.397695, size = 94, normalized size = 0.73

$$\frac{3\left(8a^2+12ab+5b^2\right)\tanh^{-1}(\sin(e+fx)) + \tan(e+fx)\sec(e+fx)\left(3\left(8a^2+12ab+5b^2\right)+2b(12a+5b)\sec^2(e+fx)+\right.}{48f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (3*(8*a^2 + 12*a*b + 5*b^2)*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]*(3*(8*a^2
+ 12*a*b + 5*b^2) + 2*b*(12*a + 5*b)*Sec[e + f*x]^2 + 8*b^2*Sec[e + f*x]^4)
*Tan[e + f*x])/(48*f)
```

Maple [A] time = 0.037, size = 191, normalized size = 1.5

$$\frac{a^2\tan(fx+e)\sec(fx+e)}{2f} + \frac{a^2\ln(\sec(fx+e)+\tan(fx+e))}{2f} + \frac{ab\tan(fx+e)(\sec(fx+e))^3}{2f} + \frac{3ab\tan(fx+e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*a^2*tan(f*x+e)*sec(f*x+e)+1/2/f*a^2*ln(sec(f*x+e)+tan(f*x+e))+1/2/f*a
*b*tan(f*x+e)*sec(f*x+e)^3+3/4/f*a*b*tan(f*x+e)*sec(f*x+e)+3/4/f*a*b*ln(sec
(f*x+e)+tan(f*x+e))+1/6/f*b^2*tan(f*x+e)*sec(f*x+e)^5+5/24/f*b^2*tan(f*x+e)
*sec(f*x+e)^3+5/16*b^2*sec(f*x+e)*tan(f*x+e)/f+5/16/f*b^2*ln(sec(f*x+e)+tan
(f*x+e))
```

Maxima [A] time = 1.0066, size = 224, normalized size = 1.74

$$\frac{3\left(8a^2+12ab+5b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(8a^2+12ab+5b^2\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\left(8a^2+12ab+5b^2\right)\sin\left(fx+e\right)^5\cdots\right.}{\sin(f\cdots}}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(sin(f*x + e) + 1) - 3*(8*a^2 + 12*a*b
+ 5*b^2)*log(sin(f*x + e) - 1) - 2*(3*(8*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)
^5 - 8*(6*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)^3 + 3*(8*a^2 + 20*a*b + 11*b^2
)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))
/f
```

Fricas [A] time = 0.528086, size = 355, normalized size = 2.75

$$\frac{3\left(8a^2+12ab+5b^2\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-3\left(8a^2+12ab+5b^2\right)\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\cdots\right.}{96f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(
8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(8*a^2
 + 12*a*b + 5*b^2)*cos(f*x + e)^4 + 2*(12*a*b + 5*b^2)*cos(f*x + e)^2 + 8*b
^2)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**3, x)
```

Giac [A] time = 1.27548, size = 263, normalized size = 2.04

$$\frac{3\left(8a^2+12ab+5b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(8a^2+12ab+5b^2\right)\log\left(-\sin\left(fx+e\right)+1\right)-\frac{2\left(24a^2\sin\left(fx+e\right)^5+36ab\sin\left(fx\right.\right.}{}}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(sin(f*x + e) + 1) - 3*(8*a^2 + 12*a*b
+ 5*b^2)*log(-sin(f*x + e) + 1) - 2*(24*a^2*sin(f*x + e)^5 + 36*a*b*sin(f*x
 + e)^5 + 15*b^2*sin(f*x + e)^5 - 48*a^2*sin(f*x + e)^3 - 96*a*b*sin(f*x +
e)^3 - 40*b^2*sin(f*x + e)^3 + 24*a^2*sin(f*x + e) + 60*a*b*sin(f*x + e) +
33*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

3.167 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3b(2a+b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)\left(-a\sin^2(e+\right.}{4f}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]])/(8*f) + (3*b*(2*a + b)*Sec[
e + f*x]*Tan[e + f*x])/(8*f) + (b*Sec[e + f*x]^3*(a + b - a*Sin[e + f*x]^2)
*Tan[e + f*x])/(4*f)
```

Rubi [A] time = 0.0739133, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4147, 413, 385, 206}

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3b(2a+b)\tan(e+fx)\sec(e+fx)}{8f}+\frac{b\tan(e+fx)\sec^3(e+fx)\left(-a\sin^2(e+\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]])/(8*f) + (3*b*(2*a + b)*Sec[
e + f*x]*Tan[e + f*x])/(8*f) + (b*Sec[e + f*x]^3*(a + b - a*Sin[e + f*x]^2)
*Tan[e + f*x])/(4*f)
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^3} dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{4f} - \frac{\text{Subst}\left(\int \frac{-(a+b)(4a+3b)+a(4}{\left(1-x^2\right)^2}\right.}{4}$$

$$= \frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f} + \frac{b\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\text{ta}}{4f}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}(\sin(e+fx))}{8f} + \frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f} +$$

Mathematica [A] time = 0.13585, size = 63, normalized size = 0.69

$$\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}(\sin(e+fx))+b\tan(e+fx)\sec(e+fx)\left(8a+2b\sec^2(e+fx)+3b\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]] + b*Sec[e + f*x]*(8*a + 3*b
+ 2*b*Sec[e + f*x]^2)*Tan[e + f*x])/(8*f)
```

Maple [A] time = 0.037, size = 125, normalized size = 1.4

$$\frac{a^2 \ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{ab\tan\left(fx+e\right)\sec\left(fx+e\right)}{f}+\frac{ab\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+\frac{b^2\tan\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*a^2*ln(sec(f*x+e)+tan(f*x+e))+1/f*a*b*tan(f*x+e)*sec(f*x+e)+1/f*a*b*ln(
sec(f*x+e)+tan(f*x+e))+1/4/f*b^2*tan(f*x+e)*sec(f*x+e)^3+3/8*b^2*sec(f*x+e)
*tan(f*x+e)/f+3/8/f*b^2*ln(sec(f*x+e)+tan(f*x+e))
```

Maxima [A] time = 1.01531, size = 161, normalized size = 1.77

$$\frac{\left(8a^2+8ab+3b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(8a^2+8ab+3b^2\right)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(\left(8ab+3b^2\right)\sin\left(fx+e\right)^3-\left(8ab+5b^2\right)\sin\right.}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*log(sin(f*x + e) + 1) - (8*a^2 + 8*a*b + 3*b^
2)*log(sin(f*x + e) - 1) - 2*((8*a*b + 3*b^2)*sin(f*x + e)^3 - (8*a*b + 5*b
^2)*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.51094, size = 284, normalized size = 3.12

$$\frac{\left(8a^2+8ab+3b^2\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-\left(8a^2+8ab+3b^2\right)\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2}{16f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (8*a^2
 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*((8*a*b + 3*b^2
)*cos(f*x + e)^2 + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec ^2\left(e+fx\right)\right)^2\sec \left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x), x)
```

Giac [A] time = 1.21823, size = 171, normalized size = 1.88

$$\frac{\left(8a^2+8ab+3b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(8a^2+8ab+3b^2\right)\log\left(-\sin\left(fx+e\right)+1\right)-\frac{2\left(8ab\sin\left(fx+e\right)^3+3b^2\sin\left(fx+e\right)^3-8ab\cdots\right.}{\left(\sin\left(fx+e\right)^2-1\cdots\right.}}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((8*a^2 + 8*a*b + 3*b^2)*log(sin(f*x + e) + 1) - (8*a^2 + 8*a*b + 3*b^
2)*log(-sin(f*x + e) + 1) - 2*(8*a*b*sin(f*x + e)^3 + 3*b^2*sin(f*x + e)^3
- 8*a*b*sin(f*x + e) - 5*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^2)/f
```

3.168 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{a^2\sin(e+fx)}{f}+\frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b^2\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] (b*(4*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*Sin[e + f*x])/f + (b^2*Sec
[e + f*x]*Tan[e + f*x])/(2*f)
```

Rubi [A] time = 0.0673528, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4147, 390, 385, 206}

$$\frac{a^2\sin(e+fx)}{f}+\frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b^2\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*Sin[e + f*x])/f + (b^2*Sec
[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{\left(1-x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a^2+\frac{b(2a+b)-2abx^2}{\left(1-x^2\right)^2}\right)dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{a^2\sin(e+fx)}{f}+\frac{\text{Subst}\left(\int \frac{b(2a+b)-2abx^2}{\left(1-x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{a^2\sin(e+fx)}{f}+\frac{b^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{(b(4a+b))\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\right.}{2f}\\
&= \frac{b(4a+b)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2\sin(e+fx)}{f}+\frac{b^2\sec(e+fx)\tan(e+fx)}{2f}
\end{aligned}
$$

Mathematica [A] time = 0.0333472, size = 80, normalized size = 1.43

$$
\frac{a^2\sin(e)\cos(fx)}{f}+\frac{a^2\cos(e)\sin(fx)}{f}+\frac{2ab\tanh^{-1}(\sin(e+fx))}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{2f}+\frac{b^2\tan(e+fx)\sec(e+fx)}{2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[e + f*x]])/f + (b^2*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2*
Cos[f*x]*Sin[e])/f + (a^2*Cos[e]*Sin[f*x])/f + (b^2*Sec[e + f*x]*Tan[e + f*
```

```
x])/(2*f)
```

Maple [A] time = 0.055, size = 78, normalized size = 1.4

$$\frac{a^2\sin(fx+e)}{f}+2\frac{ab\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{b^2\sec(fx+e)\tan(fx+e)}{2f}+\frac{b^2\ln(\sec(fx+e)+\tan(fx+e))}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] a^2*sin(f*x+e)/f+2/f*a*b*ln(sec(f*x+e)+tan(f*x+e))+1/2*b^2*sec(f*x+e)*tan(f
*x+e)/f+1/2/f*b^2*ln(sec(f*x+e)+tan(f*x+e))
```

Maxima [A] time = 1.00985, size = 117, normalized size = 2.09

$$-\frac{b^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-4ab(\log(\sin(fx+e)+1)-\log(\sin(fx+e)-1))}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)
) - 4*a^2*sin(f*x + e))/f
```

Fricas [A] time = 0.515399, size = 239, normalized size = 4.27

$$\frac{(4ab+b^2)\cos(fx+e)^2\log(\sin(fx+e)+1)-(4ab+b^2)\cos(fx+e)^2\log(-\sin(fx+e)+1)+2(2a^2\cos(fx+e$$
$$4f\cos(fx+e)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((4*a*b + b^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (4*a*b + b^2)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(2*a^2*cos(f*x + e)^2 + b^2)*sin(f*x
 + e))/(f*cos(f*x + e)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19094, size = 113, normalized size = 2.02

$$\frac{4\,a^2\sin\left(fx+e\right)+\left(4\,ab+b^2\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(4\,ab+b^2\right)\log\left(-\sin\left(fx+e\right)+1\right)-\frac{2\,b^2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*sin(f*x + e) + (4*a*b + b^2)*log(sin(f*x + e) + 1) - (4*a*b + b^
2)*log(-sin(f*x + e) + 1) - 2*b^2*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

3.169 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=49

$$-\frac{a^2\sin^3(e+fx)}{3f}+\frac{a(a+2b)\sin(e+fx)}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}$$

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (a*(a + 2*b)*Sin[e + f*x])/f - (a^2*Sin[e +
 f*x]^3)/(3*f)
```

Rubi [A] time = 0.0608709, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 390, 206}

$$-\frac{a^2\sin^3(e+fx)}{3f}+\frac{a(a+2b)\sin(e+fx)}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (a*(a + 2*b)*Sin[e + f*x])/f - (a^2*Sin[e +
 f*x]^3)/(3*f)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^2}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a(a+2b)-a^2x^2+\frac{b^2}{1-x^2}\right) dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{b^2\tanh^{-1}(\sin(e+fx))}{f}+\frac{a(a+2b)\sin(e+fx)}{f}-\frac{a^2\sin^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 0.0250815, size = 72, normalized size = 1.47

$$-\frac{a^2\sin^3(e+fx)}{3f}+\frac{a^2\sin(e+fx)}{f}+\frac{2ab\sin(e)\cos(fx)}{f}+\frac{2ab\cos(e)\sin(fx)}{f}+\frac{b^2\tanh^{-1}(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[e + f*x]])/f + (2*a*b*Cos[f*x]*Sin[e])/f + (2*a*b*Cos[e]*S
in[f*x])/f + (a^2*Sin[e + f*x])/f - (a^2*Sin[e + f*x]^3)/(3*f)
```

Maple [A] time = 0.059, size = 72, normalized size = 1.5

$$\frac{\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)a^2}{3f}+\frac{2a^2\sin\left(fx+e\right)}{3f}+2\frac{ab\sin\left(fx+e\right)}{f}+\frac{b^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/3/f*cos(f*x+e)^2*sin(f*x+e)*a^2+2/3*a^2*sin(f*x+e)/f+2/f*a*b*sin(f*x+e)+1
/f*b^2*ln(sec(f*x+e)+tan(f*x+e))
```

Maxima [A] time = 0.986582, size = 85, normalized size = 1.73

$$-\frac{2\,a^2\sin\left(fx+e\right)^3-3\,b^2\log\left(\sin\left(fx+e\right)+1\right)+3\,b^2\log\left(\sin\left(fx+e\right)-1\right)-6\left(a^2+2\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(sin(f*x + e) + 1) + 3*b^2*log(sin(f*
x + e) - 1) - 6*(a^2 + 2*a*b)*sin(f*x + e))/f
```

Fricas [A] time = 0.512509, size = 165, normalized size = 3.37

$$\frac{3\,b^2\log\left(\sin\left(fx+e\right)+1\right)-3\,b^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(a^2\cos\left(fx+e\right)^2+2\,a^2+6\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*log(sin(f*x + e) + 1) - 3*b^2*log(-sin(f*x + e) + 1) + 2*(a^2*co
s(f*x + e)^2 + 2*a^2 + 6*a*b)*sin(f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27523, size = 101, normalized size = 2.06

$$-\frac{2\,a^{2}\sin\left(fx+e\right)^{3}-3\,b^{2}\log\left(\sin\left(fx+e\right)+1\right)+3\,b^{2}\log\left(-\sin\left(fx+e\right)+1\right)-6\,a^{2}\sin\left(fx+e\right)-12\,ab\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(sin(f*x + e) + 1) + 3*b^2*log(-sin(f
*x + e) + 1) - 6*a^2*sin(f*x + e) - 12*a*b*sin(f*x + e))/f
```

3.170 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{a^2\sin^5(e+fx)}{5f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)^2\sin(e+fx)}{f}$$

```
[Out] ((a + b)^2*Sin[e + f*x])/f - (2*a*(a + b)*Sin[e + f*x]^3)/(3*f) + (a^2*Sin[
e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0662634, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4147, 194}

$$\frac{a^2\sin^5(e+fx)}{5f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{(a+b)^2\sin(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Sin[e + f*x])/f - (2*a*(a + b)*Sin[e + f*x]^3)/(3*f) + (a^2*Sin[
e + f*x]^5)/(5*f)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b-ax^2\right)^2\,dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-2a^2\left(1+\frac{b}{a}\right)x^2+a^2x^4\right)dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{(a+b)^2\sin(e+fx)}{f}-\frac{2a(a+b)\sin^3(e+fx)}{3f}+\frac{a^2\sin^5(e+fx)}{5f}$$

Mathematica [A] time = 0.0261719, size = 106, normalized size = 2.

$$\frac{a^2\sin^5(e+fx)}{5f}-\frac{2a^2\sin^3(e+fx)}{3f}+\frac{a^2\sin(e+fx)}{f}-\frac{2ab\sin^3(e+fx)}{3f}+\frac{2ab\sin(e+fx)}{f}+\frac{b^2\sin(e)\cos(fx)}{f}+\frac{b^2\cos(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (b^2*Cos[f*x]*Sin[e])/f + (b^2*Cos[e]*Sin[f*x])/f + (a^2*Sin[e + f*x])/f +
(2*a*b*Sin[e + f*x])/f - (2*a^2*Sin[e + f*x]^3)/(3*f) - (2*a*b*Sin[e + f*x]
^3)/(3*f) + (a^2*Sin[e + f*x]^5)/(5*f)
```

Maple [A] time = 0.065, size = 67, normalized size = 1.3

$$\frac{1}{f}\left(\frac{a^2\sin(fx+e)}{5}\left(\frac{8}{3}+(\cos(fx+e))^4+\frac{4(\cos(fx+e))^2}{3}\right)+\frac{2ab\left(2+(\cos(fx+e))^2\right)\sin(fx+e)}{3}+b^2\sin(fx+e)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/5*a^2*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+2/3*a*b*(2+cos(
f*x+e)^2)*sin(f*x+e)+b^2*sin(f*x+e))
```

Maxima [A] time = 0.973232, size = 74, normalized size = 1.4

$$\frac{3\,a^2\sin\left(fx+e\right)^5-10\left(a^2+ab\right)\sin\left(fx+e\right)^3+15\left(a^2+2\,ab+b^2\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*a^2*sin(f*x + e)^5 - 10*(a^2 + a*b)*sin(f*x + e)^3 + 15*(a^2 + 2*a*
b + b^2)*sin(f*x + e))/f
```

Fricas [A] time = 0.486925, size = 142, normalized size = 2.68

$$\frac{\left(3\,a^2\cos\left(fx+e\right)^4+2\left(2\,a^2+5\,ab\right)\cos\left(fx+e\right)^2+8\,a^2+20\,ab+15\,b^2\right)\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^2*cos(f*x + e)^4 + 2*(2*a^2 + 5*a*b)*cos(f*x + e)^2 + 8*a^2 + 20*
a*b + 15*b^2)*sin(f*x + e)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29164, size = 111, normalized size = 2.09

$$\frac{3\,a^2\sin\left(fx+e\right)^5-10\,a^2\sin\left(fx+e\right)^3-10\,ab\sin\left(fx+e\right)^3+15\,a^2\sin\left(fx+e\right)+30\,ab\sin\left(fx+e\right)+15\,b^2\sin\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*sin(f*x + e)^5 - 10*a^2*sin(f*x + e)^3 - 10*a*b*sin(f*x + e)^3
+ 15*a^2*sin(f*x + e) + 30*a*b*sin(f*x + e) + 15*b^2*sin(f*x + e))/f
```

3.171 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=106

$$\frac{\left(a^2+6ab+6b^2\right)\tan^5(e+fx)}{5f}+\frac{2b(a+2b)\tan^7(e+fx)}{7f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2}{}$$

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*(a + b)*(a + 2*b)*Tan[e + f*x]^3)/(3*f) + (
(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x]^7
)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rubi [A] time = 0.089263, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 373}

$$\frac{\left(a^2+6ab+6b^2\right)\tan^5(e+fx)}{5f}+\frac{2b(a+2b)\tan^7(e+fx)}{7f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*(a + b)*(a + 2*b)*Tan[e + f*x]^3)/(3*f) + (
(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5)/(5*f) + (2*b*(a + 2*b)*Tan[e + f*x]^7
)/(7*f) + (b^2*Tan[e + f*x]^9)/(9*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+b+bx^2\right)^2\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((a+b)^2+2(a+b)(a+2b)x^2+\left(a^2+6ab+6b^2\right)x^4+2b(a+2b)x^6\right.}{f}$$
$$= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f}+\frac{\left(a^2+6ab+6b^2\right)\tan^5}{5f}$$

Mathematica [A] time = 0.416444, size = 96, normalized size = 0.91

$$\frac{63\left(a^2+6ab+6b^2\right)\tan^5(e+fx)+210\left(a^2+3ab+2b^2\right)\tan^3(e+fx)+90b(a+2b)\tan^7(e+fx)+315(a+b)^2\tan(e+}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (315*(a + b)^2*Tan[e + f*x] + 210*(a^2 + 3*a*b + 2*b^2)*Tan[e + f*x]^3 + 63
*(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5 + 90*b*(a + 2*b)*Tan[e + f*x]^7 + 35*
b^2*Tan[e + f*x]^9)/(315*f)
```

Maple [A] time = 0.039, size = 134, normalized size = 1.3

$$\frac{1}{f}\left(-a^2\left(-\frac{8}{15}-\frac{\left(\sec\left(fx+e\right)\right)^4}{5}-\frac{4\left(\sec\left(fx+e\right)\right)^2}{15}\right)\tan\left(fx+e\right)-2\,ab\left(-\frac{16}{35}-1/7\left(\sec\left(fx+e\right)\right)^6-\frac{6\left(\sec\left(fx+e\right)\right)^4}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-16/
35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-1
28/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-64/315*sec(f*
x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 1.01043, size = 139, normalized size = 1.31

$$\frac{35\,b^2\tan\left(fx+e\right)^9+90\left(ab+2\,b^2\right)\tan\left(fx+e\right)^7+63\left(a^2+6\,ab+6\,b^2\right)\tan\left(fx+e\right)^5+210\left(a^2+3\,ab+2\,b^2\right)\tan\left(fx+e\right)^3+315\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*(a*b + 2*b^2)*tan(f*x + e)^7 + 63*(a^2 +
6*a*b + 6*b^2)*tan(f*x + e)^5 + 210*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^3 +
315*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [A] time = 0.508377, size = 300, normalized size = 2.83

$$\frac{\left(8\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^8+4\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+3\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^4+10\left(9\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+35\,b^2\right)\sin\left(fx+e\right)}{315\,f\cos\left(fx+e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^8 + 4*(21*a^2 + 36*a*b + 1
6*b^2)*cos(f*x + e)^6 + 3*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^4 + 10*(9
*a*b + 4*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*x + e)/(f*cos(f*x + e)^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\sec^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**6, x)
```

Giac [A] time = 1.28304, size = 221, normalized size = 2.08

$$\frac{35\, b^2 \tan\left(fx+e\right)^9 + 90\, ab \tan\left(fx+e\right)^7 + 180\, b^2 \tan\left(fx+e\right)^7 + 63\, a^2 \tan\left(fx+e\right)^5 + 378\, ab \tan\left(fx+e\right)^5 + 378\, b^2 \mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 180*b^2*tan(f*x + e)
^7 + 63*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 378*b^2*tan(f*x + e)^
5 + 210*a^2*tan(f*x + e)^3 + 630*a*b*tan(f*x + e)^3 + 420*b^2*tan(f*x + e)^
3 + 315*a^2*tan(f*x + e) + 630*a*b*tan(f*x + e) + 315*b^2*tan(f*x + e))/f
```

3.172 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] ((a + b)^2*Tan[e + f*x])/f + ((a + b)*(a + 3*b)*Tan[e + f*x]^3)/(3*f) + (b*
(2*a + 3*b)*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.0759502, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 373}

$$\frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + b)^2*Tan[e + f*x])/f + ((a + b)*(a + 3*b)*Tan[e + f*x]^3)/(3*f) + (b*
(2*a + 3*b)*Tan[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+b+bx^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int\left((a+b)^2+(a+b)(a+3b)x^2+b(2a+3b)x^4+b^2x^6\right)\,dx,x,\tan(e+\ldots\right)}{f}$$
$$= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f}+\frac{b(2a+3b)\tan^5(e+fx\ldots}{5f}$$

Mathematica [A] time = 0.38451, size = 75, normalized size = 0.94

$$\frac{35\left(a^2+4ab+3b^2\right)\tan^3(e+fx)+21b(2a+3b)\tan^5(e+fx)+105(a+b)^2\tan(e+fx)+15b^2\tan^7(e+fx)}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (105*(a + b)^2*Tan[e + f*x] + 35*(a^2 + 4*a*b + 3*b^2)*Tan[e + f*x]^3 + 21*
b*(2*a + 3*b)*Tan[e + f*x]^5 + 15*b^2*Tan[e + f*x]^7)/(105*f)
```

Maple [A] time = 0.038, size = 104, normalized size = 1.3

$$\frac{1}{f}\left(-a^2\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)-2\,ab\left(-\frac{8}{15}-1/5\,\left(\sec\left(fx+e\right)\right)^4-\frac{4\,\left(\sec\left(fx+e\right)\right)^2}{15}\right)\tan\left(fx+e\right)-b^2\left(-\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-8/15-1/5*sec(f*x+e)^4-
4/15*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^
4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 0.999666, size = 109, normalized size = 1.36

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab+3\,b^2\right)\tan\left(fx+e\right)^5+35\left(a^2+4\,ab+3\,b^2\right)\tan\left(fx+e\right)^3+105\left(a^2+2\,ab+b^2\right)\tan\left(fx+\ldots\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + 3*b^2)*tan(f*x + e)^5 + 35*(a^2
+ 4*a*b + 3*b^2)*tan(f*x + e)^3 + 105*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

Fricas [A] time = 0.490088, size = 234, normalized size = 2.92

$$\frac{\left(2\left(35\,a^2+56\,ab+24\,b^2\right)\cos\left(fx+e\right)^6+\left(35\,a^2+56\,ab+24\,b^2\right)\cos\left(fx+e\right)^4+6\left(7\,ab+3\,b^2\right)\cos\left(fx+e\right)^2+15\,b^2\right)}{105\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*a^2 + 56*a*b + 24*b^2)*cos(f*x + e)^6 + (35*a^2 + 56*a*b + 24*
b^2)*cos(f*x + e)^4 + 6*(7*a*b + 3*b^2)*cos(f*x + e)^2 + 15*b^2)*sin(f*x +
e)/(f*cos(f*x + e)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\sec^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**4, x)
```

Giac [A] time = 1.24721, size = 166, normalized size = 2.08

$$\frac{15\,b^2\tan\left(fx+e\right)^7+42\,ab\tan\left(fx+e\right)^5+63\,b^2\tan\left(fx+e\right)^5+35\,a^2\tan\left(fx+e\right)^3+140\,ab\tan\left(fx+e\right)^3+105\,b^2\,\mathrm{ta}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)^
5 + 35*a^2*tan(f*x + e)^3 + 140*a*b*tan(f*x + e)^3 + 105*b^2*tan(f*x + e)^3
 + 105*a^2*tan(f*x + e) + 210*a*b*tan(f*x + e) + 105*b^2*tan(f*x + e))/f
```

3.173 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*b*(a + b)*Tan[e + f*x]^3)/(3*f) + (b^2*Tan[
e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.06289, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 194}

$$\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan(e+fx)}{f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] ((a + b)^2*Tan[e + f*x])/f + (2*b*(a + b)*Tan[e + f*x]^3)/(3*f) + (b^2*Tan[
e + f*x]^5)/(5*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b+bx^2\right)^2 dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)+2ab\left(1+\frac{b}{a}\right)x^2+b^2x^4\right) dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a+b)^2\tan(e+fx)}{f}+\frac{2b(a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Mathematica [A] time = 0.265406, size = 48, normalized size = 0.91

$$\frac{10b(a+b)\tan^3(e+fx)+15(a+b)^2\tan(e+fx)+3b^2\tan^5(e+fx)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (15*(a + b)^2*Tan[e + f*x] + 10*b*(a + b)*Tan[e + f*x]^3 + 3*b^2*Tan[e + f*
x]^5)/(15*f)
```

Maple [A] time = 0.032, size = 71, normalized size = 1.3

$$\frac{1}{f}\left(a^2\tan\left(fx+e\right)-2\,ab\left(-2/3-1/3\,\left(\sec\left(fx+e\right)\right)^2\right)\tan\left(fx+e\right)-b^2\left(-\frac{8}{15}-\frac{\left(\sec\left(fx+e\right)\right)^4}{5}-\frac{4\,\left(\sec\left(fx+e\right)\right)^2}{15}\right)\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*tan(f*x+e)-2*a*b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-8/15-1/5
*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 0.986876, size = 96, normalized size = 1.81

$$\frac{10\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ab+\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)b^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(10*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*b + (3*tan(f*x + e)^5 + 10*tan
(f*x + e)^3 + 15*tan(f*x + e))*b^2 + 15*a^2*tan(f*x + e))/f
```

Fricas [A] time = 0.477477, size = 167, normalized size = 3.15

$$\frac{\left(\left(15\,a^2+20\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+2\left(5\,ab+2\,b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((15*a^2 + 20*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(5*a*b + 2*b^2)*cos(f*x
+ e)^2 + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

Giac [A] time = 1.35357, size = 111, normalized size = 2.09

$$\frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+10\,b^2\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+e\right)+30\,ab\tan\left(fx+e\right)+15\,b^2\tan\left(fx\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 10*b^2*tan(f*x + e)^3
+ 15*a^2*tan(f*x + e) + 30*a*b*tan(f*x + e) + 15*b^2*tan(f*x + e))/f
```

3.174 $\int \left(a + b\sec^2(e + fx)\right)^2 \, dx$

Optimal. Leaf size=40

$$a^2x + \frac{b(2a+b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0294881, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$a^2x + \frac{b(2a+b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Mathematica [B] time = 0.397328, size = 106, normalized size = 2.65

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3a^2fx\cos^3(e+fx)+2b(3a+b)\sec(e)\sin(fx)\cos^2(e+fx)+b^2\tan(e)\cos(e+fx)\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.038, size = 48, normalized size = 1.2

$$\frac{1}{f}\left(a^2\left(fx+e\right)+2\,ab\tan\left(fx+e\right)-b^2\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 1.01899, size = 59, normalized size = 1.48

$$a^2x + \frac{\left(\tan\left(fx+e\right)^3 + 3\tan\left(fx+e\right)\right)b^2}{3f} + \frac{2ab\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Fricas [A] time = 0.482485, size = 142, normalized size = 3.55

$$\frac{3a^2fx\cos\left(fx+e\right)^3 + \left(2\left(3ab+b^2\right)\cos\left(fx+e\right)^2 + b^2\right)\sin\left(fx+e\right)}{3f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a + b\sec^2\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.32292, size = 72, normalized size = 1.8

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

3.175 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=47

$$\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ax(a+4b)+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*Tan[e + f*
x])/f
```

Rubi [A] time = 0.0722403, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 203}

$$\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}ax(a+4b)+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a*(a + 4*b)*x)/2 + (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (b^2*Tan[e + f*
x])/f
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(b^2+\frac{a(a+2b)+2abx^2}{\left(1+x^2\right)^2}\right)dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{b^2\tan(e+fx)}{f}+\frac{\text{Subst}\left(\int \frac{a(a+2b)+2abx^2}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{a^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{b^2\tan(e+fx)}{f}+\frac{(a(a+4b))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{2f} \\
&= \frac{1}{2}a(a+4b)x+\frac{a^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{b^2\tan(e+fx)}{f}
\end{aligned}
$$

Mathematica [A] time = 0.15227, size = 52, normalized size = 1.11

$$
\frac{a^2(e+fx)}{2f}+\frac{a^2\sin(2(e+fx))}{4f}+2abx+\frac{b^2\tan(e+fx)}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (a^2*(e + f*x))/(2*f) + (a^2*Sin[2*(e + f*x)])/(4*f) + (b^2*Tan[e
 + f*x])/f
```

Maple [A] time = 0.05, size = 51, normalized size = 1.1

$$\frac{1}{f}\left(a^{2}\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2\,ab\left(fx+e\right)+b^{2}\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(f*x+e)+b^2*tan(f*
x+e))
```

Maxima [A] time = 1.50555, size = 72, normalized size = 1.53

$$\frac{2\,b^{2}\tan\left(fx+e\right)+\left(a^{2}+4\,ab\right)\left(fx+e\right)+\frac{a^{2}\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{2}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

Fricas [A] time = 0.488119, size = 134, normalized size = 2.85

$$\frac{\left(a^{2}+4\,ab\right)fx\cos\left(fx+e\right)+\left(a^{2}\cos\left(fx+e\right)^{2}+2\,b^{2}\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 4*a*b)*f*x*cos(f*x + e) + (a^2*cos(f*x + e)^2 + 2*b^2)*sin(f*x
+ e))/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42626, size = 77, normalized size = 1.64

$$\frac{2\,b^2\tan\left(fx+e\right)+\left(a^2+4\,ab\right)\left(fx+e\right)+\frac{a^2\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

3.176 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=81

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{a\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4f}$$

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) + (a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(4*f)
```

Rubi [A] time = 0.0863722, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 413, 385, 203}

$$\frac{1}{8}x\left(3a^2+8ab+8b^2\right)+\frac{3a(a+2b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{a\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*x)/8 + (3*a*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) + (a*Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(4*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{4f} + \frac{\text{Subst}\left(\int \frac{(a+b)(3a+4b)+b(}{\left(1+x^2\right)^2}\right.}{4}$$

$$= \frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan\right.}{4f}$$

$$= \frac{1}{8}\left(3a^2+8ab+8b^2\right)x + \frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\,\mathrm{s}}{}$$

Mathematica [A] time = 0.12574, size = 58, normalized size = 0.72

$$\frac{4\left(3a^2+8ab+8b^2\right)(e+fx)+a^2\sin(4(e+fx))+8a(a+2b)\sin(2(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 + 8*a*b + 8*b^2)*(e + f*x) + 8*a*(a + 2*b)*Sin[2*(e + f*x)] + a^2
*Sin[4*(e + f*x)])/(32*f)
```

Maple [A] time = 0.057, size = 78, normalized size = 1.

$$\frac{1}{f}\left(a^{2}\left(\frac{\sin\left(fx+e\right)}{4}\left(\left(\cos\left(fx+e\right)\right)^{3}+\frac{3\cos\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(1/2\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2fx+e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e)+2*a*b
*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b^2*(f*x+e))
```

Maxima [A] time = 1.47088, size = 117, normalized size = 1.44

$$\frac{\left(3a^{2}+8ab+8b^{2}\right)\left(fx+e\right)+\frac{\left(3a^{2}+8ab\right)\tan\left(fx+e\right)^{3}+\left(5a^{2}+8ab\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{4}+2\tan\left(fx+e\right)^{2}+1}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + ((3*a^2 + 8*a*b)*tan(f*x + e)^3 +
(5*a^2 + 8*a*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.490376, size = 143, normalized size = 1.77

$$\frac{\left(3a^{2}+8ab+8b^{2}\right)fx+\left(2a^{2}\cos\left(fx+e\right)^{3}+\left(3a^{2}+8ab\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*f*x + (2*a^2*cos(f*x + e)^3 + (3*a^2 + 8*a*b)*
cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33873, size = 126, normalized size = 1.56

$$\frac{\left(3\,a^2+8\,ab+8\,b^2\right)\left(fx+e\right)+\frac{3\,a^2\tan\left(fx+e\right)^3+8\,ab\tan\left(fx+e\right)^3+5\,a^2\tan\left(fx+e\right)+8\,ab\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^2+1\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + (3*a^2*tan(f*x + e)^3 + 8*a*b*tan(
f*x + e)^3 + 5*a^2*tan(f*x + e) + 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^
2)/f
```

3.177 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=119

$$\frac{\left(5a^2+12ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(5a^2+12ab+8b^2\right)+\frac{a(5a+8b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a\sin(e-}{}$$

```
[Out] ((5*a^2 + 12*a*b + 8*b^2)*x)/16 + ((5*a^2 + 12*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(5*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (
a*Cos[e + f*x]^5*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(6*f)
```

Rubi [A] time = 0.145638, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 413, 385, 199, 203}

$$\frac{\left(5a^2+12ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(5a^2+12ab+8b^2\right)+\frac{a(5a+8b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a\sin(e-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] ((5*a^2 + 12*a*b + 8*b^2)*x)/16 + ((5*a^2 + 12*a*b + 8*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*f) + (a*(5*a + 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) + (
a*Cos[e + f*x]^5*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(6*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f} + \frac{\text{Subst}\left(\int \frac{(a+b)(5a+6b)+3b}{\left(1+x^2\right)^3}\right.}{}$$

$$= \frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\,\text{ta}\right.}{6f}$$

$$= \frac{\left(5a^2+12ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(5a+8b)\cos^3(e+fx)\sin(e+}{24f}$$

$$= \frac{1}{16}\left(5a^2+12ab+8b^2\right)x + \frac{\left(5a^2+12ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} + \frac{a(5a}{}$$

Mathematica [A] time = 0.196163, size = 99, normalized size = 0.83

$$\frac{\left(45a^2+96ab+48b^2\right)\sin(2(e+fx))+a^2\sin(6(e+fx))+60a^2e+60a^2fx+3a(3a+4b)\sin(4(e+fx))+144abe+144}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (60*a^2*e + 144*a*b*e + 96*b^2*e + 60*a^2*f*x + 144*a*b*f*x + 96*b^2*f*x +
(45*a^2 + 96*a*b + 48*b^2)*Sin[2*(e + f*x)] + 3*a*(3*a + 4*b)*Sin[4*(e + f*
x)] + a^2*Sin[6*(e + f*x)])/(192*f)
```

Maple [A] time = 0.059, size = 116, normalized size = 1.

$$\frac{1}{f}\left(a^2\left(\frac{\sin\left(fx+e\right)}{6}\left(\left(\cos\left(fx+e\right)\right)^5+\frac{5\left(\cos\left(fx+e\right)\right)^3}{4}+\frac{15\cos\left(fx+e\right)}{8}\right)+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(1/4\left(\left(\cos\left(fx+e\right)\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(1/6*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/
16*f*x+5/16*e)+2*a*b*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+
3/8*e)+b^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.50327, size = 182, normalized size = 1.53

$$\frac{3\left(5a^2+12ab+8b^2\right)\left(fx+e\right)+\frac{3\left(5a^2+12ab+8b^2\right)\tan\left(fx+e\right)^5+8\left(5a^2+12ab+6b^2\right)\tan\left(fx+e\right)^3+3\left(11a^2+20ab+8b^2\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^6+3\tan\left(fx+e\right)^4+3\tan\left(fx+e\right)^2+1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (3*(5*a^2 + 12*a*b + 8*b^2)*ta
n(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(11*a^2 + 20*a
```

```
*b + 8*b^2)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x +
e)^2 + 1))/f
```

Fricas [A] time = 0.503977, size = 212, normalized size = 1.78

$$\frac{3\left(5a^2+12ab+8b^2\right)fx+\left(8a^2\cos\left(fx+e\right)^5+2\left(5a^2+12ab\right)\cos\left(fx+e\right)^3+3\left(5a^2+12ab+8b^2\right)\cos\left(fx+e\right)\right)\sin}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*f*x + (8*a^2*cos(f*x + e)^5 + 2*(5*a^2 + 1
2*a*b)*cos(f*x + e)^3 + 3*(5*a^2 + 12*a*b + 8*b^2)*cos(f*x + e))*sin(f*x +
e))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31132, size = 217, normalized size = 1.82

$$\frac{3\left(5a^2+12ab+8b^2\right)\left(fx+e\right)+\frac{15a^2\tan\left(fx+e\right)^5+36ab\tan\left(fx+e\right)^5+24b^2\tan\left(fx+e\right)^5+40a^2\tan\left(fx+e\right)^3+96ab\tan\left(fx+e\right)^3+48b^2\tan\left(fx+e\right)^3+3}{\left(\tan\left(fx+e\right)^2+1\right)^3}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (15*a^2*tan(f*x + e)^5 + 36*a*
b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 96*a*b*t
an(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) + 60*a*b*tan(f*
x + e) + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

3.178 $\int \left(a+b\sec^2(c+dx)\right)^3 dx$

Optimal. Leaf size=73

$$\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+a^3x+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b^2*(3*a + 2*b)*Tan[c +
 d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0444889, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+a^3x+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^3,x]
```



```
[Out] a^3*x + (b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x])/d + (b^2*(3*a + 2*b)*Tan[c +
 d*x]^3)/(3*d) + (b^3*Tan[c + d*x]^5)/(5*d)
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^3}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int \left(b\left(3a^2+3ab+b^2\right)+b^2(3a+2b)x^2+b^3x^4+\frac{a^3}{1+x^2}\right)dx,x,\tan(c+dx)\right)}{d}$$
$$=\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}+\frac{a^3\,\text{Subst}}{}$$
$$=a^3x+\frac{b\left(3a^2+3ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2(3a+2b)\tan^3(c+dx)}{3d}+\frac{b^3\tan^5(c+dx)}{5d}$$

Mathematica [B] time = 1.02891, size = 268, normalized size = 3.67

$$\frac{\sec(c)\sec^5(c+dx)\left(-360a^2b\sin(2c+dx)+360a^2b\sin(2c+3dx)-90a^2b\sin(4c+3dx)+90a^2b\sin(4c+5dx)+540a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^5*(150*a^3*d*x*Cos[d*x] + 150*a^3*d*x*Cos[2*c + d*x] +
 75*a^3*d*x*Cos[2*c + 3*d*x] + 75*a^3*d*x*Cos[4*c + 3*d*x] + 15*a^3*d*x*Cos
[4*c + 5*d*x] + 15*a^3*d*x*Cos[6*c + 5*d*x] + 540*a^2*b*Sin[d*x] + 420*a*b^
2*Sin[d*x] + 160*b^3*Sin[d*x] - 360*a^2*b*Sin[2*c + d*x] - 180*a*b^2*Sin[2*
c + d*x] + 360*a^2*b*Sin[2*c + 3*d*x] + 300*a*b^2*Sin[2*c + 3*d*x] + 80*b^3
*Sin[2*c + 3*d*x] - 90*a^2*b*Sin[4*c + 3*d*x] + 90*a^2*b*Sin[4*c + 5*d*x] +
 60*a*b^2*Sin[4*c + 5*d*x] + 16*b^3*Sin[4*c + 5*d*x]))/(480*d)
```

Maple [A] time = 0.036, size = 84, normalized size = 1.2

$$\frac{1}{d}\left(a^3(dx+c)+3a^2b\tan(dx+c)-3ab^2\left(-2/3-1/3\,(\sec(dx+c))^2\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4\,(\sec}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(d*x+c)+3*a^2*b*tan(d*x+c)-3*a*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x
+c)-b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 0.997411, size = 112, normalized size = 1.53

$$a^3x + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)ab^2}{d} + \frac{\left(3\,\tan\left(dx+c\right)^5 + 10\,\tan\left(dx+c\right)^3 + 15\,\tan\left(dx+c\right)\right)b^3}{15\,d} + \frac{3\,a^2b\tan\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a^3*x + (tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3/d + 3*a^2*b*tan(d*x + c)/d
```

Fricas [A] time = 0.497846, size = 215, normalized size = 2.95

$$\frac{15\,a^3dx\cos\left(dx+c\right)^5 + \left(\left(45\,a^2b + 30\,ab^2 + 8\,b^3\right)\cos\left(dx+c\right)^4 + 3\,b^3 + \left(15\,ab^2 + 4\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^3*d*x*cos(d*x + c)^5 + ((45*a^2*b + 30*a*b^2 + 8*b^3)*cos(d*x +
c)^4 + 3*b^3 + (15*a*b^2 + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x
+ c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c)**2)**3,x)

[Out] Integral((a + b*sec(c + d*x)**2)**3, x)

Giac [A] time = 1.24093, size = 123, normalized size = 1.68

$$\frac{3\,b^3\tan\left(dx+c\right)^5+15\,ab^2\tan\left(dx+c\right)^3+10\,b^3\tan\left(dx+c\right)^3+15\,(dx+c)a^3+45\,a^2b\tan\left(dx+c\right)+45\,ab^2\tan\left(dx+c\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/15*(3*b^3*tan(d*x + c)^5 + 15*a*b^2*tan(d*x + c)^3 + 10*b^3*tan(d*x + c)^
3 + 15*(d*x + c)*a^3 + 45*a^2*b*tan(d*x + c) + 45*a*b^2*tan(d*x + c) + 15*b
^3*tan(d*x + c))/d

3.179 $\int \left(a+b\sec^2(c+dx)\right)^4 \, dx$

Optimal. Leaf size=111

$$\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+a^4x+\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+\frac{b^4\tan^7(}{7}$$

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*a^2 +
8*a*b + 3*b^2)*Tan[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tan[c + d*x]^5)/(5*
d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0648342, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+a^4x+\frac{b^3(4a+3b)\tan^5(c+dx)}{5d}+\frac{b^4\tan^7(}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^4,x]
```

```
[Out] a^4*x + (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x])/d + (b^2*(6*a^2 +
8*a*b + 3*b^2)*Tan[c + d*x]^3)/(3*d) + (b^3*(4*a + 3*b)*Tan[c + d*x]^5)/(5*
d) + (b^4*Tan[c + d*x]^7)/(7*d)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b(2a+b)\left(2a^2+2ab+b^2\right)+b^2\left(6a^2+8ab+3b^2\right)x^2+b^3(4a+3b)x^4+b^4x^6+\right.\right.}{d}$$

$$= \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b^3(4a+3b}{}$$

$$= a^4x+\frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx)}{d}+\frac{b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx)}{3d}+\frac{b^3(4}{}$$

Mathematica [B] time = 1.61481, size = 455, normalized size = 4.1

$$\frac{\sec(c)\sec^7(c+dx)\left(-10920a^2b^2\sin(2c+dx)+15120a^2b^2\sin(2c+3dx)-2520a^2b^2\sin(4c+3dx)+5880a^2b^2\sin(4c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^7*(3675*a^4*d*x*Cos[d*x] + 3675*a^4*d*x*Cos[2*c + d*x]
 + 2205*a^4*d*x*Cos[2*c + 3*d*x] + 2205*a^4*d*x*Cos[4*c + 3*d*x] + 735*a^4*
d*x*Cos[4*c + 5*d*x] + 735*a^4*d*x*Cos[6*c + 5*d*x] + 105*a^4*d*x*Cos[6*c +
 7*d*x] + 105*a^4*d*x*Cos[8*c + 7*d*x] + 16800*a^3*b*Sin[d*x] + 18480*a^2*b
^2*Sin[d*x] + 11200*a*b^3*Sin[d*x] + 3360*b^4*Sin[d*x] - 12600*a^3*b*Sin[2*
c + d*x] - 10920*a^2*b^2*Sin[2*c + d*x] - 4480*a*b^3*Sin[2*c + d*x] + 12600
*a^3*b*Sin[2*c + 3*d*x] + 15120*a^2*b^2*Sin[2*c + 3*d*x] + 9408*a*b^3*Sin[2
*c + 3*d*x] + 2016*b^4*Sin[2*c + 3*d*x] - 5040*a^3*b*Sin[4*c + 3*d*x] - 252
0*a^2*b^2*Sin[4*c + 3*d*x] + 5040*a^3*b*Sin[4*c + 5*d*x] + 5880*a^2*b^2*Sin
[4*c + 5*d*x] + 3136*a*b^3*Sin[4*c + 5*d*x] + 672*b^4*Sin[4*c + 5*d*x] - 84
0*a^3*b*Sin[6*c + 5*d*x] + 840*a^3*b*Sin[6*c + 7*d*x] + 840*a^2*b^2*Sin[6*c
 + 7*d*x] + 448*a*b^3*Sin[6*c + 7*d*x] + 96*b^4*Sin[6*c + 7*d*x]))/(13440*d
)
```

Maple [A] time = 0.041, size = 130, normalized size = 1.2

$$\frac{1}{d}\left(a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-2/3-1/3\,(\sec(dx+c))^2\right)\tan(dx+c)-4ab^3\left(-\frac{8}{15}-1/5\,(\sec(dx+c))^4-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c)^2)^4,x)
```

```
[Out] 1/d*(a^4*(d*x+c)+4*a^3*b*tan(d*x+c)-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d
*x+c)-4*a*b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-b^4*(-1
6/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.00227, size = 181, normalized size = 1.63

$$a^4x+\frac{2\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^2b^2}{d}+\frac{4\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)ab^3}{15\,d}+\frac{\left(5\,\tan(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x + c
)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3/d + 1/35*(5*tan(d*x + c)^7
 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*b^4/d + 4*a^3*b
*tan(d*x + c)/d
```

Fricas [A] time = 0.52788, size = 317, normalized size = 2.86

$$\frac{105\,a^4dx\cos(dx+c)^7+\left(4\left(105\,a^3b+105\,a^2b^2+56\,ab^3+12\,b^4\right)\cos(dx+c)^6+2\left(105\,a^2b^2+56\,ab^3+12\,b^4\right)\cos(dx+c\right.}{105\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^4*d*x*cos(d*x + c)^7 + (4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3
+ 12*b^4)*cos(d*x + c)^6 + 2*(105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cos(d*x + c)
```

```
^4 + 15*b^4 + 6*(14*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x
 + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(c + dx\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)**2)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x)**2)**4, x)
```

Giac [A] time = 1.23412, size = 200, normalized size = 1.8

$$\frac{15\,b^4\tan\left(dx+c\right)^7 + 84\,ab^3\tan\left(dx+c\right)^5 + 63\,b^4\tan\left(dx+c\right)^5 + 210\,a^2b^2\tan\left(dx+c\right)^3 + 280\,ab^3\tan\left(dx+c\right)^3 + 105\,b}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 84*a*b^3*tan(d*x + c)^5 + 63*b^4*tan(d*x + c
)^5 + 210*a^2*b^2*tan(d*x + c)^3 + 280*a*b^3*tan(d*x + c)^3 + 105*b^4*tan(d
*x + c)^3 + 105*(d*x + c)*a^4 + 420*a^3*b*tan(d*x + c) + 630*a^2*b^2*tan(d*
x + c) + 420*a*b^3*tan(d*x + c) + 105*b^4*tan(d*x + c))/d
```

3.180 $\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a+b}}-\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{\tan(e+fx)\sec(e+fx)}{2bf}$$

```
[Out] -((2*a - b)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (a^(3/2)*ArcTanh[(Sqrt[a]*Si
n[e + f*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*f) + (Sec[e + f*x]*Tan[e + f*x])
/(2*b*f)
```

Rubi [A] time = 0.120596, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 414, 522, 206, 208}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a+b}}-\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{\tan(e+fx)\sec(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```


```
[Out] -((2*a - b)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (a^(3/2)*ArcTanh[(Sqrt[a]*Si
n[e + f*x])/Sqrt[a + b]])/(b^2*Sqrt[a + b]*f) + (Sec[e + f*x]*Tan[e + f*x])
/(2*b*f)
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+b-ax^2\right)}\,dx, x, \sin(e+fx)\right)}{f} \\
&= \frac{\sec(e+fx)\tan(e+fx)}{2bf} + \frac{\text{Subst}\left(\int \frac{-a+b-ax^2}{\left(1-x^2\right)\left(a+b-ax^2\right)}\,dx, x, \sin(e+fx)\right)}{2bf} \\
&= \frac{\sec(e+fx)\tan(e+fx)}{2bf} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{b^2 f} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{2b^2 f} \\
&= -\frac{(2a-b)\tanh^{-1}(\sin(e+fx))}{2b^2 f} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}\,f} + \frac{\sec(e+fx)\tan(e+fx)}{2bf}
\end{aligned}$$

Mathematica [C] time = 6.45738, size = 1195, normalized size = 13.9

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^2(e+fx)\left(2i\tan^{-1}\left(\frac{2\sin(e)\left(\sin(2e)a+ia-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\sin(fx)\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\sin(2e+fx)\sqrt{a}+\sqrt{a+b}\cos(\cdots\right.}{i(a+3b)\cos(e)+i(a+b)\cos(3e)+ia\cos(e+2fx)+ia\cos(3e+2fx)+3a\sin(e)+\cdots}\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(4*a*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] - 2*b*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 4*a*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*b*Log[Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]] + (a^(3/2)*Cos[e]*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)]
- (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e]
- I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
*Sin[2*e + f*x]])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) - (a^(3/2)*Cos[
e]*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2
*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x]
 + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]])/(Sqrt
[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((2*I)*a^(3/2)*ArcTan[(2*Sin[e]*(I*a
 + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*Sqrt[(Cos[e] - I
*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2
] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e +
 f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e + 2*f*x] + I*a
*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*Sin[3*e] + a*Sin
[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[e] + I*
Sin[e]))/Sqrt[a + b] - (I*a^(3/2)*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e +
 f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin
[e])^2]*Sin[2*e + f*x]]*Sin[e])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) +
 (I*a^(3/2)*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[
2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x
]]*Sin[e])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) + (2*a^(3/2)*ArcTan[((
a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*Sqrt[(Cos[e] - I*Sin[e])^2]*((
-I)*Cos[e] + Sin[e]))/Sqrt[a + b] + b/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2 - b/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(8*b^2*f*(a + b*Sec[e + f*
x]^2))
```

Maple [A] time = 0.075, size = 141, normalized size = 1.6

$$-\frac{1}{4\,fb\left(\sin\left(fx+e\right)+1\right)}-\frac{\ln\left(\sin\left(fx+e\right)+1\right)a}{2\,fb^{2}}+\frac{\ln\left(\sin\left(fx+e\right)+1\right)}{4\,fb}+\frac{a^{2}}{fb^{2}}\mathrm{Artanh}\left(a\sin\left(fx+e\right)\frac{1}{\sqrt{(a+b)\,a}}\right)\frac{}{\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/4/f/b/(sin(f*x+e)+1)-1/2/f/b^2*ln(sin(f*x+e)+1)*a+1/4/f/b*ln(sin(f*x+e)+
1)+1/f*a^2/b^2/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-1/4/f/
b/(sin(f*x+e)-1)+1/2/f/b^2*ln(sin(f*x+e)-1)*a-1/4/f/b*ln(sin(f*x+e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.602482, size = 687, normalized size = 7.99

$$\left[\frac{2\,a\sqrt{\frac{a}{a+b}}\cos\left(fx+e\right)^{2}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\,(a+b)\sqrt{\frac{a}{a+b}}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^{2}+b}\right)-(2\,a-b)\cos\left(fx+e\right)^{2}\log\left(\sin\left(fx+e\right)+1\right)+(2\,a}{4\,b^{2}f\cos\left(fx+e\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(a/(a + b))*cos(f*x + e)^2*log(-(a*cos(f*x + e)^2 - 2*(a + b)
*sqrt(a/(a + b))*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - (2*a - b
)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*a - b)*cos(f*x + e)^2*log(-sin(
f*x + e) + 1) + 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2), -1/4*(4*a*sqrt(-a
/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e))*cos(f*x + e)^2 + (2*a - b)*
cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a - b)*cos(f*x + e)^2*log(-sin(f*
x + e) + 1) - 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.25492, size = 159, normalized size = 1.85

$$-\frac{\frac{4\,a^2\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b^2}+\frac{(2\,a-b)\log\left(\sin\left(fx+e\right)+1\right)}{b^2}-\frac{(2\,a-b)\log\left(-\sin\left(fx+e\right)+1\right)}{b^2}+\frac{2\,\sin\left(fx+e\right)}{\left(\sin\left(fx+e\right)^2-1\right)b}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b^2)
+ (2*a - b)*log(sin(f*x + e) + 1)/b^2 - (2*a - b)*log(-sin(f*x + e) + 1)/b^
2 + 2*sin(f*x + e)/((sin(f*x + e)^2 - 1)*b))/f
```

3.181 $\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\sin(e+fx))}{bf}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{bf\sqrt{a+b}}$$

```
[Out] ArcTanh[Sin[e + f*x]]/(b*f) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*f)
```

Rubi [A] time = 0.0717949, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 391, 206, 208}

$$\frac{\tanh^{-1}(\sin(e+fx))}{bf}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{bf\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTanh[Sin[e + f*x]]/(b*f) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[
a + b]])/(b*Sqrt[a + b]*f)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-ax^2)}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{bf} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{bf}\\ &= \frac{\tanh^{-1}(\sin(e+fx))}{bf} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}f}\end{aligned}$$

Mathematica [C] time = 2.02713, size = 1022, normalized size = 18.58

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^2(e+fx)\left(-4\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\sqrt{a}\cos\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-(Sqrt[a]*Cos[e]*Log[a + 2*
(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e]
 + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]) + Sqrt[a]*Cos[e]*Lo
g[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b
*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]] - (2*I)*Sqr
t[a]*ArcTan[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]
*Cos[f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*
```

```
Sqrt[(Cos[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a +
b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e]
+ I*a*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin
[3*e] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(Cos[e] - I*Si
n[e]) - 4*Sqrt[a + b]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sqrt[(Cos[e]
 - I*Sin[e])^2] + 4*Sqrt[a + b]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sq
rt[(Cos[e] - I*Sin[e])^2] + I*Sqrt[a]*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*
(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sq
rt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I
*Sin[e])^2]*Sin[2*e + f*x]]*Sin[e] - I*Sqrt[a]*Log[-a - 2*(a + b)*Cos[2*e]
+ a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt
[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sin[e] + 2*Sqrt[a]*ArcTan[((a + b)*Si
n[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Co
s[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(I*Cos[e] + Sin[e])))/(8*b*Sqrt[a + b]*
f*(a + b*Sec[e + f*x]^2)*Sqrt[(Cos[e] - I*Sin[e])^2])
```

Maple [A] time = 0.059, size = 68, normalized size = 1.2

$$\frac{\ln\left(\sin\left(fx+e\right)+1\right)}{2\,fb}-\frac{a}{fb}\mathrm{Artanh}\left(a\sin\left(fx+e\right)\frac{1}{\sqrt{(a+b)\,a}}\right)\frac{1}{\sqrt{(a+b)\,a}}-\frac{\ln\left(\sin\left(fx+e\right)-1\right)}{2\,fb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2/f/b*ln(sin(f*x+e)+1)-1/f*a/b/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b
)*a)^(1/2))-1/2/f/b*ln(sin(f*x+e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.557766, size = 405, normalized size = 7.36

$$\left[\frac{\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)+\log\left(\sin\left(fx+e\right)+1\right)-\log\left(-\sin\left(fx+e\right)+1\right)}{2\,bf},\ \frac{2\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*si
n(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + log(sin(f*x + e) + 1) - log
(-sin(f*x + e) + 1))/(b*f), 1/2*(2*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))
*sin(f*x + e)) + log(sin(f*x + e) + 1) - log(-sin(f*x + e) + 1))/(b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.30815, size = 104, normalized size = 1.89

$$\frac{\frac{2\,a\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b}+\frac{\log\left(\sin\left(fx+e\right)+1\right)}{b}-\frac{\log\left(-\sin\left(fx+e\right)+1\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b) + log
(sin(f*x + e) + 1)/b - log(-sin(f*x + e) + 1)/b)/f
```

3.182 $\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}f\sqrt{a+b}}$$

```
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Rubi [A] time = 0.042072, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4147, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

Mathematica [A] time = 0.0795575, size = 36, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

Maple [A] time = 0.065, size = 28, normalized size = 0.8

$$\frac{1}{f}\text{Artanh}\left(a\sin\left(fx+e\right)\frac{1}{\sqrt{(a+b)\,a}}\right)\frac{1}{\sqrt{(a+b)\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.510653, size = 271, normalized size = 7.53

$$\left[\frac{\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{a^2+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)}{2\sqrt{a^2+ab}f}, -\frac{\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin\left(fx+e\right)}{a+b}\right)}{\left(a^2+ab\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*
cos(f*x + e)^2 + b))/(sqrt(a^2 + a*b)*f), -sqrt(-a^2 - a*b)*arctan(sqrt(-a^
2 - a*b)*sin(f*x + e)/(a + b))/((a^2 + a*b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.2437, size = 53, normalized size = 1.47

$$-\frac{\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*f)
```

3.183 $\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sin(e+fx)}{af}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}f\sqrt{a+b}}$$

```
[Out] -((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*f))
+ Sin[e + f*x]/(a*f)
```

Rubi [A] time = 0.0632139, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4147, 388, 208}

$$\frac{\sin(e+fx)}{af}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]*f))
+ Sin[e + f*x]/(a*f)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\sin(e+fx)}{af} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{af}\\ &= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}f} + \frac{\sin(e+fx)}{af}\end{aligned}$$

Mathematica [A] time = 0.112577, size = 52, normalized size = 1.

$$\frac{\sqrt{a}\sin(e+fx) - \frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*S
in[e + f*x])/(a^(3/2)*f)
```

Maple [A] time = 0.091, size = 45, normalized size = 0.9

$$\frac{1}{f}\left(\frac{\sin\left(fx+e\right)}{a} - \frac{b}{a}\text{Artanh}\left(\sin\left(fx+e\right)a\frac{1}{\sqrt{(a+b)\,a}}\right)\frac{1}{\sqrt{(a+b)\,a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(1/a*sin(f*x+e)-b/a/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2
)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.535825, size = 378, normalized size = 7.27

$$\left[\frac{\sqrt{a^2+ab}b\log\left(-\frac{a\cos\left(fx+e\right)^2+2\sqrt{a^2+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(a^2+ab\right)\sin\left(fx+e\right)}{2\left(a^3+a^2b\right)f}, \frac{\sqrt{-a^2-ab}b\arctan\left(\frac{\sqrt{-a^2-ab}\sin\left(fx+e\right)}{a+b}\right)+\left(\right.}{\left(a^3+a^2b\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + a*b)*b*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
+ e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b)*sin(f*x + e))/((a^3
 + a^2*b)*f), (sqrt(-a^2 - a*b)*b*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a +
 b)) + (a^2 + a*b)*sin(f*x + e))/((a^3 + a^2*b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.18061, size = 74, normalized size = 1.42

$$\frac{\frac{b\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a}+\frac{\sin\left(fx+e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (b*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a) + sin(f*x +
 e)/a)/f
```

3.184 $\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}f\sqrt{a+b}}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}$$

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*f) +
 ((a - b)*Sin[e + f*x])/(a^2*f) - Sin[e + f*x]^3/(3*a*f)
```

Rubi [A] time = 0.0890151, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 390, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}f\sqrt{a+b}}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]*f) +
 ((a - b)*Sin[e + f*x])/(a^2*f) - Sin[e + f*x]^3/(3*a*f)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a-b}{a^2}-\frac{x^2}{a}+\frac{b^2}{a^2\left(a+b-ax^2\right)}\right)dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{a^2f}\\ &= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}f}+\frac{(a-b)\sin(e+fx)}{a^2f}-\frac{\sin^3(e+fx)}{3af}\end{aligned}$$

Mathematica [A] time = 0.299817, size = 105, normalized size = 1.38

$$\frac{a^{3/2}\sin(3(e+fx))+\frac{6b^2\left(\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}}+3\sqrt{a}(3a-4b)\sin(e+fx)}{12a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((6*b^2*(-Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] + Log[Sqrt[a + b] + Sqrt[
a]*Sin[e + f*x]]))/Sqrt[a + b] + 3*Sqrt[a]*(3*a - 4*b)*Sin[e + f*x] + a^(3/
2)*Sin[3*(e + f*x)])/(12*a^(5/2)*f)
```

Maple [A] time = 0.097, size = 70, normalized size = 0.9

$$\frac{1}{f}\left(-\frac{1}{a^2}\left(\frac{a\left(\sin\left(fx+e\right)\right)^3}{3}-\sin\left(fx+e\right)a+\sin\left(fx+e\right)b\right)+\frac{b^2}{a^2}\text{Artanh}\left(\sin\left(fx+e\right)a\frac{1}{\sqrt{(a+b)\,a}}\right)\frac{1}{\sqrt{(a+b)\,a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(-1/a^2*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+sin(f*x+e)*b)+b^2/a^2/((a+b)*a
)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.550273, size = 517, normalized size = 6.8

$$\left[\frac{3\sqrt{a^2+ab}b^2\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{a^2+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(2a^3-a^2b-3ab^2+\left(a^3+a^2b\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}{6\left(a^4+a^3b\right)f},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 + a*b)*b^2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(
f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3 - a^2*b - 3*a*b^2 +
(a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)*f), -1/3*(3*sqrt
(-a^2 - a*b)*b^2*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (2*a^3 - a
^2*b - 3*a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)
*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26462, size = 120, normalized size = 1.58

$$-\frac{\frac{3\,b^2\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a^2}+\frac{a^2\sin\left(fx+e\right)^3-3\,a^2\sin\left(fx+e\right)+3\,ab\sin\left(fx+e\right)}{a^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^2)
+ (a^2*sin(f*x + e)^3 - 3*a^2*sin(f*x + e) + 3*a*b*sin(f*x + e))/a^3)/f
```

3.185 $\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{\left(a^2-ab+b^2\right)\sin(e+fx)}{a^3f}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}f\sqrt{a+b}}-\frac{(2a-b)\sin^3(e+fx)}{3a^2f}+\frac{\sin^5(e+fx)}{5af}$$

```
[Out] -((b^3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b]*f)
) + ((a^2 - a*b + b^2)*Sin[e + f*x])/(a^3*f) - ((2*a - b)*Sin[e + f*x]^3)/(
3*a^2*f) + Sin[e + f*x]^5/(5*a*f)
```

Rubi [A] time = 0.102014, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 390, 208}

$$\frac{\left(a^2-ab+b^2\right)\sin(e+fx)}{a^3f}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}f\sqrt{a+b}}-\frac{(2a-b)\sin^3(e+fx)}{3a^2f}+\frac{\sin^5(e+fx)}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b]*f)
) + ((a^2 - a*b + b^2)*Sin[e + f*x])/(a^3*f) - ((2*a - b)*Sin[e + f*x]^3)/(
3*a^2*f) + Sin[e + f*x]^5/(5*a*f)
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{a^3} - \frac{(2a-b)x^2}{a^2} + \frac{x^4}{a} - \frac{b^3}{a^3\left(a+b-ax^2\right)}\right) dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(a^2-ab+b^2\right)\sin(e+fx)}{a^3 f} - \frac{(2a-b)\sin^3(e+fx)}{3a^2 f} + \frac{\sin^5(e+fx)}{5af} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx,\right.}{a^3 f}$$

$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}\,f} + \frac{\left(a^2-ab+b^2\right)\sin(e+fx)}{a^3 f} - \frac{(2a-b)\sin^3(e+fx)}{3a^2 f} + \frac{\sin^5(e+f}{5af}$$

Mathematica [A] time = 0.725452, size = 136, normalized size = 1.26

$$\frac{30\sqrt{a}\left(5a^2-6ab+8b^2\right)\sin(e+fx)+5a^{3/2}(5a-4b)\sin(3(e+fx))+3a^{5/2}\sin(5(e+fx))+\frac{120b^3\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\text{lo}\right.}{\sqrt{a+b}}}{240a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((120*b^3*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a + b] + Sqrt
[a]*Sin[e + f*x]]))/Sqrt[a + b] + 30*Sqrt[a]*(5*a^2 - 6*a*b + 8*b^2)*Sin[e
+ f*x] + 5*a^(3/2)*(5*a - 4*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)
])/(240*a^(7/2)*f)
```

Maple [A] time = 0.089, size = 110, normalized size = 1.

$$\frac{1}{f}\left(\frac{1}{a^3}\left(\frac{\left(\sin\left(fx+e\right)\right)^5 a^2}{5}-\frac{2\left(\sin\left(fx+e\right)\right)^3 a^2}{3}+\frac{\left(\sin\left(fx+e\right)\right)^3 ab}{3}+a^2\sin\left(fx+e\right)-ab\sin\left(fx+e\right)+b^2\sin\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(1/a^3*(1/5*sin(f*x+e)^5*a^2-2/3*sin(f*x+e)^3*a^2+1/3*sin(f*x+e)^3*a*b+
a^2*sin(f*x+e)-a*b*sin(f*x+e)+b^2*sin(f*x+e))-b^3/a^3/((a+b)*a)^(1/2)*arcta
nh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.573552, size = 689, normalized size = 6.38

$$\left[\frac{15\sqrt{a^2+ab}b^3\log\left(-\frac{a\cos\left(fx+e\right)^2+2\sqrt{a^2+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(3\left(a^4+a^3b\right)\cos\left(fx+e\right)^4+8a^4-2a^3b+5a^2b^2+15ab^3+\right.}{30\left(a^5+a^4b\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a^2 + a*b)*b^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*si
n(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(3*(a^4 + a^3*b)*cos(f*x
+ e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a^2*b^
```

```
2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f), 1/15*(15*sqrt(-a^2 - a*
b)*b^3*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (3*(a^4 + a^3*b)*cos
(f*x + e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.237, size = 184, normalized size = 1.7

$$\frac{\frac{15\,b^{3}\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^{2}-ab}}\right)}{\sqrt{-a^{2}-ab}a^{3}}+\frac{3\,a^{4}\sin\left(fx+e\right)^{5}-10\,a^{4}\sin\left(fx+e\right)^{3}+5\,a^{3}b\sin\left(fx+e\right)^{3}+15\,a^{4}\sin\left(fx+e\right)-15\,a^{3}b\sin\left(fx+e\right)+15\,a^{2}b^{2}\sin\left(fx+e\right)}{a^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*b^3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^3)
 + (3*a^4*sin(f*x + e)^5 - 10*a^4*sin(f*x + e)^3 + 5*a^3*b*sin(f*x + e)^3 +
 15*a^4*sin(f*x + e) - 15*a^3*b*sin(f*x + e) + 15*a^2*b^2*sin(f*x + e))/a^5
)/f
```

3.186 $\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}f\sqrt{a+b}}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] (a^2*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*f) -
((a - b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rubi [A] time = 0.0880261, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 390, 205}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}f\sqrt{a+b}}-\frac{(a-b)\tan(e+fx)}{b^2f}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*f) -
((a - b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a-b}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+b+bx^2)}\right)dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a-b)\tan(e+fx)}{b^2 f}+\frac{\tan^3(e+fx)}{3bf}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{b^2 f} \\
&= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,f}-\frac{(a-b)\tan(e+fx)}{b^2 f}+\frac{\tan^3(e+fx)}{3bf}
\end{aligned}
$$

Mathematica [C] time = 2.38925, size = 224, normalized size = 2.91

$$
\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sqrt{b(\sin(e)+i\cos(e))^4}\sec(e+fx)\left(\sec(e)\sin(fx)\left(-3a+b\sec^2(e+fx)\right.\right.\right.}{6b^2 f\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-3*a^2*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*
Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(Sec[e]*(-3*a + 2*b + b*Sec[e +
f*x]^2)*Sin[f*x] + b*Sec[e + f*x]*Tan[e])))/(6*b^2*Sqrt[a + b]*f*(a + b*Sec
[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.059, size = 79, normalized size = 1.

$$\frac{\left(\tan\left(fx+e\right)\right)^{3}}{3\,fb}-\frac{\tan\left(fx+e\right)a}{fb^{2}}+\frac{\tan\left(fx+e\right)}{fb}+\frac{a^{2}}{fb^{2}}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{\left(a+b\right)b}}\right)\frac{1}{\sqrt{\left(a+b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/3*tan(f*x+e)^3/b/f-1/f/b^2*tan(f*x+e)*a+tan(f*x+e)/b/f+1/f*a^2/b^2/((a+b)
*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.565428, size = 824, normalized size = 10.7

$$\left[-\frac{3\sqrt{-ab-b^{2}}a^{2}\cos\left(fx+e\right)^{3}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a+2\,b\right)\cos\left(fx+e\right)^{3}-b\cos\left(fx+e\right)\right)\sqrt{-ab-b^{2}}\sin\left(fx+e\right)\cdots}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}}\cdots\right.}{12\left(ab^{3}+b^{4}\right)f\cos\left(fx+e\right)^{3}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a*b - b^2)*a^2*cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*co
s(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)
^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)
^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b^2 + b^3 - (3*a^2*b + a*b^2 - 2*b
```

```
^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x + e)^3), -1/6*(3
*sqrt(a*b + b^2)*a^2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b +
b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(a*b^2 + b^3 - (3*a^2*b
 + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x +
e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.28178, size = 136, normalized size = 1.77

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)a^2}{\sqrt{ab+b^2}\,b^2}+\frac{b^2\tan\left(fx+e\right)^3-3\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{b^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*a^2/(sqrt(a*b + b^2)*b^2) + (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x
+ e) + 3*b^2*tan(f*x + e))/b^3)/f
```

3.187 $\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=52

$$\frac{\tan(e+fx)}{bf}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}f\sqrt{a+b}}$$

```
[Out] -((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*f)) +
 Tan[e + f*x]/(b*f)
```

Rubi [A] time = 0.0676325, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 388, 205}

$$\frac{\tan(e+fx)}{bf}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*f)) +
 Tan[e + f*x]/(b*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\tan(e+fx)}{bf} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{bf} \\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}f} + \frac{\tan(e+fx)}{bf}
\end{aligned}$$

Mathematica [C] time = 0.804315, size = 192, normalized size = 3.69

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sec(e)\sin(fx)\sqrt{b(\sin(e)+i\cos(e))^4}\sec(e+fx)+a(\cos(2e)-i\sin(2e)\right.}{2bf\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(a*ArcTan[(Sec[f*x]*(Cos[2*e
] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*
Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Sec[e
]*Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*Sin[f*x]))/(2*b*Sqrt[a + b]*f*
(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.059, size = 47, normalized size = 0.9

$$\frac{\tan\left(fx+e\right)}{fb} - \frac{a}{fb}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] tan(f*x+e)/b/f-1/f*a/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.542119, size = 678, normalized size = 13.04

$$\left[-\frac{\sqrt{-ab-b^2}a\cos\left(fx+e\right)\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+2\,b\right)\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)\right)\sqrt{-ab-b^2}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)}{4\left(ab^2+b^3\right)f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b - b^2)*a*cos(f*x + e)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*
cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b + b^2)*sin(f*x + e))/((a*b^2 + b^3)*f*c
os(f*x + e)), 1/2*(sqrt(a*b + b^2)*a*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*(a*b + b^
2)*sin(f*x + e))/((a*b^2 + b^3)*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.18399, size = 93, normalized size = 1.79

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a}{\sqrt{ab+b^2}\,b}-\frac{\tan(fx+e)}{b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*a/(sqrt(a*b + b^2)*b) - tan(f*x + e)/b)/f
```

3.188 $\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}f\sqrt{a+b}}$$

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Rubi [A] time = 0.0565247, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4146, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f}$$

Mathematica [A] time = 0.0903977, size = 36, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

Maple [A] time = 0.054, size = 28, normalized size = 0.8

$$\frac{1}{f}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.547137, size = 498, normalized size = 13.83

$$\left[-\frac{\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a+2\,b\right)\cos\left(fx+e\right)^3-b\cos\left(fx+e\right)\right)\sqrt{-ab-b^2}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)}{4\left(ab+b^2\right)f}, -\frac{\arctan\left(\frac{\ldots}{2\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sq
rt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2))/((a*b + b^2)*f), -1/2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/
(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))/(sqrt(a*b + b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.27193, size = 68, normalized size = 1.89

$$\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2
)))/(sqrt(a*b + b^2)*f)
```

3.189 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rubi [A] time = 0.0420105, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4127, 3181, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 4127

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b\sec^2(e+fx)}\,dx &= \frac{x}{a}-\frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}\\
&= \frac{x}{a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx,x,\cot(e+fx)\right)}{af}\\
&= \frac{x}{a}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}
\end{aligned}
$$

Mathematica [C] time = 0.318623, size = 182, normalized size = 4.04

$$
\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(fx\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}+b(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\,\mathrm{se}\ldots}{2\sqrt{a+\ldots}}\right.\right.}{2af\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.069, size = 48, normalized size = 1.1

$$
\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}-\frac{b}{fa}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/a*arctan(tan(f*x+e))-1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.554304, size = 544, normalized size = 12.09

$$\left[\frac{4\,fx + \sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)}{4\,af},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\sec^2\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.34351, size = 92, normalized size = 2.04

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

3.190 $\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=75

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]
)/(a^2*Sqrt[a + b]*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rubi [A] time = 0.103376, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 414, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2f\sqrt{a+b}}+\frac{x(a-2b)}{2a^2}+\frac{\sin(e+fx)\cos(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a - 2*b)*x)/(2*a^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]
)/(a^2*Sqrt[a + b]*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
```

```
&&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\cos(e+fx)\sin(e+fx)}{2af} - \frac{\text{Subst}\left(\int \frac{-a+b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af} \\
&= \frac{\cos(e+fx)\sin(e+fx)}{2af} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{2a^2f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\right.}{a^2} \\
&= \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,f} + \frac{\cos(e+fx)\sin(e+fx)}{2af}
\end{aligned}$$

Mathematica [A] time = 0.252891, size = 67, normalized size = 0.89

$$\frac{\frac{4b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 2(a-2b)(e+fx) + a\sin(2(e+fx))}{4a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (2*(a - 2*b)*(e + f*x) + (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b]])/Sqrt[a + b] + a*Sin[2*(e + f*x)])/(4*a^2*f)
```

Maple [A] time = 0.101, size = 92, normalized size = 1.2

$$\frac{\tan\left(fx+e\right)}{2fa\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{2fa}-\frac{\arctan\left(\tan\left(fx+e\right)\right)b}{fa^{2}}+\frac{b^{2}}{fa^{2}}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2/f/a*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2/f/a*arctan(tan(f*x+e))-1/f/a^2*arct
an(tan(f*x+e))*b+1/f*b^2/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.566991, size = 663, normalized size = 8.84

$$\left[\frac{2\,(a-2\,b)fx+2\,a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}-4\left(\left(a^{2}+3\,ab+2\,b^{2}\right)\cos\left(f\right.\right.}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}}\right.}{4\,a^{2}f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a - 2*b)*f*x + 2*a*cos(f*x + e)*sin(f*x + e) + b*sqrt(-b/(a + b))*
log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqr
t(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^
2 + b^2)))/(a^2*f), 1/2*((a - 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e) - b*sq
rt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*
cos(f*x + e)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.35891, size = 134, normalized size = 1.79

$$\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b^2}{\sqrt{ab+b^2}a^2}+\frac{\left(fx+e\right)(a-2b)}{a^2}+\frac{\tan\left(fx+e\right)}{\left(\tan\left(fx+e\right)^2+1\right)a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*b^2/(sqrt(a*b + b^2)*a^2) + (f*x + e)*(a - 2*b)/a^2 + tan(f*x +
e)/((tan(f*x + e)^2 + 1)*a))/f
```

3.191 $\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f\sqrt{a+b}}+\frac{x\left(3a^2-4ab+8b^2\right)}{8a^3}+\frac{(3a-4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b]])/(a^3*Sqrt[a + b]*f) + ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x
])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rubi [A] time = 0.156664, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3f\sqrt{a+b}}+\frac{x\left(3a^2-4ab+8b^2\right)}{8a^3}+\frac{(3a-4b)\sin(e+fx)\cos(e+fx)}{8a^2f}+\frac{\sin(e+fx)\cos^3(e+fx)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*x)/(8*a^3) - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b]])/(a^3*Sqrt[a + b]*f) + ((3*a - 4*b)*Cos[e + f*x]*Sin[e + f*x
])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{\text{Subst}\left(\int \frac{-3a+b-3bx^2}{(1+x^2)^2(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} + \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^2+(3a-4b)bx^2}{(1+x^2)(a+b+bx^2)}\right.}{8a^2f}$$

$$= \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan\right.}{a^3f}$$

$$= \frac{(3a^2-4ab+8b^2)x}{8a^3} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}f} + \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e}{}$$

Mathematica [A] time = 0.463789, size = 95, normalized size = 0.81

$$\frac{4\left(3a^2-4ab+8b^2\right)(e+fx)+a^2\sin(4(e+fx)) - \frac{32b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 8a(a-b)\sin(2(e+fx))}{32a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (4*(3*a^2 - 4*a*b + 8*b^2)*(e + f*x) - (32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sin[2*(e + f*x)] + a^2*Sin[4*
(e + f*x)])/(32*a^3*f)
```

Maple [A] time = 0.097, size = 194, normalized size = 1.7

$$\frac{3\left(\tan(fx+e)\right)^3}{8fa\left(\left(\tan(fx+e)\right)^2+1\right)^2} - \frac{\left(\tan(fx+e)\right)^3 b}{2fa^2\left(\left(\tan(fx+e)\right)^2+1\right)^2} - \frac{\tan(fx+e)\,b}{2fa^2\left(\left(\tan(fx+e)\right)^2+1\right)^2} + \frac{5\tan(fx+e)}{8fa\left(\left(\tan(fx+e)\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 3/8/f/a/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3-1/2/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*
x+e)^3*b-1/2/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)*b+5/8/f/a/(tan(f*x+e)^2+1)
^2*tan(f*x+e)+1/f/a^3*arctan(tan(f*x+e))*b^2+3/8/f/a*arctan(tan(f*x+e))-1/2
/f/a^2*arctan(tan(f*x+e))*b-1/f*b^3/a^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b
/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.59508, size = 811, normalized size = 6.93

$$\left[\frac{2\,b^2\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)+\left(\right.}{8\,a^3 f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b^2*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*
(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 -
(a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x
 + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (3*a^2 - 4*a*b + 8*b^2)*f*x + (2*a
^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/
8*(4*b^2*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(
a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (3*a^2 - 4*a*b + 8*b^2)*f*x + (2*a
^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2972, size = 201, normalized size = 1.72

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{\sqrt{ab+b^2}a^3}-\frac{(3a^2-4ab+8b^2)(fx+e)}{a^3}-\frac{3a\tan(fx+e)^3-4b\tan(fx+e)^3+5a\tan(fx+e)-4b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^3/(sqrt(a*b + b^2)*a^3) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a
^3 - (3*a*tan(f*x + e)^3 - 4*b*tan(f*x + e)^3 + 5*a*tan(f*x + e) - 4*b*tan(
f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

3.192 $\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=163

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f\sqrt{a+b}}+\frac{\left(5a^2-6ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16a^3f}+\frac{x\left(-6a^2b+5a^3+8ab^2-16b^3\right)}{16a^4}+\frac{(5a-6b)\sin($$

```
[Out] ((5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*x)/(16*a^4) + (b^(7/2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*Sqrt[a + b]*f) + ((5*a^2 - 6*a*b + 8*b^2
)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((5*a - 6*b)*Cos[e + f*x]^3*Sin[e
 + f*x])/(24*a^2*f) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f)
```

Rubi [A] time = 0.244825, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4f\sqrt{a+b}}+\frac{\left(5a^2-6ab+8b^2\right)\sin(e+fx)\cos(e+fx)}{16a^3f}+\frac{x\left(-6a^2b+5a^3+8ab^2-16b^3\right)}{16a^4}+\frac{(5a-6b)\sin($$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```



```
[Out] ((5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*x)/(16*a^4) + (b^(7/2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a^4*Sqrt[a + b]*f) + ((5*a^2 - 6*a*b + 8*b^2
)*Cos[e + f*x]*Sin[e + f*x])/(16*a^3*f) + ((5*a - 6*b)*Cos[e + f*x]^3*Sin[e
 + f*x])/(24*a^2*f) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af} - \frac{\text{Subst}\left(\int \frac{-5a+b-5bx^2}{(1+x^2)^3(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af} + \frac{\text{Subst}\left(\int \frac{3(5a^2-ab+2b^2)+3(5a-}{(1+x^2)^2(a+b+bx}\right.}{24a^}$$

$$= \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+}{}$$

$$= \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+}{}$$

$$= \frac{(5a^3-6a^2b+8ab^2-16b^3)\,x}{16a^4} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4\sqrt{a+b}f} + \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin}{16a^3f}$$

Mathematica [A] time = 0.936595, size = 133, normalized size = 0.82

$$\frac{12\left(-6a^2b+5a^3+8ab^2-16b^3\right)(e+fx)+3a\left(15a^2-16ab+16b^2\right)\sin(2(e+fx))+3a^2(3a-2b)\sin(4(e+fx))+a^3\sin}{192a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (12*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*(e + f*x) + (192*b^(7/2)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/Sqrt[a + b] + 3*a*(15*a^2 - 16*a*b + 16*
b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a - 2*b)*Sin[4*(e + f*x)] + a^3*Sin[6*(e +
 f*x)])/(192*a^4*f)
```

Maple [B] time = 0.098, size = 359, normalized size = 2.2

$$\frac{5\left(\tan\left(fx+e\right)\right)^{5}}{16fa\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{3}}-\frac{3\left(\tan\left(fx+e\right)\right)^{5}b}{8fa^{2}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{3}}+\frac{\left(\tan\left(fx+e\right)\right)^{5}b^{2}}{2fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{3}}+\frac{\left(\tan\left(fx+e\right)\right)^{3}b^{2}}{fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 5/16/f/a/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5-3/8/f/a^2/(tan(f*x+e)^2+1)^3*tan(f
*x+e)^5*b+1/2/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b^2+1/f/a^3/(tan(f*x+e)
^2+1)^3*tan(f*x+e)^3*b^2+5/6/f/a/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-1/f/a^2/(t
an(f*x+e)^2+1)^3*tan(f*x+e)^3*b-5/8/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b+1
/2/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b^2+11/16/f/a/(tan(f*x+e)^2+1)^3*tan
(f*x+e)-1/f/a^4*arctan(tan(f*x+e))*b^3+5/16/f/a*arctan(tan(f*x+e))-3/8/f/a^
2*arctan(tan(f*x+e))*b+1/2/f/a^3*arctan(tan(f*x+e))*b^2+1/f*b^4/a^4/((a+b)*
b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.611514, size = 996, normalized size = 6.11

$$\left[12\,b^{3}\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}-4\left(\left(a^{2}+3\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{3}-\left(ab+b^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^{2}}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}}\right)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*b^3*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 1
6*b^3)*f*x + (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3 + 3
*(5*a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), -1/48*(24
*b^3*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))) - 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)
*f*x - (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3 + 3*(5*a^
3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27734, size = 309, normalized size = 1.9

$$\frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{\sqrt{ab+b^2}\,a^4}+\frac{3\left(5a^3-6a^2b+8ab^2-16b^3\right)(fx+e)}{a^4}+\frac{15a^2\tan(fx+e)^5-18ab\tan(fx+e)^5+24b^2\tan(fx+e)^5+40a^2\tan(}{}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^4/(sqrt(a*b + b^2)*a^4) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*
b^3)*(f*x + e)/a^4 + (15*a^2*tan(f*x + e)^5 - 18*a*b*tan(f*x + e)^5 + 24*b^
2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 48*a*b*tan(f*x + e)^3 + 48*b^2*t
an(f*x + e)^3 + 33*a^2*tan(f*x + e) - 30*a*b*tan(f*x + e) + 24*b^2*tan(f*x
```

```
+ e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

3.193 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2f(a+b)^{3/2}}-\frac{a\sin(e+fx)}{2bf(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}(\sin(e+fx))}{b^2f}$$

```
[Out] ArcTanh[Sin[e + f*x]]/(b^2*f) - (Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*f) - (a*Sin[e + f*x])/(2*b*(a +
 b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.140251, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 414, 522, 206, 208}

$$-\frac{\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2f(a+b)^{3/2}}-\frac{a\sin(e+fx)}{2bf(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}(\sin(e+fx))}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[e + f*x]]/(b^2*f) - (Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]])/(2*b^2*(a + b)^(3/2)*f) - (a*Sin[e + f*x])/(2*b*(a +
 b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-2b-ax^2}{(1-x^2)\left(a+b-ax^2\right)}\,dx, x, \sin(e+fx)\right)}{2b(a+b)f}$$

$$= -\frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(e+fx)\right)}{b^2 f} - \frac{(a(2a+3b))\,\text{Su}}{}$$

$$= \frac{\tanh^{-1}(\sin(e+fx))}{b^2 f} - \frac{\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}f} - \frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+\right.}$$

Mathematica [C] time = 4.42183, size = 980, normalized size = 9.61

$$\frac{(\cos(2(e+fx))a+a+2b)\sec^3(e+fx)\left(-8b\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan(e+fx)a^{3/2}-2i(2a+3b)\tan^{-1}\left(\frac{2\sin(e)(\sin(}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*a*(2*a + 3*b)*ArcTan
[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*S
qrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e
] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e
 + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*S
in[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f
*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - a*(2*a + 3*b)*(a + 2*b + a*Cos[2*(
e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*
e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*S
in[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]
*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + a*(2*a + 3*b)*(a + 2*b + a*Cos[2*(e + f
*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] +
 (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f
*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec
[e + f*x]*(Cos[e] - I*Sin[e]) - 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b + a*Cos[2*
(e + f*x)])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sec[e + f*x]*Sqrt[(Cos
[e] - I*Sin[e])^2] + 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b + a*Cos[2*(e + f*x)])
*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sec[e + f*x]*Sqrt[(Cos[e] - I*Sin
[e])^2] + 2*a*(2*a + 3*b)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]
*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*
x])]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]*(I*Cos[e] + Sin[e]) - 8*a^
(3/2)*b*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[e + f*x]))/(32*Sqrt[a]*
b^2*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^2*Sqrt[(Cos[e] - I*Sin[e])^2])
```

Maple [A] time = 0.087, size = 151, normalized size = 1.5

$$\frac{\ln\left(\sin\left(fx+e\right)+1\right)}{2\,fb^2}+\frac{\sin\left(fx+e\right)a}{2\,fb\left(a+b\right)\left(-a-b+a\left(\sin\left(fx+e\right)\right)^2\right)}-\frac{a^2}{fb^2\left(a+b\right)}\mathrm{Artanh}\left(\sin\left(fx+e\right)a\frac{1}{\sqrt{\left(a+b\right)a}}\right)\frac{1}{\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f/b^2*ln(sin(f*x+e)+1)+1/2/f*a/b/(a+b)*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)
-1/f*a^2/b^2/(a+b)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-3/
2/f*a/b/(a+b)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))-1/2/f/b
^2*ln(sin(f*x+e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.707565, size = 950, normalized size = 9.31

$$\left[-\frac{2\,ab\sin\left(fx+e\right)-\left(\left(2\,a^2+3\,ab\right)\cos\left(fx+e\right)^2+2\,ab+3\,b^2\right)\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)-2\left(\left(a^2\right.\right.}{4\left(\left(a^2b^2+ab^3\right)f\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b + 3*b^2
)*sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*sin(f*
x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - 2*((a^2 + a*b)*cos(f*x + e)^2 +
 a*b + b^2)*log(sin(f*x + e) + 1) + 2*((a^2 + a*b)*cos(f*x + e)^2 + a*b + b
^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 +
b^4)*f), -1/2*(a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b +
 3*b^2)*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e)) - ((a^2 + a*
b)*cos(f*x + e)^2 + a*b + b^2)*log(sin(f*x + e) + 1) + ((a^2 + a*b)*cos(f*x
```

```
+ e)^2 + a*b + b^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x +
e)^2 + (a*b^3 + b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.27204, size = 177, normalized size = 1.74

$$\frac{\frac{\left(2\,a^2+3\,ab\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(ab^2+b^3\right)\sqrt{-a^2-ab}}+\frac{a\sin\left(fx+e\right)}{\left(a\sin\left(fx+e\right)^2-a-b\right)\left(ab+b^2\right)}+\frac{\log\left(\sin\left(fx+e\right)+1\right)}{b^2}-\frac{\log\left(-\sin\left(fx+e\right)+1\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 + 3*a*b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a*b^2 + b^3)
*sqrt(-a^2 - a*b)) + a*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a*b + b^2)
) + log(sin(f*x + e) + 1)/b^2 - log(-sin(f*x + e) + 1)/b^2)/f
```

3.194 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{\sin(e+fx)}{2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}f(a+b)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*f) + S
in[e + f*x]/(2*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.0703302, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 199, 208}

$$\frac{\sin(e+fx)}{2f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)*f) + S
in[e + f*x]/(2*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2(a+b)f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}f} + \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [A] time = 0.304015, size = 88, normalized size = 1.19

$$\frac{\sqrt{a}\sqrt{a+b}\sin(e+fx) + \left(-a\sin^2(e+fx)+a+b\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}f(a+b)^{3/2}(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Sqrt[a]*Sqrt[a + b]*Sin[e + f*x] + ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]]*(a + b - a*Sin[e + f*x]^2))/(Sqrt[a]*(a + b)^(3/2)*f*(a + 2*b + a*Cos[
2*(e + f*x)]))
```

Maple [A] time = 0.068, size = 68, normalized size = 0.9

$$\frac{1}{f}\left(-\frac{\sin\left(fx+e\right)}{(2a+2b)\left(-a-b+a\left(\sin\left(fx+e\right)\right)^2\right)} + \frac{1}{2a+2b}\text{Artanh}\left(\sin\left(fx+e\right)a\frac{1}{\sqrt{(a+b)a}}\right)\frac{1}{\sqrt{(a+b)a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-1/2*sin(f*x+e)/(a+b)/(-a-b+a*sin(f*x+e)^2)+1/2/(a+b)/((a+b)*a)^(1/2)*
arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.546575, size = 601, normalized size = 8.12

$$\left[\frac{\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{a^2+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(a^2+ab\right)\sin\left(fx+e\right)}{4\left(\left(a^4+2a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^3b+2a^2b^2+ab^3\right)f\right)}, -\frac{\left(a\cos\left(fx+e\right)^2+b\right)}{2\left(\left(a^4+2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((a*cos(f*x + e)^2 + b)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqr
t(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b
)*sin(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^
2*b^2 + a*b^3)*f), -1/2*((a*cos(f*x + e)^2 + b)*sqrt(-a^2 - a*b)*arctan(sqr
t(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (a^2 + a*b)*sin(f*x + e))/((a^4 + 2*a
^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.33044, size = 107, normalized size = 1.45

$$-\frac{\frac{\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}(a+b)}+\frac{\sin\left(fx+e\right)}{\left(a\sin\left(fx+e\right)^2-a-b\right)(a+b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*(a + b)) +
sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a + b)))/f
```

3.195 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}f(a+b)^{3/2}}-\frac{b\sin(e+fx)}{2af(a+b)\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[Out] ((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*f) - (b*Sin[e + f*x])/(2*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.0683364, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4147, 385, 208}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}f(a+b)^{3/2}}-\frac{b\sin(e+fx)}{2af(a+b)\left(-a\sin^2(e+fx)+a+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b)^
(3/2)*f) - (b*Sin[e + f*x])/(2*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x, \sin(e+fx)\right)}{2a(a+b)f}$$

$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}f} - \frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

Mathematica [A] time = 0.413117, size = 82, normalized size = 0.99

$$\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{2\sqrt{a}b\sin(e+fx)}{(a+b)(a\cos(2(e+fx))+a+2b)}}{2a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (2
*Sqrt[a]*b*Sin[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*a^(3/
2)*f)
```

Maple [A] time = 0.083, size = 80, normalized size = 1.

$$\frac{1}{f}\left(\frac{\sin\left(fx+e\right)b}{(2a+2b)a\left(-a-b+a\left(\sin\left(fx+e\right)\right)^2\right)} + \frac{b+2a}{(2a+2b)a}\text{Artanh}\left(\sin\left(fx+e\right)a\frac{1}{\sqrt{(a+b)a}}\right)\frac{1}{\sqrt{(a+b)a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/2*b/(a+b)/a*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)+1/2*(b+2*a)/(a+b)/a/((a
+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.558604, size = 676, normalized size = 8.14

$$\left[\frac{\left(\left(2a^2+ab\right)\cos\left(fx+e\right)^2+2ab+b^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos\left(fx+e\right)^2-2\sqrt{a^2+ab}\sin\left(fx+e\right)-2a-b}{a\cos\left(fx+e\right)^2+b}\right)-2\left(a^2b+ab^2\right)\sin\left(fx+e\right)}{4\left(\left(a^5+2a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^4b+2a^3b^2+a^2b^3\right)f\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a^2 + a*b)*cos(f*x + e)^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)*log(-(a*
cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^
2 + b)) - 2*(a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(
f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/2*(((2*a^2 + a*b)*cos(f*x
 + e)^2 + 2*a*b + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e
)/(a + b)) + (a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos
(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.32121, size = 127, normalized size = 1.53

$$-\frac{\frac{(2\,a+b)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^{2}-ab}}\right)}{\left(a^{2}+ab\right)\sqrt{-a^{2}-ab}}-\frac{b\sin\left(fx+e\right)}{\left(a\sin\left(fx+e\right)^{2}-a-b\right)\left(a^{2}+ab\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + a*b)*sqrt(-
a^2 - a*b)) - b*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a^2 + a*b)))/f
```

3.196 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=101

$$\frac{b^2 \sin(e+fx)}{2a^2 f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}f(a+b)^{3/2}}+\frac{\sin(e+fx)}{a^2 f}$$

```
[Out] -(b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(5/2)*(a
+ b)^(3/2)*f) + Sin[e + f*x]/(a^2*f) + (b^2*Sin[e + f*x])/(2*a^2*(a + b)*f*
(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.132112, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4147, 390, 385, 208}

$$\frac{b^2 \sin(e+fx)}{2a^2 f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}f(a+b)^{3/2}}+\frac{\sin(e+fx)}{a^2 f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(5/2)*(a
+ b)^(3/2)*f) + Sin[e + f*x]/(a^2*f) + (b^2*Sin[e + f*x])/(2*a^2*(a + b)*f*
(a + b - a*Sin[e + f*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2}-\frac{b(2a+b)-2abx^2}{a^2\left(a+b-ax^2\right)^2}\right)dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\sin(e+fx)}{a^2 f}-\frac{\text{Subst}\left(\int \frac{b(2a+b)-2abx^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{a^2 f}\\
&= \frac{\sin(e+fx)}{a^2 f}+\frac{b^2\sin(e+fx)}{2a^2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}-\frac{(b(4a+3b))\,\text{Subst}\left(\int \frac{1}{a+b-ax^2}\,dx, x,\right.}{2a^2(a+b)f}\\
&= -\frac{b(4a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}f}+\frac{\sin(e+fx)}{a^2 f}+\frac{b^2\sin(e+fx)}{2a^2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}
\end{aligned}
$$

Mathematica [C] time = 3.47386, size = 945, normalized size = 9.36

$$
\frac{(\cos(2(e+fx))a+a+2b)\sec^3(e+fx)\left(8\sqrt{a}\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan(e+fx)b^2-2i(4a+3b)\tan^{-1}\left(\frac{2\sin(e)\left(\sin(2\right.}{}\right.\right.}{}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*b*(4*a + 3*b)*ArcTan
[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*S
qrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e
] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e
 + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*S
in[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f
*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - b*(4*a + 3*b)*(a + 2*b + a*Cos[2*(
e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*
e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*S
in[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]
*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + b*(4*a + 3*b)*(a + 2*b + a*Cos[2*(e + f
*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] +
 (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f
*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec
[e + f*x]*(Cos[e] - I*Sin[e]) + 8*Sqrt[a]*(a + b)^(3/2)*Cos[f*x]*(a + 2*b +
 a*Cos[2*(e + f*x)])*Sec[e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[e] + 2*b*
(4*a + 3*b)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(a + 2*b
+ a*Cos[2*(e + f*x)])*Sec[e + f*x]*(I*Cos[e] + Sin[e]) + 8*Sqrt[a]*(a + b)^
(3/2)*Cos[e]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]*Sqrt[(Cos[e] - I*S
in[e])^2]*Sin[f*x] + 8*Sqrt[a]*b^2*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*
Tan[e + f*x]))/(32*a^(5/2)*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^2*Sqrt[(C
os[e] - I*Sin[e])^2])
```

Maple [A] time = 0.095, size = 92, normalized size = 0.9

$$\frac{1}{f}\left(\frac{\sin\left(fx+e\right)}{a^2}+\frac{b}{a^2}\left(-\frac{\sin\left(fx+e\right)b}{\left(2a+2b\right)\left(-a-b+a\left(\sin\left(fx+e\right)\right)^2\right)}-\frac{4a+3b}{2a+2b}\mathrm{Artanh}\left(\sin\left(fx+e\right)a\frac{1}{\sqrt{(a+b)a}}\right)\frac{1}{\sqrt{(a+b)a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/a^2*sin(f*x+e)+b/a^2*(-1/2*b/(a+b)*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-
```

```
1/2*(4*a+3*b)/(a+b)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.589017, size = 867, normalized size = 8.58

$$\left[\frac{\left(4\,ab^2 + 3\,b^3 + \left(4\,a^2b + 3\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos\left(fx+e\right)^2 + 2\,\sqrt{a^2+ab}\sin\left(fx+e\right) - 2\,a - b}{a\cos\left(fx+e\right)^2 + b}\right) + 2\left(2\,a^3b + 5\,a^2b^2 + 3\right.}{4\left(\left(a^6 + 2\,a^5b + a^4b^2\right)f\cos\left(fx+e\right)^2 + \left(a^5b + 2\,a^4b^2 + a^3b^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((4*a*b^2 + 3*b^3 + (4*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt(a^2 + a*b
)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos
(f*x + e)^2 + b)) + 2*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b + a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(f*x
+ e)^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f), 1/2*((4*a*b^2 + 3*b^3 + (4*a^2*b
 + 3*a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*
x + e)/(a + b)) + (2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b + a^2*b
^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(f*x + e)
^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19373, size = 158, normalized size = 1.56

$$-\frac{\frac{b^2\sin\left(fx+e\right)}{\left(a^3+a^2b\right)\left(a\sin\left(fx+e\right)^2-a-b\right)}-\frac{\left(4\,ab+3\,b^2\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^3+a^2b\right)\sqrt{-a^2-ab}}-\frac{2\,\sin\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(f*x + e)/((a^3 + a^2*b)*(a*sin(f*x + e)^2 - a - b)) - (4*a*b
+ 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + a^2*b)*sqrt(-a^2 -
 a*b)) - 2*sin(f*x + e)/a^2)/f
```

3.197 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{b^3\sin(e+fx)}{2a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}f(a+b)^{3/2}}+\frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}$$

```
[Out] (b^2*(6*a + 5*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*f) + ((a - 2*b)*Sin[e + f*x])/(a^3*f) - Sin[e + f*x]^3/(3*a^2*f
) - (b^3*Sin[e + f*x])/(2*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.155194, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 390, 385, 208}

$$-\frac{b^3\sin(e+fx)}{2a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}f(a+b)^{3/2}}+\frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] (b^2*(6*a + 5*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(7/2)*(a
 + b)^(3/2)*f) + ((a - 2*b)*Sin[e + f*x])/(a^3*f) - Sin[e + f*x]^3/(3*a^2*f
) - (b^3*Sin[e + f*x])/(2*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{a^3}-\frac{x^2}{a^2}+\frac{b^2(3a+2b)-3ab^2x^2}{a^3\left(a+b-ax^2\right)^2}\right)dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}+\frac{\text{Subst}\left(\int \frac{b^2(3a+2b)-3ab^2x^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{a^3f}$$

$$= \frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}-\frac{b^3\sin(e+fx)}{2a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)}+\frac{\left(b^2(6a+5b)\right)}{}$$

$$= \frac{b^2(6a+5b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}f}+\frac{(a-2b)\sin(e+fx)}{a^3f}-\frac{\sin^3(e+fx)}{3a^2f}-\frac{b^3\sin}{2a^3(a+b)f\,(a+}$$

Mathematica [A] time = 1.15593, size = 139, normalized size = 1.1

$$\frac{a^{3/2}\sin(3(e+fx))-\frac{3b^2(6a+5b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{3/2}}+3\sqrt{a}\sin(e+fx)\left(-\frac{4b^3}{(a+b)(a\cos(2(e+fx))+a+2b)}+3a\right.}{12a^{7/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((-3*b^2*(6*a + 5*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
+ b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*b^
3)/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))*Sin[e + f*x] + a^(3/2)*Sin[3*(
e + f*x)])/(12*a^(7/2)*f)
```

Maple [A] time = 0.101, size = 120, normalized size = 1.

$$\frac{1}{f}\left(-\frac{1}{a^3}\left(\frac{a\left(\sin\left(fx+e\right)\right)^3}{3}-\sin\left(fx+e\right)a+2\sin\left(fx+e\right)b\right)-\frac{b^2}{a^3}\left(-\frac{\sin\left(fx+e\right)b}{(2a+2b)\left(-a-b+a\left(\sin\left(fx+e\right)\right)^2\right)}-\frac{6a+5}{2a+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-1/a^3*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+2*sin(f*x+e)*b)-b^2/a^3*(-1/2*
b/(a+b)*sin(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(6*a+5*b)/(a+b)/((a+b)*a)^(1/2
)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.651429, size = 1077, normalized size = 8.55

$$\left[\frac{3\left(6\,ab^{3}+5\,b^{4}+\left(6\,a^{2}b^{2}+5\,ab^{3}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a^{2}+ab}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{a^{2}+ab}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^{2}+b}\right)+2\left(4\,a^{4}b-4\,a^{3}b^{2}-2\right.}{12\left(\left(a^{7}+2\,a^{6}b+a^{5}b^{2}\right)f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(6*a*b^3 + 5*b^4 + (6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a^2
+ a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(
a*cos(f*x + e)^2 + b)) + 2*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4 + 2
*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^2 -
5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f
*x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f), -1/6*(3*(6*a*b^3 + 5*b^4 + (6
*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b
)*sin(f*x + e)/(a + b)) - (4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4 + 2*
(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^2 - 5
*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15722, size = 205, normalized size = 1.63

$$\frac{\frac{3\,b^{3}\sin\left(fx+e\right)}{\left(a^{4}+a^{3}b\right)\left(a\sin\left(fx+e\right)^{2}-a-b\right)}-\frac{3\left(6\,ab^{2}+5\,b^{3}\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^{2}-ab}}\right)}{\left(a^{4}+a^{3}b\right)\sqrt{-a^{2}-ab}}-\frac{2\left(a^{4}\sin\left(fx+e\right)^{3}-3\,a^{4}\sin\left(fx+e\right)+6\,a^{3}b\sin\left(fx+e\right)\right)}{a^{6}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*sin(f*x + e)/((a^4 + a^3*b)*(a*sin(f*x + e)^2 - a - b)) - 3*(6*a
*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4 + a^3*b)*sqrt(-
a^2 - a*b)) - 2*(a^4*sin(f*x + e)^3 - 3*a^4*sin(f*x + e) + 6*a^3*b*sin(f*x
+ e))/a^6)/f
```

3.198 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=157

$$\frac{b^4\sin(e+fx)}{2a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f}-\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}f(a+b)^{3/2}}-\frac{2(a-b)\sin^3(}{3a^3f}$$

```
[Out] -(b^3*(8*a + 7*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(9/2)*(
a + b)^(3/2)*f) + ((a^2 - 2*a*b + 3*b^2)*Sin[e + f*x])/(a^4*f) - (2*(a - b)
*Sin[e + f*x]^3)/(3*a^3*f) + Sin[e + f*x]^5/(5*a^2*f) + (b^4*Sin[e + f*x])/
(2*a^4*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.168473, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 390, 385, 208}

$$\frac{b^4\sin(e+fx)}{2a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f}-\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}f(a+b)^{3/2}}-\frac{2(a-b)\sin^3(}{3a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(b^3*(8*a + 7*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(9/2)*(
a + b)^(3/2)*f) + ((a^2 - 2*a*b + 3*b^2)*Sin[e + f*x])/(a^4*f) - (2*(a - b)
*Sin[e + f*x]^3)/(3*a^3*f) + Sin[e + f*x]^5/(5*a^2*f) + (b^4*Sin[e + f*x])/
(2*a^4*(a + b)*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2-2ab+3b^2}{a^4} - \frac{2(a-b)x^2}{a^3} + \frac{x^4}{a^2} - \frac{b^3(4a+3b)-4ab^3x^2}{a^4\left(a+b-ax^2\right)^2}\right)dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} - \frac{\operatorname{Subst}\left(\int \frac{b^3(4a+3b)}{(a+b-}\right.}{}$$

$$= \frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f} + \frac{\sin^5(e+fx)}{5a^2f} + \frac{b^4\sin(}{2a^4(a+b)f\left(a+b\right.}$$

$$= -\frac{b^3(8a+7b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}f} + \frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3f}$$

Mathematica [A] time = 2.14099, size = 171, normalized size = 1.09

$$\frac{30\sqrt{a}\sin(e+fx)\left(5a^2+8b^2\left(\frac{b^2}{(a+b)(a\cos(2(e+fx))+a+2b)}+3\right)-12ab\right)+5a^{3/2}(5a-8b)\sin(3(e+fx))+3a^{5/2}\sin(5(e+fx))}{240a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((60*b^3*(8*a + 7*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
+ b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 30*Sqrt[a]*(5*a^2 - 12*a*b +
 8*b^2*(3 + b^2/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))))*Sin[e + f*x] + 5
*a^(3/2)*(5*a - 8*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)])/(240*a^
(9/2)*f)
```

Maple [A] time = 0.105, size = 158, normalized size = 1.

$$\frac{1}{f}\left(\frac{1}{a^4}\left(\frac{\left(\sin\left(fx+e\right)\right)^5 a^2}{5}-\frac{2\left(\sin\left(fx+e\right)\right)^3 a^2}{3}+\frac{2\left(\sin\left(fx+e\right)\right)^3 ab}{3}+a^2\sin\left(fx+e\right)-2ab\sin\left(fx+e\right)+3b^2\sin\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(1/a^4*(1/5*sin(f*x+e)^5*a^2-2/3*sin(f*x+e)^3*a^2+2/3*sin(f*x+e)^3*a*b+
a^2*sin(f*x+e)-2*a*b*sin(f*x+e)+3*b^2*sin(f*x+e))+1/a^4*b^3*(-1/2*b/(a+b)*s
in(f*x+e)/(-a-b+a*sin(f*x+e)^2)-1/2*(8*a+7*b)/(a+b)/((a+b)*a)^(1/2)*arctanh
(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.689115, size = 1304, normalized size = 8.31

$$\left[\frac{15\left(8\,ab^4+7\,b^5+\left(8\,a^2b^3+7\,ab^4\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos\left(fx+e\right)^2+2\,\sqrt{a^2+ab}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^2+b}\right)+2\left(6\left(a^6+2\,a^5b+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(8*a*b^4 + 7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(a^2
 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/
(a*cos(f*x + e)^2 + b)) + 2*(6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^6 + 1
6*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a^6 + a^5
*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2 + 54*a^
3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2
)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f), 1/30*(15*(8*a*b^4 +
7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(
-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x +
e)^6 + 16*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a
^6 + a^5*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2
 + 54*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b +
 a^6*b^2)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18588, size = 266, normalized size = 1.69

$$-\frac{\frac{15\,b^4\sin\left(fx+e\right)}{\left(a^5+a^4b\right)\left(a\sin\left(fx+e\right)^2-a-b\right)}-\frac{15\left(8\,ab^3+7\,b^4\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^5+a^4b\right)\sqrt{-a^2-ab}}-\frac{2\left(3\,a^8\sin\left(fx+e\right)^5-10\,a^8\sin\left(fx+e\right)^3+10\,a^7b\sin\left(fx+e\right)^3+15\,a^8\sin\left(fx+e\right)-30\,a^7b\right.}{a^{10}}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*sin(f*x + e)/((a^5 + a^4*b)*(a*sin(f*x + e)^2 - a - b)) - 15*
(8*a*b^3 + 7*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^5 + a^4*b)*sq
rt(-a^2 - a*b)) - 2*(3*a^8*sin(f*x + e)^5 - 10*a^8*sin(f*x + e)^3 + 10*a^7*
b*sin(f*x + e)^3 + 15*a^8*sin(f*x + e) - 30*a^7*b*sin(f*x + e) + 45*a^6*b^2
*sin(f*x + e))/a^10)/f
```

3.199 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=100

$$\frac{a^2\tan(e+fx)}{2b^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{b^2f}$$

```
[Out] -(a*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(5/2)*(a +
 b)^(3/2)*f) + Tan[e + f*x]/(b^2*f) + (a^2*Tan[e + f*x])/(2*b^2*(a + b)*f*(
a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.135116, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 390, 385, 205}

$$\frac{a^2\tan(e+fx)}{2b^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(5/2)*(a +
 b)^(3/2)*f) + Tan[e + f*x]/(b^2*f) + (a^2*Tan[e + f*x])/(2*b^2*(a + b)*f*(
a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a(a+2b)+2abx^2}{b^2\left(a+b+bx^2\right)^2}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{b^2f} - \frac{\text{Subst}\left(\int \frac{a(a+2b)+2abx^2}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{b^2f}$$

$$= \frac{\tan(e+fx)}{b^2f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{(a(3a+4b))\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x,\right.}{2b^2(a+b)f}$$

$$= -\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{b^2f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 2.58448, size = 248, normalized size = 2.48

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{a(a\sin(2fx)-(a+2b)\sin(2e))}{(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))}+2\sec(e)\sin(fx)\sec(e+fx)(a\cos(2(e+fx))+a\right.}{8b^2f\left(a+b\sec^2(e+fx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((a*(3*a + 4*b)*ArcTan[(Sec[
f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2
*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])
*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2
*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x] + (a*(-((a + 2
*b)*Sin[2*e]) + a*Sin[2*f*x]))/((a + b)*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(8*b^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.069, size = 128, normalized size = 1.3

$$\frac{\tan\left(fx+e\right)}{b^2f}+\frac{a^2\tan\left(fx+e\right)}{2\,b^2\,(a+b)\,f\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)}-\frac{3\,a^2}{2\,b^2\,(a+b)\,f}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] tan(f*x+e)/b^2/f+1/2*a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)-3/2/f*
a^2/b^2/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-2/f*a/b/
(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.600593, size = 1172, normalized size = 11.72

$$\left[-\frac{\left(\left(3\,a^{3}+4\,a^{2}b\right)\cos\left(fx+e\right)^{3}+\left(3\,a^{2}b+4\,ab^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-ab-b^{2}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}-4}{a^{2}\cos\left(fx+e\right)^{4}+}\right.}{8\left(\left(a^{3}b^{3}+2\,a^{2}b^{4}+ab^{5}\right)f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e)
)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4
*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-
a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) - 4*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3)*c
os(f*x + e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x + e)^
3 + (a^2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e)), 1/4*(((3*a^3 + 4*a^2*b)*cos(
f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e))*sqrt(a*b + b^2)*arctan(1/2*(
(a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))
+ 2*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3)*cos(f*x
+ e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x + e)^3 + (a^
2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19698, size = 169, normalized size = 1.69

$$\frac{\frac{a^2\tan\left(fx+e\right)}{\left(ab^2+b^3\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3\,a^2+4\,ab\right)}{\left(ab^2+b^3\right)\sqrt{ab+b^2}}+\frac{2\,\tan\left(fx+e\right)}{b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*tan(f*x + e)/((a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)) - (pi*floo
r((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a
^2 + 4*a*b)/((a*b^2 + b^3)*sqrt(a*b + b^2)) + 2*tan(f*x + e)/b^2)/f
```

3.200 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=82

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}f(a+b)^{3/2}}-\frac{a\tan(e+fx)}{2bf(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)^(
3/2)*f) - (a*Tan[e + f*x])/(2*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.0823868, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 385, 205}

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}f(a+b)^{3/2}}-\frac{a\tan(e+fx)}{2bf(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)^(
3/2)*f) - (a*Tan[e + f*x])/(2*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2b(a+b)f}$$

$$= \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}f} - \frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 0.310233, size = 84, normalized size = 1.02

$$\frac{\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a\cos(2(e+fx))+a+2b)}}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a*
Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*b^(3
/2)*f)
```

Maple [A] time = 0.071, size = 106, normalized size = 1.3

$$-\frac{a\tan\left(fx+e\right)}{2\,(a+b)\,bf\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)} + \frac{a}{2\,(a+b)\,bf}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}} + \frac{1}{f\,(a+b)}\,\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)+1/2/f*a/b/(a+b)/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/(a+b)/((a+b)*b)^(1/2)*arctan
(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.572113, size = 936, normalized size = 11.41

$$\left[-\frac{4\left(a^{2}b+ab^{2}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+2\,ab\right)\cos\left(fx+e\right)^{2}+ab+2\,b^{2}\right)\sqrt{-ab-b^{2}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\cdots}{\cdots}\right.}{8\left(\left(a^{3}b^{2}+2\,a^{2}b^{3}+ab^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b^{3}+2\,ab^{4}+\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x
 + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b
*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^
2 + (a^2*b^3 + 2*a*b^4 + b^5)*f), -1/4*(2*(a^2*b + a*b^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arc
tan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*
x + e))))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^2*b^3 + 2*a*
```

```
b^4 + b^5)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.389, size = 126, normalized size = 1.54

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2\,b)}{(ab+b^2)^{\frac{3}{2}}}-\frac{a\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(ab+b^2)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 2*b)/(a*b + b^2)^(3/2) - a*tan(f*x + e)/((b*tan(f*x + e)^2 +
a + b)*(a*b + b^2)))/f
```

3.201 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=73

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*f) + Ta
n[e + f*x]/(2*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.0735063, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}f(a+b)^{3/2}}+\frac{\tan(e+fx)}{2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*f) + Ta
n[e + f*x]/(2*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{2(a+b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 0.964962, size = 211, normalized size = 2.89

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{a\sin(2fx)-(a+2b)\sin(2e)}{a(\cos(e)-\sin(e))(\sin(e)+\cos(e))}-\frac{(\cos(2e)-i\sin(2e))(a\cos(2(e+fx))+a+2b)\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec\cdots}{2\sqrt{a+b}\cdots}\right)}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)}{8f(a+b)\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-((ArcTan[(Sec[f*x]*(Cos[2*
e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]
*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] -
 I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) + (-((a + 2*b)*S
in[2*e]) + a*Sin[2*f*x])/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a +
b)*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.088, size = 66, normalized size = 0.9

$$\frac{\tan\left(fx+e\right)}{\left(2a+2b\right)f\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)}+\frac{1}{2f\left(a+b\right)}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{\left(a+b\right)b}}\right)\frac{1}{\sqrt{\left(a+b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2*tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)+1/2/f/(a+b)/((a+b)*b)^(1/2)*arc
tan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.565296, size = 857, normalized size = 11.74

$$\left[\frac{4\left(ab+b^{2}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(a\cos\left(fx+e\right)^{2}+b\right)\sqrt{-ab-b^{2}}\log\left(\frac{\left(a^{2}+8ab+8b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3ab+4b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\cdots\right.}{a^{2}\cos\left(fx+e\right)^{4}+2a\cdots}\right.}{8\left(\left(a^{3}b+2a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b^{2}+2ab^{3}+b^{4}\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*sqrt
(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b -
b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))
```

```
)/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^4)
*f), 1/4*(2*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*
sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*
cos(f*x + e)*sin(f*x + e))))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2
+ (a^2*b^2 + 2*a*b^3 + b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.17184, size = 117, normalized size = 1.6

$$\frac{\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}(a+b)}+\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)(a+b)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))/(sqrt(a*b + b^2)*(a + b)) + tan(f*x + e)/((b*tan(f*x + e)^2 + a +
b)*(a + b)))/f
```

3.202 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rubi [A] time = 0.0815188, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4128, 414, 522, 203, 205}

$$-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f} \\
&= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2 f} - \frac{(b(3a+2b))
\end{aligned}
$$

$$
= \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}
$$

Mathematica [C] time = 1.94418, size = 240, normalized size = 2.61

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(2x(a\cos(2(e+fx))+a+2b)+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{f(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))}+\frac{b(3a+2b)(\cos(2e)-i\sin}{}\right.}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

Maple [A] time = 0.078, size = 127, normalized size = 1.4

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^2}-\frac{b\tan\left(fx+e\right)}{2\,(a+b)\,af\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)}-\frac{3\,b}{2\,(a+b)\,af}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{}{\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f/a^2*arctan(tan(f*x+e))-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)-
3/2/f*b/a/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^
2/a^2/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.604585, size = 1027, normalized size = 11.16

$$\left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2-4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+8\left(ab+b^2\right)fx+\left(\left(3\,a^2+2\,ab\right)\cos\left(fx+e\right)^2+3\,ab+2\,b\right.}{8\left(\left(a^4+a^3b\right)f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Giac [A] time = 1.22245, size = 161, normalized size = 1.75

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

3.203 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f(a+b)^{3/2}}+\frac{b(a+2b)\tan(e+fx)}{2a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{x(a-4b)}{2a^3}+\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*
(a + b + b*Tan[e + f*x]^2)) + (b*(a + 2*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*(
a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.193785, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3f(a+b)^{3/2}}+\frac{b(a+2b)\tan(e+fx)}{2a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{x(a-4b)}{2a^3}+\frac{\sin(e+fx)\cos(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a - 4*b)*x)/(2*a^3) + (b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/
Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*
(a + b + b*Tan[e + f*x]^2)) + (b*(a + 2*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*(
a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-2(a^2-2a}{(1+}\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-4b)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b}{2a^2(a+}$$

Mathematica [A] time = 1.25881, size = 103, normalized size = 0.73

$$\frac{\frac{2b^{3/2}(5a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \sin(2(e+fx))\left(\frac{2ab^2}{(a+b)(a\cos(2(e+fx))+a+2b)} + a\right) + 2(a-4b)(e+fx)}{4a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(a - 4*b)*(e + f*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])))*Sin[2*(e + f*x)])/(4*a^3*f)
```

Maple [A] time = 0.109, size = 174, normalized size = 1.2

$$\frac{\tan(fx+e)}{2fa^2\left((\tan(fx+e))^2+1\right)} + \frac{\arctan(\tan(fx+e))}{2fa^2} - 2\frac{\arctan(\tan(fx+e))\,b}{fa^3} + \frac{b^2\tan(fx+e)}{2fa^2(a+b)\left(a+b+b(\tan(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f/a^2*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2/f/a^2*arctan(tan(f*x+e))-2/f/a^3*
arctan(tan(f*x+e))*b+1/2/f*b^2/a^2/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+5/
2/f*b^2/a^2/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+2/f*
b^3/a^3/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.660316, size = 1246, normalized size = 8.77

$$\left[\frac{4\left(a^3-3a^2b-4ab^2\right)fx\cos\left(fx+e\right)^2+4\left(a^2b-3ab^2-4b^3\right)fx+\left(5ab^2+4b^3+\left(5a^2b+4ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-\frac{b}{a+b}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 4*(a^2*b - 3*a*b^2 -
 4*b^3)*f*x + (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)) + 4*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2)*
cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b + a^3*
b^2)*f), 1/4*(2*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 2*(a^2*b - 3
*a*b^2 - 4*b^3)*f*x - (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2
)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))
```

```
/(b*cos(f*x + e)*sin(f*x + e))) + 2*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b
+ 2*a*b^2)*cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a
^4*b + a^3*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3207, size = 274, normalized size = 1.93

$$\frac{\frac{(5\,ab^2+4\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+a^3b)\sqrt{ab+b^2}}+\frac{ab\tan(fx+e)^3+2\,b^2\tan(fx+e)^3+a^2\tan(fx+e)+2\,ab\tan(fx+e)+2\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)(a^3+a^2b)}+\frac{(fx+e)(a-4}{a^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 + 4*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (a*b*tan(f*x +
 e)^3 + 2*b^2*tan(f*x + e)^3 + a^2*tan(f*x + e) + 2*a*b*tan(f*x + e) + 2*b^
2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2
+ a + b)*(a^3 + a^2*b)) + (f*x + e)*(a - 4*b)/a^3)/f
```

3.204 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=203

$$-\frac{b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f(a+b)^{3/2}}+\frac{x\left(3a^2-8ab+24b^2\right)}{8a^4}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{3(a-2b)\sin(e+f}{8a^2f\left(a+b\tan^2(\right.}$$

```
[Out] ((3*a^2 - 8*a*b + 24*b^2)*x)/(8*a^4) - (b^(5/2)*(7*a + 6*b)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*(a + b)^(3/2)*f) + (3*(a - 2*b)*Cos[e +
 f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*
Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) + ((a - 3*b)*b*(3*a + 4*b)
*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.289948, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$-\frac{b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4f(a+b)^{3/2}}+\frac{x\left(3a^2-8ab+24b^2\right)}{8a^4}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{8a^3f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{3(a-2b)\sin(e+f}{8a^2f\left(a+b\tan^2(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((3*a^2 - 8*a*b + 24*b^2)*x)/(8*a^4) - (b^(5/2)*(7*a + 6*b)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(2*a^4*(a + b)^(3/2)*f) + (3*(a - 2*b)*Cos[e +
 f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)) + (Cos[e + f*x]^3*
Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)) + ((a - 3*b)*b*(3*a + 4*b)
*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-3a+b-5bx^2}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{3a^2+ab+6b^2+9(}{\left(1+x^2\right)\left(a+b+\right.}\right.}{8}$$

$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-3b)b(3a+4b)\,\text{t}}{8a^3(a+b)f\left(a+b+b\right.}$$

$$= \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a-3b)b(3a+4b)\,\text{t}}{8a^3(a+b)f\left(a+b+b\right.}$$

$$= \frac{\left(3a^2-8ab+24b^2\right)x}{8a^4} - \frac{b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4(a+b)^{3/2}f} + \frac{3(a-2b)\cos(e+fx)\sin(e+f}{8a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [A] time = 1.58873, size = 138, normalized size = 0.68

$$\frac{4\left(3a^2-8ab+24b^2\right)(e+fx)+a^2\sin(4(e+fx)) - \frac{16b^{5/2}(7a+6b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{16ab^3\sin(2(e+fx))}{(a+b)(a\cos(2(e+fx))+a+2b)} + 8a(a-2b)\sin(}{32a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(3*a^2 - 8*a*b + 24*b^2)*(e + f*x) - (16*b^(5/2)*(7*a + 6*b)*ArcTan[(Sqr
t[b]*Tan[e + f*x]])/Sqrt[a + b]])/(a + b)^(3/2) + 8*a*(a - 2*b)*Sin[2*(e + f
*x)] - (16*a*b^3*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))
 + a^2*Sin[4*(e + f*x)])/(32*a^4*f)
```

Maple [A] time = 0.106, size = 276, normalized size = 1.4

$$\frac{3\left(\tan\left(fx+e\right)\right)^{3}}{8fa^{2}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{\left(\tan\left(fx+e\right)\right)^{3}b}{fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}-\frac{\tan\left(fx+e\right)b}{fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}+\frac{5\tan\left(fx+e\right)}{8fa^{2}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 3/8/f/a^2/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3-1/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*
x+e)^3*b-1/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e)*b+5/8/f/a^2/(tan(f*x+e)^2+1)
^2*tan(f*x+e)+3/f/a^4*arctan(tan(f*x+e))*b^2+3/8/f/a^2*arctan(tan(f*x+e))-1
/f/a^3*arctan(tan(f*x+e))*b-1/2/f*b^3/a^3/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e
)^2)-7/2/f*b^3/a^3/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2
))-3/f*b^4/a^4/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.710846, size = 1492, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*cos(f*x + e)^2 + (3*a^3
*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + (7*a*b^3 + 6*b^4 + (7*a^2*b^2 + 6
*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f
```

```
*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/
(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*(a^4 + a^3*b)*cos(f
*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 5*a^2*b
^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^2
+ (a^5*b + a^4*b^2)*f), 1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*
cos(f*x + e)^2 + (3*a^3*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + 2*(7*a*b^3
 + 6*b^4 + (7*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2
*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e
))) + (2*(a^4 + a^3*b)*cos(f*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x
 + e)^3 + (3*a^3*b - 5*a^2*b^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^
6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25679, size = 277, normalized size = 1.36

$$-\frac{\frac{4\,b^3\tan\left(fx+e\right)}{\left(a^4+a^3b\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{4\left(7\,ab^3+6\,b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+a^4b\right)\sqrt{ab+b^2}}-\frac{\left(3\,a^2-8\,ab+24\,b^2\right)\left(fx+e\right)}{a^4}-\frac{3\,a\tan\left(fx+e\right)^3-8\,b\tan\left(fx+e\right)^3+}{\left(\tan\left(fx+e\right)\right.}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*tan(f*x + e)/((a^4 + a^3*b)*(b*tan(f*x + e)^2 + a + b)) + 4*(7*
a*b^3 + 6*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)
/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) - (3*a^2 - 8*a*b + 24*b^
2)*(f*x + e)/a^4 - (3*a*tan(f*x + e)^3 - 8*b*tan(f*x + e)^3 + 5*a*tan(f*x +
 e) - 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

3.205 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=278

$$\frac{b^{7/2}(9a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f(a+b)^{3/2}}+\frac{b\left(-7a^2b+5a^3+12ab^2+32b^3\right)\tan(e+fx)}{16a^4f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{\left(15a^2-26ab+48b^2\right)\sin(e+fx)\,\mathrm{co}}{48a^3f\left(a+b\tan^2(e+fx)+\right.}$$

```
[Out] ((5*a^3 - 12*a^2*b + 24*a*b^2 - 64*b^3)*x)/(16*a^5) + (b^(7/2)*(9*a + 8*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*(a + b)^(3/2)*f) + ((15*
a^2 - 26*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan[
e + f*x]^2)) + ((5*a - 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b +
 b*Tan[e + f*x]^2)) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e
 + f*x]^2)) + (b*(5*a^3 - 7*a^2*b + 12*a*b^2 + 32*b^3)*Tan[e + f*x])/(16*a^
4*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.346716, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{7/2}(9a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f(a+b)^{3/2}}+\frac{b\left(-7a^2b+5a^3+12ab^2+32b^3\right)\tan(e+fx)}{16a^4f(a+b)\left(a+b\tan^2(e+fx)+b\right)}+\frac{\left(15a^2-26ab+48b^2\right)\sin(e+fx)\,\mathrm{co}}{48a^3f\left(a+b\tan^2(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((5*a^3 - 12*a^2*b + 24*a*b^2 - 64*b^3)*x)/(16*a^5) + (b^(7/2)*(9*a + 8*b)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^5*(a + b)^(3/2)*f) + ((15*
a^2 - 26*a*b + 48*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*f*(a + b + b*Tan[
e + f*x]^2)) + ((5*a - 8*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b +
 b*Tan[e + f*x]^2)) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e
 + f*x]^2)) + (b*(5*a^3 - 7*a^2*b + 12*a*b^2 + 32*b^3)*Tan[e + f*x])/(16*a^
4*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^4(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{-5a+b-7bx^2}{(1+x^2)^3(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{15a^2-ab+8b}{(1+x^2)^2(}\right.}{}$$

$$= \frac{\left(15a^2-26ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{c}{6a}$$

$$= \frac{\left(15a^2-26ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{c}{6a}$$

$$= \frac{\left(15a^2-26ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{c}{6a}$$

$$= \frac{\left(5a^3-12a^2b+24ab^2-64b^3\right)x}{16a^5} + \frac{b^{7/2}(9a+8b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5(a+b)^{3/2}f} + \frac{\left(15a^2-26ab+48\right.}{48a^3f\left(a+\right.}$$

Mathematica [C] time = 4.49534, size = 499, normalized size = 1.79

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(12x\left(-12a^2b+5a^3+24ab^2-64b^3\right)(a\cos(2(e+fx))+a+2b)+\frac{3a\left(15a^2-32ab+4\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(12*(5*a^3 - 12*a^2*b + 24*a
*b^2 - 64*b^3)*x*(a + 2*b + a*Cos[2*(e + f*x)]) - (96*b^4*(9*a + 8*b)*ArcTa
```

```
n[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*
x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
 f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e]
)^4]) + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*f*x]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sin[2*e])/f + (3*a^2*(3*a - 4*b)*Cos[4*f*x]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sin[4*e])/f + (a^3*Cos[6*f*x]*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[6*e
])/f + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*e]*(a + 2*b + a*Cos[2*(e + f*x
)])*Sin[2*f*x])/f - (96*b^4*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f
*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) + (3*a^2*(3*a - 4*b)*Cos[4*e]*(a + 2*
b + a*Cos[2*(e + f*x)])*Sin[4*f*x])/f + (a^3*Cos[6*e]*(a + 2*b + a*Cos[2*(e
 + f*x)])*Sin[6*f*x])/f))/(768*a^5*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.109, size = 442, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 5/16/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5-3/4/f/a^3/(tan(f*x+e)^2+1)^3*tan
(f*x+e)^5*b+3/2/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b^2+3/f/a^4/(tan(f*x+
e)^2+1)^3*tan(f*x+e)^3*b^2+5/6/f/a^2/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-2/f/a^
3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b-5/4/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)
*b+3/2/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b^2+11/16/f/a^2/(tan(f*x+e)^2+1)
^3*tan(f*x+e)-4/f/a^5*arctan(tan(f*x+e))*b^3+5/16/f/a^2*arctan(tan(f*x+e))-
3/4/f/a^3*arctan(tan(f*x+e))*b+3/2/f/a^4*arctan(tan(f*x+e))*b^2+1/2/f*b^4/a
^4/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+9/2/f*b^4/a^4/(a+b)/((a+b)*b)^(1/2
)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+4/f*b^5/a^5/(a+b)/((a+b)*b)^(1/2)*ar
ctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.772644, size = 1801, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(5*a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x
 + e)^2 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x + 6*
(9*a*b^4 + 8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt
(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2
 + b^2)) + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*a^3*b^2
)*cos(f*x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*cos(f*x +
e)^3 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e))*sin(f*
x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f), 1/48*(3*(5*
a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x + e)^2 + 3*
(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x - 12*(9*a*b^4 +
8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((
a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))
 + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*a^3*b^2)*cos(f*
x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*cos(f*x + e)^3 + 3
*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e))*sin(f*x + e))/
((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28722, size = 385, normalized size = 1.38

$$\frac{\frac{24 b^4 \tan(fx+e)}{(a^5+a^4b)(b\tan(fx+e)^2+a+b)} + \frac{24(9ab^4+8b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+a^5b)\sqrt{ab+b^2}} + \frac{3(5a^3-12a^2b+24ab^2-64b^3)(fx+e)}{a^5} + \frac{15a^2\tan(fx+e)^5-36}{}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/48*(24*b^4*tan(f*x + e)/((a^5 + a^4*b)*(b*tan(f*x + e)^2 + a + b)) + 24*(
9*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x +
e)/sqrt(a*b + b^2)))/((a^6 + a^5*b)*sqrt(a*b + b^2)) + 3*(5*a^3 - 12*a^2*b
+ 24*a*b^2 - 64*b^3)*(f*x + e)/a^5 + (15*a^2*tan(f*x + e)^5 - 36*a*b*tan(f*
x + e)^5 + 72*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 96*a*b*tan(f*x +
 e)^3 + 144*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 60*a*b*tan(f*x + e)
+ 72*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

3.206 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{3\sin(e+fx)}{8f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}f(a+b)^{5/2}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*f)
 + Sin[e + f*x]/(4*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*Sin[e + f*x
])/(8*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.0993723, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4147, 199, 208}

$$\frac{3\sin(e+fx)}{8f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*Sqrt[a]*(a + b)^(5/2)*f)
 + Sin[e + f*x]/(4*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*Sin[e + f*x
])/(8*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\sin(e+fx)}{8(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{}{a+b}\right.}{8}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\sin(e+fx)}{8(a+b)^2f\left(a+b-a\sin^2(e\right.}$$

Mathematica [A] time = 0.534259, size = 128, normalized size = 1.19

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{4\sin(e+fx)\left(5(a+b)-3a\sin^2(e+fx)\right)}{(a+b)^2(a\cos(2(e+fx))+a+2b)^2}\right)}{64f\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((3*ArcTanh[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b)
 - 3*a*Sin[e + f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(64*
```

```
f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.074, size = 108, normalized size = 1.

$$\frac{1}{f}\left(\frac{\sin\left(fx+e\right)}{\left(4a+4b\right)\left(-a-b+a\left(\sin\left(fx+e\right)\right)^{2}\right)^{2}}+\frac{3}{4a+4b}\left(-\frac{\sin\left(fx+e\right)}{\left(2a+2b\right)\left(-a-b+a\left(\sin\left(fx+e\right)\right)^{2}\right)}+\frac{1}{2a+2b}\mathrm{Artanh}\left(\mathrm{s}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(1/4*sin(f*x+e)/(a+b)/(-a-b+a*sin(f*x+e)^2)^2+3/4/(a+b)*(-1/2*sin(f*x+e
)/(a+b)/(-a-b+a*sin(f*x+e)^2)+1/2/(a+b)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e
)/((a+b)*a)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.608319, size = 1053, normalized size = 9.75

$$\left[\frac{3\left(a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{a^{2}+ab}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{a^{2}+ab}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^{2}+b}\right)+2\left(2\,a^{3}+7\,a^{2}b+5\,ab\right.}{16\left(\left(a^{6}+3\,a^{5}b+3\,a^{4}b^{2}+a^{3}b^{3}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{5}b+3\,a^{4}b^{2}+3\,a^{3}b^{3}+a^{2}b^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a^2 + a*b)*
log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f
*x + e)^2 + b)) + 2*(2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2
+ 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f), -1/8*(3*(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(
a + b)) - (2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x + e)^2)*sin(
f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 + 2*(a^5*
b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2 + 3*a^3*b^
3 + 3*a^2*b^4 + a*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35633, size = 165, normalized size = 1.53

$$-\frac{\frac{3\,\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^2+2\,ab+b^2\right)\sqrt{-a^2-ab}}+\frac{3\,a\sin\left(fx+e\right)^3-5\,a\sin\left(fx+e\right)-5\,b\sin\left(fx+e\right)}{\left(a\sin\left(fx+e\right)^2-a-b\right)^2\left(a^2+2\,ab+b^2\right)}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + 2*a*b + b^2)*sqrt(-
a^2 - a*b)) + (3*a*sin(f*x + e)^3 - 5*a*sin(f*x + e) - 5*b*sin(f*x + e))/((
a*sin(f*x + e)^2 - a - b)^2*(a^2 + 2*a*b + b^2)))/f
```

3.207 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=125

$$\frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}f(a+b)^{5/2}}+\frac{(4a+b)\sin(e+fx)}{8af(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\sin(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

```
[Out] ((4*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*f) - (b*Sin[e + f*x])/(4*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) +
((4*a + b)*Sin[e + f*x])/(8*a*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.106025, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4147, 385, 199, 208}

$$\frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}f(a+b)^{5/2}}+\frac{(4a+b)\sin(e+fx)}{8af(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\sin(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^(3/2)*(a + b)^
(5/2)*f) - (b*Sin[e + f*x])/(4*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) +
((4*a + b)*Sin[e + f*x])/(8*a*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\,\text{Subst}\left(\int \frac{1}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} + \frac{(4a+b)\,\text{Su}}{}$$

$$= \frac{(4a+b)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}f} - \frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\sin}{8a(a+b)^2f\left(a+b\right.}$$

Mathematica [A] time = 0.619619, size = 163, normalized size = 1.3

$$-\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\frac{8\sin(e+fx)}{\left(-a\sin^2(e+fx)+a+b\right)^2} - (4a+b)\left(\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{4\sin(e+fx)\left(5(a+b)-3a\sin^2(e+fx)\right)}{(a+b)^2(a\cos(2(e+fx))+a+2b)^2}\right.\right.}{192af\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((8*Sin[e + f*x])/(a + b
- a*Sin[e + f*x]^2)^2 - (4*a + b)*((3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
 + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b) - 3*a*Sin[e +
f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))))/(192*a*f*(a + b*Se
c[e + f*x]^2)^3)
```

Maple [A] time = 0.078, size = 124, normalized size = 1.

$$\frac{1}{f}\left(\frac{1}{\left(-a-b+a\left(\sin\left(fx+e\right)\right)^{2}\right)^{2}}\left(-\frac{(4a+b)\left(\sin\left(fx+e\right)\right)^{3}}{8a^{2}+16ab+8b^{2}}+\frac{(4a-b)\sin\left(fx+e\right)}{(8a+8b)a}\right)+\frac{4a+b}{\left(8a^{2}+16ab+8b^{2}\right)a}\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*((-1/8*(4*a+b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(4*a-b)/(a+b)/a*sin(f*x
+e))/(-a-b+a*sin(f*x+e)^2)^2+1/8*(4*a+b)/(a^2+2*a*b+b^2)/a/((a+b)*a)^(1/2)*
arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.651157, size = 1183, normalized size = 9.46

$$\left[\frac{\left(\left(4\,a^{3}+a^{2}b\right)\cos\left(fx+e\right)^{4}+4\,ab^{2}+b^{3}+2\left(4\,a^{2}b+ab^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a^{2}+ab}\,\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{a^{2}+ab}\sin\left(fx+e\right)-2\,a-b}{a\cos\left(fx+e\right)^{2}+b}\right.}{16\left(\left(a^{7}+3\,a^{6}b+3\,a^{5}b^{2}+a^{4}b^{3}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{6}b+3\,a^{5}b^{2}+3\,a^{4}b^{3}+a^{3}b^{4}\right)f\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*b + a*b^2
)*cos(f*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b
)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3*b + a^2*b^2 -
a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 3
*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a
^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2
*b^5)*f), -1/8*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*
b + a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x
 + e)/(a + b)) - (2*a^3*b + a^2*b^2 - a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f
*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 +
(a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30061, size = 219, normalized size = 1.75

$$-\frac{\frac{(4\,a+b)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^{2}-ab}}\right)}{\left(a^{3}+2\,a^{2}b+ab^{2}\right)\sqrt{-a^{2}-ab}}+\frac{4\,a^{2}\sin\left(fx+e\right)^{3}+ab\sin\left(fx+e\right)^{3}-4\,a^{2}\sin\left(fx+e\right)-3\,ab\sin\left(fx+e\right)+b^{2}\sin\left(fx+e\right)}{\left(a^{3}+2\,a^{2}b+ab^{2}\right)\left(a\sin\left(fx+e\right)^{2}-a-b\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + 2*a^2*b + a
*b^2)*sqrt(-a^2 - a*b)) + (4*a^2*sin(f*x + e)^3 + a*b*sin(f*x + e)^3 - 4*a^
2*sin(f*x + e) - 3*a*b*sin(f*x + e) + b^2*sin(f*x + e))/((a^3 + 2*a^2*b + a
*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

3.208 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}f(a+b)^{5/2}}-\frac{3b(2a+b)\sin(e+fx)}{8a^2f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\sin(e+fx)\cos^2(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*f) - (b*Cos[e + f*x]^2*Sin[e + f*x])/(4*a*(a + b)*f*(a
+ b - a*Sin[e + f*x]^2)^2) - (3*b*(2*a + b)*Sin[e + f*x])/(8*a^2*(a + b)^2*
f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.131073, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4147, 413, 385, 208}

$$\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}f(a+b)^{5/2}}-\frac{3b(2a+b)\sin(e+fx)}{8a^2f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{b\sin(e+fx)\cos^2(e+fx)}{4af(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(8*a^
(5/2)*(a + b)^(5/2)*f) - (b*Cos[e + f*x]^2*Sin[e + f*x])/(4*a*(a + b)*f*(a
+ b - a*Sin[e + f*x]^2)^2) - (3*b*(2*a + b)*Sin[e + f*x])/(8*a^2*(a + b)^2*
f*(a + b - a*Sin[e + f*x]^2))
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-4a-b+(4a+3b)x^2}{\left(a+b-ax^2\right)^2}\,dx, x, \sin(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{3b(2a+b)\sin(e+fx)}{8a^2(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)} + \frac{\left(8a^2+8\right.}{}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}f} - \frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{3}{8a^2(a+}$$

Mathematica [C] time = 7.36192, size = 2256, normalized size = 15.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((
(I/128)*ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] +
a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Si
n[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] +
a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]
*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
 + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])
/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Co
s[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e]
- I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(5/2)*Sqrt[a + b]*
f*Sqrt[Cos[2*e] - I*Sin[2*e]]) + (ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*
Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e -
 f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt
[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*S
qrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Co
s[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I
*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e]
 + a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*
a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin
[e])/(128*a^(5/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(
a + b*Sec[e + f*x]^2)^3) + ((-8*a^2 - 8*a*b - 3*b^2)*(a + 2*b + a*Cos[2*e +
 2*f*x])^3*Sec[e + f*x]^6*((ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (
2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]
] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt
[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a
 + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(5/2)*Sqr
t[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/128)*ArcTanh[(2*(a + b)*Sin[e
])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqr
t[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e]
 - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e -
f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*S
in[e])/(a^(5/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a
+ b*Sec[e + f*x]^2)^3) + ((8*a^2 + 8*a*b + 3*b^2)*(a + 2*b + a*Cos[2*e + 2*
f*x])^3*Sec[e + f*x]^6*((Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos
[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]
*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e]
 - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(5/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] -
I*Sin[2*e]]) - ((I/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2
*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Co
s[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin
[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(5/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin
[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((-8*a^2 - 8*a*b - 3*b^2)*
```

```
(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((Cos[e]*Log[-a - 2*a*Cos[2
*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*
e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]
*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(5/2)*Sqrt
[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256)*Log[-a - 2*a*Cos[2*e] - 2
*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a
 + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(5/2)*Sqrt[a +
 b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) +
 ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^6*(-8*a*b*Sin[e + f*x] - 5*
b^2*Sin[e + f*x]))/(32*a^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + (b^2*(a
+ 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[e + f*x])/(8*a^2*(a + b)*f*(
a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.08, size = 142, normalized size = 1.

$$\frac{1}{f}\left(-\frac{1}{\left(-a-b+a\left(\sin\left(fx+e\right)\right)^2\right)^2}\left(-\frac{b\left(8a+5b\right)\left(\sin\left(fx+e\right)\right)^3}{8a\left(a^2+2ab+b^2\right)}+\frac{\left(8a+3b\right)b\sin\left(fx+e\right)}{8a^2\left(a+b\right)}\right)+\frac{8a^2+8ab+3b^2}{\left(8a^2+16ab+8b^2\right)a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(-(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(8*a+3*b)/a^2*b/
(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*a*
b+b^2)/a^2/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.659955, size = 1331, normalized size = 9.24

$$\left[\frac{\left(\left(8\,a^4+8\,a^3b+3\,a^2b^2\right)\cos\left(fx+e\right)^4+8\,a^2b^2+8\,ab^3+3\,b^4+2\left(8\,a^3b+8\,a^2b^2+3\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\right.}{16\left(\left(a^8+3\,a^7b+3\,a^6b^2+a^5b^3\right)f\cos\left(fx+e\right)^4+2\left(a^7b+3\,a^6b^2+3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3
+ 3*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(a^2 + a*b)
*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(
f*x + e)^2 + b)) - 2*(6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (8*a^4*b + 13*a^3*b
^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a^7*b + 3*a^6*b^2 +
 a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*
cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f), -1/8*(((8*
a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3 + 3*b^4 + 2
*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sq
rt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (
8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a
^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5
*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b
^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31612, size = 250, normalized size = 1.74

$$-\frac{\frac{\left(8 a^{2}+8 a b+3 b^{2}\right) \arctan\left(\frac{a \sin\left(f x+e\right)}{\sqrt{-a^{2}-a b}}\right)}{\left(a^{4}+2 a^{3} b+a^{2} b^{2}\right) \sqrt{-a^{2}-a b}}-\frac{8 a^{2} b \sin\left(f x+e\right)^{3}+5 a b^{2} \sin\left(f x+e\right)^{3}-8 a^{2} b \sin\left(f x+e\right)-11 a b^{2} \sin\left(f x+e\right)-3 b^{3} \sin\left(f x+e\right)}{\left(a^{4}+2 a^{3} b+a^{2} b^{2}\right)\left(a \sin\left(f x+e\right)^{2}-a-b\right)^{2}}}{8 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4
 + 2*a^3*b + a^2*b^2)*sqrt(-a^2 - a*b)) - (8*a^2*b*sin(f*x + e)^3 + 5*a*b^2
*sin(f*x + e)^3 - 8*a^2*b*sin(f*x + e) - 11*a*b^2*sin(f*x + e) - 3*b^3*sin(
f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

3.209 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{b^3\sin(e+fx)}{4a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sin(e+fx)}{8a^3f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tanh^{-1}\left(\right.}{8a^{7/2}f(a+b)^{5/2}}$$

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*f) + Sin[e + f*x]/(a^3*f) - (b^3*Sin[e + f*x])
/(4*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin[e
+ f*x])/(8*a^3*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.193503, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4147, 390, 1157, 385, 208}

$$-\frac{b^3\sin(e+fx)}{4a^3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3b^2(4a+3b)\sin(e+fx)}{8a^3f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tanh^{-1}\left(\right.}{8a^{7/2}f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(7/2)*(a + b)^(5/2)*f) + Sin[e + f*x]/(a^3*f) - (b^3*Sin[e + f*x])
/(4*a^3*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin[e
+ f*x])/(8*a^3*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a+b-ax^2\right)^3}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3}-\frac{b\left(3a^2+3ab+b^2\right)-3ab(2a+b)x^2+3a^2bx^4}{a^3\left(a+b-ax^2\right)^3}\right)dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{a^3f}-\frac{\text{Subst}\left(\int \frac{b\left(3a^2+3ab+b^2\right)-3ab(2a+b)x^2+3a^2bx^4}{\left(a+b-ax^2\right)^3}\,dx,x,\sin(e+fx)\right)}{a^3f}$$

$$= \frac{\sin(e+fx)}{a^3f}-\frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}+\frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2+12ab(a+b)x^2}{\left(a+b-ax^2\right)^2}\,dx,x,\right.}{4a^3(a+b)f}$$

$$= \frac{\sin(e+fx)}{a^3f}-\frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}+\frac{3b^2(4a+3b)\sin(e+fx)}{8a^3(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

$$= -\frac{3b\left(4(a+b)^2+(2a+b)^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}f}+\frac{\sin(e+fx)}{a^3f}-\frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin\right.}$$

Mathematica [C] time = 7.52371, size = 2382, normalized size = 15.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[e])/(8*a^3*f*
(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[
2*e + 2*f*x])^3*Sec[e + f*x]^6*((((-3*I)/128)*ArcTan[((-I)*a*Cos[e] - I*b*C
os[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a
+ b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e
 + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*S
qrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a
+ b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[
Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e]
+ b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b
```

```
*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e +
2*f*x])]*Cos[e])/(a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (3*A
rcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e]
+ b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] +
 Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e
] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e -
f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] +
 I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e
] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2
*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin
[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin[e])/(128*a^(7/2)*Sqrt[a + b]*f*Sqr
t[Cos[2*e] - I*Sin[2*e]]))))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*
b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((3*A
rcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[
a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3
*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e]
 - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2
*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*
Sin[2*e]]) - (((3*I)/128)*ArcTanh[(2*(a + b)*Sin[e])/((-2*I)*a*Cos[e] - (2*
I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]]
+ Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sqrt[a
]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt[a +
 b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Sin[e])/(a^(7/2)*Sqrt[a +
b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]))))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) +
((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]
^6*((-3*Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2*f*x] - (
2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] -
I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Si
n[2*e + f*x]])/(256*a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) + ((
(3*I)/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e] - a*Cos[2*e + 2*f*x] - (2*I)
*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Si
n[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*
e + f*x]]*Sin[e])/(a^(7/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]))))/((a
 + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((8*a^2*b + 12*a*b^2 + 5*b^3)*(a + 2*b
+ a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((3*Cos[e]*Log[-a - 2*a*Cos[2*e] - 2
*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*
Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a
 + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(7/2)*Sqrt[a + b]
*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (((3*I)/256)*Log[-a - 2*a*Cos[2*e] - 2*b*
Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqr
t[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a +
b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(7/2)*Sqrt[a + b]
*f*Sqrt[Cos[2*e] - I*Sin[2*e]]))))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + (C
os[e]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[f*x])/(8*a^3*f*(a
 + b*Sec[e + f*x]^2)^3) + (3*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^
```

```
6*(4*a*b^2*Sin[e + f*x] + 3*b^3*Sin[e + f*x]))/(32*a^3*(a + b)^2*f*(a + b*S
ec[e + f*x]^2)^3) - (b^3*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[
e + f*x])/(8*a^3*(a + b)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.107, size = 149, normalized size = 1.

$$\frac{1}{f}\left(\frac{\sin\left(fx+e\right)}{a^3}+\frac{b}{a^3}\left(\frac{1}{\left(-a-b+a\left(\sin\left(fx+e\right)\right)^2\right)^2}\left(-\frac{3\,ab\,(4\,a+3\,b)\left(\sin\left(fx+e\right)\right)^3}{8\,a^2+16\,ab+8\,b^2}+\frac{(12\,a+7\,b)\,b\sin\left(fx+e\right)}{8\,a+8\,b}\right)-\frac{24}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(1/a^3*sin(f*x+e)+1/a^3*b*((-3/8*a*b*(4*a+3*b)/(a^2+2*a*b+b^2)*sin(f*x+
e)^3+1/8*(12*a+7*b)*b/(a+b)*sin(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2-3/8*(8*a^2+
12*a*b+5*b^2)/(a^2+2*a*b+b^2)/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a
)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.738315, size = 1594, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3)
*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x + e)^2)*sqrt
(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a -
 b)/(a*cos(f*x + e)^2 + b)) + 2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a
*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b +
 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9
+ 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 +
3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 +
a^4*b^5)*f), 1/8*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 +
 5*a^2*b^3)*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x +
 e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (8*
a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2
 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b + 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b
^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 + 3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)
^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23937, size = 277, normalized size = 1.78

$$\frac{\frac{3\left(8a^2b+12ab^2+5b^3\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{-a^2-ab}}-\frac{12a^2b^2\sin(fx+e)^3+9ab^3\sin(fx+e)^3-12a^2b^2\sin(fx+e)-19ab^3\sin(fx+e)-7b^4\sin(fx+e)}{\left(a^5+2a^4b+a^3b^2\right)\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{8\sin(fx+e)}{a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(-a^2 - a*b)) - (12*a^2*b^2*sin(f*x + e)^3
```

```
+ 9*a*b^3*sin(f*x + e)^3 - 12*a^2*b^2*sin(f*x + e) - 19*a*b^3*sin(f*x + e)
- 7*b^4*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*(a*sin(f*x + e)^2 - a - b)
^2) + 8*sin(f*x + e)/a^3)/f
```

3.210 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=181

$$\frac{b^4\sin(e+fx)}{4a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{b^2\left(48a^2+80ab+35b^2\right)\tanh^{-1}\left(\sqrt{a}\right.}{8a^{9/2}f(a+b)^{5/2}}$$

```
[Out] (b^2*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
)/(8*a^(9/2)*(a + b)^(5/2)*f) + ((a - 3*b)*Sin[e + f*x])/(a^4*f) - Sin[e +
f*x]^3/(3*a^3*f) + (b^4*Sin[e + f*x])/(4*a^4*(a + b)*f*(a + b - a*Sin[e + f
*x]^2)^2) - (b^3*(16*a + 13*b)*Sin[e + f*x])/(8*a^4*(a + b)^2*f*(a + b - a*
Sin[e + f*x]^2))
```

Rubi [A] time = 0.242773, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 390, 1157, 385, 208}

$$\frac{b^4\sin(e+fx)}{4a^4f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{b^2\left(48a^2+80ab+35b^2\right)\tanh^{-1}\left(\sqrt{a}\right.}{8a^{9/2}f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] (b^2*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
)/(8*a^(9/2)*(a + b)^(5/2)*f) + ((a - 3*b)*Sin[e + f*x])/(a^4*f) - Sin[e +
f*x]^3/(3*a^3*f) + (b^4*Sin[e + f*x])/(4*a^4*(a + b)*f*(a + b - a*Sin[e + f
*x]^2)^2) - (b^3*(16*a + 13*b)*Sin[e + f*x])/(8*a^4*(a + b)^2*f*(a + b - a*
Sin[e + f*x]^2))
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{a^4} - \frac{x^2}{a^3} + \frac{b^2\left(6a^2+8ab+3b^2\right)-4ab^2(3a+2b)x^2+6a^2b^2x^4}{a^4\left(a+b-ax^2\right)^3}\right) dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a-3b)\sin(e+fx)}{a^4 f} - \frac{\sin^3(e+fx)}{3a^3 f} + \frac{\text{Subst}\left(\int \frac{b^2\left(6a^2+8ab+3b^2\right)-4ab^2(3a+2b)x^2+6a^2b^2x^4}{\left(a+b-ax^2\right)^3}\,dx, x, \text{s}\right.}{a^4 f}$$

$$= \frac{(a-3b)\sin(e+fx)}{a^4 f} - \frac{\sin^3(e+fx)}{3a^3 f} + \frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{(a-3b)\sin(e+fx)}{a^4 f} - \frac{\sin^3(e+fx)}{3a^3 f} + \frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{b^3(16}{8a^4(a+b)}$$

$$= \frac{b^2\left(48a^2+80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}(a+b)^{5/2}f} + \frac{(a-3b)\sin(e+fx)}{a^4 f} - \frac{\sin^3(e+fx)}{3a^3 f} + \frac{}{4a}$$

Mathematica [A] time = 4.45507, size = 194, normalized size = 1.07

$$\frac{-\frac{3b^2\left(48a^2+80ab+35b^2\right)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{5/2}} + 4a^{3/2}\sin(3(e+fx)) + 12\sqrt{a}\sin(e+fx)\left(-\frac{b^4(13a\cos(2(e+}{(a+b)^2(a\cos(2(e}\right.}{48a^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((-3*b^2*(48*a^2 + 80*a*b + 35*b^2)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]
] - Log[Sqrt[a + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(5/2) + 12*Sqrt[a]*(-
12*b - (b^4*(9*a + 22*b + 13*a*Cos[2*(e + f*x)]))/((a + b)^2*(a + 2*b + a*C
os[2*(e + f*x)])^2) + a*(3 - (16*b^3)/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*
x)])))))*Sin[e + f*x] + 4*a^(3/2)*Sin[3*(e + f*x)])/(48*a^(9/2)*f)
```

Maple [A] time = 0.102, size = 177, normalized size = 1.

$$\frac{1}{f}\left(-\frac{1}{a^4}\left(\frac{a\left(\sin\left(fx+e\right)\right)^3}{3}-\sin\left(fx+e\right)a+3\sin\left(fx+e\right)b\right)-\frac{b^2}{a^4}\left(\frac{1}{\left(-a-b+a\left(\sin\left(fx+e\right)\right)^2\right)^2}\left(-\frac{ab\left(16a+13b\right)\left(\sin\right.}{8a^2+16ab\,-}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(-1/a^4*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+3*sin(f*x+e)*b)-1/a^4*b^2*((-1
/8*a*b*(16*a+13*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(16*a+11*b)*b/(a+b)*sin
(f*x+e))/(-a-b+a*sin(f*x+e)^2)^2-1/8*(48*a^2+80*a*b+35*b^2)/(a^2+2*a*b+b^2)
/((a+b)*a)^(1/2)*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.778927, size = 1908, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(48*a^2*b^4 + 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^
2*b^4)*cos(f*x + e)^4 + 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)
^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e)
 - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(16*a^5*b^2 - 24*a^4*b^3 - 210*a^3*
b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos
```

```
(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4)*cos(f
*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175*a^2*b^
5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*f*
cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos(f*x + e)
^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f), -1/24*(3*(48*a^2*b^4 +
 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4
+ 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*a
rctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (16*a^5*b^2 - 24*a^4*b^3 - 2
10*a^3*b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b
^3)*cos(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4
)*cos(f*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175
*a^2*b^5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*
b^3)*f*cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos(f
*x + e)^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29686, size = 323, normalized size = 1.78

$$-\frac{\frac{3\left(48\,a^2b^2+80\,ab^3+35\,b^4\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\sqrt{-a^2-ab}}-\frac{3\left(16\,a^2b^3\sin\left(fx+e\right)^3+13\,ab^4\sin\left(fx+e\right)^3-16\,a^2b^3\sin\left(fx+e\right)-27\,ab^4\sin\left(fx+e\right)-11\,b^5\sin\left(fx+e\right)\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\left(a\sin\left(fx+e\right)^2-a-b\right)^2}+\cdots}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 -
 a*b))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a^2 - a*b)) - 3*(16*a^2*b^3*sin(f*x
 + e)^3 + 13*a*b^4*sin(f*x + e)^3 - 16*a^2*b^3*sin(f*x + e) - 27*a*b^4*sin(
```

```
f*x + e) - 11*b^5*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(a*sin(f*x + e)^
2 - a - b)^2) + 8*(a^6*sin(f*x + e)^3 - 3*a^6*sin(f*x + e) + 9*a^5*b*sin(f*
x + e))/a^9)/f
```

3.211 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

Optimal. Leaf size=214

$$-\frac{b^5\sin(e+fx)}{4a^5f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{b^4(20a+17b)\sin(e+fx)}{8a^5f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f}-\frac{b^3}{}$$

```
[Out] -(b^3*(80*a^2 + 140*a*b + 63*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(11/2)*(a + b)^(5/2)*f) + ((a^2 - 3*a*b + 6*b^2)*Sin[e + f*x])/(a^
5*f) - ((2*a - 3*b)*Sin[e + f*x]^3)/(3*a^4*f) + Sin[e + f*x]^5/(5*a^3*f) -
(b^5*Sin[e + f*x])/(4*a^5*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (b^4*(2
0*a + 17*b)*Sin[e + f*x])/(8*a^5*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.258366, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4147, 390, 1157, 385, 208}

$$-\frac{b^5\sin(e+fx)}{4a^5f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{b^4(20a+17b)\sin(e+fx)}{8a^5f(a+b)^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f}-\frac{b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] -(b^3*(80*a^2 + 140*a*b + 63*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(11/2)*(a + b)^(5/2)*f) + ((a^2 - 3*a*b + 6*b^2)*Sin[e + f*x])/(a^
5*f) - ((2*a - 3*b)*Sin[e + f*x]^3)/(3*a^4*f) + Sin[e + f*x]^5/(5*a^3*f) -
(b^5*Sin[e + f*x])/(4*a^5*(a + b)*f*(a + b - a*Sin[e + f*x]^2)^2) + (b^4*(2
0*a + 17*b)*Sin[e + f*x])/(8*a^5*(a + b)^2*f*(a + b - a*Sin[e + f*x]^2))
```


Rule 4147

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m +
 n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && Int
egerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^5}{\left(a+b-ax^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+6b^2}{a^5} - \frac{(2a-3b)x^2}{a^4} + \frac{x^4}{a^3} - \frac{b^3\left(10a^2+15ab+6b^2\right)-5ab^3(4a+3b)x^2+10a^2b^3x^4}{a^5\left(a+b-ax^2\right)^3}\right)dx, x, \sin(e\right.}{f}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{\text{Subst}\left(\int \frac{b^3\left(10a^2\right.}{}\right.}{}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{b^5\sin}{4a^5(a+b)f\,(a+}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f} - \frac{(2a-3b)\sin^3(e+fx)}{3a^4f} + \frac{\sin^5(e+fx)}{5a^3f} - \frac{b^5\sin}{4a^5(a+b)f\,(a+}$$

$$= -\frac{b^3\left(80a^2+140ab+63b^2\right)\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}(a+b)^{5/2}f} + \frac{\left(a^2-3ab+6b^2\right)\sin(e+fx)}{a^5f} - \frac{(2a-}{}$$

Mathematica [C] time = 7.61834, size = 2670, normalized size = 12.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((5*a^2 - 18*a*b + 48*b^2)*Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6*Sin[e])/(64*a^5*f*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 140*
a*b^4 - 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((I/128)*A
rcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e]
+ b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] +
 Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e
] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e -
f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] +
 I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e
] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2
```

*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e] - I*a*Sin
[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[C
os[2*e] - I*Sin[2*e]]) + (ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e]
 + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sq
rt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e
] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[
2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] -
 I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e
]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos
[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*
e] - I*b*Sin[3*e] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Sin[e])/(12
8*a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*S
ec[e + f*x]^2)^3) + ((80*a^2*b^3 + 140*a*b^4 + 63*b^5)*(a + 2*b + a*Cos[2*e
 + 2*f*x])^3*Sec[e + f*x]^6*((ArcTanh[(2*(a + b)*Sin[e])/(((-2*I)*a*Cos[e] -
 (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*
e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] - I*Sq
rt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] + I*Sqrt[a]*Sqrt
[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])]*Cos[e])/(128*a^(11/2)*
Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/128)*ArcTanh[(2*(a + b)*Si
n[e])/((-2*I)*a*Cos[e] - (2*I)*b*Cos[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*
Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2
*e] - I*Sin[2*e]] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
 - f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])
]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2
*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 140*a*b^4 - 63*b^5)*(a + 2*b +
 a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((Cos[e]*Log[a + 2*a*Cos[2*e] + 2*b*C
os[2*e] - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt
[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b
]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]])/(256*a^(11/2)*Sqrt[a + b]*f*
Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256)*Log[a + 2*a*Cos[2*e] + 2*b*Cos[2*e]
 - a*Cos[2*e + 2*f*x] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqr
t[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[
Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt
[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((80*a^2*
b^3 + 140*a*b^4 + 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(
(Cos[e]*Log[-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a
*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[
2*e]]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e
+ f*x]])/(256*a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - ((I/256
)*Log[-a - 2*a*Cos[2*e] - 2*b*Cos[2*e] + a*Cos[2*e + 2*f*x] + (2*I)*a*Sin[2
*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*
Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[2*e + f*x]
]*Sin[e])/(a^(11/2)*Sqrt[a + b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((a + b)^2
*(a + b*Sec[e + f*x]^2)^3) + ((5*a - 12*b)*Cos[3*f*x]*(a + 2*b + a*Cos[2*e
+ 2*f*x])^3*Sec[e + f*x]^6*Sin[3*e])/(384*a^4*f*(a + b*Sec[e + f*x]^2)^3) +

```
(Cos[5*f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[5*e])/(640
*a^3*f*(a + b*Sec[e + f*x]^2)^3) + ((5*a^2 - 18*a*b + 48*b^2)*Cos[e]*(a + 2
*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[f*x])/(64*a^5*f*(a + b*Sec[e
+ f*x]^2)^3) + ((5*a - 12*b)*Cos[3*e]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[
e + f*x]^6*Sin[3*f*x])/(384*a^4*f*(a + b*Sec[e + f*x]^2)^3) + (Cos[5*e]*(a
+ 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*Sin[5*f*x])/(640*a^3*f*(a + b*
Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^6*(20*a
*b^4*Sin[e + f*x] + 17*b^5*Sin[e + f*x]))/(32*a^5*(a + b)^2*f*(a + b*Sec[e
+ f*x]^2)^3) - (b^5*(a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x]^5*Tan[e + f
*x])/(8*a^5*(a + b)*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.118, size = 214, normalized size = 1.

$$\frac{1}{f}\left(\frac{1}{a^5}\left(\frac{\left(\sin\left(fx+e\right)\right)^5 a^2}{5}-\frac{2\left(\sin\left(fx+e\right)\right)^3 a^2}{3}+\left(\sin\left(fx+e\right)\right)^3 ab+a^2\sin\left(fx+e\right)-3\,ab\sin\left(fx+e\right)+6\,b^2\sin\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f*(1/a^5*(1/5*sin(f*x+e)^5*a^2-2/3*sin(f*x+e)^3*a^2+sin(f*x+e)^3*a*b+a^2*
sin(f*x+e)-3*a*b*sin(f*x+e)+6*b^2*sin(f*x+e))+1/a^5*b^3*((-1/8*a*b*(20*a+17
*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+5/8*(4*a+3*b)*b/(a+b)*sin(f*x+e))/(-a-b+a*
sin(f*x+e)^2)^2-1/8*(80*a^2+140*a*b+63*b^2)/(a^2+2*a*b+b^2)/((a+b)*a)^(1/2)
*arctanh(a*sin(f*x+e)/((a+b)*a)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.845971, size = 2271, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(80*a^2*b^5 + 140*a*b^6 + 63*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 6
3*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x
 + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
 + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(24*(a^8 + 3*a^7*b + 3*a^6*b^2
 + a^5*b^3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1774*a
^3*b^5 + 2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 - 23*a^
5*b^3 - 9*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2 + 131*a
^5*b^3 + 167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64*a^6*b^2
 + 360*a^5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f*x + e)^2
)*sin(f*x + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*x + e)^4 +
 2*(a^10*b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 + (a^9*b^2 +
 3*a^8*b^3 + 3*a^7*b^4 + a^6*b^5)*f), 1/120*(15*(80*a^2*b^5 + 140*a*b^6 + 6
3*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 63*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*
b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(
-a^2 - a*b)*sin(f*x + e)/(a + b)) + (24*(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1774*a^3*b^5 +
2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 - 23*a^5*b^3 - 9
*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2 + 131*a^5*b^3 +
167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64*a^6*b^2 + 360*a^
5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f*x + e)^2)*sin(f*x
 + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*x + e)^4 + 2*(a^10*
b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 + (a^9*b^2 + 3*a^8*b^
3 + 3*a^7*b^4 + a^6*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32138, size = 383, normalized size = 1.79

$$\frac{\frac{15\left(80\,a^2b^3+140\,ab^4+63\,b^5\right)\arctan\left(\frac{a\sin\left(fx+e\right)}{\sqrt{-a^2-ab}}\right)}{\left(a^7+2\,a^6b+a^5b^2\right)\sqrt{-a^2-ab}}-\frac{15\left(20\,a^2b^4\sin\left(fx+e\right)^3+17\,ab^5\sin\left(fx+e\right)^3-20\,a^2b^4\sin\left(fx+e\right)-35\,ab^5\sin\left(fx+e\right)-15\,b^6\sin\left(fx+e\right)\right)}{\left(a^7+2\,a^6b+a^5b^2\right)\left(a\sin\left(fx+e\right)^2-a-b\right)^2}+\frac{8}{}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*arctan(a*sin(f*x + e)/sqrt(-a^2
 - a*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt(-a^2 - a*b)) - 15*(20*a^2*b^4*sin(
f*x + e)^3 + 17*a*b^5*sin(f*x + e)^3 - 20*a^2*b^4*sin(f*x + e) - 35*a*b^5*s
in(f*x + e) - 15*b^6*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*(a*sin(f*x +
e)^2 - a - b)^2) + 8*(3*a^12*sin(f*x + e)^5 - 10*a^12*sin(f*x + e)^3 + 15*a
^11*b*sin(f*x + e)^3 + 15*a^12*sin(f*x + e) - 45*a^11*b*sin(f*x + e) + 90*a
^10*b^2*sin(f*x + e))/a^15)/f
```

3.212 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{\left(3a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}f(a+b)^{5/2}}-\frac{3a(a+2b)\tan(e+fx)}{8b^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)\sec^2(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(
5/2)*(a + b)^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(4*b*(a + b)*f*(a +
 b + b*Tan[e + f*x]^2)^2) - (3*a*(a + 2*b)*Tan[e + f*x])/(8*b^2*(a + b)^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.156647, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 413, 385, 205}

$$\frac{\left(3a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}f(a+b)^{5/2}}-\frac{3a(a+2b)\tan(e+fx)}{8b^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)\sec^2(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] ((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(
5/2)*(a + b)^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(4*b*(a + b)*f*(a +
 b + b*Tan[e + f*x]^2)^2) - (3*a*(a + 2*b)*Tan[e + f*x])/(8*b^2*(a + b)^2*f
*(a + b + b*Tan[e + f*x]^2))
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+4b+(3a+4b)x^2}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3a(a+2b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(3a^2 +$$

$$= \frac{\left(3a^2+8ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3}{8b^2(a+}$$

Mathematica [A] time = 0.99519, size = 125, normalized size = 0.88

$$\frac{\frac{(3a^2+8ab+8b^2)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{a\sqrt{b}\sin(2(e+fx))\left(3a^2+3a(a+2b)\cos(2(e+fx))+16ab+16b^2\right)}{(a+b)^2(a\cos(2(e+fx))+a+2b)^2}}{8b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a +
b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f
*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))/(8*b^
(5/2)*f)
```

Maple [B] time = 0.073, size = 294, normalized size = 2.1

$$-\frac{5a^2\left(\tan\left(fx+e\right)\right)^3}{8f\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)^2b\left(a^2+2ab+b^2\right)}-\frac{a\left(\tan\left(fx+e\right)\right)^3}{f\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)^2\left(a^2+2ab+b^2\right)}-\frac{3a^2}{8f\left(a+b+b\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -5/8/f/(a+b+b*tan(f*x+e)^2)^2*a^2/b/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/f/(a+b+b
*tan(f*x+e)^2)^2*a/(a^2+2*a*b+b^2)*tan(f*x+e)^3-3/8/f/(a+b+b*tan(f*x+e)^2)^
2*a^2/b^2/(a+b)*tan(f*x+e)-a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/
8/f/(a^2+2*a*b+b^2)/b^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2)
)*a^2+1/f/(a^2+2*a*b+b^2)/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(
1/2))*a+1/f/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(
1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.697212, size = 1597, normalized size = 11.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3
 + 8*b^4 + 2*(3*a^3*b + 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(3*(a
^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^3 + 8*
a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*
b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(f
*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f), -1/16*(((3*a^4 + 8*a^
3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 2*(3*a^3*b
+ 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b
)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(
a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^3 + 8
*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2
*b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)*f*cos(
f*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4028, size = 261, normalized size = 1.84

$$\frac{\frac{\left(\pi \left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+8ab+8b^2)}{(a^2b^2+2ab^3+b^4)\sqrt{ab+b^2}}-\frac{5a^2b\tan(fx+e)^3+8ab^2\tan(fx+e)^3+3a^3\tan(fx+e)+11a^2b\tan(fx+e)+8ab^2\tan(fx+e)}{(a^2b^2+2ab^3+b^4)(b\tan(fx+e)^2+a+b)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(3*a^2 + 8*a*b + 8*b^2)/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(a*b + b^2)
) - (5*a^2*b*tan(f*x + e)^3 + 8*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) +
 11*a^2*b*tan(f*x + e) + 8*a*b^2*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^4)*(
b*tan(f*x + e)^2 + a + b)^2))/f
```

3.213 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal. Leaf size=123

$$\frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}f(a+b)^{5/2}}+\frac{(a+4b)\tan(e+fx)}{8bf(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(
5/2)*f) - (a*Tan[e + f*x])/(4*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (
(a + 4*b)*Tan[e + f*x])/(8*b*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.101683, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 385, 199, 205}

$$\frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}f(a+b)^{5/2}}+\frac{(a+4b)\tan(e+fx)}{8bf(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{a\tan(e+fx)}{4bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*b^(3/2)*(a + b)^(
5/2)*f) - (a*Tan[e + f*x])/(4*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (
(a + 4*b)*Tan[e + f*x])/(8*b*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4b(a+b)f}$$

$$= -\frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{(a+4b)\,\text{S}}{}$$

$$= \frac{(a+4b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}f} - \frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\tan}{8b(a+b)^2f\left(a+b+\right.}$$

Mathematica [C] time = 3.8305, size = 283, normalized size = 2.3

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{((a+4b)\sin(2e)-(a-2b)\sin(2fx))(a\cos(2(e+fx))+a+2b)}{b(\cos(e)-\sin(e))(\sin(e)+\cos(e))} - \frac{4(a+b)((a+2b)\sin(2e)-a\sin(2fx))}{a(\cos(e)-\sin(e))(\sin(e)+\cos(e))} - \frac{(a+4b)(}{}\right.}{64f(a+b)^2\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-(((a + 4*b)*ArcTan[(Sec[f*
x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*S
qrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2
*(Cos[2*e] - I*Sin[2*e]))/(b*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) -
(4*(a + b)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e
] + Sin[e])) + ((a + 2*b + a*Cos[2*(e + f*x)])*((a + 4*b)*Sin[2*e] - (a - 2
*b)*Sin[2*f*x]))/(b*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(64*(a + b)^2*f*
(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.071, size = 238, normalized size = 1.9

$$\frac{a\left(\tan\left(fx+e\right)\right)^{3}}{8f\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}\left(a^{2}+2ab+b^{2}\right)}+\frac{\left(\tan\left(fx+e\right)\right)^{3}b}{2f\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}\left(a^{2}+2ab+b^{2}\right)}-\frac{a\,\mathrm{ta}}{8\left(a+b\right)bf\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/8/f/(a+b+b*tan(f*x+e)^2)^2*a/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/2/f/(a+b+b*ta
n(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3*b-1/8*a*tan(f*x+e)/b/(a+b)/f/(a+
b+b*tan(f*x+e)^2)^2+1/2*tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/8/f/(a^
2+2*a*b+b^2)/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*a+1/2/f
/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.667593, size = 1449, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b^
2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)
^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos
(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b
*cos(f*x + e)^2 + b^2)) + 4*((a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^3 - (
a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^
3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3 + 3*a^3*b^4 + 3*a^2*
b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f), -
1/16*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b^2)
*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/
(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*((a^3*b - a^2*b^2 - 2*a*b^
3)*cos(f*x + e)^3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))
/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3
 + 3*a^3*b^4 + 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 +
 3*a*b^6 + b^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39014, size = 231, normalized size = 1.88

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+4b)}{(a^2b+2ab^2+b^3)\sqrt{ab+b^2}}+\frac{ab\tan(fx+e)^3+4b^2\tan(fx+e)^3-a^2\tan(fx+e)+3ab\tan(fx+e)+4b^2\tan(fx+e)}{(a^2b+2ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 4*b)/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b + b^2)) + (a*b*tan(f*x
 + e)^3 + 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) + 3*a*b*tan(f*x + e) + 4*
b^2*tan(f*x + e))/((a^2*b + 2*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2))/f
```

3.214 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=106

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}f(a+b)^{5/2}}+\frac{3\tan(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\tan(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] (3*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*f)
+ Tan[e + f*x]/(4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (3*Tan[e + f*x]
)/(8*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.0818153, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 199, 205}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}f(a+b)^{5/2}}+\frac{3\tan(e+fx)}{8f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{\tan(e+fx)}{4f(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*Sqrt[b]*(a + b)^(5/2)*f)
+ Tan[e + f*x]/(4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (3*Tan[e + f*x]
)/(8*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\tan(e+fx)}{8(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}(a+b)^{5/2}f} + \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\tan(e+fx)}{8(a+b)^2 f\left(a+b+b\tan^2\right.}$$

Mathematica [C] time = 2.74883, size = 265, normalized size = 2.5

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\sec(2e)\left(a(5a+2b)\sin(2fx)-\left(5a^2+16ab+8b^2\right)\sin(2e)\right)(a\cos(2(e+fx))+a+2b)}{a^2} + \frac{4b(a+b)\sec(2e)((a+2b)}{a^2}\right.}{64f(a+b)^2\left(a+b\sec^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((-3*ArcTan[(Sec[f*x]*(Cos[2
*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b
```

```
]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e
] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*b*(a + b)
*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/a^2 + ((a + 2*b + a*Cos[2*(e
 + f*x)])*Sec[2*e]*(-((5*a^2 + 16*a*b + 8*b^2)*Sin[2*e]) + a*(5*a + 2*b)*Si
n[2*f*x]))/a^2))/(64*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.067, size = 97, normalized size = 0.9

$$\frac{\tan\left(fx+e\right)}{\left(4a+4b\right)f\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}}+\frac{3\tan\left(fx+e\right)}{8\left(a+b\right)^{2}f\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)}+\frac{3}{8\left(a+b\right)^{2}f}\arctan\left(\tan\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/4*tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/8*tan(f*x+e)/(a+b)^2/f/(a+b
+b*tan(f*x+e)^2)+3/8/f/(a+b)^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.645417, size = 1319, normalized size = 12.44

$$\left[-\frac{3\left(a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\sqrt{-ab-b^{2}}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a+2\,b\right)\cos\left(fx+e\right.\right.}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}}\right.}{32\left(\left(a^{5}b+3\,a^{4}b^{2}+3\,a^{3}b^{3}+a^{2}b^{4}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{4}b^{2}+3\,a^{3}b^{3}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a*b - b^2
)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e
)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*
x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((5*a^
2*b + 7*a*b^2 + 2*b^3)*cos(f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f
*x + e))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a
^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2
*b^4 + 3*a*b^5 + b^6)*f), -1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a
*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^2*b + 7*a*b^2 + 2*b^3)*cos(
f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f*x + e))/((a^5*b + 3*a^4*b^
2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*
b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33375, size = 176, normalized size = 1.66

$$\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^2+2ab+b^2)\sqrt{ab+b^2}}+\frac{3b\tan(fx+e)^3+5a\tan(fx+e)+5b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2(a^2+2ab+b^2)}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^3 + 5*
a*tan(f*x + e) + 5*b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*(a^2 + 2*a
*b + b^2)))/f
```

3.215 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e+fx}{4af(a+b)\left(a+b\tan^2(e\right.}$$

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.174835, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e+fx}{4af(a+b)\left(a+b\tan^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}$$

Mathematica [C] time = 5.76198, size = 332, normalized size = 2.31

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b\left(\left(9a^2+28ab+16b^2\right)\sin(2e)-3a(3a+2b)\sin(2fx)\right)(a\cos(2(e+fx))+a+2b)}{f(a+b)^2(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + \frac{b\left(15a^2+20ab+8b^2\right)(\cos(2e)}{}\right.}{64a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.086, size = 321, normalized size = 2.2

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f a^{3}}-\frac{7 b^{2} \left(\tan\left(fx+e\right)\right)^{3}}{8 f a \left(a+b+b \left(\tan\left(fx+e\right)\right)^{2}\right)^{2} \left(a^{2}+2 a b+b^{2}\right)}-\frac{b^{3} \left(\tan\left(fx+e\right)\right)^{3}}{2 f a^{2} \left(a+b+b \left(\tan\left(fx+e\right)\right)^{2}\right)^{2} \left(a^{2}+2 a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f/a^3*arctan(tan(f*x+e))-7/8/f/a*b^2/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^
2)*tan(f*x+e)^3-1/2/f/a^2*b^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*
x+e)^3-9/8*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/2/f/a^2*b^2/(a+b
+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)-15/8/f/a*b/(a^2+2*a*b+b^2)/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-5/2/f/a^2*b^2/(a^2+2*a*b+b^2)/((a+
b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/a^3*b^3/(a^2+2*a*b+b^2
)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.716671, size = 1854, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
```

```
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27303, size = 277, normalized size = 1.92

$$-\frac{\frac{\left(15\,a^2b+20\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan(fx+e)^3+4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+13\,ab^2\tan(fx+e)+4\,b^3\tan(fx+}{\left(a^4+2\,a^3b+a^2b^2\right)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

3.216 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=201

$$\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{5/2}}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b(2a+3b)\tan(e+fx)}{4a^2f(a+b)\left(a+b\tan^2(e+f\right.}$$

```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*f) + (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(2*a + 3*b)*Tan[e + f*x])
/(4*a^2*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(4*a + 3*b)*(a + 4*b)*
Tan[e + f*x])/(8*a^3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.335745, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4f(a+b)^{5/2}}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b(2a+3b)\tan(e+fx)}{4a^2f(a+b)\left(a+b\tan^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```



```
[Out] ((a - 6*b)*x)/(2*a^4) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(5/2)*f) + (Cos[e + f*x]*Sin[e
+ f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(2*a + 3*b)*Tan[e + f*x])
/(4*a^2*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(4*a + 3*b)*(a + 4*b)*
Tan[e + f*x])/(8*a^3*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a+b-5bx^2}{\left(1+x^2\right)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \text{Subst}\left(\int \frac{-2(2a^2\right.$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4a+3b)(a}{8a^3(a+b)^2f\,(a}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4a+3b)(a}{8a^3(a+b)^2f\,(a}$$

$$= \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx\right.}$$

Mathematica [A] time = 3.7653, size = 156, normalized size = 0.78

$$\frac{\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + a\sin(2(e+fx))\left(\frac{2b^3(5a\cos(2(e+fx))+3a+8b)}{(a+b)^2(a\cos(2(e+fx))+a+2b)^2} + \frac{13ab^2}{(a+b)^2(a\cos(2(e+fx))+a+2b)} + 2\right) + 4(a-6b}{8a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (4*(a - 6*b)*(e + f*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^2*
(a + 2*b + a*Cos[2*(e + f*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cos[2*(e + f*x)])
)/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))*Sin[2*(e + f*x)])/(8*a^4*f)
```

Maple [A] time = 0.113, size = 366, normalized size = 1.8

$$\frac{\tan\left(fx+e\right)}{2\,fa^{3}\left(\left(\tan\left(fx+e\right)\right)^{2}+1\right)}+\frac{\arctan\left(\tan\left(fx+e\right)\right)}{2\,fa^{3}}-3\,\frac{\arctan\left(\tan\left(fx+e\right)\right)b}{fa^{4}}+\frac{11\,b^{3}\left(\tan\left(fx+e\right)\right)^{3}}{8\,fa^{2}\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/2/f/a^3*tan(f*x+e)/(tan(f*x+e)^2+1)+1/2/f/a^3*arctan(tan(f*x+e))-3/f/a^4*
arctan(tan(f*x+e))*b+11/8/f/a^2*b^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*
tan(f*x+e)^3+1/f*b^4/a^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^
3+13/8/f/a^2*b^2/(a+b+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)+1/f*b^3/a^3/(a+b+b
*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)+35/8/f/a^2*b^2/(a^2+2*a*b+b^2)/((a+b)*b)^
(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+7/f/a^3*b^3/(a^2+2*a*b+b^2)/((a+
b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/f*b^4/a^4/(a^2+2*a*b+b^2
)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.785094, size = 2188, normalized size = 10.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 32*
(a^4*b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^3*b^2
```

```
 - 4*a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x + (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (3
5*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*
b^3 + 24*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^
2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e)
 + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(4*(a^5 + 2*
a^4*b + a^3*b^2)*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*cos(f
*x + e)^3 + (4*a^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*x + e))
/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^5*b
^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), 1/16*(8*(a^5 - 4
*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 16*(a^4*b - 4*a^3*b^2
 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 8*(a^3*b^2 - 4*a^2*b^3 - 11*a
*b^4 - 6*b^5)*f*x - (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (35*a^4*b + 56*a^3*b^
2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*b^3 + 24*a*b^4)*cos
(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt
(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(4*(a^5 + 2*a^4*b + a^3*b^2)
*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + (4*a
^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^8 + 2*a^7*b
 + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x +
e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35934, size = 323, normalized size = 1.61

$$\frac{\frac{\left(35\,a^2b^2+56\,ab^3+24\,b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^6+2\,a^5b+a^4b^2\right)\sqrt{ab+b^2}}+\frac{11\,ab^3\tan\left(fx+e\right)^3+8\,b^4\tan\left(fx+e\right)^3+13\,a^2b^2\tan\left(fx+e\right)+21\,ab^3\tan\left(fx+e\right)+8\,b^4\tan(\cdots}{\left(a^5+2\,a^4b+a^3b^2\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(a
*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 8*b^4*tan(f*x + e)^3 + 13*a^2*b^2*t
an(f*x + e) + 21*a*b^3*tan(f*x + e) + 8*b^4*tan(f*x + e))/((a^5 + 2*a^4*b +
 a^3*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 4*(f*x + e)*(a - 6*b)/a^4 + 4*tan
(f*x + e)/((tan(f*x + e)^2 + 1)*a^3))/f
```

3.217 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=269

$$-\frac{3b^{5/2}\left(21a^2+36ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f(a+b)^{5/2}}+\frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b\left(3a^2-7ab-12b^2\right)\text{ta}}{8a^3f(a+b)\left(a+b\tan^2(e\right.}$$

```
[Out] (3*(a^2 - 4*a*b + 16*b^2)*x)/(8*a^5) - (3*b^(5/2)*(21*a^2 + 36*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*(a + b)^(5/2)*f) + ((3
*a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*
(3*a^2 - 7*a*b - 12*b^2)*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e +
f*x]^2)^2) + (3*b*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Tan[e + f*x])/(8*a^4*(a +
 b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.376786, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$-\frac{3b^{5/2}\left(21a^2+36ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5f(a+b)^{5/2}}+\frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b\left(3a^2-7ab-12b^2\right)\text{ta}}{8a^3f(a+b)\left(a+b\tan^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (3*(a^2 - 4*a*b + 16*b^2)*x)/(8*a^5) - (3*b^(5/2)*(21*a^2 + 36*a*b + 16*b^2
)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^5*(a + b)^(5/2)*f) + ((3
*a - 8*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f*(a + b + b*Tan[e + f*x]^2)^2)
 + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*
(3*a^2 - 7*a*b - 12*b^2)*Tan[e + f*x])/(8*a^3*(a + b)*f*(a + b + b*Tan[e +
f*x]^2)^2) + (3*b*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Tan[e + f*x])/(8*a^4*(a +
 b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
```

```
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-3a+b-7bx^2}{(1+x^2)^2(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{3a^2+3ab+8b}{(1+x^2)(}\right.}{}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b\left(3a^2-7ab-12\right.}{8a^3(a+b)f\left(a+b+\right.}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b\left(3a^2-7ab-12\right.}{8a^3(a+b)f\left(a+b+\right.}$$

$$= \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b\left(3a^2-7ab-12\right.}{8a^3(a+b)f\left(a+b+\right.}$$

$$= \frac{3\left(a^2-4ab+16b^2\right)x}{8a^5} - \frac{3b^{5/2}\left(21a^2+36ab+16b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5(a+b)^{5/2}f} + \frac{(3a-8b)\cos(e}{8a^2f\left(a+b+\right.}$$

Mathematica [C] time = 6.57259, size = 1430, normalized size = 5.32

$$\frac{\left(21a^2+36ba+16b^2\right)(\cos(2e+2fx)a+a+2b)^3\left(\frac{3b^3\tan^{-1}\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right)(-a\sin(fx)-2b\sin(\right.}{64a^5\sqrt{a+b}f\sqrt{b\cos(4e)-ib\sin(4e)}}\right.}{(a+b)^2\left(b\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((21*a^2 + 36*a*b + 16*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*((3*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin
```

```
[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(
-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^5*Sqrt[a
+ b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - (((3*I)/64)*b^3*ArcTan[Sec[f*x]*(
Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])
/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*
x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*
Sin[4*e]]))))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e +
 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(144*a^6*f*x*Cos[2*e] + 96*a^5*b*f*x*Cos[2
*e] + 912*a^4*b^2*f*x*Cos[2*e] + 6720*a^3*b^3*f*x*Cos[2*e] + 16512*a^2*b^4*
f*x*Cos[2*e] + 16896*a*b^5*f*x*Cos[2*e] + 6144*b^6*f*x*Cos[2*e] + 96*a^6*f*
x*Cos[2*f*x] + 480*a^4*b^2*f*x*Cos[2*f*x] + 4416*a^3*b^3*f*x*Cos[2*f*x] + 6
912*a^2*b^4*f*x*Cos[2*f*x] + 3072*a*b^5*f*x*Cos[2*f*x] + 96*a^6*f*x*Cos[4*e
 + 2*f*x] + 480*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 4416*a^3*b^3*f*x*Cos[4*e + 2
*f*x] + 6912*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 3072*a*b^5*f*x*Cos[4*e + 2*f*x]
 + 24*a^6*f*x*Cos[2*e + 4*f*x] - 48*a^5*b*f*x*Cos[2*e + 4*f*x] + 216*a^4*b^
2*f*x*Cos[2*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[2*e + 4*f*x] + 384*a^2*b^4*f*x
*Cos[2*e + 4*f*x] + 24*a^6*f*x*Cos[6*e + 4*f*x] - 48*a^5*b*f*x*Cos[6*e + 4*
f*x] + 216*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[6*e + 4*f*x]
+ 384*a^2*b^4*f*x*Cos[6*e + 4*f*x] + 816*a^3*b^3*Sin[2*e] + 2848*a^2*b^4*Si
n[2*e] + 3968*a*b^5*Sin[2*e] + 1792*b^6*Sin[2*e] + 44*a^6*Sin[2*f*x] + 104*
a^5*b*Sin[2*f*x] - 180*a^4*b^2*Sin[2*f*x] - 1696*a^3*b^3*Sin[2*f*x] - 3264*
a^2*b^4*Sin[2*f*x] - 1664*a*b^5*Sin[2*f*x] + 44*a^6*Sin[4*e + 2*f*x] + 104*
a^5*b*Sin[4*e + 2*f*x] - 180*a^4*b^2*Sin[4*e + 2*f*x] - 608*a^3*b^3*Sin[4*e
 + 2*f*x] - 192*a^2*b^4*Sin[4*e + 2*f*x] + 128*a*b^5*Sin[4*e + 2*f*x] + 38*
a^6*Sin[2*e + 4*f*x] + 60*a^5*b*Sin[2*e + 4*f*x] - 170*a^4*b^2*Sin[2*e + 4*
f*x] - 640*a^3*b^3*Sin[2*e + 4*f*x] - 400*a^2*b^4*Sin[2*e + 4*f*x] + 38*a^6
*Sin[6*e + 4*f*x] + 60*a^5*b*Sin[6*e + 4*f*x] - 170*a^4*b^2*Sin[6*e + 4*f*x
] - 368*a^3*b^3*Sin[6*e + 4*f*x] - 176*a^2*b^4*Sin[6*e + 4*f*x] + 12*a^6*Si
n[4*e + 6*f*x] + 8*a^5*b*Sin[4*e + 6*f*x] - 20*a^4*b^2*Sin[4*e + 6*f*x] - 1
6*a^3*b^3*Sin[4*e + 6*f*x] + 12*a^6*Sin[8*e + 6*f*x] + 8*a^5*b*Sin[8*e + 6*
f*x] - 20*a^4*b^2*Sin[8*e + 6*f*x] - 16*a^3*b^3*Sin[8*e + 6*f*x] + a^6*Sin[
6*e + 8*f*x] + 2*a^5*b*Sin[6*e + 8*f*x] + a^4*b^2*Sin[6*e + 8*f*x] + a^6*Si
n[10*e + 8*f*x] + 2*a^5*b*Sin[10*e + 8*f*x] + a^4*b^2*Sin[10*e + 8*f*x]))/(
2048*a^5*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.126, size = 470, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 3/8/f/a^3/(tan(f*x+e)^2+1)^2*tan(f*x+e)^3-3/2/f/a^4/(tan(f*x+e)^2+1)^2*tan(
f*x+e)^3*b-3/2/f/a^4/(tan(f*x+e)^2+1)^2*tan(f*x+e)*b+5/8/f/a^3/(tan(f*x+e)^
2+1)^2*tan(f*x+e)+6/f/a^5*arctan(tan(f*x+e))*b^2+3/8/f/a^3*arctan(tan(f*x+e
))-3/2/f/a^4*arctan(tan(f*x+e))*b-15/8/f*b^4/a^3/(a+b+b*tan(f*x+e)^2)^2/(a^
2+2*a*b+b^2)*tan(f*x+e)^3-3/2/f*b^5/a^4/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b
^2)*tan(f*x+e)^3-17/8/f*b^3/a^3/(a+b+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)-3/2
/f*b^4/a^4/(a+b+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)-63/8/f/a^3*b^3/(a^2+2*a*
b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-27/2/f*b^4/a^4/
(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-6/f*b^
5/a^5/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.880947, size = 2531, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x
 + e)^4 + 24*(a^5*b - 2*a^4*b^2 + 9*a^3*b^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*co
s(f*x + e)^2 + 12*(a^4*b^2 - 2*a^3*b^3 + 9*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x
 + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^
4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2*b^4 + 16*a*b^5)*cos(f*x + e)^2)*
sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b
^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*
cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)) + 4*(2*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7
+ (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3*b^3)*cos(f*x + e)^5 + (6*a^5*b - 10
*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos(f*x + e)^3 + 3*(a^4*b^2 - 2*a^3*b^3
```

```
 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*sin(f*x + e))/((a^9 + 2*a^8*b + a^7*
b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^2 +
(a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), 1/16*(6*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28
*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x + e)^4 + 12*(a^5*b - 2*a^4*b^2 + 9*a^3*b
^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*cos(f*x + e)^2 + 6*(a^4*b^2 - 2*a^3*b^3 + 9
*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21
*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2
*b^4 + 16*a*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(
f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*(a^6
+ 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3
*b^3)*cos(f*x + e)^5 + (6*a^5*b - 10*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos
(f*x + e)^3 + 3*(a^4*b^2 - 2*a^3*b^3 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*
sin(f*x + e))/((a^9 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^
7*b^2 + a^6*b^3)*f*cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37784, size = 663, normalized size = 2.46

$$-\frac{\frac{3\left(21\,a^2b^3+36\,ab^4+16\,b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^7+2\,a^6b+a^5b^2\right)\sqrt{ab+b^2}}-\frac{3\,a^3b^2\tan\left(fx+e\right)^7-6\,a^2b^3\tan\left(fx+e\right)^7-36\,ab^4\tan\left(fx+e\right)^7-24\,b^5\tan\left(fx+e\right)^7+6\,a^4b\,\mathrm{t}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 2*a^6*b + a^5*b^2)*sqr
t(a*b + b^2)) - (3*a^3*b^2*tan(f*x + e)^7 - 6*a^2*b^3*tan(f*x + e)^7 - 36*a
```

```
*b^4*tan(f*x + e)^7 - 24*b^5*tan(f*x + e)^7 + 6*a^4*b*tan(f*x + e)^5 - a^3*
b^2*tan(f*x + e)^5 - 73*a^2*b^3*tan(f*x + e)^5 - 144*a*b^4*tan(f*x + e)^5 -
 72*b^5*tan(f*x + e)^5 + 3*a^5*tan(f*x + e)^3 + 10*a^4*b*tan(f*x + e)^3 - 2
4*a^3*b^2*tan(f*x + e)^3 - 136*a^2*b^3*tan(f*x + e)^3 - 180*a*b^4*tan(f*x +
 e)^3 - 72*b^5*tan(f*x + e)^3 + 5*a^5*tan(f*x + e) + 8*a^4*b*tan(f*x + e) -
 18*a^3*b^2*tan(f*x + e) - 69*a^2*b^3*tan(f*x + e) - 72*a*b^4*tan(f*x + e)
- 24*b^5*tan(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)^4 + a*tan
(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2) - 3*(a^2 - 4*a*b + 16*b^2)*(f*
x + e)/a^5)/f
```

3.218 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=352

$$\frac{b^{7/2}\left(99a^2+176ab+80b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f(a+b)^{5/2}}+\frac{b\left(17a^2b^2-8a^3b+5a^4+116ab^3+80b^4\right)\tan(e+fx)}{16a^5f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b\left(-29a^2b+15\right.}{48a^4f(a+}$$

```
[Out] ((5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*x)/(16*a^6) + (b^(7/2)*(99*a^2 + 1
76*a*b + 80*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*(a + b)
^(5/2)*f) + ((15*a^2 - 34*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*
f*(a + b + b*Tan[e + f*x]^2)^2) + (5*(a - 2*b)*Cos[e + f*x]^3*Sin[e + f*x])
/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^5*Sin[e + f*x])/(6
*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(15*a^3 - 29*a^2*b + 64*a*b^2 + 120
*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(5
*a^4 - 8*a^3*b + 17*a^2*b^2 + 116*a*b^3 + 80*b^4)*Tan[e + f*x])/(16*a^5*(a
+ b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.475289, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4146, 414, 527, 522, 203, 205}

$$\frac{b^{7/2}\left(99a^2+176ab+80b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6f(a+b)^{5/2}}+\frac{b\left(17a^2b^2-8a^3b+5a^4+116ab^3+80b^4\right)\tan(e+fx)}{16a^5f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{b\left(-29a^2b+15\right.}{48a^4f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] ((5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*x)/(16*a^6) + (b^(7/2)*(99*a^2 + 1
76*a*b + 80*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(8*a^6*(a + b)
^(5/2)*f) + ((15*a^2 - 34*a*b + 80*b^2)*Cos[e + f*x]*Sin[e + f*x])/(48*a^3*
f*(a + b + b*Tan[e + f*x]^2)^2) + (5*(a - 2*b)*Cos[e + f*x]^3*Sin[e + f*x])
/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^2) + (Cos[e + f*x]^5*Sin[e + f*x])/(6
*a*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(15*a^3 - 29*a^2*b + 64*a*b^2 + 120
*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) + (b*(5
*a^4 - 8*a^3*b + 17*a^2*b^2 + 116*a*b^3 + 80*b^4)*Tan[e + f*x])/(16*a^5*(a
+ b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-5a+b-9bx^2}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{15a^2+ab+10b}{\left(1+x^2\right)^2(}\right.}{}$$

$$= \frac{\left(15a^2-34ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{co}}{6af}$$

$$= \frac{\left(15a^2-34ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{co}}{6af}$$

$$= \frac{\left(15a^2-34ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{co}}{6af}$$

$$= \frac{\left(15a^2-34ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{co}}{6af}$$

$$= \frac{\left(5a^3-18a^2b+48ab^2-160b^3\right)x}{16a^6} + \frac{b^{7/2}\left(99a^2+176ab+80b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6(a+b)^{5/2}f} + \frac{(15a}{}$$

Mathematica [C] time = 6.65895, size = 1770, normalized size = 5.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

[Out] ((99*a^2 + 176*a*b + 80*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-(b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^6*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^6*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(720*a^7*f*x*Cos[2*e] + 768*a^6*b*f*x*Cos[2*e] + 1296*a^5*b^2*f*x*Cos[2*e] - 8352*a^4*b^3*f*x*Cos[2*e] - 64128*a^3*b^4*f*x*Cos[2*e] - 158976*a^2*b^5*f*x*Cos[2*e] - 165888*a*b^6*f*x*Cos[2*e] - 61440*b^7*f*x*Cos[2*e] + 480*a^7*f*x*Cos[2*f*x] + 192*a^6*b*f*x*Cos[2*f*x] + 96*a^5*b^2*f*x*Cos[2*f*x] - 4608*a^4*b^3*f*x*Cos[2*f*x] - 41856*a^3*b^4*f*x*Cos[2*f*x] - 67584*a^2*b^5*f*x*Cos[2*f*x] - 30720*a*b^6*f*x*Cos[2*f*x] + 480*a^7*f*x*Cos[4*e + 2*f*x] + 192*a^6*b*f*x*Cos[4*e + 2*f*x] + 96*a^5*b^2*f*x*Cos[4*e + 2*f*x] - 4608*a^4*b^3*f*x*Cos[4*e + 2*f*x] - 41856*a^3*b^4*f*x*Cos[4*e + 2*f*x] - 67584*a^2*b^5*f*x*Cos[4*e + 2*f*x] - 30720*a*b^6*f*x*Cos[4*e + 2*f*x] + 120*a^7*f*x*Cos[2*e + 4*f*x] - 192*a^6*b*f*x*Cos[2*e + 4*f*x] + 408*a^5*b^2*f*x*Cos[2*e + 4*f*x] - 1968*a^4*b^3*f*x*Cos[2*e + 4*f*x] - 6528*a^3*b^4*f*x*Cos[2*e + 4*f*x] - 3840*a^2*b^5*f*x*Cos[2*e + 4*f*x] + 120*a^7*f*x*Cos[6*e + 4*f*x] - 192*a^6*b*f*x*Cos[6*e + 4*f*x] + 408*a^5*b^2*f*x*Cos[6*e + 4*f*x] - 1968*a^4*b^3*f*x*Cos[6*e + 4*f*x] - 6528*a^3*b^4*f*x*Cos[6*e + 4*f*x] - 3840*a^2*b^5*f*x*Cos[6*e + 4*f*x] - 6048*a^3*b^4*Sin[2*e] - 21312*a^2*b^5*Sin[2*e] - 29952*a*b^6*Sin[2*e] - 13824*b^7*Sin[2*e] + 262*a^7*Sin[2*f*x] + 524*a^6*b*Sin[2*f*x] - 26*a^5*b^2*Sin[2*f*x] + 1728*a^4*b^3*Sin[2*f*x] + 14976*a^3*b^4*Sin[2*f*x] + 28416*a^2*b^5*Sin[2*f*x] + 14592*a*b^6*Sin[2*f*x] + 262*a^7*Sin[4*e + 2*f*x] + 524*a^6*b*Sin[4*e + 2*f*x] - 26*a^5*b^2*Sin[4*e + 2*f*x] + 1728*a^4*b^3*Sin[4*e + 2*f*x] + 6912*a^3*b^4*Sin[4*e + 2*f*x] + 5376*a^2*b^5*Sin[4*e + 2*f*x] + 768*a*b^6*Sin[4*e + 2*f*x] + 238*a^7*Sin[2*e + 4*f*x] + 304*a^6*b*Sin[2*e + 4*f*x] - 250*a^5*b^2*Sin[2*e + 4*f*x] + 1556*a^4*b^3*Sin[2*e + 4*f*x] + 5904*a^3*b^4*Sin[2*e + 4*f*x] + 3744*a^2*b^5*Sin[2*e + 4*f*x] + 238*a^7*Sin[6*e + 4*f*x] + 304*a^6*b*Sin[6*e + 4*f*x] - 250*a^5*b^2*Sin[6*e + 4*f*x] + 1556*a^4*b^3*Sin[6*e + 4*f*x] + 3888*a^3*b^4*Sin[6*e + 4*f*x] + 2016*a^2*b^5*Sin[6*e + 4*f*x] + 87*a^7*Sin[4*e + 6*f*x] + 46*a^6*b*Sin[4*e + 6*f*x] - 9*a^5*b^2*Sin[4*e + 6*f*x] + 192*a^4*b^3*Sin[4*e + 6*f*x] + 160*a^3*b^4*Sin[4*e + 6*f*x] + 87*a^7*Sin[8*e + 6*f*x] + 46*a^6*b*Sin[8*e + 6*f*x] - 9*a^5*b^2*Sin[8*e + 6*f*x] + 192*a^4*b^3*Sin[8*e + 6*f*x] + 160*a^3*b^4*Sin[8*e + 6*f*x] + 13*a^7*Sin[6*e + 8*f*x] + 16*a^6*b*Sin[6*e + 8*f*x] - 7*a^5*b^2*Sin[6*e + 8*f*x] - 10*a^4*b^3*Sin[6*e + 8*f*x] + 13*a^7*Sin[10*e + 8*f*x] + 16*a^6*b*Sin[10*e + 8*f*x] - 7*a^5*b^2*Sin[10*e + 8*f*x] - 10*a^4*b^3*Sin[10*e + 8*f*x] + a^7*Sin[8*e + 10*f*x] + 2*a^6*b*Sin[8*e + 10*f*x] + a^5*b^2*Sin[8*e + 10*f*x] + a^7*Sin[12*e + 10*f*x] + 2*a^6*b*Sin[12*e + 10*f*x] + a^5*b^2*Sin[12*e + 10*f*x]))/(12288*a^6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)

Maple [A] time = 0.114, size = 636, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 5/16/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5-9/8/f/a^4/(tan(f*x+e)^2+1)^3*tan
(f*x+e)^5*b+3/f/a^5/(tan(f*x+e)^2+1)^3*tan(f*x+e)^5*b^2+6/f/a^5/(tan(f*x+e)
^2+1)^3*tan(f*x+e)^3*b^2+5/6/f/a^3/(tan(f*x+e)^2+1)^3*tan(f*x+e)^3-3/f/a^4/
(tan(f*x+e)^2+1)^3*tan(f*x+e)^3*b-15/8/f/a^4/(tan(f*x+e)^2+1)^3*tan(f*x+e)*
b+3/f/a^5/(tan(f*x+e)^2+1)^3*tan(f*x+e)*b^2+11/16/f/a^3/(tan(f*x+e)^2+1)^3*
tan(f*x+e)-10/f/a^6*arctan(tan(f*x+e))*b^3+5/16/f/a^3*arctan(tan(f*x+e))-9/
8/f/a^4*arctan(tan(f*x+e))*b+3/f/a^5*arctan(tan(f*x+e))*b^2+19/8/f*b^5/a^4/
(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3+2/f*b^6/a^5/(a+b+b*tan(
f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*x+e)^3+21/8/f*b^4/a^4/(a+b+b*tan(f*x+e)^2
)^2/(a+b)*tan(f*x+e)+2/f*b^5/a^5/(a+b+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)+99
/8/f*b^4/a^4/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^
(1/2))+22/f*b^5/a^5/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a
+b)*b)^(1/2))+10/f*b^6/a^6/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(tan(f*x+e
)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.999743, size = 2954, normalized size = 8.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(5*a^7 - 8*a^6*b + 17*a^5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2
*b^5)*f*x*cos(f*x + e)^4 + 12*(5*a^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^
4 - 272*a^2*b^5 - 160*a*b^6)*f*x*cos(f*x + e)^2 + 6*(5*a^5*b^2 - 8*a^4*b^3
+ 17*a^3*b^4 - 82*a^2*b^5 - 272*a*b^6 - 160*b^7)*f*x + 3*(99*a^2*b^5 + 176*
a*b^6 + 80*b^7 + (99*a^4*b^3 + 176*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^4 + 2
*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*b^6)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log
(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 -
 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-
b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
 b^2)) + 2*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 10*(a^7 - 3*a^5*b^
2 - 2*a^4*b^3)*cos(f*x + e)^7 + (15*a^7 - 4*a^6*b + 27*a^5*b^2 + 126*a^4*b^
3 + 80*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 19*a^5*b^2 + 43*a^4*b^3 + 26
6*a^3*b^4 + 180*a^2*b^5)*cos(f*x + e)^3 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3
*b^4 + 116*a^2*b^5 + 80*a*b^6)*cos(f*x + e))*sin(f*x + e))/((a^10 + 2*a^9*b
 + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x +
e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f), 1/48*(3*(5*a^7 - 8*a^6*b + 17*a^
5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2*b^5)*f*x*cos(f*x + e)^4 + 6*(5*a
^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^4 - 272*a^2*b^5 - 160*a*b^6)*f*x*c
os(f*x + e)^2 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 - 82*a^2*b^5 - 272*a*
b^6 - 160*b^7)*f*x - 3*(99*a^2*b^5 + 176*a*b^6 + 80*b^7 + (99*a^4*b^3 + 176
*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^4 + 2*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*
b^6)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (8*(a^7 + 2*a^6*b + a^
5*b^2)*cos(f*x + e)^9 + 10*(a^7 - 3*a^5*b^2 - 2*a^4*b^3)*cos(f*x + e)^7 + (
15*a^7 - 4*a^6*b + 27*a^5*b^2 + 126*a^4*b^3 + 80*a^3*b^4)*cos(f*x + e)^5 +
2*(15*a^6*b - 19*a^5*b^2 + 43*a^4*b^3 + 266*a^3*b^4 + 180*a^2*b^5)*cos(f*x
+ e)^3 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 + 116*a^2*b^5 + 80*a*b^6)*co
s(f*x + e))*sin(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*
(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6
*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36004, size = 498, normalized size = 1.41

$$\frac{\frac{6\left(99\,a^{2}b^{4}+176\,ab^{5}+80\,b^{6}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{8}+2\,a^{7}b+a^{6}b^{2}\right)\sqrt{ab+b^{2}}}+\frac{6\left(19\,ab^{5}\tan\left(fx+e\right)^{3}+16\,b^{6}\tan\left(fx+e\right)^{3}+21\,a^{2}b^{4}\tan\left(fx+e\right)+37\,ab^{5}\tan\left(fx+e\right)+16\,b^{6}\right.}{\left(a^{7}+2\,a^{6}b+a^{5}b^{2}\right)\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 2*a^7*b + a^6*b^2)*sq
rt(a*b + b^2)) + 6*(19*a*b^5*tan(f*x + e)^3 + 16*b^6*tan(f*x + e)^3 + 21*a^
2*b^4*tan(f*x + e) + 37*a*b^5*tan(f*x + e) + 16*b^6*tan(f*x + e))/((a^7 + 2
*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 3*(5*a^3 - 18*a^2*b + 48*
a*b^2 - 160*b^3)*(f*x + e)/a^6 + (15*a^2*tan(f*x + e)^5 - 54*a*b*tan(f*x +
e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 144*a*b*tan(f*x + e
)^3 + 288*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 90*a*b*tan(f*x + e) +
144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

3.219 $\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{b}\left(70a^2b+35a^3+56ab^2+16b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{16a^3d(a+b)^3\left(a+b\tan^2(c+dx)+b\right)}-\frac{b(11a+6b}{24a^2d(a+b)^2\left(a+\right.}$$

```
[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTan[(Sqrt[b]*Ta
n[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(6*a*
(a + b)*d*(a + b + b*Tan[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tan[c + d*x])/(24
*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b + 8*b^
2)*Tan[c + d*x])/(16*a^3*(a + b)^3*d*(a + b + b*Tan[c + d*x]^2))
```

Rubi [A] time = 0.334845, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(70a^2b+35a^3+56ab^2+16b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4d(a+b)^{7/2}}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{16a^3d(a+b)^3\left(a+b\tan^2(c+dx)+b\right)}-\frac{b(11a+6b}{24a^2d(a+b)^2\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] x/a^4 - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*ArcTan[(Sqrt[b]*Ta
n[c + d*x])/Sqrt[a + b]])/(16*a^4*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(6*a*
(a + b)*d*(a + b + b*Tan[c + d*x]^2)^3) - (b*(11*a + 6*b)*Tan[c + d*x])/(24
*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^2) - (b*(19*a^2 + 22*a*b + 8*b^
2)*Tan[c + d*x])/(16*a^3*(a + b)^3*d*(a + b + b*Tan[c + d*x]^2))
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} + \frac{\text{Subst}\left(\int \frac{6a+b-5bx^2}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(c+dx)\right)}{6a(a+b)d}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} + \text{Subs}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} - \frac{b(1}{16a^3(}$$

$$= -\frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} - \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} - \frac{b(1}{16a^3(}$$

$$= \frac{x}{a^4} - \frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} - \frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2\right.}$$

Mathematica [C] time = 6.85346, size = 1411, normalized size = 6.92

$$\frac{\left(35a^3+70ba^2+56b^2a+16b^3\right)(\cos(2c+2dx)a+a+2b)^4\left(\frac{b\tan^{-1}\left(\sec(dx)\left(\frac{\cos(2c)}{2\sqrt{a+b}\sqrt{b\cos(4c)-ib\sin(4c)}}-\frac{i\sin(2c)}{2\sqrt{a+b}\sqrt{b\cos(4c)-ib\sin(4c)}}\right)(-a\sin(\right.}{256a^4\sqrt{a+b}d\sqrt{b\cos(4c)-ib\sin(4c)}}\right.}{(a+b)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(-4),x]
```

```
[Out] ((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cos[2*c + 2*d*x])^4*S
ec[c + d*x]^8*((b*ArcTan[Sec[d*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c]
- I*b*Sin[4*c]]) - ((I/2)*Sin[2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[
4*c]]))*(-(a*Sin[d*x]) - 2*b*Sin[d*x] + a*Sin[2*c + d*x])]*Cos[2*c])/(256*a
^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/256)*b*ArcTan[Sec[d
*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/2)*Sin[
```

```
2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]))*(-(a*Sin[d*x]) - 2*b*S
in[d*x] + a*Sin[2*c + d*x])]*Sin[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] -
 I*b*Sin[4*c]])))/((a + b)^3*(a + b*Sec[c + d*x]^2)^4) + ((a + 2*b + a*Cos[
2*c + 2*d*x])*Sec[2*c]*Sec[c + d*x]^8*(480*a^6*d*x*Cos[2*c] + 3168*a^5*b*d*
x*Cos[2*c] + 8928*a^4*b^2*d*x*Cos[2*c] + 14112*a^3*b^3*d*x*Cos[2*c] + 13248
*a^2*b^4*d*x*Cos[2*c] + 6912*a*b^5*d*x*Cos[2*c] + 1536*b^6*d*x*Cos[2*c] + 3
60*a^6*d*x*Cos[2*d*x] + 2232*a^5*b*d*x*Cos[2*d*x] + 5688*a^4*b^2*d*x*Cos[2*
d*x] + 7272*a^3*b^3*d*x*Cos[2*d*x] + 4608*a^2*b^4*d*x*Cos[2*d*x] + 1152*a*b
^5*d*x*Cos[2*d*x] + 360*a^6*d*x*Cos[4*c + 2*d*x] + 2232*a^5*b*d*x*Cos[4*c +
 2*d*x] + 5688*a^4*b^2*d*x*Cos[4*c + 2*d*x] + 7272*a^3*b^3*d*x*Cos[4*c + 2*
d*x] + 4608*a^2*b^4*d*x*Cos[4*c + 2*d*x] + 1152*a*b^5*d*x*Cos[4*c + 2*d*x]
+ 144*a^6*d*x*Cos[2*c + 4*d*x] + 720*a^5*b*d*x*Cos[2*c + 4*d*x] + 1296*a^4*
b^2*d*x*Cos[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[2*c + 4*d*x] + 288*a^2*b^4*
d*x*Cos[2*c + 4*d*x] + 144*a^6*d*x*Cos[6*c + 4*d*x] + 720*a^5*b*d*x*Cos[6*c
 + 4*d*x] + 1296*a^4*b^2*d*x*Cos[6*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[6*c +
4*d*x] + 288*a^2*b^4*d*x*Cos[6*c + 4*d*x] + 24*a^6*d*x*Cos[4*c + 6*d*x] + 7
2*a^5*b*d*x*Cos[4*c + 6*d*x] + 72*a^4*b^2*d*x*Cos[4*c + 6*d*x] + 24*a^3*b^3
*d*x*Cos[4*c + 6*d*x] + 24*a^6*d*x*Cos[8*c + 6*d*x] + 72*a^5*b*d*x*Cos[8*c
+ 6*d*x] + 72*a^4*b^2*d*x*Cos[8*c + 6*d*x] + 24*a^3*b^3*d*x*Cos[8*c + 6*d*x
] + 870*a^5*b*Sin[2*c] + 4292*a^4*b^2*Sin[2*c] + 8792*a^3*b^3*Sin[2*c] + 99
36*a^2*b^4*Sin[2*c] + 5824*a*b^5*Sin[2*c] + 1408*b^6*Sin[2*c] - 870*a^5*b*S
in[2*d*x] - 3792*a^4*b^2*Sin[2*d*x] - 6432*a^3*b^3*Sin[2*d*x] - 4608*a^2*b^
4*Sin[2*d*x] - 1248*a*b^5*Sin[2*d*x] + 435*a^5*b*Sin[4*c + 2*d*x] + 2124*a^
4*b^2*Sin[4*c + 2*d*x] + 3972*a^3*b^3*Sin[4*c + 2*d*x] + 3072*a^2*b^4*Sin[4
*c + 2*d*x] + 864*a*b^5*Sin[4*c + 2*d*x] - 435*a^5*b*Sin[2*c + 4*d*x] - 137
4*a^4*b^2*Sin[2*c + 4*d*x] - 1248*a^3*b^3*Sin[2*c + 4*d*x] - 384*a^2*b^4*Si
n[2*c + 4*d*x] + 87*a^5*b*Sin[6*c + 4*d*x] + 366*a^4*b^2*Sin[6*c + 4*d*x] +
 408*a^3*b^3*Sin[6*c + 4*d*x] + 144*a^2*b^4*Sin[6*c + 4*d*x] - 87*a^5*b*Sin
[4*c + 6*d*x] - 116*a^4*b^2*Sin[4*c + 6*d*x] - 44*a^3*b^3*Sin[4*c + 6*d*x])
)/(3072*a^4*(a + b)^3*d*(a + b*Sec[c + d*x]^2)^4)
```

Maple [B] time = 0.092, size = 649, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)^2)^4,x)
```

```
[Out] 1/d/a^4*arctan(tan(d*x+c))-19/16/d*b^3/a/(a+b+b*tan(d*x+c)^2)^3/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(d*x+c)^5-11/8/d*b^4/a^2/(a+b+b*tan(d*x+c)^2)^3/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(d*x+c)^5-1/2/d*b^5/a^3/(a+b+b*tan(d*x+c)^2)^3/(a^3+3*a
```

```
^2*b+3*a*b^2+b^3)*tan(d*x+c)^5-17/6/d*b^2/a/(a+b+b*tan(d*x+c)^2)^3/(a^2+2*a
*b+b^2)*tan(d*x+c)^3-3/d*b^3/a^2/(a+b+b*tan(d*x+c)^2)^3/(a^2+2*a*b+b^2)*tan
(d*x+c)^3-1/d*b^4/a^3/(a+b+b*tan(d*x+c)^2)^3/(a^2+2*a*b+b^2)*tan(d*x+c)^3-2
9/16*b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(d*x+c)^2)^3-13/8/d*b^2/a^2/(a+b+b*ta
n(d*x+c)^2)^3/(a+b)*tan(d*x+c)-1/2/d*b^3/a^3/(a+b+b*tan(d*x+c)^2)^3/(a+b)*t
an(d*x+c)-35/16/d*b/a/(a^3+3*a^2*b+3*a*b^2+b^3)/((a+b)*b)^(1/2)*arctan(tan(
d*x+c)*b/((a+b)*b)^(1/2))-35/8/d*b^2/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/((a+b)*b
)^(1/2)*arctan(tan(d*x+c)*b/((a+b)*b)^(1/2))-7/2/d*b^3/a^3/(a^3+3*a^2*b+3*a
*b^2+b^3)/((a+b)*b)^(1/2)*arctan(tan(d*x+c)*b/((a+b)*b)^(1/2))-1/d*b^4/a^4/
(a^3+3*a^2*b+3*a*b^2+b^3)/((a+b)*b)^(1/2)*arctan(tan(d*x+c)*b/((a+b)*b)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.883143, size = 2967, normalized size = 14.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(192*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 576*
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*x*cos(d*x + c)^4 + 576*(a^4*b^2
 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*x*cos(d*x + c)^2 + 192*(a^3*b^3 + 3*a^2
*b^4 + 3*a*b^5 + b^6)*d*x + 3*((35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3
)*cos(d*x + c)^6 + 35*a^3*b^3 + 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35*a^5*
b + 70*a^4*b^2 + 56*a^3*b^3 + 16*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*b^2 +
70*a^3*b^3 + 56*a^2*b^4 + 16*a*b^5)*cos(d*x + c)^2)*sqrt(-b/(a + b))*log(((
a^2 + 8*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a*b + 4*b^2)*cos(d*x + c)^2 + 4*
((a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^3 - (a*b + b^2)*cos(d*x + c))*sqrt(-b/(
a + b))*sin(d*x + c) + b^2)/(a^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^2 + b^
```

```
2)) - 4*((87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 + 2*(68*a^4*b
^2 + 83*a^3*b^3 + 30*a^2*b^4)*cos(d*x + c)^3 + 3*(19*a^3*b^3 + 22*a^2*b^4 +
 8*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^
3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x
 + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 +
(a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d), 1/96*(96*(a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 288*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3
 + a^2*b^4)*d*x*cos(d*x + c)^4 + 288*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b
^5)*d*x*cos(d*x + c)^2 + 96*(a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*d*x + 3*(
(35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3)*cos(d*x + c)^6 + 35*a^3*b^3 +
 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35*a^5*b + 70*a^4*b^2 + 56*a^3*b^3 + 1
6*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*b^2 + 70*a^3*b^3 + 56*a^2*b^4 + 16*a*
b^5)*cos(d*x + c)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(d*x + c)^2 -
 b)*sqrt(b/(a + b))/(b*cos(d*x + c)*sin(d*x + c))) - 2*((87*a^5*b + 116*a^4
*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 + 2*(68*a^4*b^2 + 83*a^3*b^3 + 30*a^2*b^4
)*cos(d*x + c)^3 + 3*(19*a^3*b^3 + 22*a^2*b^4 + 8*a*b^5)*cos(d*x + c))*sin(
d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9
*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7
*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5
*b^5 + a^4*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32772, size = 437, normalized size = 2.14

$$-\frac{\frac{3\left(35\,a^3b+70\,a^2b^2+56\,ab^3+16\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)\sqrt{ab+b^2}}+\frac{57\,a^2b^3\tan(dx+c)^5+66\,ab^4\tan(dx+c)^5+24\,b^5\tan(dx+c)^5+136\,a^3b^2\tan(dx+c)\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] -1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*(pi*floor((d*x + c)/pi
 + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b + b^2)))/((a^7 + 3*a^6*b +
3*a^5*b^2 + a^4*b^3)*sqrt(a*b + b^2)) + (57*a^2*b^3*tan(d*x + c)^5 + 66*a*b
^4*tan(d*x + c)^5 + 24*b^5*tan(d*x + c)^5 + 136*a^3*b^2*tan(d*x + c)^3 + 28
0*a^2*b^3*tan(d*x + c)^3 + 192*a*b^4*tan(d*x + c)^3 + 48*b^5*tan(d*x + c)^3
 + 87*a^4*b*tan(d*x + c) + 252*a^3*b^2*tan(d*x + c) + 267*a^2*b^3*tan(d*x +
 c) + 126*a*b^4*tan(d*x + c) + 24*b^5*tan(d*x + c))/((a^6 + 3*a^5*b + 3*a^4
*b^2 + a^3*b^3)*(b*tan(d*x + c)^2 + a + b)^3) - 48*(d*x + c)/a^4)/d
```

3.220 $\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}+\frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a^3\cot(c+$$

```
[Out] -((a^3*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^3*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^3*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d) - (a^3*Tan[c + d*x]^5*Sqrt[-(a*Tan[c + d*x]^2)])/
(6*d)
```

Rubi [A] time = 0.0623407, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}+\frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a^3\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

```
[Out] -((a^3*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^3*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^3*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d) - (a^3*Tan[c + d*x]^5*Sqrt[-(a*Tan[c + d*x]^2)])/
(6*d)
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a-a\sec^2(c+dx)\right)^{7/2}\,dx=\int \left(-a\tan^2(c+dx)\right)^{7/2}\,dx$$
$$=-\left(\left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^7(c+dx)\,dx\right)$$
$$=-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}+\left(a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^5(c+dx$$
$$=\frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}-\left(a^3\cot(c+dx\right.$$
$$=-\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^3\tan^5(c+dx}{}$$
$$=-\frac{a^3\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-$$

Mathematica [A] time = 2.20579, size = 70, normalized size = 0.52

$$\frac{\cot^7(c+dx)\left(-a\tan^2(c+dx)\right)^{7/2}\left(2\tan^6(c+dx)-3\tan^4(c+dx)+6\tan^2(c+dx)+12\log(\cos(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

```
[Out] (Cot[c + d*x]^7*(-(a*Tan[c + d*x]^2))^(7/2)*(12*Log[Cos[c + d*x]] + 6*Tan[c
 + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

Maple [A] time = 0.337, size = 167, normalized size = 1.3

$$\frac{\cos(dx+c)}{12\,d\,(\sin(dx+c))^7}\left(12\,(\cos(dx+c))^6\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+12\,(\cos(dx+c))^6\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(7/2),x)
```

```
[Out] 1/12/d*(12*cos(d*x+c)^6*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*cos(d*x
+c)^6*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-12*cos(d*x+c)^6*ln(2/(cos(
d*x+c)+1))-11*cos(d*x+c)^6+18*cos(d*x+c)^4-9*cos(d*x+c)^2+2)*cos(d*x+c)*(-a
*sin(d*x+c)^2/cos(d*x+c)^2)^(7/2)/sin(d*x+c)^7
```

Maxima [A] time = 1.46337, size = 109, normalized size = 0.81

$$-\frac{2\sqrt{-a}a^3\tan(dx+c)^6-3\sqrt{-a}a^3\tan(dx+c)^4+6\sqrt{-a}a^3\tan(dx+c)^2-6\sqrt{-a}a^3\log\left(\tan(dx+c)^2+1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/12*(2*sqrt(-a)*a^3*tan(d*x + c)^6 - 3*sqrt(-a)*a^3*tan(d*x + c)^4 + 6*sq
rt(-a)*a^3*tan(d*x + c)^2 - 6*sqrt(-a)*a^3*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.528424, size = 244, normalized size = 1.82

$$-\frac{\left(12\,a^3\cos(dx+c)^6\log(-\cos(dx+c))+18\,a^3\cos(dx+c)^4-9\,a^3\cos(dx+c)^2+2\,a^3\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{12\,d\cos(dx+c)^5\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a^3*cos(d*x + c)^6*log(-cos(d*x + c)) + 18*a^3*cos(d*x + c)^4 - 9
*a^3*cos(d*x + c)^2 + 2*a^3)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d
*cos(d*x + c)^5*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.70824, size = 293, normalized size = 2.19

$$-\frac{6\sqrt{-a}a^3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+2\right)-6\sqrt{-a}a^3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}-2\right)+\frac{11\left(\tan\left(\frac{1}{2}c\right.\right.}{}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 + 2) - 6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^
2 - 2) + (11*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^3*sqrt(-a)
*a^3 - 90*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a)*a^
3 + 276*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^3 -
408*sqrt(-a)*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)^3
)/d
```

3.221 $\int \left(a - a\sec^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=101

$$\frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[Out] -((a^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^2*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^2*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d)
```

Rubi [A] time = 0.0513671, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$\frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}-\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] -((a^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a^2*
Tan[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d) + (a^2*Tan[c + d*x]^3*Sqrt[-(
a*Tan[c + d*x]^2)])/(4*d)
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2} dx = \int \left(-a\tan^2(c+dx)\right)^{5/2} dx$$
$$= \left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^5(c+dx)\,dx$$
$$= \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \left(a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,d$$
$$= -\frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} + \left(a^2\cot(c+dx\right.$$
$$= -\frac{a^2\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}$$

Mathematica [A] time = 0.554348, size = 60, normalized size = 0.59

$$-\frac{\cot^5(c+dx)\left(-a\tan^2(c+dx)\right)^{5/2}\left(-\tan^4(c+dx)+2\tan^2(c+dx)+4\log(\cos(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] -(Cot[c + d*x]^5*(-(a*Tan[c + d*x]^2))^(5/2)*(4*Log[Cos[c + d*x]] + 2*Tan[c
 + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

Maple [A] time = 0.304, size = 157, normalized size = 1.6

$$-\frac{\cos(dx+c)}{4\,d\,(\sin(dx+c))^5}\left(4\,(\cos(dx+c))^4\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+4\,(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(5/2),x)
```

```
[Out] -1/4/d*(4*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*cos(d*x+c
)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*cos(d*x+c)^4*ln(2/(cos(d*x
+c)+1))-3*cos(d*x+c)^4+4*cos(d*x+c)^2-1)*cos(d*x+c)*(-a*sin(d*x+c)^2/cos(d*
x+c)^2)^(5/2)/sin(d*x+c)^5
```

Maxima [A] time = 1.51641, size = 84, normalized size = 0.83

$$\frac{\sqrt{-a}a^2\tan(dx+c)^4-2\sqrt{-a}a^2\tan(dx+c)^2+2\sqrt{-a}a^2\log\left(\tan(dx+c)^2+1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(-a)*a^2*tan(d*x + c)^4 - 2*sqrt(-a)*a^2*tan(d*x + c)^2 + 2*sqrt(-
a)*a^2*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.510939, size = 207, normalized size = 2.05

$$-\frac{\left(4\,a^2\cos(dx+c)^4\log(-\cos(dx+c))+4\,a^2\cos(dx+c)^2-a^2\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{4\,d\cos(dx+c)^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*a^2*cos(d*x + c)^2 - a^2)
*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c)
```

)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63818, size = 246, normalized size = 2.44

$$-\frac{2\sqrt{-a}a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-2\sqrt{-a}a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)+\frac{3\left(\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
+ 2) - 2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 - 2) + (3*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a)*a
^2 - 20*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^2 +
44*sqrt(-a)*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)^2)
/d
```

3.222 $\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=64

$$-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a*Tan[
c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d)
```

Rubi [A] time = 0.0407729, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}-\frac{a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

```
[Out] -((a*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d) - (a*Tan[
c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)])/(2*d)
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sec^2(c+dx)\right)^{3/2}\,dx &= \int \left(-a\tan^2(c+dx)\right)^{3/2}\,dx\\ &= -\left(\left(a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx\right)\\ &= -\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}+\left(a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}-\frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}\end{aligned}$$

Mathematica [A] time = 0.107481, size = 48, normalized size = 0.75

$$\frac{\cot^3(c+dx)\left(-a\tan^2(c+dx)\right)^{3/2}\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

```
[Out] (Cot[c + d*x]^3*(-(a*Tan[c + d*x]^2))^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c +
d*x]^2))/(2*d)
```

Maple [B] time = 0.264, size = 145, normalized size = 2.3

$$-\frac{\cos(dx+c)}{2\,d\,(\sin(dx+c))^3}\left(2\,(\cos(dx+c))^2\ln\left(2\,(\cos(dx+c)+1)^{-1}\right)-2\,(\cos(dx+c))^2\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(3/2),x)
```

```
[Out] -1/2/d*(2*cos(d*x+c)^2*ln(2/(cos(d*x+c)+1))-2*cos(d*x+c)^2*ln((1-cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))-2*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(
d*x+c))+cos(d*x+c)^2-1)*cos(d*x+c)*(-a*sin(d*x+c)^2/cos(d*x+c)^2)^(3/2)/sin
(d*x+c)^3
```

Maxima [A] time = 1.48758, size = 54, normalized size = 0.84

$$-\frac{\sqrt{-a}a\tan\left(dx+c\right)^{2}-\sqrt{-a}a\log\left(\tan\left(dx+c\right)^{2}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(-a)*a*tan(d*x + c)^2 - sqrt(-a)*a*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.528953, size = 167, normalized size = 2.61

$$-\frac{\left(2\,a\cos\left(dx+c\right)^{2}\log\left(-\cos\left(dx+c\right)\right)+a\right)\sqrt{\frac{a\cos(dx+c)^{2}-a}{\cos(dx+c)^{2}}}}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(d*x + c)^2*log(-cos(d*x + c)) + a)*sqrt((a*cos(d*x + c)^2 - a
)/cos(d*x + c)^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a\sec^{2}\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(3/2), x)
```

Giac [B] time = 1.40452, size = 185, normalized size = 2.89

$$-\frac{\sqrt{-a}a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-\sqrt{-a}a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)+\frac{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\cdots\right.}{\tan\left(\frac{1}{2}\,dx+\cdots\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 + 2)
 - sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2) +
((tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a - 6*sqrt(-a
)*a)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2))/d
```

3.223 $\int \sqrt{a - a\sec^2(c + dx)}\, dx$

Optimal. Leaf size=33

$$-\frac{\cot(c + dx)\sqrt{-a\tan^2(c + dx)}\log(\cos(c + dx))}{d}$$

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Rubi [A] time = 0.0300599, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4121, 3658, 3475}

$$-\frac{\cot(c + dx)\sqrt{-a\tan^2(c + dx)}\log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a-a\sec^2(c+dx)}\,dx = \int \sqrt{-a\tan^2(c+dx)}\,dx$$
$$= \left(\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\right)\int \tan(c+dx)\,dx$$
$$= -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

Mathematica [A] time = 0.0368466, size = 33, normalized size = 1.

$$-\frac{\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

Maple [B] time = 0.347, size = 108, normalized size = 3.3

$$-\frac{\cos(dx+c)}{d\sin(dx+c)}\left(-\ln\left(2\left(\cos(dx+c)+1\right)^{-1}\right)+\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c)^2)^(1/2),x)
```

```
[Out] -1/d*(-ln(2/(cos(d*x+c)+1))+ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+ln(-(-
1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c)))*cos(d*x+c)*(-a*sin(d*x+c)^2/cos(d*x+c
)^2)^(1/2)/sin(d*x+c)
```

Maxima [A] time = 1.46501, size = 28, normalized size = 0.85

$$\frac{\sqrt{-a}\log\left(\tan(dx+c)^2+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a)*log(tan(d*x + c)^2 + 1)/d
```

Fricas [A] time = 0.50536, size = 128, normalized size = 3.88

$$-\frac{\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}\cos(dx+c)\log(-\cos(dx+c))}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(-cos(d*x + c)
)/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\sec^2(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sec(c + d*x)**2 + a), x)
```

Giac [B] time = 1.49128, size = 190, normalized size = 5.76

$$-\frac{\left(\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)\mathrm{sgn}\left(-\tan\left(\frac{1}{2}d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(-tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x
 + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(-tan(1/2*d*x + 1/2*c)^3
 - tan(1/2*d*x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(-tan(1/2*
d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))*sqrt(-a)*sgn(cos(d*x + c))/d
```

3.224 $\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rubi [A] time = 0.0349387, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4121, 3658, 3475}

$$\frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx &= \int \frac{1}{\sqrt{-a\tan^2(c+dx)}}\,dx\\ &= \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{\sqrt{-a\tan^2(c+dx)}}\\ &= \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0451237, size = 40, normalized size = 1.25

$$\frac{\tan(c+dx)(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

```
[Out] ((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c +
d*x]^2)])
```

Maple [B] time = 0.33, size = 75, normalized size = 2.3

$$\frac{\sin(dx+c)}{d\cos(dx+c)}\left(-\ln\left(2\left(\cos(dx+c)+1\right)^{-1}\right)+\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right)\frac{1}{\sqrt{-\frac{a(\sin(dx+c))^2}{(\cos(dx+c))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(1/2),x)
```

```
[Out] 1/d*(-ln(2/(cos(d*x+c)+1))+ln(-(-1+cos(d*x+c))/sin(d*x+c)))*sin(d*x+c)/(-a*
sin(d*x+c)^2/cos(d*x+c)^2)^(1/2)/cos(d*x+c)
```

Maxima [A] time = 1.51489, size = 50, normalized size = 1.56

$$-\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}}-\frac{2\log(\tan(dx+c))}{\sqrt{-a}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/sqrt(-a) - 2*log(tan(d*x + c))/sqrt(-a))/d
```

Fricas [A] time = 0.495317, size = 135, normalized size = 4.22

$$-\frac{\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}\cos\left(dx+c\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)}{ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(1/2*sin(d*x +
 c))/(a*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-a\sec^2\left(c+dx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sec(c + d*x)**2 + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\sec\left(dx+c\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-a*sec(d*x + c)^2 + a), x)
```

3.225 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{ad\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] Cot[c + d*x]/(2*a*d*Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c +
 d*x])/(a*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rubi [A] time = 0.0415855, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$\frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{ad\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

```
[Out] Cot[c + d*x]/(2*a*d*Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c +
 d*x])/(a*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx &= \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{3/2}}\,dx \\
&= -\frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a\sqrt{-a\tan^2(c+dx)}} \\
&= \frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{a\sqrt{-a\tan^2(c+dx)}} \\
&= \frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{ad\sqrt{-a\tan^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.135554, size = 57, normalized size = 0.85

$$-\frac{\tan^3(c+dx)\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d\left(-a\tan^2(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

```
[Out] -((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c + d*x]
^3)/(2*d*(-(a*Tan[c + d*x]^2))^(3/2))
```

Maple [B] time = 0.24, size = 141, normalized size = 2.1

$$\frac{\sin(dx+c)}{4\,d\,(\cos(dx+c))^{3}}\left(4\,(\cos(dx+c))^{2}\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-4\,(\cos(dx+c))^{2}\ln\left(2\,(\cos(dx+c)+1)^{-1}\right)-(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(3/2),x)
```

```
[Out] 1/4/d*(4*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))/sin(d*x+c))-4*cos(d*x+c)^2*ln(2/(
cos(d*x+c)+1))-cos(d*x+c)^2-4*ln(-(-1+cos(d*x+c))/sin(d*x+c))+4*ln(2/(cos(d
*x+c)+1))-1)*sin(d*x+c)/cos(d*x+c)^3/(-a*sin(d*x+c)^2/cos(d*x+c)^2)^(3/2)
```

Maxima [A] time = 1.48285, size = 81, normalized size = 1.21

$$-\frac{\frac{\log\left(\tan(dx+c)^{2}+1\right)}{\sqrt{-a}a}-\frac{2\log(\tan(dx+c))}{\sqrt{-a}a}+\frac{\sqrt{-a}}{a^{2}\tan(dx+c)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a) - 2*log(tan(d*x + c))/(sqrt(-a)*
a) + sqrt(-a)/(a^2*tan(d*x + c)^2))/d
```

Fricas [A] time = 0.503698, size = 228, normalized size = 3.4

$$-\frac{\left(2\left(\cos(dx+c)^{3}-\cos(dx+c)\right)\log\left(\frac{1}{2}\,\sin(dx+c)\right)-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)^{2}-a}{\cos(dx+c)^{2}}}}{2\left(a^{2}d\cos(dx+c)^{2}-a^{2}d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) - cos(d*x + c
))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^2*d*cos(d*x + c)^2 - a^2
```

```
*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(-3/2), x)
```

Giac [B] time = 1.81844, size = 285, normalized size = 4.25

$$-\frac{\frac{\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{8\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sqrt{-a}a\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{4\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}a\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{\cdots}{\sqrt{-a}as\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-a)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(-tan(1/2*d*x + 1/2*c)^3 + ta
n(1/2*d*x + 1/2*c))) + 8*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a*sgn(-t
an(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) - 4*log(tan(1/2*d*x + 1/2*c)
^2)/(sqrt(-a)*a*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) + (4*t
an(1/2*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/
2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^2))/d
```

3.226 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] Cot[c + d*x]/(2*a^2*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^2*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^2*d*Sqrt[-
(a*Tan[c + d*x]^2)])
```

Rubi [A] time = 0.0505803, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$-\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(2*a^2*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^2*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^2*d*Sqrt[-
(a*Tan[c + d*x]^2)])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{5/2}}\,dx &= \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{5/2}}\,dx\\
&= \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}\\
&= -\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}-\frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\int \cot(c+dx)\,dx}{a^2\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{a^2d\sqrt{-a\tan^2(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.259141, size = 69, normalized size = 0.69

$$\frac{\tan^5(c+dx)\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d\left(-a\tan^2(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

```
[Out] ((2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d
*x]])*Tan[c + d*x]^5)/(4*d*(-(a*Tan[c + d*x]^2))^(5/2))
```

Maple [B] time = 0.247, size = 203, normalized size = 2.

$$\frac{\sin(dx+c)}{32\,d\,(\cos(dx+c))^5}\left(32\,(\cos(dx+c))^4\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-32\,(\cos(dx+c))^4\ln\left(2\,(\cos(dx+c)+1)^{-1}\right)-13\,(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/32/d*(32*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))/sin(d*x+c))-32*cos(d*x+c)^4*ln(
2/(cos(d*x+c)+1))-13*cos(d*x+c)^4-64*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))/sin(d
*x+c))+64*cos(d*x+c)^2*ln(2/(cos(d*x+c)+1))-6*cos(d*x+c)^2+32*ln(-(-1+cos(d
*x+c))/sin(d*x+c))-32*ln(2/(cos(d*x+c)+1))+11)*sin(d*x+c)/cos(d*x+c)^5/(-a*
sin(d*x+c)^2/cos(d*x+c)^2)^(5/2)
```

Maxima [A] time = 1.47495, size = 107, normalized size = 1.07

$$-\frac{\frac{2\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a^2}-\frac{4\log(\tan(dx+c))}{\sqrt{-a}a^2}+\frac{2\sqrt{-a}\tan(dx+c)^2-\sqrt{-a}}{a^3\tan(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^2) - 4*log(tan(d*x + c))/(sqrt(
-a)*a^2) + (2*sqrt(-a)*tan(d*x + c)^2 - sqrt(-a))/(a^3*tan(d*x + c)^4))/d
```

Fricas [A] time = 0.512133, size = 315, normalized size = 3.15

$$\frac{\left(4\,\cos(dx+c)^3-4\left(\cos(dx+c)^5-2\,\cos(dx+c)^3+\cos(dx+c)\right)\log\left(\frac{1}{2}\,\sin(dx+c)\right)-3\,\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{4\left(a^3d\cos(dx+c)^4-2\,a^3d\cos(dx+c)^2+a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*cos(d*x + c)^3 - 4*(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c)
)*log(1/2*sin(d*x + c)) - 3*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d
*x + c)^2)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x
 + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a\sec^2\left(c+dx\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((-a*sec(c + d*x)**2 + a)**(-5/2), x)
```

Giac [B] time = 1.879, size = 369, normalized size = 3.69

$$-\frac{\frac{64\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{32\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{\sqrt{-a}a^3\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\cdots}{\cdots}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(64*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2*sgn(-tan(1/2*d*x +
1/2*c)^3 + tan(1/2*d*x + 1/2*c))) - 32*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a
)*a^2*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) - (sqrt(-a)*a^3*
sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*tan(1/2*d*x + 1/2*c)^4
- 12*sqrt(-a)*a^3*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*tan(1
/2*d*x + 1/2*c)^2)/a^6 + (48*tan(1/2*d*x + 1/2*c)^4 - 12*tan(1/2*d*x + 1/2*
c)^2 + 1)/(sqrt(-a)*a^2*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
*tan(1/2*d*x + 1/2*c)^4))/d
```

3.227 $\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

```
[Out] Cot[c + d*x]/(2*a^3*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^3*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + Cot[c + d*x]^5/(6*a^3*d*Sqrt[-(a*Tan[c + d*x]^
2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^3*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rubi [A] time = 0.0612302, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4121, 3658, 3473, 3475}

$$\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] Cot[c + d*x]/(2*a^3*d*Sqrt[-(a*Tan[c + d*x]^2)]) - Cot[c + d*x]^3/(4*a^3*d*
Sqrt[-(a*Tan[c + d*x]^2)]) + Cot[c + d*x]^5/(6*a^3*d*Sqrt[-(a*Tan[c + d*x]^
2)]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(a^3*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

Rule 4121

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{7/2}}\,dx &= \int \frac{1}{\left(-a\tan^2(c+dx)\right)^{7/2}}\,dx\\
&= -\frac{\tan(c+dx)\int \cot^7(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}\\
&= -\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{a^3\sqrt{-a\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\tan(c+d}{a^3\sqrt{-}}\\
&= \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}}+\frac{\log(\sin(c}{a^3d\sqrt{-}}
\end{aligned}
$$

Mathematica [A] time = 0.337888, size = 79, normalized size = 0.59

$$
-\frac{\tan^7(c+dx)\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+dx)+12\log(\tan(c+dx))+12\log(\cos(c+dx))\right)}{12d\left(-a\tan^2(c+dx)\right)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] -((6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos[c +
d*x]] + 12*Log[Tan[c + d*x]])*Tan[c + d*x]^7)/(12*d*(-(a*Tan[c + d*x]^2))^(
7/2))
```

Maple [B] time = 0.261, size = 265, normalized size = 2.

$$-\frac{\sin(dx+c)}{48\,d\,(\cos(dx+c))^{7}}\left(48\,(\cos(dx+c))^{6}\ln\left(2\,(\cos(dx+c)+1)^{-1}\right)-48\,(\cos(dx+c))^{6}\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+25\,(\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c)^2)^(7/2),x)
```

```
[Out] -1/48/d*(48*cos(d*x+c)^6*ln(2/(cos(d*x+c)+1))-48*cos(d*x+c)^6*ln(-(-1+cos(d
*x+c))/sin(d*x+c))+25*cos(d*x+c)^6-144*cos(d*x+c)^4*ln(2/(cos(d*x+c)+1))+14
4*cos(d*x+c)^4*ln(-(-1+cos(d*x+c))/sin(d*x+c))-3*cos(d*x+c)^4+144*cos(d*x+c
)^2*ln(2/(cos(d*x+c)+1))-144*cos(d*x+c)^2*ln(-(-1+cos(d*x+c))/sin(d*x+c))-3
3*cos(d*x+c)^2-48*ln(2/(cos(d*x+c)+1))+48*ln(-(-1+cos(d*x+c))/sin(d*x+c))+1
9)*sin(d*x+c)/cos(d*x+c)^7/(-a*sin(d*x+c)^2/cos(d*x+c)^2)^(7/2)
```

Maxima [A] time = 1.49992, size = 127, normalized size = 0.95

$$-\frac{\frac{6\,\log\left(\tan(dx+c)^{2}+1\right)}{\sqrt{-a}a^{3}}-\frac{12\,\log(\tan(dx+c))}{\sqrt{-a}a^{3}}+\frac{6\,\sqrt{-a}\tan(dx+c)^{4}-3\,\sqrt{-a}\tan(dx+c)^{2}+2\,\sqrt{-a}}{a^{4}\tan(dx+c)^{6}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^3) - 12*log(tan(d*x + c))/(sqr
t(-a)*a^3) + (6*sqrt(-a)*tan(d*x + c)^4 - 3*sqrt(-a)*tan(d*x + c)^2 + 2*sqr
t(-a))/(a^4*tan(d*x + c)^6))/d
```

Fricas [A] time = 0.525282, size = 406, normalized size = 3.05

$$\frac{\left(18\cos\left(dx+c\right)^{5}-27\cos\left(dx+c\right)^{3}-12\left(\cos\left(dx+c\right)^{7}-3\cos\left(dx+c\right)^{5}+3\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\left(d\right.\right.}{12\left(a^{4}d\cos\left(dx+c\right)^{6}-3a^{4}d\cos\left(dx+c\right)^{4}+3a^{4}d\cos\left(dx+c\right)^{2}-a^{4}d\right)\sin\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/12*(18*cos(d*x + c)^5 - 27*cos(d*x + c)^3 - 12*(cos(d*x + c)^7 - 3*cos(d*
x + c)^5 + 3*cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) + 11*cos(
d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^4*d*cos(d*x + c)^
6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.96491, size = 383, normalized size = 2.88

$$-\frac{\frac{384\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)}{\sqrt{-a}a^{3}\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{192\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)}{\sqrt{-a}a^{3}\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{352\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-87\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+12\tan\left(\right.}{\sqrt{-a}a^{3}\mathrm{sgn}\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\tan}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/384*(384*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^3*sgn(-tan(1/2*d*x
+ 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) - 192*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt
```

```
(-a)*a^3*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))) + (352*tan(1/
2*d*x + 1/2*c)^6 - 87*tan(1/2*d*x + 1/2*c)^4 + 12*tan(1/2*d*x + 1/2*c)^2 -
1)/(sqrt(-a)*a^3*sgn(-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*tan(1/
2*d*x + 1/2*c)^6) + (sqrt(-a)*a^8*tan(1/2*d*x + 1/2*c)^6 - 12*sqrt(-a)*a^8*
tan(1/2*d*x + 1/2*c)^4 + 87*sqrt(-a)*a^8*tan(1/2*d*x + 1/2*c)^2)/(a^12*sgn(
-tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))))/d
```

3.228 $\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=372

$$-\frac{(a-8b)(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{\ }{a}\right.}{15bf\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[Out] -((2*a^2 - 3*a*b - 8*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])/(15*b^2*f) + ((2*a^2 - 3*a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*Ell
ipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[
e + f*x]^2)])/(15*b^2*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*
(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(15*b*f*(a + b - a*Sin[e + f*x]^2)) + ((a + 4*b)*Sec[e + f*x]*Sqrt[
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(15*b*f) + (Sec[e
+ f*x]^3*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(5*f
)
```

Rubi [A] time = 0.688479, antiderivative size = 471, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 412, 527, 524, 426, 424, 421, 419}

$$-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{15b^2f\sqrt{a\cos^2(e+fx)+b}}+\frac{\left(2a^2-3ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{-a\,\mathrm{si}\ldots}}{15b^2f\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((2*a^2 - 3*a*b - 8*b^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a +
b - a*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[b + a*Cos[e + f*x]^2]) + ((2*a^2 - 3*
a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)
]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*b^2*f*Sqrt
[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*(
a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*b*f*Sqrt[b
 + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + ((a + 4*b)*Sec[e + f
```

```
*x]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])
/(15*b*f*Sqrt[b + a*Cos[e + f*x]^2]) + (Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f
*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(5*f*Sqrt[b + a*Cos[e +
 f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{(1-x^2)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a(1-x^2)}}{(1-x^2)^{7/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{(1-x^2)^{7/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{5f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\cos^2\right.}{}$$

$$= \frac{(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{15bf\sqrt{b+a\cos^2(e+fx)}} +$$

$$= -\frac{(2a^2-3ab-8b^2)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15b^2f\sqrt{b+a\cos^2(e+fx)}} + \cdots$$

$$= -\frac{(2a^2-3ab-8b^2)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15b^2f\sqrt{b+a\cos^2(e+fx)}} + \cdots$$

$$= -\frac{(2a^2-3ab-8b^2)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15b^2f\sqrt{b+a\cos^2(e+fx)}} + \cdots$$

$$= -\frac{(2a^2-3ab-8b^2)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15b^2f\sqrt{b+a\cos^2(e+fx)}} + \cdots$$

Mathematica [F] time = 26.6862, size = 0, normalized size = 0.

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.766, size = 6562, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \sec\left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec\left(fx+e\right)^2 + a} \sec\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\sec^5\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\sec\left(fx + e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

3.229 $\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=288

$$\frac{2(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3f\left(-a\sin^2(e+fx)+a+b\right)}+\frac{(a}{}$$

```
[Out] ((a + 2*b)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3
*b*f) - ((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(
a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*b*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) + (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f*x]^2)) +
(Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])
/(3*f)
```

Rubi [A] time = 0.509675, antiderivative size = 364, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 412, 527, 524, 426, 424, 421, 419}

$$\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{3bf\sqrt{a\cos^2(e+fx)+b}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{}}{3f\sqrt{a\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f
*x]^2])/(3*b*f*Sqrt[b + a*Cos[e + f*x]^2]) - ((a + 2*b)*Sqrt[Cos[e + f*x]^2
]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqr
t[a + b - a*Sin[e + f*x]^2])/(3*b*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)]) + (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSi
n[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e +
f*x]^2)/(a + b)])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*
x]^2]) + (Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*
x]^2]*Tan[e + f*x])/(3*f*Sqrt[b + a*Cos[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{\left(1-x^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a\left(1-x^2\right)}}{\left(1-x^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\left(1-x^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{3f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(\right.}{}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3bf\sqrt{b+a\cos^2(e+fx)}} + \frac{\sec(e+fx)}{}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3bf\sqrt{b+a\cos^2(e+fx)}} + \frac{\sec(e+fx)}{}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3bf\sqrt{b+a\cos^2(e+fx)}} + \frac{\sec(e+fx)}{}$$

$$= \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3bf\sqrt{b+a\cos^2(e+fx)}} - \frac{(a+2b)\sqrt{\text{co}}}{}$$

<u>Mathematica</u> [F] time = 11.4977, size = 0, normalized size = 0.

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.462, size = 4737, normalized size = 16.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/6/f/b/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
(4*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a^2*b+5*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a*b^2-4*I*b^(5/2)*a^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-
6*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a^2*b-6*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))
^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)
^(1/2))*a*b^2+4*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a^2*b+5*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
```

```
-b^2)/(a+b)^2)^(1/2))*a*b^2-6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+4*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a*b^2-2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b^2+8*I*cos(f*x+e)^5*b^(3/2)*a^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)+4*I*cos(f*x+e)^5*b^(1/2)*a^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)-4*I*cos(f*x+e)^4*b^(3/2)*a^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)-4*I*cos(f*x+e)^4*b^(1/2)*a^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)+8*I*cos(f*x+e)^3*b^(5/2)*a^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)+4*I*cos(f*x+e)^3*b^(3/2)*a^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)-4*I*cos(f*x+e)^2*b^(5/2)*a^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-
8*I*cos(f*x+e)^2*b^(3/2)*a^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-2*cos(f*x+e)^2*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*b^3+4*I*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*b^(5/2)*a^(1/2)+2*I*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2)*a^(3/2)-2*I*cos(f*x+e)^4*sin(f*x+
e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(1/2)*a^(5/2)+4*I*co
s(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(
5/2)*a^(1/2)+2*I*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
```

```
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b^(3/2)*a^(3/2)-2*I*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(1/2)*a^(5/2)-2*cos(f*x+e)^5*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a^3+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*cos(f*
x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^2+4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+cos
(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellip
ticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(
4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+
2*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*b^3+cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a^3+2*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^3)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/co
s(f*x+e)^2/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^{2}+a} \sec \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)^{2}+a}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec^{2}\left(e+fx\right)}\sec^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)^{2}+a}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

3.230 $\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=218

$$\frac{(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{f\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin}{}$$

```
[Out] (Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/f - (Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^
2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (
(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.398086, antiderivative size = 271, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4148, 6722, 1974, 412, 12, 493, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a\cos^2(e+fx)+b}}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e+fx)}F}{f\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(f
*Sqrt[b + a*Cos[e + f*x]^2]) - (Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e
 + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]
^2])/(f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) +
((a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sq
rt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(f*Sqrt[b +
a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
```

```
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{b}{1-x^2}}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a(1-x^2)}}{(1-x^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{(1-x^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+\right.}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} + \frac{\left(a\sqrt{\cos^2(e+fx)}\sqrt{a+\right.}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+\right.}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+\right.}{}$$

$$= \frac{\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-}\right.}{}$$

Mathematica [F] time = 11.5677, size = 0, normalized size = 0.

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.421, size = 3454, normalized size = 15.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(2
*cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
a*b+2*I*cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(1/2)*2^(1/
2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e)))^(1/2)+2*I*cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^
(1/2)*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)+2*I*cos(f*x+e)*sin(f*x+e)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a^(3/2)*b^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)+2*I*cos(f*x+e)*s
in(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
+2*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
```

```
a*b-2*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*a*b+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2*cos(f*x
+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2*sin(f*x+e)*cos(f*x+e)^2
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-4*I*a^(1/2)*b^(3/2)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a^2-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+2*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2+cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2+cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2+cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2-2*cos(f*x+e)*sin(f*x+e)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2+4*I*cos(f*x+e)^3*a^(3/2)*b^(
1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^2*a^(3/2)*b^(1/
2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)*a^(1/2)*b^(3/2)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^2+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*cos(f*
```

```
x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2)*((
b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

3.231 $\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

```
[Out] (Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e
 + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)])
```

Rubi [A] time = 0.150185, antiderivative size = 103, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 426, 424}

$$\frac{\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b
*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(f*Sqrt[b + a*Cos[e + f*x]
^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
```

```
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{b+a(1-x^2)}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a+b-ax^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{ax^2}{a+b}}}{\sqrt{1-x^2}}\right.}{f\sqrt{b+a\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$= \frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e}}{f\sqrt{b+a\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

Mathematica [A] time = 0.289721, size = 69, normalized size = 0.86

$$\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}E\left(e+fx\middle|\frac{a}{a+b}\right)}{f\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*EllipticE[e + f*x, a/(a + b)]*Sqrt[a + b*Sec[e + f*x]
^2])/(f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [C] time = 0.447, size = 3408, normalized size = 42.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)*(2
*I*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(1/2)*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)+2*I*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(
3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)-2*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b+2*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^
(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b^2*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^2*sin(f*x+e)+2*I*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
a^(3/2)*b^(1/2)*sin(f*x+e)+2*I*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^(1/2)*b^(3/2)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
```

```
^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*a*b*sin(f*x+e)-2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a*b+2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b^2+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-4*I*a^(1/2)*
b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*cos(f*x+e)^3*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^2-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)+2*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2
)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2-cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2+4*I*cos(f*x+e)^3*a^(3/2)*b^(1/2)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^2*a^(3/2)*b^(1/2)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)*a^(1/2)*b^(3/2)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2
*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*c
os(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

3.232 $\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=246

$$-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3af\left(-a\sin^2(e+fx)+a+b\right)}+\text{si}$$

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
])/(3*f) + ((2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]],
a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*a*f*Sqrt[1 -
 (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[A
rcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a*f*(a + b - a*Sin[e + f*x]^2
))
```

Rubi [A] time = 0.389358, antiderivative size = 299, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4148, 6722, 1974, 417, 524, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\cos^2(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{3f\sqrt{a\cos^2(e+fx)+b}}-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\,\text{se}}}{3af\sqrt{-a\sin^2(e+fx)+a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[
e + f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]) + ((2*a + b)*Sqrt[Cos[e + f*x
]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*
Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[Ar
cSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e
 + f*x]^2)/(a + b)])/(3*a*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\sqrt{b+a\left(1-x^2\right)}\,dx, x, \text{s}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\sqrt{a+b-ax^2}\,dx, x, \sin\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} + \frac{\left(2\sqrt{\text{co}\right.}{}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} - \frac{((-2a}{}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left((-2a\right.}{}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} + \frac{(2a+}{}$$

Mathematica [C] time = 8.52001, size = 539, normalized size = 2.19

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{(2a+2b)\sqrt{\frac{a\cos(2e+2fx)+a+2b}{2a+2b}}E\left(\frac{1}{2}(2e+2fx)|\frac{2a}{2a+2b}\right)}{f\sqrt{a\cos(2e+2fx)+a+2b}} + \frac{\sin(2e+2fx)\cos(2(e+fx))\sec\left(2\left(\frac{1}{2}\left(\cos^{-1}(\cos(2e+2fx))-2e\right)\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(((2*a + 2*b)*Sqrt[(a + 2*b + a*Co
s[2*e + 2*f*x])/(2*a + 2*b)]*EllipticE[(2*e + 2*f*x)/2, (2*a)/(2*a + 2*b)])
```

```
/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + (Cos[2*(e + f*x)]*(-(Sqrt[-(a + b
)^(-1)]*(-a + a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x])*Sqrt[a + 2*b + a
*Cos[2*e + 2*f*x]]) - I*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*
Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(
a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - I*a
*b*Sqrt[(4*a + 4*b - 2*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a
 + 2*b + a*Cos[2*e + 2*f*x])/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sq
rt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sec[2*(e + (-2*e +
ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e + 2*f*x])/(3*a^2*Sqrt[-(a + b)^(-1)]*
f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[1 - Co
s[2*e + 2*f*x]^2])))/(2*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [C] time = 0.721, size = 4623, normalized size = 18.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/6/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)/a*
(6*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
a^2*b-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)-2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-2*cos(f*x+e)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*I*b^(5/2)*a^(1/2)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
```

)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2)+2*I*sin(f*x+e)*cos(f*x+e)*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)-2*I*sin(f
*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*
b^(5/2)-4*I*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^
(3/2)+4*I*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2
)+4*I*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(5/2)+4*
I*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*b^(1/2)+8*I*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)-8*I*co
s(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(5/2)*b^(1/2)+4*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+4*I*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2)*sin(f*x+e)+2*I*2^(1/2)*(1/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)*sin(f*x+e)-2*I*2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
x+e)))^(1/2)(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)*sin(f*x+e)+6*sin(f*x+e)
*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-5*sin(f
*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2

```
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+6*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2
)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-8*I*b^(3/2)*a^
(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)^5*b^(1/2)*a^(5
/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*b^3-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+2
*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-2*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b^2+2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^2*b-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b
+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*cos(f*x+e)^
2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^2*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^3+4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^
3-2*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a^3-sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*b^3-5*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-
4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2)*cos(f
*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

3.233 $\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=338

$$-\frac{2b(2a-b)(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx))\right.}{15a^2f\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[Out] (2*(2*a - b)*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)])/(15*a*f) + (Cos[e + f*x]^2*Sin[e + f*x]*(a + b - a*Sin[e + f
*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(5*a*f) + ((8*a^2 +
 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
 b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(15*a^2*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) - (2*(2*a - b)*b*(a + b)*Sqrt[Cos[e + f*x]^2]*El
lipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^2*f*(a + b - a*Si
n[e + f*x]^2))
```

Rubi [A] time = 0.571286, antiderivative size = 400, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 416, 528, 524, 426, 424, 421, 419}

$$\frac{\left(8a^2+3ab-2b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}}-\frac{2b(2a-b)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (2*(2*a - b)*Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a
+ b - a*Sin[e + f*x]^2])/(15*a*f*Sqrt[b + a*Cos[e + f*x]^2]) + (Cos[e + f*x
]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2)^(3/2
))/(5*a*f*Sqrt[b + a*Cos[e + f*x]^2]) + ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e
 + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*
x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*a^2*f*Sqrt[b + a*Cos[e + f*x]^2]*
Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (2*(2*a - b)*b*(a + b)*Sqrt[Cos[e +
 f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]
^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^2*f*Sqrt[b + a*Cos[e + f*x]
```

```
^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\sqrt{a+\frac{b}{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \left(1-x^2\right)^{3/2}\sqrt{b+a\left(1-x^2\right)}\,dx, x,\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \left(1-x^2\right)^{3/2}\sqrt{a+b-ax^2}\,dx, x, \text{si}\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)^{3/2}}{5af\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\text{co}}\right.}{}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{2(2a-b)\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15af\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F] time = 11.436, size = 0, normalized size = 0.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.747, size = 6392, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx+e\right)^2 + a} \cos\left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b \sec\left(fx+e\right)^2 + a} \cos\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

3.234 $\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=186

$$\frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{(a+b)\left(a^2-2ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{(3a-5b)}{}$$

```
[Out] ((a + b)*(a^2 - 2*a*b + 5*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) + ((a^2 - 2*a*b + 5*b^2)*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) - ((3*a - 5*b)*Tan[e + f*x]*(a + b
 + b*Tan[e + f*x]^2)^(3/2))/(24*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*(a +
b + b*Tan[e + f*x]^2)^(3/2))/(6*b*f)
```

Rubi [A] time = 0.17012, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 416, 388, 195, 217, 206}

$$\frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{(a+b)\left(a^2-2ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{(3a-5b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] ((a + b)*(a^2 - 2*a*b + 5*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) + ((a^2 - 2*a*b + 5*b^2)*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) - ((3*a - 5*b)*Tan[e + f*x]*(a + b
 + b*Tan[e + f*x]^2)^(3/2))/(24*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*(a +
b + b*Tan[e + f*x]^2)^(3/2))/(6*b*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf} + \frac{\text{Subst}\left(\int \left(-a+5b-(\right.\right.}{}$$

$$= -\frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} + \frac{\sec^2(e+fx)\tan(e+fx}{}$$

$$= \frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-5b)\tan(e+fx}{}$$

$$= \frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} - \frac{(3a-5b)\tan(e+fx}{}$$

$$= \frac{(a+b)\left(a^2-2ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} + \frac{\left(a^2-2ab+5b^2\right)\tan(e}{}$$

Mathematica [C] time = 11.5441, size = 968, normalized size = 5.2

$$-\frac{ie^{i(e+fx)}\sqrt{ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2+4b}\left(\frac{15(a+b)\tan^{-1}\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}}\right)a^2}{8b^{3/2}} + \frac{15\left(-1+e^{2i(e+fx)}\right)\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}\,a^2}{8b\left(1+e^{2i(e+fx)}\right)^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-I/15)*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*((-15*a*(-1 + E^((2*I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2])/(4*(1 + E^((2*I)*(e + f*x)))^2) + (15*
a^2*(-1 + E^((2*I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2])/(8*b*(1 + E^((2*I)*(e + f*x)))^2) + (75*b*(-1 + E^((2*I)
*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])
```

```
/(8*(1 + E^((2*I)*(e + f*x)))^2) + (40*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^
((2*I)*(e + f*x)))^2)^(3/2))/(1 + E^((2*I)*(e + f*x)))^6 - (60*(4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2))/(1 + E^((2*I)*(e + f*x
)))^5 - (24*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2)
)/(1 + E^((2*I)*(e + f*x)))^4 + (3*(5*a + 21*b)*(4*b*E^((2*I)*(e + f*x)) +
a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2))/(2*b*(1 + E^((2*I)*(e + f*x)))^4) + (
50*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2))/(1 + E^
((2*I)*(e + f*x)))^3 + (7*(7*a + 15*b)*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^
((2*I)*(e + f*x)))^2)^(3/2))/(4*b*(1 + E^((2*I)*(e + f*x)))^3) - (2*(8*a +
35*b)*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^(3/2))/(b*(
1 + E^((2*I)*(e + f*x)))^3) + (15*a^2*(a + b)*ArcTan[(Sqrt[b]*(-1 + E^((2*I
)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2
]])/(8*b^(3/2)) - (15*a*(a + b)*ArcTan[(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))))
/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(4*Sqrt[b]
) + (75*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))))/Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/8)*Cos[e + f*x]*Sq
rt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [C] time = 0.607, size = 2518, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/48/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2*sin(f*x+e)*((b+a*cos(f*x
+e)^2)/cos(f*x+e)^2)^(1/2)*(3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+14*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a*b^2-14*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a*b^2+4*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+15*co
s(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-4*cos(f*x+e)^6*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-15*cos(f*x+e)^6*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b^2+10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*b^3-10*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-
8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+15*cos(f*x+e)^5*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-15*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(
```

a+b))^(1/2)*b^3-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^3-15*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
(-2/(a+b)(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*b^3+6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+
30*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3-3*cos(f*x+e)^7*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a^3+8*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-c
os(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+cos(f*x+e)^4*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b^2-10*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a*b^2-9*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a*b^2-6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+18
*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2)/(-1+cos(f*x+e))/(b+a*cos(
f*x+e)^2)/cos(f*x+e)^5

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.42859, size = 1138, normalized size = 6.12

$$\left[\frac{3\left(a^3-a^2b+3ab^2+5b^3\right)\sqrt{b}\cos\left(fx+e\right)^5\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\,c}{}}}{\cos\left(fx+e\right)^4}\right.}{192\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2
- 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*c
os(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b - 4*a*b^2 -
15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1
/96*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2*((3*a^
2*b - 4*a*b^2 - 15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*co
s(f*x + e)^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec^2 \left(e + fx\right)} \sec^6 \left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx + e\right)^2 + a} \sec \left(fx + e\right)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^6, x)
```

3.235 $\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=122

$$-\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4bf}-\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2}}{8bf}$$

```
[Out] -((a - 3*b)*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(8*b^(3/2)*f) - ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(8*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b*f)
```

Rubi [A] time = 0.106159, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4146, 388, 195, 217, 206}

$$-\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4bf}-\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((a - 3*b)*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(8*b^(3/2)*f) - ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(8*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)\sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a-3b)\,\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx,\right.}{4bf}$$

$$= -\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(\right.}{4bf}$$

$$= -\frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(\right.}{4bf}$$

$$= -\frac{(a-3b)(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b}}{8bf}$$

Mathematica [C] time = 8.25721, size = 390, normalized size = 3.2

$$-\frac{ie^{i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{\left(a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}\right)^{3/2}}{\left(1+e^{2i(e+fx)}\right)^3}-\frac{2\left(a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}\right)^{3/2}}{\left(1+e^{2i(e+fx)}\right)^4}+\frac{1}{2}(3b-a)\left(\cdots\right.\right.}{2\sqrt{2}bf\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}\sqrt{a\cos(2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((-I/2)*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)
*(e + f*x))]*((-2*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)
^(3/2))/(1 + E^((2*I)*(e + f*x)))^4 + (4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2)^(3/2)/(1 + E^((2*I)*(e + f*x)))^3 + ((-a + 3*b)*(((-1
+ E^((2*I)*(e + f*x)))*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2])/(1 + E^((2*I)*(e + f*x)))^2 + ((a + b)*ArcTan[(Sqrt[b]*(-1 + E^(
(2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]])/Sqrt[b]))/2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*Sq
rt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f*Sqrt[a + 2*b
+ a*Cos[2*e + 2*f*x]])
```

Maple [C] time = 0.387, size = 1770, normalized size = 14.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e
)^2)/cos(f*x+e)^2)^(1/2)*(2*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(
1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-4
*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
```

```
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2
)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-6*sin(f*x+e)*cos(f*x+e)^4*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*b^2-sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*a^2+2*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a*b+3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^2-3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+c
os(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f*x+e)^4*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*
x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*cos(f*x+e)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/cos(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.12055, size = 972, normalized size = 7.97

$$\left[-\frac{\left(a^2-2\,ab-3\,b^2\right)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2 \cdots}{\cos\left(fx+e\right)^2}}}{\cos\left(fx+e\right)^4}\right.}{32\,b^2f\cos\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a*b + 3*b^2)*cos(f*x + e)^2 + 2*
b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f
*x + e)^3), -1/16*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*((a*
b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\sec^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^{2}+a}\sec \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

3.236 $\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}f}$$

```
[Out] ((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2
*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0814039, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4146, 195, 217, 206}

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] ((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2
*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+}{\sqrt{a+b+b\tan}}\right.}{2f}$$

$$= \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

Mathematica [B] time = 1.66582, size = 210, normalized size = 2.76

$$\frac{\tan(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{\frac{b\sin^2(e+fx)}{a+b}}(a\cos(2(e+fx))+a+2b)+\sqrt{2}(a+b)\cos^2(e+\right.}{\sqrt{2}f\sqrt{\frac{b\sin^2(e+fx)}{a+b}}(a\cos(2(e+fx))+a+2b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*(Sqrt[2]*(a + b)
*ArcTanh[Sqrt[(b*Sin[e + f*x]^2)/(a + b)]/Sqrt[(a + b - a*Sin[e + f*x]^2)/(
a + b)]]*Cos[e + f*x]^2*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)] + (a +
 2*b + a*Cos[2*(e + f*x)])*Sqrt[(b*Sin[e + f*x]^2)/(a + b)])*Tan[e + f*x])/
(Sqrt[2]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sqrt[(b*Sin[e + f*x]^2)/(a
+ b)])
```

Maple [C] time = 0.304, size = 1098, normalized size = 14.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)^2
)/cos(f*x+e)^2)^(1/2)*(2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a+2*sin(
f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/cos(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.703574, size = 813, normalized size = 10.7

$$\left[\frac{(a+b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)+8}{\cos\left(fx+e\right)^4}\right.}{8\,bf\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(b*f*cos(f*x + e)), 1/4*((a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 2*b*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

3.237 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rubi [A] time = 0.0493114, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int\sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int\frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\,\text{Subst}\left(\int\frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{f}+\frac{b\,\text{Subst}\left(\int\frac{1}{\sqrt{a+b+bx^2}}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\,\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{b\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\ &= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [F] time = 0.0894982, size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.453, size = 589, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*2^(1/2)*(2*EllipticPi((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*a+2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b-
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a-EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*b)*cos(f*x+e)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)^2*((b
+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.1934, size = 2984, normalized size = 37.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

3.238 $\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
f)
```

Rubi [A] time = 0.0919363, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4146, 378, 377, 203}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```



```
[Out] ((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
Sqrt[a]*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+}}\right.}{2f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,\right.}{2f}$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e}}{2f}$$

Mathematica [A] time = 0.71287, size = 136, normalized size = 1.66

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{a+b}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{2}\sqrt{a}\sin(e+fx)\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}\right)}{2\sqrt{2}\sqrt{a}f\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin
[e + f*x])/Sqrt[a + b]] + Sqrt[2]*Sqrt[a]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)
])/(a + b)]*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[(a + 2*b + a*Cos[2*(e
+ f*x)])/(a + b)])
```

Maple [C] time = 0.315, size = 1066, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(2*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin(f
*x+e)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(
f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*cos
(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/
(b+a*cos(f*x+e)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Fricas [B] time = 0.878981, size = 1210, normalized size = 14.76

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{-a}(a+b)\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) - sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)
*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^
5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e)))/(a*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)*sin(f*x + e) - (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)
^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2
*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\cos^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**2, x)
```

Giac [A] time = 1.42896, size = 112, normalized size = 1.37

$$-\frac{\left(\frac{(a+b)\log\left(\left|-\sqrt{-a}\sin\left(fx+e\right)+\sqrt{-a\sin\left(fx+e\right)^2+a+b}\right|\right)}{\sqrt{-a}} - \sqrt{-a\sin\left(fx+e\right)^2 + a + b}\sin\left(fx+e\right)\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((a + b)*log(abs(-sqrt(-a)*sin(f*x + e) + sqrt(-a*sin(f*x + e)^2 + a +
 b)))/sqrt(-a) - sqrt(-a*sin(f*x + e)^2 + a + b)*sin(f*x + e))*sgn(cos(f*x
+ e))/f
```

3.239 $\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=140

$$\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4af}+\frac{(3a-b)\sin(e+fx)\,\mathrm{c}}{}$$

```
[Out] ((3*a - b)*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(8*a^(3/2)*f) + ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(8*a*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e +
f*x]^2)^(3/2))/(4*a*f)
```

Rubi [A] time = 0.122573, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4146, 382, 378, 377, 203}

$$\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{3/2}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4af}+\frac{(3a-b)\sin(e+fx)\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a - b)*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(8*a^(3/2)*f) + ((3*a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(8*a*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e +
f*x]^2)^(3/2))/(4*a*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af} + \frac{(3a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+b}}{(1+}\right.}{4}$$

$$= \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3(e+fx)\sin(e}{}$$

$$= \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} + \frac{\cos^3(e+fx)\sin(e}{}$$

$$= \frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a}}{8af}$$

Mathematica [A] time = 1.28061, size = 152, normalized size = 1.09

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}(3a-b)\sqrt{a+b}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a}\sin(e+fx)(a\cos(2(e+fx))+4a+b)\sqrt{\frac{a\,\mathrm{c}}{}}\right.}{8a^{3/2}f\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*(3*a - b)*Sqrt[a + b]*Arc
Sin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]] + Sqrt[a]*(4*a + b + a*Cos[2*(e + f
*x)])*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(8*a^(3/2
)*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [C] time = 0.39, size = 1713, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(-2*cos(f*x+e)^5*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*a^2*sin(f*x+e)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a*b*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*si
n(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-4*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f
*x+e)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a^2-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(
f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f*x+e)^2*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+3
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*b^2)*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2
)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Fricas [A] time = 1.51429, size = 1355, normalized size = 9.68

$$\left[\frac{\left(3\,a^{2}+2\,ab-b^{2}\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((3*a^2 + 2*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + a*b)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f),
 -1/32*((3*a^2 + 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*
(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*
b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x + e
)^3 + (3*a^2 + a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
```

```
2)*sin(f*x + e))/(a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39471, size = 166, normalized size = 1.19

$$-\frac{\left(\sqrt{-a\sin\left(fx+e\right)^{2}+a+b}\left(2\,\sin\left(fx+e\right)^{2}-\frac{5\,a^{2}+ab}{a^{2}}\right)\sin\left(fx+e\right)+\frac{\left(3\,a^{2}+2\,ab-b^{2}\right)\log\left(\left|-\sqrt{-a}\sin\left(fx+e\right)+\sqrt{-a\sin\left(fx+e\right)^{2}+a+b}\right|\right)}{\sqrt{-a}a}\right)\mathrm{sg}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-a*sin(f*x + e)^2 + a + b)*(2*sin(f*x + e)^2 - (5*a^2 + a*b)/a^2
)*sin(f*x + e) + (3*a^2 + 2*a*b - b^2)*log(abs(-sqrt(-a)*sin(f*x + e) + sqr
t(-a*sin(f*x + e)^2 + a + b)))/(sqrt(-a)*a))*sgn(cos(f*x + e))/f
```

3.240 $\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=196

$$\frac{(a+b)\left(5a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}+\frac{(3a-b)(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^2f}$$

```
[Out] ((a + b)*(5*a^2 - 2*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + ((3*a - b)*(5*a + 3*b)*Cos[e + f*x]*Sin
[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^2*f) + ((5*a + b)*Cos[e + f
*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) + (Cos[e + f*x]
^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rubi [A] time = 0.20586, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 412, 527, 12, 377, 203}

$$\frac{(a+b)\left(5a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{5/2}f}+\frac{(3a-b)(5a+3b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{48a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a + b)*(5*a^2 - 2*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/(16*a^(5/2)*f) + ((3*a - b)*(5*a + 3*b)*Cos[e + f*x]*Sin
[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^2*f) + ((5*a + b)*Cos[e + f
*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a*f) + (Cos[e + f*x]
^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{-5(a+b)-4bx^2}{\left(1+x^2\right)^3\sqrt{a+b+bx^2}}\right.}{6f}$$

$$= \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} + \frac{\cos^5(e+fx)\sin(}{}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} + \frac{(5a+b)}{}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} + \frac{(5a+b)}{}$$

$$= \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} + \frac{(5a+b)}{}$$

$$= \frac{(a+b)\left(5a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{(3a-b)(5a+3b)\cos(e+}{}$$

Mathematica [C] time = 16.6238, size = 1902, normalized size = 9.7

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^10*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sqrt[a + b*Sec[
e + f*x]^2]*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a +
 b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]^5*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/(3*(a + b)*AppellF1[1/
```

```
2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1
[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)
*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2) - (12*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2) + (3*(a + b)*Cos[e
 + f*x]^4*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sin[e + f*x]*(-(a*f*AppellF1[3
/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*
Sin[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(f*(3*(a + b)
*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]]*Sin[e + f*x]*(-2*f*(a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*
(a + b)*(-(a*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1,
 -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e
+ f*x])/3) - Sin[e + f*x]^2*(a*((3*a*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b))
- (12*f*AppellF1[5/2, -1, 1/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]*Sin[e + f*x])/5) + 4*(a + b)*((-3*a*f*AppellF1[5/2, -1, 1
/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f
*x])/(5*(a + b)) - (6*f*Cos[e + f*x]*Sin[e + f*x]*(1 - (a*Sin[e + f*x]^2)/(
a + b))^(3/2)*(5/(6*(1 - (a*Sin[e + f*x]^2)/(a + b))) + (5*(a + b)^3*Csc[e
+ f*x]^6*((-2*a*Sin[e + f*x]^2)/(a + b) - (4*a^2*Sin[e + f*x]^4)/(3*(a + b)
^2) + (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(
Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])))/(32*a^3*(1 - (a*Sin[e +
 f*x]^2)/(a + b)))))/5))))/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2
, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) - (3
*a*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2*(e + f*x)])/(Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [C] time = 0.563, size = 2436, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/48/f/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(3*2^(1/2)*(1/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+15*2^(1/2)*(1/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)-15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^2*b+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+9*2
^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-3*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-18*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)-3*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I
```

```
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-8*cos(
f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+8*cos(f*x+e)^6*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^3+10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a^3+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+4*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^2*b+10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b-14*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+co
s(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+14*cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a*b^2-4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a*b^2-15*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+15*
cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-30*2^(1/2)*(1/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*a^3*sin(f*x+e))*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)
^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

Fricas [A] time = 3.94642, size = 1527, normalized size = 7.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*
cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*
(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5 + 2*(5*
a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), -1/192*(3*
(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos(f*x +
 e)^5 + 2*(5*a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

3.241 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=450

$$-\frac{(a+b)\left(a^2-16ab-16b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e\right.}{35bf\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[Out] (-2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)])/(35*b^2*f) + (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt
[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*
x]^2*(a + b - a*Sin[e + f*x]^2)])/(35*b^2*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)]) - ((a + b)*(a^2 - 16*a*b - 16*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticF[Ar
cSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(35*b*f*(a + b - a*Sin[e + f*x]^2
)) + ((a^2 + 11*a*b + 8*b^2)*Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Si
n[e + f*x]^2)]*Tan[e + f*x])/(35*b*f) + (2*(4*a + 3*b)*Sec[e + f*x]^3*Sqrt[
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(35*f) + (b*Sec[e
+ f*x]^5*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(7*f
)
```

Rubi [A] time = 0.893631, antiderivative size = 572, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 413, 527, 524, 426, 424, 421, 419}

$$-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{35b^2f\sqrt{a\cos^2(e+fx)+b}}+\frac{\left(a^2+11ab+8b^2\right)\tan(e+fx)\text{se}}{35}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]
*Sqrt[a + b - a*Sin[e + f*x]^2])/(35*b^2*f*Sqrt[b + a*Cos[e + f*x]^2]) + (2
*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[
e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x
]^2])/(35*b^2*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
 b)]) - ((a + b)*(a^2 - 16*a*b - 16*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticF[Arc
```

```
Sin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e
+ f*x]^2)/(a + b)])/(35*b*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2]) + ((a^2 + 11*a*b + 8*b^2)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^
2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(35*b*f*Sqrt[b + a*Cos[e +
f*x]^2]) + (2*(4*a + 3*b)*Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a
+ b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(35*f*Sqrt[b + a*Cos[e + f*x]^2]) + (
b*Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*
Tan[e + f*x])/(7*f*Sqrt[b + a*Cos[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{\left(1-x^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(b+a\left(1-x^2\right)\right)^{3/2}}{\left(1-x^2\right)^{9/2}}\,dx, x, \sin(e+f\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^{3/2}}{\left(1-x^2\right)^{9/2}}\,dx, x, \sin(e+fx\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{7f\sqrt{b+a\cos^2(e+fx)}} - \frac{(}{}$$

$$= \frac{2(4a+3b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e}{35f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(a^2+11ab+8b^2\right)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\,\text{t}}{35bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin}}{35b^2f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F] time = 10.3495, size = 0, normalized size = 0.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] time = 1.169, size = 8000, normalized size = 17.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^7+a\sec\left(fx+e\right)^5\right)\sqrt{b\sec\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^7 + a*sec(f*x + e)^5)*sqrt(b*sec(f*x + e)^2 + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

3.242 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=371

$$\frac{(a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{15f\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[Out] ((3*a^2 + 13*a*b + 8*b^2)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])/(15*b*f) - ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])/(15*b*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(9*a
+ 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)])/(15*f*(a + b - a*Sin[e + f*x]^2)) + (2*(3*a + 2*b)*Sec[e + f*x]*Sqrt
[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(15*f) + (b*Sec[e
 + f*x]^3*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Tan[e + f*x])/(5*
f)
```

Rubi [A] time = 0.700338, antiderivative size = 470, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 413, 527, 524, 426, 424, 421, 419}

$$\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{15bf\sqrt{a\cos^2(e+fx)+b}}-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{-a\sin}}{15bf\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a +
b - a*Sin[e + f*x]^2])/(15*b*f*Sqrt[b + a*Cos[e + f*x]^2]) - ((3*a^2 + 13*a
*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]
*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*b*f*Sqrt[b
+ a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(9*a +
 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[
a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*f*Sqrt[b +
a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + (2*(3*a + 2*b)*Sec[e +
```

```
f*x]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x]
)/(15*f*Sqrt[b + a*Cos[e + f*x]^2]) + (b*Sec[e + f*x]^3*Sqrt[a + b*Sec[e +
f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(5*f*Sqrt[b + a*Cos[e
+ f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{\left(1-x^2\right)^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(b+a\left(1-x^2\right)\right)^{3/2}}{\left(1-x^2\right)^{7/2}}dx, x, \sin(e+f\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^{3/2}}{\left(1-x^2\right)^{7/2}}dx, x, \sin(e+fx\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{5f\sqrt{b+a\cos^2(e+fx)}} - -$$

$$= \frac{2(3a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx}}{15bf\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [F] time = 16.4127, size = 0, normalized size = 0.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] time = 0.654, size = 6562, normalized size = 17.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^5+a\sec\left(fx+e\right)^3\right)\sqrt{b\sec\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^5 + a*sec(f*x + e)^3)*sqrt(b*sec(f*x + e)^2 + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

3.243 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=290

$$\frac{(a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3f\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[Out] (2*(2*a + b)*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/
(3*f) - (2*(2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*f*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(3*a + 2*b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f
*x]^2)) + (b*Sec[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*T
an[e + f*x])/(3*f)
```

Rubi [A] time = 0.529156, antiderivative size = 366, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4148, 6722, 1974, 413, 527, 524, 426, 424, 421, 419}

$$\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{3f\sqrt{a\cos^2(e+fx)+b}}+\frac{b\tan(e+fx)\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3f\sqrt{a\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(2*a + b)*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e +
 f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]) - (2*(2*a + b)*Sqrt[Cos[e + f*x]
^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*S
qrt[a + b - a*Sin[e + f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)]) + ((a + b)*(3*a + 2*b)*Sqrt[Cos[e + f*x]^2]*Ellip
ticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (
a*Sin[e + f*x]^2)/(a + b)])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*
Sin[e + f*x]^2]) + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b -
a*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f*Sqrt[b + a*Cos[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^{3/2}}{1-x^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(b+a(1-x^2)\right)^{3/2}}{(1-x^2)^{5/2}}dx, x, \sin(e+fx\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\left(a+b-ax^2\right)^{3/2}}{(1-x^2)^{5/2}}dx, x, \sin(e+fx)\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{3f\sqrt{b+a\cos^2(e+fx)}} - \frac{(\sqrt{c}}{}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} + \frac{b\sec(e}{}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} + \frac{b\sec(e}{}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} + \frac{b\sec(e}{}$$

$$= \frac{2(2a+b)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} - \frac{2(2a+}{}$$

Mathematica [F] time = 11.5943, size = 0, normalized size = 0.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] time = 0.406, size = 5185, normalized size = 17.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec\left(fx + e\right)^{2} + a\right)^{\frac{3}{2}} \sec\left(fx + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec\left(fx + e\right)^{3} + a \sec\left(fx + e\right)\right) \sqrt{b \sec\left(fx + e\right)^{2} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^3 + a*sec(f*x + e))*sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

3.244 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=224

$$\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{f\left(-a\sin^2(e+fx)+a+b\right)}+\frac{b\sin\cdots}{}$$

```
[Out] (b*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/f + ((a -
b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec
[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)]) + (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/
(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e
+ f*x]^2)/(a + b)])/(f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.300054, antiderivative size = 277, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4148, 6722, 1974, 413, 524, 426, 424, 421, 419}

$$\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a\cos^2(e+fx)+b}}+\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e+fx)}\cdots}{f\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/
(f*Sqrt[b + a*Cos[e + f*x]^2]) + ((a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[Ar
cSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Si
n[e + f*x]^2])/(f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)]) + (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/
(f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int\left(a+\frac{b}{1-x^2}\right)^{3/2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{\left(b+a(1-x^2)\right)^{3/2}}{(1-x^2)^{3/2}}dx, x, \sin(e+fx\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{\left(a+b-ax^2\right)^{3/2}}{(1-x^2)^{3/2}}dx, x, \sin(e+fx)\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left((-a+b)\sqrt{\cos^2}\right.}{}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} - \frac{\left(\left((-a+b)\sqrt{\cos^2}\right.\right.}{}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{f\sqrt{b+a\cos^2(e+fx)}} + \frac{(a-b)\sqrt{\cos^2(e}}{}$$

Mathematica [F] time = 12.611, size = 0, normalized size = 0.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] time = 0.348, size = 3632, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((
b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^2*(2*cos(f*x+e)*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*I*b^(5/2)*a^(1/2)*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*a^3+sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*
b-b^2)/(a+b)^2)^(1/2))*b^3-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+2*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)+4*I*cos(f*x+e)^4*a^(5
/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^3*a^(5/2
)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*a^(3/2)*b^(3/2)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)+4*I*cos(f*x+e)^3*b^(3/2)*a^(3
/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(5/2)+sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
```

cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^3-2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3
+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+2*I*sin(f*x+e)*co
s(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2
)+4*I*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a^(3/2)*b^(3/2)+2*I*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)+2*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2)+4*I*sin(f*x+e)*cos
(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(3/2)+2
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+2*cos(f*x+e)^4*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a^2*b+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
*b^2-4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+2*cos(f*x+e
)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-sin(f*x+e)*cos(f*x+e)*2^(1/
2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^

```
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+sin(f*x+e)*cos(f*x+e)*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3)/sin(f*x+e)/(b+a*cos(f*x+
e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \cos \left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \cos \left(fx+e\right) \sec \left(fx+e\right)^{2}+a \cos \left(fx+e\right)\right) \sqrt{b \sec \left(fx+e\right)^{2}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)*sec(f*x + e)^2 + a*cos(f*x + e))*sqrt(b*sec(f*x +
e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

3.245 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=241

$$-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3f\left(-a\sin^2(e+fx)+a+b\right)}+\frac{a\,s}{}$$

```
[Out] (a*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)])/(3*f) + (2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x
]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(3*f*Sqrt[1
 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF
[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*f*(a + b - a*Sin[e + f*x]^2
))
```

Rubi [A] time = 0.421035, antiderivative size = 294, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4148, 6722, 1974, 416, 524, 426, 424, 421, 419}

$$\frac{a\sin(e+fx)\cos^2(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}{3f\sqrt{a\cos^2(e+fx)+b}}-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b}}{3f\sqrt{-a\sin^2(e+fx)+a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Si
n[e + f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]) + (2*(a + 2*b)*Sqrt[Cos[e +
 f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]
^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[1
- (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[
ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin
[e + f*x]^2)/(a + b)])/(3*f*Sqrt[b + a*Cos[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+\frac{b}{1-x^2}\right)^{3/2}dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{\left(b+a\left(1-x^2\right)\right)^{3/2}}{\sqrt{1-x^2}}dx,x,\sin(e+f\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{\left(a+b-ax^2\right)^{3/2}}{\sqrt{1-x^2}}dx,x,\sin(e+fx\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} - \frac{(}{}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} - \frac{(b}{}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} + \frac{(2}{}$$

$$= \frac{a\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3f\sqrt{b+a\cos^2(e+fx)}} + \frac{2}{}$$

<u>Mathematica</u> [A] time = 1.96235, size = 179, normalized size = 0.74

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(-2\sqrt{2}b(a+b)\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}\text{EllipticF}\left(e+fx,\frac{a}{a+b}\right)+4\sqrt{2}\left(a^2+3ab+2b^2\right)\sqrt{\frac{a\cos}{}}\right.}{6f(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(4*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*S
qrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] -
 2*Sqrt[2]*b*(a + b)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF
```

```
[e + f*x, a/(a + b)] + a*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/
(6*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Maple [C] time = 0.439, size = 5069, normalized size = 21.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^3\sec\left(fx+e\right)^2+a\cos\left(fx+e\right)^3\right)\sqrt{b\sec\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^3*sec(f*x + e)^2 + a*cos(f*x + e)^3)*sqrt(b*sec(f*
x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

3.246 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=319

$$-\frac{b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\right.}{15af\left(-a\sin^2(e+fx)+a+b\right)}$$

```
[Out] (-2*(a - 3*(a + b))*Cos[e + f*x]^2*Sin[e + f*x]*Sqrt[Sec[e + f*x]^2*(a + b
- a*Sin[e + f*x]^2)])/(15*f) + (a*Cos[e + f*x]^4*Sin[e + f*x]*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)])/(5*f) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[
Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[Sec[e + f*x
]^2*(a + b - a*Sin[e + f*x]^2)])/(15*a*f*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b
)]) - (b*(a + b)*(4*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e +
f*x]], a/(a + b)]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)])/(15*a*f*(a + b - a*Sin[e + f*x]^2))
```

Rubi [A] time = 0.641289, antiderivative size = 395, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 416, 528, 524, 426, 424, 421, 419}

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}}+\frac{a\sin(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - 3*(a + b))*Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*
Sqrt[a + b - a*Sin[e + f*x]^2])/(15*f*Sqrt[b + a*Cos[e + f*x]^2]) + (a*Cos[
e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f
*x]^2])/(5*f*Sqrt[b + a*Cos[e + f*x]^2]) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[C
os[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e
+ f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*a*f*Sqrt[b + a*Cos[e + f*x]^2
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(4*a + 3*b)*Sqrt[Cos[e
 + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b*Sec[e + f*
x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a*f*Sqrt[b + a*Cos[e + f*x]
```

```
^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\left(b+a\left(1-x^2\right)\right)^{3/2}\,d\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\right)\text{Subst}\left(\int \sqrt{1-x^2}\left(a+b-ax^2\right)^{3/2}\,dx\right.}{f\sqrt{b+a\cos^2(e+fx)}}$$

$$= \frac{a\cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{5f\sqrt{b+a\cos^2(e+fx)}} - \frac{(}{}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

$$= -\frac{2(a-3(a+b))\cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\,\text{si}}}{15f\sqrt{b+a\cos^2(e+fx)}}$$

Mathematica [C] time = 9.31625, size = 350, normalized size = 1.1

$$\frac{\cos^3(e+fx)\csc(2(e+fx))\left(a+b\sec^2(e+fx)\right)^{3/2}\left(a\left(a\sqrt{-\frac{1}{a+b}}\sin^2(2(e+fx))\sqrt{a\cos(2(e+fx))+a+2b}(3a\cos(2(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]^3*Csc[2*(e + f*x)]*(a + b*Sec[e + f*x]^2)^(3/2)*((-8*I)*Sqrt[
2]*b*(8*a^2 + 13*a*b + 3*b^2)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticE[I*Arc
Sinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], (a
+ b)/b]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)] + a*((16*I)*Sqrt[2]*b*(2*a + 3*b)*
Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt
[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], (a + b)/b]*Sqrt[(a*Sin[e + f*x]^2
)/(a + b)] + a*Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(11*a
 + 12*b + 3*a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]^2)))/(30*a^2*Sqrt[-(a + b)
^(-1)]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

Maple [C] time = 0.623, size = 6396, normalized size = 20.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec\left(fx+e\right)^2 + a\right)^{\frac{3}{2}} \cos\left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^5 \sec\left(fx+e\right)^2 + a\cos\left(fx+e\right)^5\right)\sqrt{b\sec\left(fx+e\right)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^5*sec(f*x + e)^2 + a*cos(f*x + e)^5)*sqrt(b*sec(f*
x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

3.247 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=243

$$\frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{192b^2f}+\frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+f}}{128b^2f}$$

```
[Out] ((a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) + ((a + b)*(3*a^2 - 10*a*b + 35*b
^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(128*b^2*f) + ((3*a^2 - 10
*a*b + 35*b^2)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(192*b^2*f) -
 ((3*a - 7*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(48*b^2*f) + (
Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(8*b*f)
```

Rubi [A] time = 0.22289, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 416, 388, 195, 217, 206}

$$\frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{192b^2f}+\frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+f}}{128b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) + ((a + b)*(3*a^2 - 10*a*b + 35*b
^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(128*b^2*f) + ((3*a^2 - 10
*a*b + 35*b^2)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(192*b^2*f) -
 ((3*a - 7*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(48*b^2*f) + (
Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(8*b*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)^2\left(a+b+bx^2\right)^{3/2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf} + \frac{\text{Subst}\left(\int\left(-a+7b-\right.\right.}{}$$

$$= -\frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\sec^2(e+fx)\tan(e+f}{}$$

$$= \frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} - \frac{(3a-7b)\tan}{}$$

$$= \frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2-1\right.}{}$$

$$= \frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2-1\right.}{}$$

$$= \frac{(a+b)^2\left(3a^2-10ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} + \frac{(a+b)\left(3a^2-10a\right.}{}$$

Mathematica [C] time = 11.0491, size = 512, normalized size = 2.11

$$\frac{e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(3a^2b\left(18e^{2i(e+fx)}+5e^{4i(e+fx)}+5\right)\left(1+e^{2i(e+fx)}\right)^4-9a^3\left(1+e^{2i(e+fx)}\right)^6+ab^2\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-9*a^3*(1 +
 E^((2*I)*(e + f*x)))^6 + 3*a^2*b*(1 + E^((2*I)*(e + f*x)))^4*(5 + 18*E^((2
*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x))) + a*b^2*(1 + E^((2*I)*(e + f*x)))^2
```

```
*(145 + 948*E^((2*I)*(e + f*x)) + 2758*E^((4*I)*(e + f*x)) + 948*E^((6*I)*(
e + f*x)) + 145*E^((8*I)*(e + f*x))) + b^3*(105 + 910*E^((2*I)*(e + f*x)) +
 3591*E^((4*I)*(e + f*x)) + 8644*E^((6*I)*(e + f*x)) + 3591*E^((8*I)*(e + f
*x)) + 910*E^((10*I)*(e + f*x)) + 105*E^((12*I)*(e + f*x)))))/(1 + E^((2*I)
*(e + f*x)))^8 - (3*(a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*Log[(-4*Sqrt[b]*(-1
 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(96*S
qrt[2]*b^(5/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.836, size = 3343, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/384/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2*sin(f*x+e)*(24*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^3*b-70*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*b^4+70*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*b^4+56*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+48*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-120*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^3*a*b^3-48*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*c
os(f*x+e)*b^4-15*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b
-145*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-105*cos(f
*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+15*cos(f*x+e)^8*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+145*cos(f*x+e)^8*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+105*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a*b^3-108*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^2*b^2-36
0*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
```

1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a*b^3-12*2^(1/2)*(1/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^3*b+54*2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
x+e)))^(1/2)(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^2*b^2+180*2^(
1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+
b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a*b^
3+9*cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-9*cos(f*x+e)^8
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-105*cos(f*x+e)^7*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+105*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^4-56*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^
4+3*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-107*cos(f*x+
e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-215*cos(f*x+e)^7*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^3*b+107*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^2*b^2+215*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
*b^3-78*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-148*co
s(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+78*cos(f*x+e)^4*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+148*cos(f*x+e)^4*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+120*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a*b^3+9*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*s
in(f*x+e)*cos(f*x+e)^8*a^4+105*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*sin(f*x+e)*cos(f*x+e)^8*b^4-18*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+

```
e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e
)^8*a^4-210*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^8*b^4)*((b+a*cos(f*x+e)
^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 13.3166, size = 1381, normalized size = 5.68

$$\left[\frac{3\left(3\,a^4-4\,a^3b+18\,a^2b^2+60\,ab^3+35\,b^4\right)\sqrt{b}\cos\left(fx+e\right)^7\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+\cdots}{\cos\left(fx+e\right)^4}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*sqrt(b)*cos(f
*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x +
 e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f
```

```
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((
9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 +
 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 + 7*b^4)*cos(f*x +
 e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos
(f*x + e)^7), 1/768*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*s
qrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e)^7 - 2*((9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*
cos(f*x + e)^6 - 2*(3*a^2*b^2 + 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4
- 8*(9*a*b^3 + 7*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^6, x)
```

3.248 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6bf}-\frac{(a-5b)\tan(e+fx)\left(a+b\,\mathrm{ta}\right.}{24bf}$$

```
[Out] -((a - 5*b)*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(16*b^(3/2)*f) - ((a - 5*b)*(a + b)*Tan[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(16*b*f) - ((a - 5*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]
^2)^(3/2))/(24*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*b*
f)
```

Rubi [A] time = 0.130678, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4146, 388, 195, 217, 206}

$$-\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6bf}-\frac{(a-5b)\tan(e+fx)\left(a+b\,\mathrm{ta}\right.}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - 5*b)*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]])/(16*b^(3/2)*f) - ((a - 5*b)*(a + b)*Tan[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(16*b*f) - ((a - 5*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]
^2)^(3/2))/(24*b*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*b*
f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+b+bx^2\right)^{3/2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf} - \frac{(a-5b)\,\text{Subst}\left(\int \left(a+b+bx^2\right)^{3}\right.}{6bf}$$

$$= -\frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf} + \frac{\tan(e+fx)\left(a+b+b\right.}{6bf}$$

$$= -\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} - \frac{(a-5b)\tan(e+f}{}$$

$$= -\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} - \frac{(a-5b)\tan(e+f}{}$$

$$= -\frac{(a-5b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a-5b)(a+b)\tan(e+fx)}{16bf}$$

Mathematica [C] time = 9.38514, size = 400, normalized size = 2.42

$$\frac{e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{3(a-5b)(a+b)^2\log\left(\frac{4if\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}-4\sqrt{b}f\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}\right)}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}} - \frac{i\sqrt{b}\left(-1+e^{2i(e\right.}}{}\right.}{12\sqrt{2}b^{3/2}f(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(3*a^2*(1 +
E^((2*I)*(e + f*x)))^4 + 2*a*b*(1 + E^((2*I)*(e + f*x)))^2*(11 + 50*E^((2*I
)*(e + f*x)) + 11*E^((4*I)*(e + f*x))) + b^2*(15 + 100*E^((2*I)*(e + f*x))
+ 298*E^((4*I)*(e + f*x)) + 100*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x)
))))/(1 + E^((2*I)*(e + f*x)))^6 + (3*(a - 5*b)*(a + b)^2*Log[(-4*Sqrt[b]*(
-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E
```

```
^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e
+ f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(12
*Sqrt[2]*b^(3/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.502, size = 2519, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/48/f/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(-9*sin(f*x+e)*
cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+32*cos
(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-32*cos(f*x+e)^4*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+22*cos(f*x+e)^7*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^2*b+15*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b^2-22*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^
2*b-15*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+10*cos(f*
x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-10*cos(f*x+e)^2*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*b^3+15*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-15*cos(f
*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+3*sin(f*x+e)*cos(f*x+e)
^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-15*sin(f*x+e)*cos(
f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-6*sin(f*x+e)
*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
```

```
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+30*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*b^3+3*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-3*cos(
f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+8*cos(f*x+e)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+17*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a^2*b-17*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a^2*b+22*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-22*c
os(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-27*sin(f*x+e)*cos
(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+18*sin(f*
x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+54*sin(f*x+e)*cos(f*x+e)^6*2^(1/2
)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(
a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2))*a*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/
(b+a*cos(f*x+e)^2)^2/cos(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.4534, size = 1154, normalized size = 6.99

$$\left[-\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{b}\cos\left(fx+e\right)^5\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2b\cos\left(fx+e\right)\right)\sqrt{b}}{\cos\left(fx+e\right)^4}\right.}{19}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a
^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b
)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b + 22*a*b^
2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b^3)*cos(f*x + e)^2)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)
^5), -1/96*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a -
b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 -
 2*((3*a^2*b + 22*a*b^2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b
^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(b^2*f*cos(f*x + e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

3.249 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$\frac{3(a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+f}{\sqrt{a+b\tan^2(e+}}\right.}{8\sqrt{b}f}$$

```
[Out] (3*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(8*Sqrt[b]*f) + (3*(a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
8*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.102451, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4146, 195, 217, 206}

$$\frac{3(a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8f}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+f}{\sqrt{a+b\tan^2(e+}}\right.}{8\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(8*Sqrt[b]*f) + (3*(a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(
8*f) + (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int\left(a+b+bx^2\right)^{3/2}dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+b))\,\text{Subst}\left(\int\sqrt{a+b+bx}\right.}{4f}$$

$$= \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+b+b\tan\right.}{4f}$$

$$= \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+b+b\tan\right.}{4f}$$

$$= \frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan}}{8f}$$

Mathematica [C] time = 0.32554, size = 84, normalized size = 0.76

$$\frac{(a+b)^2\sin(2(e+fx))\sqrt{a+b\sec^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{b\sin^2(e+fx)}{-a\sin^2(e+fx)+a+b}\right)}{f(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]

[Out] ((a + b)^2*Hypergeometric2F1[1/2, 3, 3/2, (b*Sin[e + f*x]^2)/(a + b - a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(f*(a + 2*b + a*Cos[2*(e + f*x)]))

Maple [C] time = 0.344, size = 1768, normalized size = 15.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)

[Out] -1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-12*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+

```
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-5*cos(
f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^5*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^2+3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a*b-7*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e
)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+7*cos(f*x+e)^2*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*b^2-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)
^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.1217, size = 969, normalized size = 8.73

$$\left[\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2 \ldots}{\cos\left(fx+e\right)^2 \ldots}}}{\cos\left(fx+e\right)^4}\right.}{32\,bf\cos\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2
)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3
 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b + 3*b^2)*cos(f*x + e)^2 + 2
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*
x + e)^3), 1/16*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((5*a*
b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\sec^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx + e\right)^2 + a\right)^{\frac{3}{2}}\sec\left(fx + e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

3.250 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0920687, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4128, 416, 523, 217, 206, 377, 203}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{1+x^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}}dx, x, \tan(e+fx\right.}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1+ax^2}dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(b}{}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{}}{}$$

Mathematica [C] time = 2.05414, size = 527, normalized size = 4.47

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-ia^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+ia^{3/2}\log\left(\sqrt{a}\sqrt{a(1+\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
```

```
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.293, size = 1557, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(6*sin(f*x+e)*cos(f*x+e)^2*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*a*b+2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-2*cos(f
*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-3*
sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*b^2+4*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
```

```
/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+cos(f*x+e)^3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/
cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 1.77804, size = 3602, normalized size = 30.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
```

```
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx + e\right)^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.251 $\int \cos^2(e + fx) \left(a + b \sec^2(e + fx)\right)^{3/2} dx$

Optimal. Leaf size=124

$$\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{a}(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{a\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{2f}$$

```
[Out] (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/f + (a*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*f)
```

Rubi [A] time = 0.136319, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4146, 413, 523, 217, 206, 377, 203}

$$\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{a}(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}+\frac{a\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```



```
[Out] (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/f + (a*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2
])/(2*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(a+2b)+2b^2 \ldots}{\left(1+x^2\right)\sqrt{a+b+bx \ldots}}\right.}{2f \ldots}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,d \ldots\right.}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \ldots\right.}{f}$$

$$= \frac{\sqrt{a}(a+3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{a\,\mathrm{c}\ldots}{\ldots}$$

Mathematica [C] time = 7.03179, size = 466, normalized size = 3.76

$$\frac{e^{-i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{2e^{2i(e+fx)}\left(2a^{3/2}fx-4b^{3/2}\log\left(-\frac{e^{3ie}f\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}\right)}{2b^2\left(1+e^{2i(e+fx)}\right)}\right)-i\sqrt{a}\ldots\right.}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f*
x]^3*((-I)*a*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a^(3/2)
*f*x + 6*Sqrt[a]*b*f*x - I*Sqrt[a]*(a + 3*b)*Log[(a + 2*b + a*E^((2*I)*(e +
 f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
^2])/E^((2*I)*e)] + I*Sqrt[a]*(a + 3*b)*Log[(a + a*E^((2*I)*(e + f*x)) + 2*
b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])/E^((2*I)*e)] - 4*b^(3/2)*Log[-(E^((3*I)*e)*(Sqrt[b]*(-1
```

```
 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*
(e + f*x)))^2])*f)/(2*b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(
2*Sqrt[2]*E^(I*(e + f*x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.306, size = 1511, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(2^(1/2)*(1/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-6*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*a*b*sin(f*x+e)-4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
```

```
(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-cos(f*x+e)
^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e)^2*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b)*cos(f*x+e)^3*((b+a*co
s(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Fricas [B] time = 1.74004, size = 3456, normalized size = 27.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28
*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^
3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e
)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*
b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e)) + 4*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a
*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos
(f*x + e)^4))/f, 1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)*sin(f*x + e) + 8*sqrt(-b)*b*arctan(-1/2*((a - b)*cos(f*x + e)^3 +
2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*
```

```
b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, 1/8*(4*a*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - (a + 3*b)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(
a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/co
s(f*x + e)^4))/f, 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(
f*x + e)*sin(f*x + e) - (a + 3*b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*sqrt(-b)*b*arc
tan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/
f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

3.252 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)\sin(e+fx)\cos(e-}{}$$

```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*Sqrt[a]*f) + (3*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(4*f)
```

Rubi [A] time = 0.120005, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4146, 378, 377, 203}

$$\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{a}f}+\frac{\sin(e+fx)\cos^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4f}+\frac{3(a+b)\sin(e+fx)\cos(e-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(8*Sqrt[a]*f) + (3*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*f) + (Cos[e + f*x]^3*Sin[e + f*x]*(a + b + b*Tan[e + f*x]^2)^
(3/2))/(4*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a+b))\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\cos^3(e+fx)\sin}{}$$

$$= \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\cos^3(e+fx)\sin}{}$$

$$= \frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b}}{8f}$$

Mathematica [A] time = 1.03027, size = 191, normalized size = 1.53

$$\frac{\cos(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\left(a\cos^2(e+fx)+b\right)\sqrt{a+b\sec^2(e+fx)}\left(3(a+b)^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a}\sin(\ldots\right.}{2\sqrt{a}f(a\cos(2(e+fx))+a+2b)^{3/2}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b
- a*Sin[e + f*x]^2]*(3*(a + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]] + Sqrt[a]*(4*a + 5*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]*Sqrt[(a + b -
a*Sin[e + f*x]^2)/(a + b)]))/(2*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3
/2)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

Maple [C] time = 0.347, size = 1713, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)^
2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3*(-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^2+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
^2*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f
*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
```

```
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-6*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)
+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-12*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-6*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*b^2*sin(f*x+e)+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^2-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-
7*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)^2*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+7*cos(f*x+e)^2*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b-5*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Fricas [B] time = 1.52381, size = 1357, normalized size = 10.86

$$\left[-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + 5*a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f)
, -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*x +
e)^3 + (3*a^2 + 5*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

3.253 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=193

$$\frac{(5a-b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}+\frac{\sin(e+fx)\cos^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6af}+\frac{(5a-b)\sin(e+fx)\,\cdots}{\cdots}$$

```
[Out] ((5*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(16*a^(3/2)*f) + ((5*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a - b)*Cos[e + f*x]^3*Sin[e + f*
x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(24*a*f) + (Cos[e + f*x]^5*Sin[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*a*f)
```

Rubi [A] time = 0.162324, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4146, 382, 378, 377, 203}

$$\frac{(5a-b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{3/2}f}+\frac{\sin(e+fx)\cos^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6af}+\frac{(5a-b)\sin(e+fx)\,\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] ((5*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(16*a^(3/2)*f) + ((5*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*a*f) + ((5*a - b)*Cos[e + f*x]^3*Sin[e + f*
x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(24*a*f) + (Cos[e + f*x]^5*Sin[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*a*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af} + \frac{(5a-b)\,\text{Subst}\left(\int \frac{(}{}\right.}{}$$

$$= \frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af} + \frac{\cos^5(e+f}{}$$

$$= \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a-b)}{}$$

$$= \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a-b)}{}$$

$$= \frac{(5a-b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(5a-b)(a+b)\cos(e+fx)\sin}{16}$$

Mathematica [A] time = 1.84457, size = 165, normalized size = 0.85

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{3\sqrt{2}\sqrt{a+b}\left(5a^2+4ab-b^2\right)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}} + \sqrt{a}\sin(e+fx)\left(a^2\cos(4(e+fx))+23a^2+a(9a+\right.\right.}{48a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((3*Sqrt[2]*Sqrt[a + b]*(5*a^2 + 4
*a*b - b^2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)] + Sqrt[a]*(23*a^2 + 29*a*b + 3*b^2 + a*(9*a + 7*b
)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Sin[e + f*x]))/(48*a^(3/2)*f)
```

Maple [C] time = 0.49, size = 2439, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/48/f/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*((b+a*cos(f*x+
e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^3*(-3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)+15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*a^3*sin(f*x+e)-15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+27*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+9*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-54*2^(1/2)*(1/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(
1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*
b*sin(f*x+e)-18*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+3*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*b^3+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e
```

```
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-8*cos(f*x+e)^7*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+8*cos(f*x+e)^6*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^3-10*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a^3+10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-3*c
os(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+22*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a*b^2-22*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a^2*b+22*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*
b-32*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-17*cos(f*x+
e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+32*cos(f*x+e)^2*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+17*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^2-22*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a*b^2-15*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+15*co
s(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-30*2^(1/2)*(1/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a^3*sin(f*x+e))/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

Fricas [A] time = 4.04061, size = 1539, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)
^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 + 2*(5
*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f), -1/192
*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e
)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt
(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^
2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*cos
(f*x + e)^5 + 2*(5*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a
*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

3.254 $\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{8d}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right.}{4d}$$

```
[Out] (a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]])/d +
 (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a +
 b + b*Tan[c + d*x]^2]])/(8*d) + (b*(7*a + 3*b)*Tan[c + d*x]*Sqrt[a + b + b
*Tan[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b + b*Tan[c + d*x]^2)^(3/2))
/(4*d)
```

Rubi [A] time = 0.171885, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4128, 416, 528, 523, 217, 206, 377, 203}

$$\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{8d}+\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{d}+\frac{b\tan(c+dx)\left(a+b\tan^2(c+dx)\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^(5/2),x]
```


```
[Out] (a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]])/d +
 (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a +
 b + b*Tan[c + d*x]^2]])/(8*d) + (b*(7*a + 3*b)*Tan[c + d*x]*Sqrt[a + b + b
*Tan[c + d*x]^2])/(8*d) + (b*Tan[c + d*x]*(a + b + b*Tan[c + d*x]^2)^(3/2))
/(4*d)
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{5/2}}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$=\frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}\left((a+b)(4a+3b)+b(7a+3b)x^2\right)}{1+x^2}\,dx,\right.}{4d}$$

$$=\frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right.}{4d}$$

$$=\frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right.}{4d}$$

$$=\frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d}+\frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right.}{4d}$$

$$=\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d}+\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{8d}+\frac{b(}{}$$

Mathematica [C] time = 9.73198, size = 706, normalized size = 4.25

$$e^{i(c+dx)}\cos^5(c+dx)\sqrt{4b+ae^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2}\left(\frac{-15a^2\sqrt{b}\log\left(\frac{4id\sqrt{a\left(1+e^{2i(c+dx)}\right)^2+4be^{2i(c+dx)}}-4\sqrt{b}d\left(-1+e^{2i(c+dx)}\right)}{b\left(15a^2+10ab+3b^2\right)\left(1+e^{2i(c+dx)}\right)}\right)-10ab^{3/2}\log\left(\frac{4id\sqrt{a\left(1+e^{2}\right.}}{}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

```
[Out] (E^(I*(c + d*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(c + d*x)))^2)/E^((2*I)*(c + d
*x))]*Cos[c + d*x]^5*(((-I)*b*(-1 + E^((2*I)*(c + d*x)))*(9*a*(1 + E^((2*I)
```

```
*(c + d*x)))^2 + b*(3 + 14*E^((2*I)*(c + d*x)) + 3*E^((4*I)*(c + d*x)))))/(
1 + E^((2*I)*(c + d*x)))^4 + (8*a^(5/2)*d*x - (4*I)*a^(5/2)*Log[a + 2*b + a
*E^((2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*
I)*(c + d*x)))^2]] + (4*I)*a^(5/2)*Log[a + a*E^((2*I)*(c + d*x)) + 2*b*E^((
2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c
 + d*x)))^2]] - 15*a^2*Sqrt[b]*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x)))
 + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(
b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))] - 10*a*b^(3/2)*Log[
(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x))) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d
*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^
((2*I)*(c + d*x))))] - 3*b^(5/2)*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x)
)) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])
/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))])/Sqrt[4*b*E^((2*I
)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])*(a + b*Sec[c + d*x]^2)^(5/2)
)/(Sqrt[2]*d*(a + 2*b + a*Cos[2*c + 2*d*x])^(5/2))
```

Maple [C] time = 0.622, size = 2231, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c)^2)^(5/2),x)
```

```
[Out] -1/8/d/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(8*sin(d*x+c)*cos(d*x+c)^4*2
^(1/2)*(1/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(d*x+c
)+b)/(cos(d*x+c)+1))^(1/2)*(-2/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-a*cos(d*x+c)-b)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+15*cos(d*x+c)^4*sin(d*
x+c)*2^(1/2)*(1/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(d*x+c)+b)/(cos(d*x+c)+1))^(1/2)*(-2/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+10*cos(d*x+c)^
4*sin(d*x+c)*2^(1/2)*(1/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(d*x+c)+b)/(cos(d*x+c)+1))^(1/2)*(-2/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+3*sin(
d*x+c)*cos(d*x+c)^4*2^(1/2)*(1/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(d*x+c)+b)/(cos(d*x+c)+1))^(1/2)*(-2/(a+b)*(I*cos(d*x+c)*a^(
```

```
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-3
0*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*(1/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(d*x+c)+b)/(cos(d*x+c)+1))^(1/2)*(-2/(a+b)*(I*cos(d*x+
c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+
c),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-20*sin(d*x+c)*cos(d*x+c)^
4*2^(1/2)*(1/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(d*
x+c)+b)/(cos(d*x+c)+1))^(1/2)*(-2/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-a*cos(d*x+c)-b)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+c),1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*a*b^2-6*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*(1/(a+b)*(I*cos
(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(d*x+c)+b)/(cos(d*x+c)+1))^(
1/2)*(-2/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(d*x+c)
-b)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(d*x+c),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3-1
6*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)*(1/(a+b)*(I*cos(d*x+c)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(d*x+c)+b)/(cos(d*x+c)+1))^(1/2)*(-2/(a+b)*(I*cos(d*x+
c)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(d*x+c)-b)/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(d*x+
c),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3-9*cos(d*x+c)^5*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-3*cos(d*x+c)^5*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b^2+9*cos(d*x+c)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^2*b+3*cos(d*x+c)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-11*cos(
d*x+c)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*cos(d*x+c)^3*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+11*cos(d*x+c)^2*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a*b^2+3*cos(d*x+c)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^3-2*cos(d*x+c)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+2*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)*cos(d*x+c)*((a*cos(d*x+c)^2+b)/cos(d
*x+c)^2)^(5/2)*sin(d*x+c)/(-1+cos(d*x+c))/(a*cos(d*x+c)^2+b)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)^{2}+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

Fricas [B] time = 4.28011, size = 3947, normalized size = 23.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*a^2*cos(d*x + c)^3*log(128*a^4*cos(d*x + c)^8 - 256*(a^4
- a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*c
os(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos
(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10*a*b + 3*b^2)*sqrt(b)*cos(d*x + c)
^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 8*(a*b - b^2)*cos(d*x + c)^2 +
 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b)*sqrt((a*cos(d*x + c)
^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b^2)/cos(d*x + c)^4) + 4*(3*(3*a*b
 + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)
*sin(d*x + c))/(d*cos(d*x + c)^3), 1/16*(2*sqrt(-a)*a^2*cos(d*x + c)^3*log(
128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*
a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3
+ b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*c
os(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a
*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-
a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10
*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x +
c))*sqrt(-b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)/((a*b*cos(d*x + c)
^2 + b^2)*sin(d*x + c)))*cos(d*x + c)^3 + 2*(3*(3*a*b + b^2)*cos(d*x + c)^2
 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^3), -1/32*(8*a^(5/2)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a
*b)*cos(d*x + c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c)^2 + b)/cos(d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(d*x + c)^2)*sin(d*x + c)))*cos(d*x + c)^3 - (15*a^2 + 10*
a*b + 3*b^2)*sqrt(b)*cos(d*x + c)^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4
 + 8*(a*b - b^2)*cos(d*x + c)^2 + 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x +
 c))*sqrt(b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b
^2)/cos(d*x + c)^4) - 4*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*co
```

```
s(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3), -1/16*(
4*a^(5/2)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a*b)*cos(d*x + c)^3 +
 (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)^2 + b)/cos(
d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(d*
x + c)^2)*sin(d*x + c)))*cos(d*x + c)^3 - (15*a^2 + 10*a*b + 3*b^2)*sqrt(-b
)*arctan(-1/2*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(-b)*sqrt((a*
cos(d*x + c)^2 + b)/cos(d*x + c)^2)/((a*b*cos(d*x + c)^2 + b^2)*sin(d*x + c
)))*cos(d*x + c)^3 - 2*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos
(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

3.255 $\int \left(1+\sec^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=42

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}+2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

```
[Out] 2*ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]] + (Tan[x]*Sqr
t[2 + Tan[x]^2])/2
```

Rubi [A] time = 0.0368925, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4128, 416, 523, 215, 377, 203}

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}+2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sec[x]^2)^(3/2),x]
```


```
[Out] 2*ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]] + (Tan[x]*Sqr
t[2 + Tan[x]^2])/2
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(1+\sec^2(x)\right)^{3/2} dx = \text{Subst}\left(\int \frac{\left(2+x^2\right)^{3/2}}{1+x^2} dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{6+4x^2}{\left(1+x^2\right)\sqrt{2+x^2}} dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}} dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{2+x^2}} dx, x\right.$$
$$= 2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)} + \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$
$$= 2\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}$$

Mathematica [C] time = 0.172578, size = 109, normalized size = 2.6

$$\frac{\left(\cos^2(x)+1\right)\sec(x)\sqrt{\sec^2(x)+1}\left(\sin(x)\sqrt{\cos(2x)+3}-2i\sqrt{2}\cos^2(x)\log\left(\sqrt{\cos(2x)+3}+i\sqrt{2}\sin(x)\right)+4\sqrt{2}\cos^2(x)\tan\right.}{(\cos(2x)+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sec[x]^2)^(3/2),x]
```

```
[Out] ((1 + Cos[x]^2)*Sec[x]*Sqrt[1 + Sec[x]^2]*(4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sin[x
])/Sqrt[3 + Cos[2*x]]]*Cos[x]^2 - (2*I)*Sqrt[2]*Cos[x]^2*Log[Sqrt[3 + Cos[2
*x]] + I*Sqrt[2]*Sin[x]] + Sqrt[3 + Cos[2*x]]*Sin[x]))/(3 + Cos[2*x])^(3/2)
```

Maple [C] time = 0.296, size = 429, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sec(x)^2)^(3/2),x)
```

```
[Out] (-1/4+1/4*I)*(8*(-1)^(3/4)*sin(x)*cos(x)^2*(-(I*cos(x)-1-I-cos(x))/(cos(x)+
1))^(1/2)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),-I,I)*((I*cos(x)+1-I+cos
(x))/(cos(x)+1))^(1/2)+4*(-1)^(3/4)*sin(x)*cos(x)^2*(-(I*cos(x)-1-I-cos(x))
/(cos(x)+1))^(1/2)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),I,I)*((I*cos(x)
+1-I+cos(x))/(cos(x)+1))^(1/2)-8*(-1)^(1/4)*sin(x)*cos(x)^2*(-(I*cos(x)-1-I
-cos(x))/(cos(x)+1))^(1/2)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),-I,I)*(
(I*cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)-4*(-1)^(1/4)*sin(x)*cos(x)^2*(-(I*c
os(x)-1-I-cos(x))/(cos(x)+1))^(1/2)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x
),I,I)*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)+6*sin(x)*cos(x)^2*2^(1/2)*(
-(I*cos(x)-1-I-cos(x))/(cos(x)+1))^(1/2)*EllipticF((1/2+1/2*I)*2^(1/2)*(-1+
cos(x))/sin(x),I)*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)-I*cos(x)^3+I*cos
(x)^2-cos(x)^3-I*cos(x)+cos(x)^2+1+I-cos(x))*((cos(x)^2+1)/cos(x)^2)^(3/2)*
sin(x)*cos(x)/(-1+cos(x))/(cos(x)^2+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sec(x)^2 + 1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sec(x)^2 + 1)^(3/2), x)
```

Fricas [B] time = 0.542438, size = 536, normalized size = 12.76

$$\frac{\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)\cos(x) - \arctan\left(\frac{\sin(x)}{\cos(x)}\right)\cos(x) + 2\cos(x)\log\left(\cos(x)^2 + \cos(x)\sin(x) + \left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))
/(cos(x)^4 + cos(x)^2 - 1))*cos(x) - arctan(sin(x)/cos(x))*cos(x) + 2*cos(x
)*log(cos(x)^2 + cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2
```

```
+ 1)/cos(x)^2) + 1) - 2*cos(x)*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - c
os(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) + 1) + sqrt((cos(x)^2 + 1)/cos(
x)^2)*sin(x))/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sec^2(x)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)**2)**(3/2),x)
```

```
[Out] Integral((sec(x)**2 + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sec(x)^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sec(x)^2 + 1)^(3/2), x)
```

3.256 $\int \sqrt{1+\sec^2(x)}\,dx$

Optimal. Leaf size=24

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

```
[Out] ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Rubi [A] time = 0.0185345, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4128, 402, 215, 377, 203}

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)+\sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sec[x]^2],x]
```

```
[Out] ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1+\sec^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{2+x^2}}{1+x^2}\,dx, x, \tan(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{2+x^2}}\,dx, x, \tan(x)\right) \\
&= \sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right) \\
&= \sinh^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.0500783, size = 57, normalized size = 2.38

$$
\frac{\sqrt{2}\cos(x)\sqrt{\sec^2(x)+1}\left(\sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{\cos(2x)+3}}\right)\right)}{\sqrt{\cos(2x)+3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sec[x]^2],x]
```

```
[Out] (Sqrt[2]*(ArcSin[Sin[x]/Sqrt[2]] + ArcTanh[(Sqrt[2]*Sin[x])/Sqrt[3 + Cos[2*
x]]])*Cos[x]*Sqrt[1 + Sec[x]^2])/Sqrt[3 + Cos[2*x]]
```

Maple [C] time = 0.299, size = 190, normalized size = 7.9

$$\frac{(-1+i)\cos(x)(\sin(x))^2}{(-1+\cos(x))\left((\cos(x))^2+1\right)}\left((-1)^{\frac{3}{4}}\operatorname{EllipticPi}\left(\frac{\sqrt[4]{-1}\,(-1+\cos(x))}{\sin(x)},i,i\right)+(-1)^{\frac{3}{4}}\operatorname{EllipticPi}\left(\frac{\sqrt[4]{-1}\,(-1+\cos(x))}{\sin(x)},-i,i\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sec(x)^2)^(1/2),x)
```

```
[Out] (-1+I)*((-1)^(3/4)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),I,I)+(-1)^(3/4)
*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),-I,I)-(-1)^(1/4)*EllipticPi((-1)^
(1/4)*(-1+cos(x))/sin(x),I,I)-(-1)^(1/4)*EllipticPi((-1)^(1/4)*(-1+cos(x))/
sin(x),-I,I)+2^(1/2)*EllipticF((1/2+1/2*I)*2^(1/2)*(-1+cos(x))/sin(x),I))*c
os(x)*sin(x)^2*((cos(x)^2+1)/cos(x)^2)^(1/2)*((I*cos(x)+1-I+cos(x))/(cos(x)
+1))^(1/2)*(-(I*cos(x)-1-I-cos(x))/(cos(x)+1))^(1/2)/(-1+cos(x))/(cos(x)^2+
1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 0.526813, size = 444, normalized size = 18.5

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)+\frac{1}{2}\log\left(\cos(x)^2+\cos(x)\sin(x)+\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))/
(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x)) + 1/2*log(cos(x)^2 +
 cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) +
 1) - 1/2*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - cos(x)*sin(x))*sqrt((c
os(x)^2 + 1)/cos(x)^2) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sec(x)**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(x)^2 + 1), x)
```

3.257 $\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=330

$$-\frac{(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{2(a-b)\tan(e+fx)\sec(e+fx)\left(-a\sin^2(e+fx)+a+\right.}{3b^2f\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] (2*(a - b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*
x]^2))/(3*b^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e +
 f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 2*b)*EllipticF[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*b*f*S
qrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (2*(
a - b)*Sec[e + f*x]*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x])/(3*b^2*f*Sqrt[
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (Sec[e + f*x]^3*(a + b - a*Si
n[e + f*x]^2)*Tan[e + f*x])/(3*b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f
*x]^2)])
```

Rubi [A] time = 0.57176, antiderivative size = 380, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 414, 527, 524, 426, 424, 421, 419}

$$-\frac{2(a-b)\tan(e+fx)\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}}{3b^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a}}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (2*(a - b)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a
+ b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a
+ b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 2*b)*Sqrt
[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)])/(3*b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e +
 f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) - (2*(a - b)*Sqrt[b + a*Cos[e + f*
x]^2]*Sec[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(3*b^2*f*Sq
rt[a + b*Sec[e + f*x]^2]) + (Sqrt[b + a*Cos[e + f*x]^2]*Sec[e + f*x]^3*Sqrt
```

[a + b - a*Sin[e + f*x]^2]*Tan[e + f*x])/(3*b*f*Sqrt[a + b*Sec[e + f*x]^2])

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sec^3(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{3bf\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx}}{3bf\sqrt{}}$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{3b^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{b+a\cos}}{}$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{3b^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{b+a\cos}}{}$$

$$= -\frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{3b^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{b+a\cos}}{}$$

$$= \frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|\frac{a}{a+b}\right)\sqrt{a+b-a\sin^2(e+fx)}}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{(a-2b)\sqrt{}}{3bf\sqrt{}}$$

Mathematica [F] time = 11.9051, size = 0, normalized size = 0.

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.564, size = 4753, normalized size = 14.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f/b^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)
*(cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a^2*b+cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*a^3-cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^3+2*I*a^(1/2)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*
cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+cos(f*x+e)^4*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+4*I*cos(f*x+e)^3*b^(3/2)*a^(3/2)
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-cos(f*x+e)^3*sin(f*x+e)*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-2*cos(f*x+e)^3*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*b^3+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+4*I*cos(f*x+e)^5*b
```

^(1/2)*a^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^4*b^(
1/2)*a^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^3*a^(1/
2)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*I*cos(f*x+e)^2*a^(3/2)
b^(3/2)((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+2*I*cos(f*x+e)^2*a^(1/2)*b
^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*cos(f*x+e)^5*a^(3/2)*b^(
3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+2*I*cos(f*x+e)^4*a^(3/2)*b^(3/
2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*
(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-3*cos(f*x+e)^4*sin(f*x+e)*2
^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*
I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*cos(f*x+e)^4*sin(f
*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+cos(f*x+e)^3*
sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-cos(f*x+
e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-3*c
os(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^
2*b+3*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a*b^2-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+2*cos
(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^2+4*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(

```
1/2)*a^2*b-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-2*c
os(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^3*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*a*b^2-2*I*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)+2*I*cos(f*x+e)^4*sin(f*x+e)*2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2)*a^(3/2)-2*I*cos(f*x+e)^4*sin(f*
x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(1/2)*a^(5/2)+2*I*
cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b
^(3/2)*a^(3/2)-2*I*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b^(1/2)*a^(5/2)-2*I*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2))/cos(f*x+e)^4/((b+a*co
s(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.258 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=170

$$\frac{\tan(e+fx)\sec(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{bf\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{\sqrt{a}\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] -((Sqrt[a]*Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]
, (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(b*f*Sqrt[Cos[e + f*x]^2
]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])) + (Sec[e + f*x]*(a + b
- a*Sin[e + f*x]^2)*Tan[e + f*x])/(b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e
 + f*x]^2)])
```

Rubi [A] time = 0.370607, antiderivative size = 202, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4148, 6722, 1974, 414, 21, 427, 424}

$$\frac{\tan(e+fx)\sec(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}}{bf\sqrt{a+b\sec^2(e+fx)}}-\frac{\sqrt{a}\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}E}{bf\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[a + b]*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]], (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)
])/(b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[
e + f*x]^2])) + (Sqrt[b + a*Cos[e + f*x]^2]*Sec[e + f*x]*Sqrt[a + b - a*Sin
[e + f*x]^2]*Tan[e + f*x])/(b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}\sqrt{b+a\left(1-x^2\right)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}}{bf\sqrt{\text{co}}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} - \frac{\left(a\sqrt{b+a\cos^2(e+f}\right.}{bf\sqrt{\cos}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}} - \frac{\left(a\sqrt{b+a\cos^2(e+f}\right.}{bf\sqrt{\cos^2(e+}}$$

$$= -\frac{\sqrt{a}\sqrt{a+b}\sqrt{b+a\cos^2(e+fx)}E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(}}{}$$

Mathematica [F] time = 10.6787, size = 0, normalized size = 0.

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.405, size = 3033, normalized size = 17.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b/(2*I*a^(1/2)*b^(1/2)-a+b)*(
4*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*I*cos(f*x+e)^
2*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*I*sin(f*x+e)*co
s(f*x+e)*a^(1/2)*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+
2*I*cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(1/2)*2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)-4*I*cos(f*x+e)^3*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)+4*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a*b-cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
E((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*a^2-2*cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*a*b-cos(f*x+e)^2*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2-4*I*cos(f*x+e)*a^(1/2)*b^(3/2)*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)+4*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e
```

```
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*a*b-cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2-2*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b-cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1
/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2+2*I*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*a^(3/2)*b^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)-2*I*sin(f*x+e)*cos(f*
x+e)^2*a^(1/2)*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-2*cos(f*x+e)^3*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-2*cos(f*x
+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/((b+a*cos
(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)^2/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^{3}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{3}\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(fx+e\right)^{3}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.259 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] (EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
+ b)])/(f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)])
```

Rubi [A] time = 0.271798, antiderivative size = 103, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 421, 419}

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
[1 - (a*Sin[e + f*x]^2)/(a + b)])/(f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e
+ f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
```

```
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\left(\sqrt{b+a\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,F\left(\sin^{-1}(\sin(e+fx))\middle|\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}}$$

Mathematica [A] time = 0.22415, size = 69, normalized size = 0.86

$$\frac{\sec(e+fx)\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}\,\text{EllipticF}\left(e+fx, \frac{a}{a+b}\right)}{\sqrt{2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)]
*Sec[e + f*x])/(Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.399, size = 269, normalized size = 3.4

$$\frac{\left(\sin\left(fx+e\right)\right)^2\sqrt{2}}{f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{1}{(a+b)\left(1+\cos\left(fx+e\right)\right)}\left(i\cos\left(fx+e\right)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+a\cos\left(fx+e\right)+b\right)}\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)^2*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos
(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(fx+e\right)}{\sqrt{b\sec \left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.260 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{\sqrt{a}f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] (Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]], (a + b)/
a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(Sqrt[a]*f*Sqrt[Cos[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.166239, antiderivative size = 128, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 427, 424}

$$\frac{\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{\sqrt{a}f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b]*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f
*x])/Sqrt[a + b]], (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(Sqrt[a
]*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e +
f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\left(\sqrt{b+a\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{1-\frac{ax^2}{a+b}}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}}$$

$$= \frac{\sqrt{a+b}\sqrt{b+a\cos^2(e+fx)}E\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{a}f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}}$$

<u>Mathematica</u> [C] time = 5.27637, size = 279, normalized size = 2.66

$$\frac{\sin(e+fx)\csc(2(e+fx))\left(a^2\sqrt{-\frac{1}{a+b}}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}\text{EllipticF}\left(e+fx, \frac{a}{a+b}\right) - 2i\csc(2(e+fx))\sqrt{\frac{a\sin^2(e+fx)}{a+b}}\sqrt{-\frac{a\cos^2(e}{b}\right.}{\sqrt{2}a^2f\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Csc[2*(e + f*x)]*Sin[e + f*x]*(a^2*Sqrt[-(a + b)^(-1)]*Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)] - (2*I)*Sqrt[-((a*C
os[e + f*x]^2)/b)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Csc[2*(e + f*x)]*(2*b
*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
])/Sqrt[2]], (a + b)/b] + a*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a
 + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], (a + b)/b])*Sqrt[(a*Sin[e + f*x]^2)
/(a + b)]))/(Sqrt[2]*a^2*Sqrt[-(a + b)^(-1)]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.424, size = 2985, normalized size = 28.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a/(2*I*a^(1/2)*b^(1/2)-a+b)*
(-2*I*sin(f*x+e)*cos(f*x+e)*a^(1/2)*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))+2*I*a^(3/2)*b^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*sin(f*x+e)-2*I*a^(1/2)*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b
^2)/(a+b)^2)^(1/2))*sin(f*x+e)+4*I*cos(f*x+e)*a^(1/2)*b^(3/2)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)-4*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)+2*I*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(3/2)*b^(1/2)
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)-cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*a^2-2*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b-cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
```

```
*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2+4*cos(f*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b+4*I*cos(f*x+e)^3*a^(3/2)*b^(1/2)*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*a^2*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a*b*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(
f*x+e)+4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)-4
*I*cos(f*x+e)^2*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*c
os(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)^3*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a^2-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a*b-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+2*cos(f*x+
e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/sin(f*x+e)
/cos(f*x+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(e+fx\right)}{\sqrt{a+b\sec^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.261 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=255

$$-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{2(a-b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\sin^{-1}(\sin\right.}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\right.}}$$

```
[Out] (Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(3*a*f*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)]) + (2*(a - b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) -
((a - 2*b)*b*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
 f*x]^2)/(a + b)])/(3*a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.406797, antiderivative size = 296, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4148, 6722, 1974, 416, 524, 426, 424, 421, 419}

$$-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e\right.}}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(\right.}{a+\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3
*a*f*Sqrt[a + b*Sec[e + f*x]^2]) + (2*(a - b)*Sqrt[b + a*Cos[e + f*x]^2]*El
lipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3
*a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e +
f*x]^2)/(a + b)]) - ((a - 2*b)*b*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSi
n[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^2*f*
Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]
^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\sqrt{b+a\left(1-x^2\right)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{-2}{\sqrt{}}\right.}{3af\sqrt{\cos^2(e+fx)}\sqrt{}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{\left(2(a-b)\sqrt{b+a\cos^2(e+fx)}\right)\text{Su}}{3a^2f\sqrt{\cos^2(e+fx}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{\left(2(a-b)\sqrt{b+a\cos^2(e+fx)}\sqrt{a}\right.}{3a^2f\sqrt{\cos^2(e+}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{3af\sqrt{a+b\sec^2(e+fx)}} + \frac{2(a-b)\sqrt{b+a\cos^2(e+fx)}E\left(\text{si}\right.}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{}}$$

Mathematica [F] time = 7.36504, size = 0, normalized size = 0.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.534, size = 4640, normalized size = 18.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)/a^
2*(3*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a^2*b-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-cos(f*x+e)*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*I*sin(f*x+e)*cos(f*x+e)*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a
^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(1/2)*b^(5/2)-2*I*a^(3/2)*b
^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*c
os(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^
(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)-4*I*a^(
3/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*I*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)*cos(f*x+e)^3+4*I*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(3/2)*cos(f*x+e)^2-4*I*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^(1/2)*b^(5/2)*cos(f*x+e)+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a^2*b+4*I*a^(1/2)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)+2*I*a^(5/2)*b^(1/2)*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
+2*I*a^(5/2)*b^(1/2)*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-4
*I*a^(5/2)*b^(1/2)*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+2*I
```

*a^(3/2)*b^(3/2)*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)+sin(f*x
+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-3*si
n(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipt
icF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2
-sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^
2*b+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-3*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+2*I*a^(5
/2)*b^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)+2*
I*a^(1/2)*b^(5/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x
+e)+2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-2*I*a^(3/2)*b^(3/2)*cos(f
*x+e)*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*
b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)-cos(f*x+
e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-2*cos(f*x+e)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+2*I*sin(f*x+e)*cos(f*x+e)*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e

```
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^(5/2)*b^(1/2)-4*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b^2+cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^2*b+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-cos(f*x+e
)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-4*cos(f*x+e)^2*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a*b^2+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a*b^2-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+2*cos(f*x+e)
^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-sin(f*x+e)*cos(f*x+e)*2^(1/2
)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+sin(f*x+e)*cos(f*x+e)*2^(1/
2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(
1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+
6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2)/cos(f*x+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)
^2)^(1/2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.262 $\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=345

$$-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(8a^2-7ab+8b^2\right)\left(-a\sin^2(e+fx)+a\right.}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2}}$$

```
[Out] (4*(a - b)*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(15*a^2*f*Sqrt[Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (Cos[e + f*x]^2*Sin[e + f*x]*(a + b -
 a*Sin[e + f*x]^2))/(5*a*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
 + ((8*a^2 - 7*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a +
 b - a*Sin[e + f*x]^2))/(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*
(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a
^2 - 3*a*b + 8*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)])/(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^
2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.573637, antiderivative size = 395, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 416, 528, 524, 426, 424, 421, 419}

$$-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{\left(8a^2-7ab+8b^2\right)\sqrt{-a\sin^2(e+}}{15a^3f\sqrt{\cos^2(e}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (4*(a - b)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f
*x]^2])/(15*a^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + (Cos[e + f*x]^2*Sqrt[b + a*
Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*a*f*Sqrt[a
+ b*Sec[e + f*x]^2]) + ((8*a^2 - 7*a*b + 8*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/
(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e
 + f*x]^2)/(a + b)]) - (b*(4*a^2 - 3*a*b + 8*b^2)*Sqrt[b + a*Cos[e + f*x]^2
]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
```

```
 + b)])/(15*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a +
b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{\sqrt{a+\frac{b}{1-x^2}}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{\sqrt{b+a(1-x^2)}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{5/2}}{\sqrt{a+b-ax^2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{5af\sqrt{a+b\sec^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}}{5af}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(e+fx)\sqrt{b+a\cos^2}}{5a}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(e+fx)\sqrt{b+a\cos^2}}{5a}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(e+fx)\sqrt{b+a\cos^2}}{5a}$$

$$= \frac{4(a-b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)\sqrt{a+b-a\sin^2(e+fx)}}{15a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^2(e+fx)\sqrt{b+a\cos^2}}{5a}$$

Mathematica [F] time = 12.874, size = 0, normalized size = 0.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.744, size = 6382, normalized size = 18.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.263 $\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=137

$$\frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\frac{\tan(e+fx)\sec^2(e+fx}{4}$$

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/(8*b^(5/2)*f) - (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
f*x]^2])/(4*b*f)
```

Rubi [A] time = 0.134542, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4146, 416, 388, 217, 206}

$$\frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\frac{\tan(e+fx)\sec^2(e+fx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/(8*b^(5/2)*f) - (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(8*b^2*f) + (Sec[e + f*x]^2*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
f*x]^2])/(4*b*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} + \frac{\text{Subst}\left(\int \frac{-a+3b-3(a-b)x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+\right.}{4bf}$$

$$= -\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan}}{4bf}$$

$$= -\frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan}}{4bf}$$

$$= \frac{(3a^2-2ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+f}}{8b^2f}$$

Mathematica [C] time = 9.03508, size = 326, normalized size = 2.38

$$\frac{e^{i(e+fx)}\sec(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(-\frac{(3a^2-2ab+3b^2)\log\left(\frac{4if\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}-4\sqrt{b}f\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}\right)}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}} - \frac{i\sqrt{b}\left(-1+e^{2i(\right.}}{}\right.}{8\sqrt{2}b^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*(-3*a*(1 + E^((2*I)*(e + f*x)))^2 + b*(3 + 14*E^((2*I)*(e + f*x)) +
 3*E^((4*I)*(e + f*x)))))/(1 + E^((2*I)*(e + f*x)))^4 - ((3*a^2 - 2*a*b + 3
*b^2)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))]
)/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec[e + f*
x])/(8*Sqrt[2]*b^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.474, size = 1756, normalized size = 12.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2*sin(f*x+e)*(-6*sin(f*x+e
)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+4*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*a*b-6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+3*sin(f*x+e)*cos(
f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-2*sin(f*x+e)
*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+3*sin(f
*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2+3*
cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^5*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/
```

```
2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x
+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-cos(f*x+e)^2*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*b^2-2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/cos(f*x+e)^5/(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27261, size = 975, normalized size = 7.12

$$\left[\frac{\left(3\,a^2-2\,ab+3\,b^2\right)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+\cdots\right.}{\cos\left(fx+\cdots\right.}}}{\cos\left(fx+e\right)^4}\right.}{32\,b^3f\cos\left(fx+e\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b
^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)
^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(3*(a*b - b^2)*cos(f*x + e)^2 - 2
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(
f*x + e)^3), 1/16*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*co
```

```
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(3
*(a*b - b^2)*cos(f*x + e)^2 - 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.264 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=81

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}$$

```
[Out] -((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rubi [A] time = 0.0912598, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4146, 388, 217, 206}

$$\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2bf}\\
&= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2bf}\\
&= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}
\end{aligned}
$$

Mathematica [C] time = 10.1666, size = 326, normalized size = 4.02

$$
\frac{\tan(e+fx)\sec^4(e+fx)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\sqrt{a\cos(2e+2fx)+a+2b}\left(\frac{16b^2\tan^4(e+fx)\left(a\cos^2(e+fx)+b\right)\text{Hypergeometric2F1}\left(2,3,\frac{7}{2},-\frac{b\,\mathrm{t}\cdots}{}\right.}{(a+b)^3}\right.}{30\sqrt{2}f\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{b\tan^2(e\cdots}{a+b}\right.}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]^4*(1 - (a*Sin[e + f*x]^2)/
(a + b))*Tan[e + f*x]*((16*b^2*(b + a*Cos[e + f*x]^2)*Hypergeometric2F1[2,
3, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4*Sqrt[-((b*Sec[e + f*x
]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)]])/(a + b)^3 + (15
*(3*b + a*(3 - 2*Sin[e + f*x]^2))*(ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b)
)]] - Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(
a + b)^2)]]))/(a + b)))/(30*Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[(a + b
*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Tan[e + f*x]
^2)/(a + b)))^(3/2))
```

Maple [C] time = 0.382, size = 1086, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(sin(f*x+e)*cos(
f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a-sin(f*x+e)*cos
(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b-2*sin(f*x+e)*
cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(
1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a+2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*b+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b
```

```
)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))
/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.773648, size = 821, normalized size = 10.14

$$\left[-\frac{(a-b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left((a-b)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)+8}{\cos\left(fx+e\right)^4}\right.}{8\,b^2 f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)), -1/4*((a - b)*sqrt(-b)*arctan(-1/2*((a - b)*
cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 2*b*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x +
e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.265 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]*f)
```

Rubi [A] time = 0.0714095, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4146, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

Mathematica [B] time = 0.147321, size = 87, normalized size = 2.23

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{b}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[b]*f*Sqrt[a + b*Sec[e +
 f*x]^2])
```

Maple [C] time = 0.411, size = 379, normalized size = 9.7

$$-\frac{\sqrt{2}\left(\sin\left(fx+e\right)\right)^2}{f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{1}{(a+b)\left(1+\cos\left(fx+e\right)\right)}\left(i\cos\left(fx+e\right)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+a\cos\left(fx+e\right)+b\right)}\sqrt{$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*(EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x
+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.655339, size = 543, normalized size = 13.92

$$\left[\frac{\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a-b\right)\cos\left(fx+e\right)^3+2\,b\cos\left(fx+e\right)\right)\sqrt{b}\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\sin\left(fx+e\right)+8\,b^2}{\cos\left(fx+e\right)^4}\right)}{4\sqrt{b}f}, \frac{\sqrt{-b}\arctan\left(-\frac{\left((a-\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)/(sqrt(b)*f)
, 1/2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2
)*sin(f*x + e)))/(b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.266 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rubi [A] time = 0.0299862, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4128, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [B] time = 0.0827838, size = 87, normalized size = 2.23

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [C] time = 0.407, size = 380, normalized size = 9.7

$$-\frac{\sqrt{2}\left(\sin\left(fx+e\right)\right)^2}{f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{1}{(a+b)\left(1+\cos\left(fx+e\right)\right)}\left(i\cos\left(fx+e\right)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+a\cos\left(fx+e\right)+b\right)}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*(EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*
x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.7916, size = 976, normalized size = 25.03

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+a^4-28\,a^3b\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
```

```
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sec^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx + e\right)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.267 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=87

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rubi [A] time = 0.0992395, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4146, 382, 377, 203}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}f}+\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*
a^(3/2)*f) + (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
a*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \text{t}\ldots\right.}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+\ldots}{\sqrt{a+b+b\tan\ldots}}\right.}{2af}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

Mathematica [A] time = 0.256906, size = 126, normalized size = 1.45

$$\frac{\sqrt{a\cos(2(e+fx))+a+2b}\left(\sqrt{a}\tan(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}+(a-b)\sec(e+fx)\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)\right)}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*((a - b)*ArcTan[(Sqrt[a]*Sin[e + f*x])/
Sqrt[a + b - a*Sin[e + f*x]^2]]*Sec[e + f*x] + Sqrt[a]*Sqrt[a + b - a*Sin[e
 + f*x]^2]*Tan[e + f*x]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.379, size = 1056, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a*sin(f*x+e)*(2*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*a*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2
+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/
(a+b)^2)^(1/2))*b*sin(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*b)/(-1+cos(f*x+e))/cos(f*x+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 0.894785, size = 1215, normalized size = 13.97

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-a}(a-b)\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) + sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)
*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^
5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e)))/(a^2*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e)*sin(f*x + e) - (a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.268 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=143

$$\frac{(3a^2-2ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}+\frac{3(a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\sin(e+fx)\,\mathrm{co}}{}$$

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(5/2)*f) + (3*(a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b
+ b*Tan[e + f*x]^2])/(4*a*f)
```

Rubi [A] time = 0.141503, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{(3a^2-2ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{5/2}f}+\frac{3(a-b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8a^2f}+\frac{\sin(e+fx)\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(5/2)*f) + (3*(a - b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a +
b + b*Tan[e + f*x]^2])/(8*a^2*f) + (Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b
+ b*Tan[e + f*x]^2])/(4*a*f)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af} - \frac{\text{Subst}\left(\int \frac{-3a+b-2bx^2}{(1+x^2)^2\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{4af}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+}}{4af}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+}}{4af}$$

$$= \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+}}{4af}$$

$$= \frac{(3a^2-2ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} + \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b}}{8a^2f}$$

Mathematica [C] time = 16.3486, size = 1840, normalized size = 12.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*
Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a + b)*
AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]^5)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -
2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2,
 -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appe
llF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
```

```
+ f*x]^2)) - (12*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[
2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*x
]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*AppellF1[
3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
*Sin[e + f*x])/3))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF
1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*Appel
lF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a +
 b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
)*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*(2*f*(a*AppellF1[3
/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*A
ppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos
[e + f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)
) - (4*f*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*(a*((9*a*f*AppellF1[5/
2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 4*(a + b)*((3
*a*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (9*(a + b)^3*f*Cot[e + f*x]*Csc[
e + f*x]^4*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*((-2*a*Sin[e + f*x]^2)/(a +
 b) - (4*a^2*Sin[e + f*x]^4)/(3*(a + b)^2) + (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin
[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]
^2)/(a + b)])))/(8*a^3)))))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b
)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
 (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 4*(a + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2)^2) + (3*a*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*Sin[2
*(e + f*x)])/((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(3*(a + b)*AppellF1[1/2,
 -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/
2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Ap
pellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2))))
```

Maple [C] time = 0.455, size = 1701, normalized size = 11.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2*sin(f*x+e)*(-2*cos(f*x+e
)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+4*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*a*b*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+2*cos(f*x+e)^4*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a*b+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e
```

```
)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(f*x+e)*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/cos(f*x+e)/((b+a*cos(f*x+e)^2
)/cos(f*x+e)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A] time = 1.50788, size = 1362, normalized size = 9.52

$$\left[-\frac{\left(3\,a^{2}-2\,ab+3\,b^{2}\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
```

```
/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + 3*(a^2 - a*b)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*
f), -1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
 + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2*cos(f*
x + e)^3 + 3*(a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.269 $\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=204

$$\frac{(a-b)\left(5a^2+2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{\left(15a^2-14ab+15b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{48a^3f}$$

```
[Out] ((a - b)*(5*a^2 + 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b +
 b*Tan[e + f*x]^2]])/(16*a^(7/2)*f) + ((15*a^2 - 14*a*b + 15*b^2)*Cos[e + f
*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + (5*(a - b)*Co
s[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos
[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```

Rubi [A] time = 0.206883, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{(a-b)\left(5a^2+2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{7/2}f}+\frac{\left(15a^2-14ab+15b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{48a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```



```
[Out] ((a - b)*(5*a^2 + 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b +
 b*Tan[e + f*x]^2]])/(16*a^(7/2)*f) + ((15*a^2 - 14*a*b + 15*b^2)*Cos[e + f
*x]*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(48*a^3*f) + (5*(a - b)*Co
s[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(24*a^2*f) + (Cos
[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af} - \frac{\text{Subst}\left(\int \frac{-5a+b-4bx^2}{(1+x^2)^3\sqrt{a+b+bx^2}}\,dx, x, \tan(e+\right.}{6af}$$

$$= \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b}}{6af}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b)\cos^3(e+f}{}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b)\cos^3(e+f}{}$$

$$= \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} + \frac{5(a-b)\cos^3(e+f}{}$$

$$= \frac{(a-b)(5a^2+2ab+5b^2)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} + \frac{(15a^2-14ab+15b^2)\cos(e+fx)\sin(}{48a^3f}$$

<u>Mathematica</u> [C] time = 16.6176, size = 1739, normalized size = 8.52

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a + b)
*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Co
s[e + f*x]^7)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2,
```

```
-3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2
, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*App
ellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
 + f*x]^2)) - (18*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos
[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*
x]^6*Sin[e + f*x]*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*AppellF1[
3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
*Sin[e + f*x]))/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1
/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1
[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)
*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*S
in[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(a*AppellF1[3/2,
 -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*Appe
llF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e
+ f*x]*Sin[e + f*x] + 3*(a + b)*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) -
 2*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(a*((9*a*f*AppellF1[5/2, -3,
5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
f*x])/(5*(a + b)) - (18*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Si
n[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 6*(a + b)*((3*a*f*Ap
pellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
 + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 1/2, 7/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(f*
Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, S
in[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2,
1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) +
 (3*a*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*Sin[2*(e + f*x)])/((a + 2*b + a*Cos[
2*(e + f*x)])^(3/2)*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [C] time = 0.587, size = 2425, normalized size = 11.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/48/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^3*sin(f*x+e)*(15*2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-15*2^(1/2)*(1/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)+15*cos(f*x+e)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^2*b+9*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e
)-9*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-18*2
^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+18*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)-1
5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-30*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
```

```
/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin
(f*x+e)+8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*
x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+10*cos(f*x+e)^5*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-10*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^3+15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^3+14*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-2*cos(f*x+e)^5*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b+5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+4*c
os(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-5*cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-14*cos(f*x+e)*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a*b^2+15*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^3-15*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+3
0*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e))/(-1+cos(f*x+e))/cos(f*x+e)/((b+a*cos(f
*x+e)^2)/cos(f*x+e)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [A] time = 3.86418, size = 1539, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(-a)*log(128*a^4*cos(f*x
+ e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5 + 10
*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f), -1/192
*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
 e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*a^3*c
os(f*x + e)^5 + 10*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a
*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^4*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.270 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{a(2a+b)\sin(e+fx)}{b^2f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{}{b^2f(a}$$

```
[Out] (a*(2*a + b)*Sin[e + f*x])/(b^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Si
n[e + f*x]^2)]) - ((2*a + b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a
+ b - a*Sin[e + f*x]^2))/(b^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (
EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
 b)])/(b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)]) + (Sec[e + f*x]*Tan[e + f*x])/(b*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)])
```

Rubi [A] time = 0.590233, antiderivative size = 367, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 414, 527, 524, 426, 424, 421, 419}

$$\frac{a(2a+b)\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{b^2f(a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}-\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}E\left(\sin^{-1}\right.}{b^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(2*a + b)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(b^2*(a + b)*f*Sqrt[a
 + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) - ((2*a + b)*Sqrt[b +
a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a
*Sin[e + f*x]^2])/(b^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*
x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (Sqrt[b + a*Cos[e + f*x]^2]*E
llipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)])/(b*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Si
n[e + f*x]^2]) + (Sqrt[b + a*Cos[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x])/(b*
```

```
f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}\left(b+a(1-x^2)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sec(e+fx)\tan(e+fx)}{bf\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{-a}{\sqrt{1-x^2}(a\ldots}\right.}{bf\sqrt{\cos^2(e+fx)}\sqrt{a+b\ldots}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\sec(e+\ldots}{bf\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b\ldots}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\sec(e+\ldots}{bf\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b\ldots}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\sec(e+\ldots}{bf\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b\ldots}}$$

$$= \frac{a(2a+b)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{b^2(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{(2a+b)\sqrt{b+a\cos^2(e+fx)}E\ldots}{b^2(a+b)f\sqrt{\cos^2(e+\ldots}}$$

<u>Mathematica</u> [F] time = 20.1776, size = 0, normalized size = 0.

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] time = 0.65, size = 12514, normalized size = 43.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)^5}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.271 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{\left(-a\sin^2(e+fx)+a+b\right)E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{a\sin(e+fx)}{bf(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+\right.}}$$

```
[Out] -((a*Sin[e + f*x])/(b*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)])) + (EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x
]^2))/(b*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[
e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
```

Rubi [A] time = 0.383954, antiderivative size = 182, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4148, 6722, 1974, 414, 21, 426, 424}

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e+fx)}}-\frac{a\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{bf(a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(b*(a + b)*f*Sqrt[a + b*Sec[e
 + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])) + (Sqrt[b + a*Cos[e + f*x]^2]*E
llipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(
b*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin
[e + f*x]^2)/(a + b)])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(b+a\left(1-x^2\right)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(a+b-ax^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \right.}{b(a+b)f\sqrt{\cos^2(e+f}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \right.}{b(a+b)f\sqrt{\cos^2(e+fx)}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(\sqrt{b+a\cos^2(e+fx)}\sqrt{a+b-}\right.}{b(a+b)f\sqrt{\cos^2(e+}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{b(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}E\left(\sin^{-1}(\text{si}\right.}{b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{}}$$

Mathematica [A] time = 2.41617, size = 113, normalized size = 0.75

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{2}(a+b)\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}E\left(e+fx\middle|\frac{a}{a+b}\right)-a\sin(2(e+fx))\right)}{4bf(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[2]*(a + b)*Sqrt[(a + 2
*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] - a*Sin[2*(
e + f*x)]))/(4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.49, size = 6593, normalized size = 44.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^3}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(e + fx)}{(a + b\sec^2(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx + e)^3}{\left(b\sec(fx + e)^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.272 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=229

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\sin(e+fx)}{f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{}{af(a+}$$

```
[Out] Sin[e + f*x]/((a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) -
(EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(a*
(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^
2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (EllipticF[ArcSin[Sin[e + f*x]]
, a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*f*Sqrt[Cos[e + f*x]^2
]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.447886, antiderivative size = 284, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4148, 6722, 1974, 412, 493, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{f(a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{af\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/((a + b)*f*Sqrt[a + b*Sec[e + f*x
]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) - (Sqrt[b + a*Cos[e + f*x]^2]*Elliptic
E[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*(a +
b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*
x]^2)/(a + b)]) + (Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]
], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*f*Sqrt[Cos[e + f*x]^
2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
```

```
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(b+a(1-x^2)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(a+b-ax^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\cdots}{\sqrt{1-\cdots}}\cdots\right.}{(a+b)f\sqrt{\cos^2(e+fx)}\cdots}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\cdots}{\sqrt{1-\cdots}}\cdots\right.}{af\sqrt{\cos^2(e+fx)}\sqrt{a\cdots}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left(\sqrt{b+a\cos^2(e+fx)}\sqrt{a+b-a\,\text{s}\cdots}\right.}{a(a+b)f\sqrt{\cos^2(e+f\cdots}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}E\left(\sin^{-1}(\sin(\cdots\right.}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{a+\cdots}}$$

Mathematica [C] time = 9.4885, size = 822, normalized size = 3.59

$$\frac{(\cos(2e+2fx)a+a+2b)^{3/2}\sec^3(e+fx)\left(\frac{\sqrt{-\frac{1}{a+b}}\cos(2(e+fx))\left(\sqrt{-\frac{1}{a+b}}(\cos(2e+2fx)a+a)\left(-2a^2+(\cos(2e+2fx)a+a-4b)a+2b(\cos(2e+2fx)a+a)\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3*(-(((2*(a - a*Cos[2*e
+ 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/(b*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
+ ((2*I)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[4 - (2*(a + 2*b + a*Co
s[2*e + 2*f*x]))/b]*(EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b
+ a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] - EllipticF[I*ArcSinh[(Sqrt[-(a
 + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b]))/Sqrt
[-(a + b)^(-1)])*Sin[2*e + 2*f*x])/(8*a*(a + b)*f*Sqrt[((a - a*Cos[2*e + 2*
f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*Sqrt[1 - Cos[2*e + 2*f*x]^2]) + (Sqrt[
-(a + b)^(-1)]*Cos[2*(e + f*x)]*(Sqrt[-(a + b)^(-1)]*(a + a*Cos[2*e + 2*f*x
])*(-2*a^2 + 2*b*(a + a*Cos[2*e + 2*f*x]) + a*(a - 4*b + a*Cos[2*e + 2*f*x]
)) - I*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b + a*
Cos[2*e + 2*f*x]]*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[
I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]]
, (a + b)/b] - I*a*b*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[(4*a + 4*b - 2
*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e + 2
*f*x])/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e
 + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sec[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f*x
]])/2)]*Sin[2*e + 2*f*x])/(4*a^2*b*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*
Cos[2*e + 2*f*x]))/a^2]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[1 - Cos[2*e
 + 2*f*x]^2])))/(2*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.5, size = 6593, normalized size = 28.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)/(b^2*sec(f*x + e)^4 + 2*a*
b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.273 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{(a+2b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\sin^{-1}(\sin\right.}{a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+fx)\left(\right.}}$$

```
[Out] -((b*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]
^2)])) + ((a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*S
in[e + f*x]^2))/(a^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a
+ b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (2*b*Ellip
ticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
/(a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
])
```

Rubi [A] time = 0.337121, antiderivative size = 295, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4148, 6722, 1974, 413, 524, 426, 424, 421, 419}

$$-\frac{2b\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+}}{a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sec[e
 + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])) + ((a + 2*b)*Sqrt[b + a*Cos[e +
 f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e +
f*x]^2])/(a^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqr
t[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (2*b*Sqrt[b + a*Cos[e + f*x]^2]*Ellipt
icF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/
(a^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(b+a\left(1-x^2\right)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(a+b-ax^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int\right.}{a(a+b)f\sqrt{\cos^2(e+}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\left(2b\sqrt{b+a\cos^2(e+fx)}\right)\text{Sub}}{a^2f\sqrt{\cos^2(e+}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left((a+2b)\sqrt{b+a\cos^2(e+fx)}\right.}{a^2(a+b)f\sqrt{\text{co}}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+2b)\sqrt{b+a\cos^2(e+fx)}E}{a^2(a+b)f\sqrt{\cos^2(e+}}$$

Mathematica [F] time = 14.2684, size = 0, normalized size = 0.

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] time = 0.508, size = 8684, normalized size = 36.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)}{b^{2}\sec\left(fx+e\right)^{4}+2\,ab\sec\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^2*sec(f*x + e)^4 + 2*a*
b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.274 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{b(a-8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\left(2a^2-3ab-8b^2\right)\left(-a\sin^2(e+fx)+a+b\right)E\left(\sin\right.}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2(e+f}}$$

```
[Out] -((b*Cos[e + f*x]^2*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b -
 a*Sin[e + f*x]^2)])) + ((a + 4*b)*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))
/(3*a^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((2*a^
2 - 3*a*b - 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Si
n[e + f*x]^2))/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a
 + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b
)*b*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/
(a + b)])/(3*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[
e + f*x]^2)])
```

Rubi [A] time = 0.56084, antiderivative size = 399, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 413, 528, 524, 426, 424, 421, 419}

$$\frac{\left(2a^2-3ab-8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e+fx)}}+\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin}}{3a^2f(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Cos[e + f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(a*(a + b)*f*S
qrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])) + ((a + 4*b)*Sqr
t[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a^2
*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((2*a^2 - 3*a*b - 8*b^2)*Sqrt[b +
a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a
*Sin[e + f*x]^2])/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e +
f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*b*Sqrt[b + a*Cos
[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
```

```
f*x]^2)/(a + b)])/(3*a^3*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]
*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(b+a\left(1-x^2\right)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(a+b-ax^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\Big(\int}{a(a+b)f\sqrt{\cos^2(e}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+a\cos^2(e+fx)}\,\text{s}}{3a^2(a+b)f\sqrt{}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+a\cos^2(e+fx)}\,\text{s}}{3a^2(a+b)f\sqrt{}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+a\cos^2(e+fx)}\,\text{s}}{3a^2(a+b)f\sqrt{}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+4b)\sqrt{b+a\cos^2(e+fx)}\,\text{s}}{3a^2(a+b)f\sqrt{}}$$

Mathematica [F] time = 16.0059, size = 0, normalized size = 0.

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] time = 0.542, size = 11939, normalized size = 35.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)^{3}}{b^{2}\sec\left(fx+e\right)^{4}+2\,ab\sec\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.275 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=436

$$-\frac{4b\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(4a^2-5ab-24b^2\right)\sin(e+fx)\left(-a\sin^2(e+\right.}{15a^3f(a+b)\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+f\right.}}$$

```
[Out] -((b*Cos[e + f*x]^4*Sin[e + f*x])/(a*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b -
 a*Sin[e + f*x]^2)])) + ((4*a^2 - 5*a*b - 24*b^2)*Sin[e + f*x]*(a + b - a*S
in[e + f*x]^2))/(15*a^3*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]) + ((a + 6*b)*Cos[e + f*x]^2*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2)
)/(5*a^2*(a + b)*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((8*a
^3 - 9*a^2*b + 16*a*b^2 + 48*b^3)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)
]*(a + b - a*Sin[e + f*x]^2))/(15*a^4*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[S
ec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)]) - (4*b*(a^2 - 2*a*b + 12*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b
)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^4*f*Sqrt[Cos[e + f*x]^2]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.751063, antiderivative size = 509, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 413, 528, 524, 426, 424, 421, 419}

$$\frac{\left(4a^2-5ab-24b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}}{15a^3f(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{4b\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\,\mathrm{c}}}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Cos[e + f*x]^4*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(a*(a + b)*f*S
qrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])) + ((4*a^2 - 5*a*
b - 24*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e +
f*x]^2])/(15*a^3*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((a + 6*b)*Cos[e +
 f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]
^2])/(5*a^2*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((8*a^3 - 9*a^2*b + 16*
```

```
a*b^2 + 48*b^3)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]],
a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*a^4*(a + b)*f*Sqrt[Cos[e + f
*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (
4*b*(a^2 - 2*a*b + 12*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[
e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^4*f*Sqrt[
Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+\frac{b}{1-x^2}\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{7/2}}{\left(b+a\left(1-x^2\right)\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{7/2}}{\left(a+b-ax^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int\right.}{a(a+b)f\sqrt{\cos^2(}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{(a+6b)\cos^2(e+fx)\sqrt{b+a\,\text{c}}}{5a^2(a\,-}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(4a^2-5ab-24b^2\right)\sqrt{b+a\,\text{co}}}{15a^3(a\,+}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(4a^2-5ab-24b^2\right)\sqrt{b+a\,\text{co}}}{15a^3(a\,+}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(4a^2-5ab-24b^2\right)\sqrt{b+a\,\text{co}}}{15a^3(a\,+}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}} + \frac{\left(4a^2-5ab-24b^2\right)\sqrt{b+a\,\text{co}}}{15a^3(a\,+}$$

Mathematica [F] time = 16.2668, size = 0, normalized size = 0.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [C] time = 0.92, size = 15199, normalized size = 34.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^5}{b^2\sec\left(fx+e\right)^4+2\,ab\sec\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^2*sec(f*x + e)^4 + 2*
a*b*sec(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.276 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b^2f(a+b)}-\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}-\frac{a\tan(e+fx)\sec^2(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] -((3*a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(2*b^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b +
b*Tan[e + f*x]^2]) + ((3*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(2*b^2*(a + b)*f)
```

Rubi [A] time = 0.14593, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4146, 413, 388, 217, 206}

$$\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b^2f(a+b)}-\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}-\frac{a\tan(e+fx)\sec^2(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```



```
[Out] -((3*a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(2*b^(5/2)*f) - (a*Sec[e + f*x]^2*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b +
b*Tan[e + f*x]^2]) + ((3*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(2*b^2*(a + b)*f)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b+(3a+b)x^2}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f} - \frac{(3\cdots}{\cdots}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f} - \frac{(3\cdots}{\cdots}$$

$$= -\frac{(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(\cdots}{\cdots}$$

Mathematica [C] time = 15.0701, size = 336, normalized size = 2.43

$$\frac{ie^{-i(e+fx)}\sec^3(e+fx)(a\cos(2e+2fx)+a+2b)^{3/2}\left(\left(3a^2+2ab-b^2\right)\left(1+e^{2i(e+fx)}\right)^2\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}\tan^{-1}\cdots\right.}{4b^{5/2}f(a+b)\left(1+e^{2i(e+fx)}\right)^2\sqrt{8b+2ae^{-2i(e+fx}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((I/4)*(-(Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(4*b^2*E^((2*I)*(e + f*x)) + 3
*a^2*(1 + E^((2*I)*(e + f*x)))^2 + a*b*(1 + 6*E^((2*I)*(e + f*x)) + E^((4*I
)*(e + f*x))))) + (3*a^2 + 2*a*b - b^2)*(1 + E^((2*I)*(e + f*x)))^2*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*ArcTan[(Sqrt[b]*(-1
+ E^((2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2]])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(b^(5/2)*
(a + b)*E^(I*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^2*Sqrt[8*b + (2*a*(1 + E^
((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.435, size = 4338, normalized size = 31.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/f/(a+b)/b^(5/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(4*sin(f*x+e)*c
os(f*x+e)^4*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipt
icF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+1
3*sin(f*x+e)*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+
e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^3-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*b^(7/2)+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(7/2)+13*s
in(f*x+e)*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))
*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b+13*sin(f*x+e)*cos(f*x+e)^4*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arcta
nh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(
1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^2-13*
sin(f*x+e)*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*b^(1/2)*(-1+cos(f*x+e))*(cos(f
*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2))*a^2*b-13*sin(f*x+e)*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*
b^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x
+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^2+13*sin(f*x+e)*cos(
f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4
^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2))*a*b^2-13*sin(f*x+e)*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*b^(1/2)
*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^2-2*sin(f*x+e)*cos(f*x+e)^2
*b^(7/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
```

```
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+4*sin(f*x+e)*cos
(f*x+e)^2*b^(7/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*
I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*b^(5/2)*a-8*sin(f*x+e)*cos(f*x+e)^4*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+6*sin(
f*x+e)*cos(f*x+e)^4*b^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*a^3-12*sin(f*x+e)*cos(f*x+e)^4*b^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+4*sin(f*x+e
)*cos(f*x+e)^2*b^(5/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(
-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a-
8*sin(f*x+e)*cos(f*x+e)^2*b^(5/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a
+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a+6*sin(f*x+e)*cos(f*x+
e)^2*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-12*sin(f
*x+e)*cos(f*x+e)^2*b^(3/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
```

```
+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-2*sin(f*x+e)*cos(f*x+e)^4*
b^(5/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a+4*sin(f*x+e)*co
s(f*x+e)^4*b^(5/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a+2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*b^(3/2)*a^2+6*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*b^(1/2)*a^3-2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*b^(3/2)*a^2+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(5/
2)*a-6*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1/2)*a^3+8*c
os(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(3/2)*a^2-4*cos(f*x+e
)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(5/2)*a-8*cos(f*x+e)^2*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(3/2)*a^2+2*cos(f*x+e)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*b^(5/2)*a-13*sin(f*x+e)*cos(f*x+e)^2*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arcta
nh(1/4*b^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/
sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^3)*sin(f*x+e)/(
-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43538, size = 1245, normalized size = 9.02

$$\left[-\frac{\left(\left(3a^3+2a^2b-ab^2\right)\cos\left(fx+e\right)^3+\left(3a^2b+2ab^2-b^3\right)\cos\left(fx+e\right)\right)\sqrt{b}\log\left(\frac{\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+8\left(ab-b^2\right)\cos\left(fx+e\right)^2+4}{}\right.}{8\left(\left(a^2b^3+ab^4\right)f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a^3 + 2*a^2*b - a*b^2)*cos(f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3
)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
+ e)^4) - 4*(a*b^2 + b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x +
e)^3 + (a*b^4 + b^5)*f*cos(f*x + e)), -1/4*(((3*a^3 + 2*a^2*b - a*b^2)*cos(
f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e))*sqrt(-b)*arctan(-1/2*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 2*(a*b^2
+ b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^3 + (a*b^4 + b^5
)*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.277 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
- (a*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0969246, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4146, 385, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{a\tan(e+fx)}{bf(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
- (a*Tan[e + f*x])/(b*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{bf} \\
&= -\frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{bf} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 7.10777, size = 405, normalized size = 5.26

$$
-\frac{\tan(e+fx)\sec^4(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}(a\cos(2e+2fx)+a+2b)^{3/2}\Big(4b(a+b)\sin^2(e+fx)\text{Hypergeometric2F1}\,(}{}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^4*Sqrt[1 - (2*a*Sin[e +
 f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*(15*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a
+ b))]]*Sec[e + f*x]^2*(3*b^2 + a*b*(6 - 5*Sin[e + f*x]^2) + a^2*(3 - 5*Sin
[e + f*x]^2 + 2*Sin[e + f*x]^4)) + 15*(a + b)*(-3*b + a*(-3 + 2*Sin[e + f*x
]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(
a + b)^2)] + 4*b*(a + b)*Hypergeometric2F1[2, 2, 7/2, -((b*Tan[e + f*x]^2)/
(a + b))]*Sin[e + f*x]^2*(-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Ta
n[e + f*x]^2)/(a + b)^2))^(3/2)))/(15*(a + b)^2*(2*a + 2*b)*f*(a + b*Sec[e
+ f*x]^2)^(3/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a + 2*b - 2*a*S
in[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*(-((b*Tan[e + f*x]^2)/(
a + b)))^(3/2))
```

Maple [C] time = 0.472, size = 3068, normalized size = 39.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/b^(3/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)*(2*2^(1/2)*sin
(f*x+e)*cos(f*x+e)^2*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1
+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3
/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2)*a-4*
2^(1/2)*sin(f*x+e)*cos(f*x+e)^2*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(3/2)*a+2*2^(1/2)*sin(f*x+e)*co
s(f*x+e)^2*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2
)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(1/2)*a^2-4*2^(1/2)*
sin(f*x+e)*cos(f*x+e)^2*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi
```

```
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(1/2)*a^2+2*2^(1/2)*sin(f*x+e)*(1/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/
2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(5/2)-4*2^(1/2)*sin(f*x+e)*(1/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^
(5/2)+2*2^(1/2)*sin(f*x+e)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2
)*a-4*2^(1/2)*sin(f*x+e)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticP
i((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(3/2)*a-sin(f*x+e)*cos(f*x+e)^2*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f
*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a
^2-sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arct
anh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^
(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*b^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e
)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+sin(f*x+e)*cos
(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*b^(1/2)*(
-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a*b+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1/2)*a
^2-sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/8*b^(1/
2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(
f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b-sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*
x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+sin(f*x+e)*((b+a*cos(f*x+e)^2)/(
```

```
1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*b^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1
/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+sin(f*x+e)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*b^(1/2)*(-1+cos(f*x+e))*(cos
(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*cos(f*
x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1/2)*a^2+2*cos(f*x+e)*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(3/2)*a-2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*b^(3/2)*a)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(
f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.805869, size = 1007, normalized size = 13.08

$$\left[-\frac{4\,ab\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}+ab+b^{2}\right)\sqrt{b}\log\left(\frac{\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+8\left(ab-b^{2}\right)}{}\right.}{4\left(\left(a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{2}+\left(ab^{3}+b^{4}\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*
b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
```

```
 + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/((a^2*b^2 + a*b^3)*f*cos(f*x +
e)^2 + (a*b^3 + b^4)*f), -1/2*(2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e)*sin(f*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*
sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 + b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.278 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] Tan[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0745116, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {4146, 191}

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Tan[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] time = 0.656438, size = 57, normalized size = 1.78

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)}{2f(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*(a + b)*f*(
a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.23, size = 59, normalized size = 1.8

$$\frac{\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)\sin\left(fx+e\right)}{f\left(a+b\right)\left(\cos\left(fx+e\right)\right)^3}\left(\frac{b+a\left(\cos\left(fx+e\right)\right)^2}{\left(\cos\left(fx+e\right)\right)^2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f/(a+b)*(b+a*cos(f*x+e)^2)*sin(f*x+e)/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/co
s(f*x+e)^2)^(3/2)
```

Maxima [A] time = 1.07825, size = 41, normalized size = 1.28

$$\frac{\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}(a+b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*f)
```

Fricas [B] time = 0.560278, size = 159, normalized size = 4.97

$$\frac{\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)\sin\left(fx+e\right)}{\left(a^2+ab\right)f\cos\left(fx+e\right)^2+\left(ab+b^2\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e)/((a^2
 + a*b)*f*cos(f*x + e)^2 + (a*b + b^2)*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [B] time = 1.94885, size = 157, normalized size = 4.91

$$-\frac{2\,a^2b^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a^3b^2+a^2b^3\right)\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^2*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*tan(1/2*f*x + 1/2*e)/((a^3*b^2 +
 a^2*b^3)*sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*f)
```

3.279 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0486906, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4128, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] time = 1.46327, size = 168, normalized size = 2.18

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\sqrt{2}\sqrt{a}b\sin(e+fx)\sqrt{\cdots}\right.}{4a^{3/2}f(a+b)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

Maple [C] time = 0.401, size = 1007, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a*(b+a*cos(f*x+e)^2)*(2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin
(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*sin(f*x+e)/(
```

```
-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.05488, size = 1438, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

3.280 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}+\frac{b(a+3b)\tan(e+fx)}{2a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] ((a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) + (b*(a + 3*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e +
 f*x]^2])
```

Rubi [A] time = 0.155055, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{5/2}f}+\frac{b(a+3b)\tan(e+fx)}{2a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\sin(e+fx)\cos(e+fx)}{2af\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(5/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) + (b*(a + 3*b)*Tan[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e +
 f*x]^2])
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{(a-3}{(1+x^2)}\right.}{2}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)\,\text{Subst}\,(}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)\,\text{Subst}\,(}{}$$

$$= \frac{(a-3b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\,\text{ta}}{2a^2(a+b)f\sqrt{a+b}}$$

Mathematica [C] time = 15.4625, size = 2059, normalized size = 15.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^6*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Appell
F1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*Ap
pellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(
a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]
```

```
^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*
b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*AppellF1[1/
2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF
1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b
)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5)/(2*Sqrt[a + 2*b + a*Cos[2*(e + f
*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1,
3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (
6*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(
a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a +
b)*Cos[e + f*x]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f
*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*Sin[e + f*x])/3))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
 - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*Ap
pellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
 + f*x]^4*Sin[e + f*x]*(2*f*(3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a +
b)*((a*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, 3/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3)
+ Sin[e + f*x]^2*(3*a*((3*a*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (12*f*Appel
lF1[5/2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
f*x]*Sin[e + f*x])/5) - 4*(a + b)*((9*a*f*AppellF1[5/2, -1, 5/2, 7/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b
)) - (6*f*Cos[e + f*x]*Hypergeometric2F1[3/2, 5/2, 7/2, (a*Sin[e + f*x]^2)/
(a + b)]*Sin[e + f*x])/5))))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
 - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2) + (3*a*(a + b
)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^4*Sin[e + f*x]*Sin[2*(e + f*x)])/(2*(a + 2*b + a*Cos[2*(e + f*x
```

```
)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -2, 5/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2))
))
```

Maple [C] time = 0.342, size = 1646, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2*(b+a*cos(f*x+e)^2)*
(2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-6*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)
+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a^2*sin(f*x+e)+2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a*b*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
```

```
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin
(f*x+e)+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e)
^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^2*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a*b+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x
+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*sin(f*x+e)/
(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 1.93877, size = 1640, normalized size = 12.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^2)
*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 3
2*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b
^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*
x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))
```

```
+ 8*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x
 + e)^2 + (a^4*b + a^3*b^2)*f), -1/8*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a
^2*b - 3*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((a^3 + a^2*b)*c
os(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b + a
^3*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.281 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{3\left(a^2-2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}+\frac{b(a-3b)(3a+5b)\tan(e+fx)}{8a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-5b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+$$

```
[Out] (3*(a^2 - 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(7/2)*f) + ((3*a - 5*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2
*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*S
qrt[a + b + b*Tan[e + f*x]^2]) + ((a - 3*b)*b*(3*a + 5*b)*Tan[e + f*x])/(8*
a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.225588, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{3\left(a^2-2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{7/2}f}+\frac{b(a-3b)(3a+5b)\tan(e+fx)}{8a^3f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-5b)\sin(e+fx)\cos(e+fx)}{8a^2f\sqrt{a+b\tan^2(e+fx)+b}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] (3*(a^2 - 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*a^(7/2)*f) + ((3*a - 5*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a^2
*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a*f*S
qrt[a + b + b*Tan[e + f*x]^2]) + ((a - 3*b)*b*(3*a + 5*b)*Tan[e + f*x])/(8*
a^3*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-3a+b-4bx^2}{(1+x^2)^2(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3a^2+5b^2+2(3a}{(1+x^2)(a+b+}\right.}{}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)}{8a^3(a+b)f\sqrt{a+b+}}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)}{8a^3(a+b)f\sqrt{a+b+}}$$

$$= \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)}{8a^3(a+b)f\sqrt{a+b+}}$$

$$= \frac{3\left(a^2-2ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} + \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3}{4af\sqrt{}}$$

Mathematica [C] time = 16.9212, size = 2046, normalized size = 10.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Cos[e + f*x]^10*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*((a + b)*AppellF1[
```

```
1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF
1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b
)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*
Sin[e + f*x]^2)*((a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*C
os[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*((a + b)*AppellF1[1/2, -3, 3/
2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3,
5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[
3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x
]^2)) + ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)]*Cos[e + f*x]^7)/(2*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b
- a*Sin[e + f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF
1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*
x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e +
 f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*
x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + ((a + b)*Cos[e + f*x]^6*Sin[e
 + f*x]*((a*f*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - 2*f*AppellF1[3/2, -2, 3/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])
)/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a +
b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
+ (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) - ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(a*Ap
pellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(
a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Cos[e + f*x]*Sin[e + f*x] + (a + b)*((a*f*AppellF1[3/2, -3, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a +
 b) - 2*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(a*((3*a*f*AppellF1[5/2,
 -3, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin
[e + f*x])/(a + b) - (18*f*AppellF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 2*(a + b)*((9*a*f*A
ppellF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1, 3/2, 7/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(2
*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a + b)*A
ppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a
*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
```

```
2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)])*Sin[e + f*x]^2)^2) + (a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*Sin[2*(e +
 f*x)])/(2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*
((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (a*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 2*(a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Sin[e + f*x]^2))))
```

Maple [C] time = 0.461, size = 2372, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/8/f/(a+b)/a^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(b+a*cos(f*x+e)^2)
*(-2*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-2*cos(f*x+e)^
5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+3*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*
a*b-b^2)/(a+b)^2)^(1/2))*a^3*sin(f*x+e)-3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+9*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*a*b^2*sin(f*x+e)+15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(
f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(
1/2))*b^3*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
```

```
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*sin(f*x+e)+6*2^(1/2)*(1/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2
*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
))*a^2*b*sin(f*x+e)-18*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ell
ipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)
/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)-30*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*b^3*sin(f*x+e)+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*a^3+2*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-3*cos(f*x
+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3+2*cos(f*x+e)^3*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a*b^2+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^3-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-5*cos(f*
x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*cos(f*x+e)*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b^2+15*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b^3+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^2-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)*si
n(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A] time = 5.06442, size = 1875, normalized size = 9.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^3*b - a^2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5
*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^
3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f
*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - 8*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4 - 2*a^3
*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*
b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f), -1/32*(3*(a^3*b - a^2*b^2 + 3*a
*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a)*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*
a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*
sin(f*x + e))) - 4*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4 - 2*a^3*b - 5*a
^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*cos(f*x + e))*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos
(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.282 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{\left(-9a^2b+5a^3+15ab^2-35b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}+\frac{b\left(-17a^2b+15a^3+25ab^2+105b^3\right)\tan(e+fx)}{48a^4f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\left(15a^2-22a\right.}{48a^3}$$

```
[Out] ((5*a^3 - 9*a^2*b + 15*a*b^2 - 35*b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
 + b + b*Tan[e + f*x]^2]])/(16*a^(9/2)*f) + ((15*a^2 - 22*a*b + 35*b^2)*Cos
[e + f*x]*Sin[e + f*x])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((5*a -
 7*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
(b*(15*a^3 - 17*a^2*b + 25*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f
*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.315395, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{\left(-9a^2b+5a^3+15ab^2-35b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{9/2}f}+\frac{b\left(-17a^2b+15a^3+25ab^2+105b^3\right)\tan(e+fx)}{48a^4f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\left(15a^2-22a\right.}{48a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] ((5*a^3 - 9*a^2*b + 15*a*b^2 - 35*b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
 + b + b*Tan[e + f*x]^2]])/(16*a^(9/2)*f) + ((15*a^2 - 22*a*b + 35*b^2)*Cos
[e + f*x]*Sin[e + f*x])/(48*a^3*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((5*a -
 7*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*a^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) + (Cos[e + f*x]^5*Sin[e + f*x])/(6*a*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
(b*(15*a^3 - 17*a^2*b + 25*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f
*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-5a+b-6bx^2}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{15a^2-2ab+7b}{\left(1+x^2\right)^2(}\right.}{}$$

$$= \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{c}{6af}$$

$$= \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{c}{6af}$$

$$= \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{c}{6af}$$

$$= \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{c}{6af}$$

$$= \frac{\left(5a^3-9a^2b+15ab^2-35b^3\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} + \frac{\left(15a^2-22ab+35b^2\right)\cos(e+fx}{48a^3f\sqrt{a+b+b\tan^2(e}}$$

Mathematica [C] time = 20.2506, size = 2068, normalized size = 7.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^14*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Appel
lF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*A
ppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*
(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(Sqrt[a + 2
*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*AppellF1[1
/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*Appell
F1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a +
b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])
*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9)/(2*Sqrt[a + 2*b + a*Cos[2*(e +
f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3,
 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) -
(12*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a
+ b)*Cos[e + f*x]^8*Sin[e + f*x]*((a*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (8
*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
Cos[e + f*x]*Sin[e + f*x])/3))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(a +
 b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*
AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]^8*Sin[e + f*x]*(2*f*(3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a
+ b)*((a*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (8*f*AppellF1[3/2, -3, 3/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3
) + Sin[e + f*x]^2*(3*a*((3*a*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (24*f*App
ellF1[5/2, -3, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e
+ f*x]*Sin[e + f*x])/5) - 8*(a + b)*((9*a*f*AppellF1[5/2, -3, 5/2, 7/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a +
 b)) - (18*f*AppellF1[5/2, -2, 3/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
```

```
/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(2*f*Sqrt[a + 2*b + a*Cos[2*(e +
 f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, S
in[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/2, -4, 5/2, 5/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3
, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)
 + (3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^8*Sin[e + f*x]*Sin[2*(e + f*x)])/(2*(a + 2*b + a*
Cos[2*(e + f*x)])^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2,
 -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[
3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*
AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Si
n[e + f*x]^2))))
```

Maple [C] time = 0.666, size = 3171, normalized size = 11.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/48/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)/a^4*(b+a*cos(f*x+e)^2
)*(8*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4-8*cos(f*x+e)^
7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+15*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^4*sin(f*x+e)-105*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*b^4*sin(f*x+e)+105*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*b^4-35*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^3-7*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2*a^3*b-15*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a^3*b+17*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*cos(f*x+e)*a^2*b^2-25*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)*a*b^3+25*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-10*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^4-15*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*cos(f*x+e)^3*a^4+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*co
s(f*x+e)^2*a^4-105*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^4+1
```

```
0*a^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4-12*sin(f*x+e)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3*b+18*sin(f*x+e)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1
/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b^2-60*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^3*sin(f*x+e)+120*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^3*
sin(f*x+e)+24*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*b*sin(f*x+e)-36*2^(1/2)*(1/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^
(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e
)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(
1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2
*b^2*sin(f*x+e)+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-17*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^3*a^3*b-13*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)^3*a^2*b^2-30*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^4*sin(f*x+e)+210*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
```

```
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*b^4*sin(f*x+e)-8*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*
b+8*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+4*cos(f*x+e)
^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+14*cos(f*x+e)^5*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-4*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*a^3*b-14*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b^2+35*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+
13*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2)*sin(f*x+e)
/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [A] time = 14.9062, size = 2190, normalized size = 8.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 + (5*a^5 - 4
*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*(a^5 + a^
4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b - 7*a^3*b^2)*cos(f*x + e)^5 + (15*
```

```
a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*cos(f*x + e)^3 + (15*a^4*b - 17*a^
3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5
*b^2)*f), -1/192*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 +
(5*a^5 - 4*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) - 4*(8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b
- 7*a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*
cos(f*x + e)^3 + (15*a^4*b - 17*a^3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*
b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.283 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=321

$$-\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3bf(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{2a(a+2b)\sin(e+fx)}{3b^2f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+\right.}}$$

```
[Out] (-2*a*(a + 2*b)*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)]) - (a*Sin[e + f*x])/(3*b*(a + b)*f*(a + b - a*Sin[e +
 f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + (2*(a + 2*b)*El
lipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*b^2
*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (EllipticF[ArcSin[Sin[e + f*
x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*b*(a + b)*f*Sqrt[C
os[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.636959, antiderivative size = 383, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 414, 527, 524, 426, 424, 421, 419}

$$-\frac{2a(a+2b)\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{3b^2f(a+b)^2\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}E\left(\right.}{3b^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(a*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*b*(a + b)*f*Sqrt[a + b*Sec[
e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - (2*a*(a + 2*b)*Sqrt[b + a*C
os[e + f*x]^2]*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]*
Sqrt[a + b - a*Sin[e + f*x]^2]) + (2*(a + 2*b)*Sqrt[b + a*Cos[e + f*x]^2]*E
llipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(
3*b^2*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)]) - (Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin
[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*b*(a +
b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e +
```

```
f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(b+a\left(1-x^2\right)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(a+b-ax^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subs}}{3b(a+b)f\sqrt{\cos}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2a(a+2b)\sqrt{b+a\,c}}{3b^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2a(a+2b)\sqrt{b+a\,c}}{3b^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2a(a+2b)\sqrt{b+a\,c}}{3b^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

$$= -\frac{a\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3b(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2a(a+2b)\sqrt{b+a\,c}}{3b^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

Mathematica [A] time = 2.81984, size = 167, normalized size = 0.52

$$\frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{2}(a+b)^2\left(\frac{a\cos(2(e+fx))+a+2b}{a+b}\right)^{3/2}\left(2(a+2b)E\left(e+fx\middle|\frac{a}{a+b}\right)-b\text{EllipticF}\left(e+f\right.\right.\right.}{24b^2f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(Sqrt[2]*(a + b)^2*((a + 2*b
 + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(2*(a + 2*b)*EllipticE[e + f*x, a/(a
+ b)] - b*EllipticF[e + f*x, a/(a + b)]) - 2*a*(a^2 + 5*a*b + 5*b^2 + a*(a
+ 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(24*b^2*(a + b)^2*f*(a + b*Sec[
e + f*x]^2)^(5/2))
```

Maple [C] time = 0.614, size = 14357, normalized size = 44.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{5}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^{2}+a}\sec\left(fx+e\right)^{5}}{b^{3}\sec\left(fx+e\right)^{6}+3\,ab^{2}\sec\left(fx+e\right)^{4}+3\,a^{2}b\sec\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.284 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=319

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3af(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{(a-b)\sin(e+fx)}{3bf(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}$$

```
[Out] -((a - b)*Sin[e + f*x])/(3*b*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]) + Sin[e + f*x]/(3*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqrt[
Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) + ((a - b)*EllipticE[ArcSin[Sin
[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*a*b*(a + b)^2*f*Sqrt[
Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a
*Sin[e + f*x]^2)/(a + b)]) + (EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqr
t[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.564733, antiderivative size = 381, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4148, 6722, 1974, 412, 527, 524, 426, 424, 421, 419}

$$-\frac{(a-b)\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{3bf(a+b)^2\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}+\frac{\sin(e+fx)\sqrt{a\cos^2(e+fx)+b}}{3f(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*(a + b)*f*Sqrt[a + b*Sec[e + f
*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - ((a - b)*Sqrt[b + a*Cos[e + f*x]
^2]*Sin[e + f*x])/(3*b*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b -
a*Sin[e + f*x]^2]) + ((a - b)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[S
in[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a*b*(a + b)^2*f
*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2
)/(a + b)]) + (Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a*(a + b)*f*Sqrt[Cos[e +
```

```
f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(b+a(1-x^2)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\Big(}{3(a+b)f\sqrt{\cos^2(e}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\sqrt{b+a\cos^2(}}{3b(a+b)^2f\sqrt{a+b\sec^2(e+}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\sqrt{b+a\cos^2(}}{3b(a+b)^2f\sqrt{a+b\sec^2(e+}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\sqrt{b+a\cos^2(}}{3b(a+b)^2f\sqrt{a+b\sec^2(e+}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\sqrt{b+a\cos^2(}}{3b(a+b)^2f\sqrt{a+b\sec^2(e+}}$$

Mathematica [C] time = 10.2832, size = 1156, normalized size = 3.62

$$(\cos(2e+2fx)a+a+2b)^{5/2}\sec^5(e+fx)\left(\frac{\cos(2(e+fx))\left(2ib\left(a^2+ba+b^2\right)\sqrt{\frac{a-a\cos(2e+2fx)}{a+b}}\sqrt{4-\frac{2(\cos(2e+2fx)a+a+2b)}{b}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a+b}}\sqrt{\cos(2e+2f}}{\sqrt{2}}\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-((-2*Sqrt[-(a + b)^(
-1)]*(-a - a*Cos[2*e + 2*f*x])*(2*a^2*(a + 3*b + a*Cos[2*e + 2*f*x]) + b*(2
*b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - 2*(a + 2*b + a*Cos[2*e + 2*f*x]
)^2) + a*(4*b^2 + 5*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b + a*Cos[2*e
 + 2*f*x])^2)) + (2*I)*b*(a + 2*b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*(
a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*
x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e +
 2*f*x]])/Sqrt[2]], (a + b)/b] - I*b*(a + 3*b)*Sqrt[(a - a*Cos[2*e + 2*f*x]
)/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b + a*Co
s[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqrt[-(a + b)^(-1)]*Sqrt[a + 2*b +
 a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Sin[2*e + 2*f*x])/(24*a*b^2*Sqr
t[-(a + b)^(-1)]*(a + b)^2*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e
+ 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[1 - Cos[2*e + 2*f
*x]^2]) + (Cos[2*(e + f*x)]*(-2*Sqrt[-(a + b)^(-1)]*(-a - a*Cos[2*e + 2*f*x
])*(4*b^4 - b^2*(a + 2*b + a*Cos[2*e + 2*f*x])^2 + 2*a^3*(a + 3*b + a*Cos[2
*e + 2*f*x]) + a*b*(10*b^2 + b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b +
a*Cos[2*e + 2*f*x])^2) + a^2*(8*b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x]) -
(a + 2*b + a*Cos[2*e + 2*f*x])^2)) + (2*I)*b*(a^2 + a*b + b^2)*Sqrt[(a - a*
Cos[2*e + 2*f*x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2
*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-(a + b)^(-1)
]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b] + I*a*b*(-a + b)
*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2
)*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(Sqrt[
-(a + b)^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], (a + b)/b])*Se
c[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e + 2*f*x])/(24*a^2*b^
2*Sqrt[-(a + b)^(-1)]*(a + b)^2*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos
[2*e + 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[1 - Cos[2*e
+ 2*f*x]^2])))/(2*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] time = 0.604, size = 10271, normalized size = 32.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\sec\left(fx+e\right)^3}{b^3\sec\left(fx+e\right)^6+3\,ab^2\sec\left(fx+e\right)^4+3\,a^2b\sec\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.285 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=327

$$\frac{(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}-\frac{2(2a+b)\left(-a\sin^2(e+fx)+a+b\right)E\left(\right.}{3a^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\sec^2\left(\right.}}$$

```
[Out] (2*(2*a + b)*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*
Sin[e + f*x]^2)]) - (b*Sin[e + f*x])/(3*a*(a + b)*f*(a + b - a*Sin[e + f*x]
^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (2*(2*a + b)*Ellipti
cE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^2*(a +
 b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + ((3*a + 2*b)*EllipticF[ArcSin[Sin
[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^2*(a + b)
*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.579403, antiderivative size = 389, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4148, 6722, 1974, 413, 527, 524, 426, 424, 421, 419}

$$\frac{(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\cos^2(e+fx)+b}\,F\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^2f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a+b\sec^2(e+fx)}}-\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e}}{3a^2f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\,s}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)*f*Sqrt[a + b*Sec[
e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (2*(2*a + b)*Sqrt[b + a*Cos
[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt
[a + b - a*Sin[e + f*x]^2]) - (2*(2*a + b)*Sqrt[b + a*Cos[e + f*x]^2]*Ellip
ticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a^
2*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*S
in[e + f*x]^2)/(a + b)]) + ((3*a + 2*b)*Sqrt[b + a*Cos[e + f*x]^2]*Elliptic
F[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3
*a^2*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b -
```

```
a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{\left(b+a(1-x^2)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{(a+b-ax^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}}{3a(a+b)f\sqrt{\cos^2}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \frac{2(2a+b)\sqrt{b+a\cos}}{3a(a+b)^2f\sqrt{a+b\sec^2(e+}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \frac{2(2a+b)\sqrt{b+a\cos}}{3a(a+b)^2f\sqrt{a+b\sec^2(e+}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \frac{2(2a+b)\sqrt{b+a\cos}}{3a(a+b)^2f\sqrt{a+b\sec^2(e+}}$$

$$= -\frac{b\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} + \frac{2(2a+b)\sqrt{b+a\cos}}{3a(a+b)^2f\sqrt{a+b\sec^2(e+}}$$

Mathematica [F] time = 12.9911, size = 0, normalized size = 0.

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] time = 0.612, size = 14353, normalized size = 43.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^{2}+a}\sec\left(fx+e\right)}{b^{3}\sec\left(fx+e\right)^{6}+3\,ab^{2}\sec\left(fx+e\right)^{4}+3\,a^{2}b\sec\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)/(b^3*sec(f*x + e)^6 + 3*a*
b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.286 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{b(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^3f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(3a^2+13ab+8b^2\right)\left(-a\sin^2(e+fx)+a+\right.}{3a^3f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{\text{se}}}$$

```
[Out] (-2*b*(3*a + 2*b)*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a +
 b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^2*Sin[e + f*x])/(3*a*(a + b)*f*(
a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
+ ((3*a^2 + 13*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a +
 b - a*Sin[e + f*x]^2))/(3*a^3*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e
+ f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])
- (b*(9*a + 8*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin
[e + f*x]^2)/(a + b)])/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f
*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.487539, antiderivative size = 411, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4148, 6722, 1974, 413, 526, 524, 426, 424, 421, 419}

$$\frac{\left(3a^2+13ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}\,E\left(\sin^{-1}(\sin(e+fx))|\frac{a}{a+b}\right)}{3a^3f(a+b)^2\sqrt{\cos^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a+b\sec^2(e+fx)}}-\frac{2b(3a+2b)\sin(e}{3a^2f(a+b)^2\sqrt{-a\sin^2(e}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Cos[e + f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - (2*b*(3*a +
2*b)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b
*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + ((3*a^2 + 13*a*b + 8*b^2
)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqr
t[a + b - a*Sin[e + f*x]^2])/(3*a^3*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a
 + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(9*a + 8*b)
*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt
```

```
[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^3*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqr
t[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(b+a\left(1-x^2\right)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(a+b-ax^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}}{3a(a+b)f\sqrt{\text{co}}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(3a+2b)\sqrt{b+a\,\text{c}}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(3a+2b)\sqrt{b+a\,\text{c}}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(3a+2b)\sqrt{b+a\,\text{c}}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

$$= -\frac{b\cos^2(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(3a+2b)\sqrt{b+a\,\text{c}}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

Mathematica [F] time = 16.1559, size = 0, normalized size = 0.

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] time = 0.899, size = 17494, normalized size = 50.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)}{b^{3}\sec\left(fx+e\right)^{6}+3\,ab^{2}\sec\left(fx+e\right)^{4}+3\,a^{2}b\sec\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^3*sec(f*x + e)^6 + 3*a*
b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.287 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=441

$$-\frac{b\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{3a^4f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\left(-a\sin^2(e+\right.}{3a^3f(a+b)^2\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+f\right.}}$$

```
[Out] (-2*b*(4*a + 3*b)*Cos[e + f*x]^2*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[
e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^4*Sin[e + f*x])/(
3*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]) + ((a^2 + 11*a*b + 8*b^2)*Sin[e + f*x]*(a + b - a*Sin[e + f*
x]^2))/(3*a^3*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
+ (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a +
 b)]*(a + b - a*Sin[e + f*x]^2))/(3*a^4*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(
a + b)]) - (b*(a^2 - 16*a*b - 16*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a
+ b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^4*(a + b)*f*Sqrt[Cos[e + f
*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.730507, antiderivative size = 512, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {4148, 6722, 1974, 413, 526, 528, 524, 426, 424, 421, 419}

$$\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}}{3a^3f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{b\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\sqrt{a\,c}}{3a^4f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{-a\sin^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Cos[e + f*x]^4*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - (2*b*(4*a +
3*b)*Cos[e + f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a^2*(a + b)
^2*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + ((a^2 + 1
1*a*b + 8*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqrt[a + b - a*Sin[e
 + f*x]^2])/(3*a^3*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + (2*(a + 2*b)*(
```

```
a^2 - 4*a*b - 4*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*
x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a^4*(a + b)^2*f*Sqrt[Cos
[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)
]) - (b*(a^2 - 16*a*b - 16*b^2)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin
[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(3*a^4*(a
+ b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e
 + f*x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
```

```
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

```
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{7/2}}{\left(b+a\left(1-x^2\right)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{7/2}}{\left(a+b-ax^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}}{3a(a+b)f\sqrt{c}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos^2(e+fx)\sqrt{b}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos^2(e+fx)\sqrt{b}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos^2(e+fx)\sqrt{b}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos^2(e+fx)\sqrt{b}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

$$= -\frac{b\cos^4(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(4a+3b)\cos^2(e+fx)\sqrt{b}}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e}}$$

Mathematica [F] time = 13.261, size = 0, normalized size = 0.

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] time = 1.43, size = 20922, normalized size = 47.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^3}{b^3\sec\left(fx+e\right)^6+3\,ab^2\sec\left(fx+e\right)^4+3\,a^2b\sec\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.288 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=559

$$-\frac{b\left(-9a^2b+4a^3+120ab^2+128b^3\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\text{EllipticF}\left(\sin^{-1}(\sin(e+fx)),\frac{a}{a+b}\right)}{15a^5f(a+b)\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}}+\frac{2\left(-3a^2b+2a^3-42ab^2-32\right.}{15a^4f(a+b)^2\sqrt{\sec^2(}}$$

```
[Out] (-2*b*(5*a + 4*b)*Cos[e + f*x]^4*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[Sec[
e + f*x]^2*(a + b - a*Sin[e + f*x]^2)]) - (b*Cos[e + f*x]^6*Sin[e + f*x])/(
3*a*(a + b)*f*(a + b - a*Sin[e + f*x]^2)*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]) + (2*(2*a^3 - 3*a^2*b - 42*a*b^2 - 32*b^3)*Sin[e + f*x]*(a +
 b - a*Sin[e + f*x]^2))/(15*a^4*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b - a*
Sin[e + f*x]^2)]) + ((3*a^2 + 61*a*b + 48*b^2)*Cos[e + f*x]^2*Sin[e + f*x]*
(a + b - a*Sin[e + f*x]^2))/(15*a^3*(a + b)^2*f*Sqrt[Sec[e + f*x]^2*(a + b
- a*Sin[e + f*x]^2)]) + ((8*a^4 - 11*a^3*b + 27*a^2*b^2 + 184*a*b^3 + 128*b
^4)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*(a + b - a*Sin[e + f*x]^2))/
(15*a^5*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]^2*(a + b - a*Sin
[e + f*x]^2)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a^3 - 9*a^2*b +
 120*a*b^2 + 128*b^3)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (
a*Sin[e + f*x]^2)/(a + b)])/(15*a^5*(a + b)*f*Sqrt[Cos[e + f*x]^2]*Sqrt[Sec
[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)])
```

Rubi [A] time = 0.905856, antiderivative size = 639, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {4148, 6722, 1974, 413, 526, 528, 524, 426, 424, 421, 419}

$$\frac{\left(3a^2+61ab+48b^2\right)\sin(e+fx)\cos^2(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\sqrt{a\cos^2(e+fx)+b}}{15a^3f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}+\frac{2\left(-3a^2b+2a^3-42ab^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Cos[e + f*x]^6*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a*(a + b)*f*
Sqrt[a + b*Sec[e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(3/2)) - (2*b*(5*a +
4*b)*Cos[e + f*x]^4*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x])/(3*a^2*(a + b)
```

```
^2*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]) + (2*(2*a^3
 - 3*a^2*b - 42*a*b^2 - 32*b^3)*Sqrt[b + a*Cos[e + f*x]^2]*Sin[e + f*x]*Sqr
t[a + b - a*Sin[e + f*x]^2])/(15*a^4*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]
) + ((3*a^2 + 61*a*b + 48*b^2)*Cos[e + f*x]^2*Sqrt[b + a*Cos[e + f*x]^2]*Si
n[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(15*a^3*(a + b)^2*f*Sqrt[a + b*S
ec[e + f*x]^2]) + ((8*a^4 - 11*a^3*b + 27*a^2*b^2 + 184*a*b^3 + 128*b^4)*Sq
rt[b + a*Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a
+ b - a*Sin[e + f*x]^2])/(15*a^5*(a + b)^2*f*Sqrt[Cos[e + f*x]^2]*Sqrt[a +
b*Sec[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a^3 - 9*a^2
*b + 120*a*b^2 + 128*b^3)*Sqrt[b + a*Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e
 + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(15*a^5*(a + b)*
f*Sqrt[Cos[e + f*x]^2]*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*
x]^2])
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+\frac{b}{1-x^2}\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{9/2}}{\left(b+a\left(1-x^2\right)\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{9/2}}{\left(a+b-ax^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{\cos^2(e+fx)}\sqrt{a+b\sec^2(e+fx)}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b+a\cos^2(e+fx)}\,\text{Subs}}{3a(a+b)f\sqrt{}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos^4(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos^4(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos^4(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos^4(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos^4(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

$$= -\frac{b\cos^6(e+fx)\sqrt{b+a\cos^2(e+fx)}\sin(e+fx)}{3a(a+b)f\sqrt{a+b\sec^2(e+fx)}\left(a+b-a\sin^2(e+fx)\right)^{3/2}} - \frac{2b(5a+4b)\cos^4(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(}}$$

Mathematica [F] time = 23.4481, size = 0, normalized size = 0.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [C] time = 2.248, size = 26983, normalized size = 48.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)^5}{b^3\sec\left(fx+e\right)^6+3\,ab^2\sec\left(fx+e\right)^4+3\,a^2b\sec\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^3*sec(f*x + e)^6 + 3*
a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.289 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{a(3a+5b)\tan(e+fx)}{3b^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{a\tan(e+fx)\sec^2(e+fx)}{3bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)*f)
- (a*Sec[e + f*x]^2*Tan[e + f*x])/(3*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)
^(3/2)) - (a*(3*a + 5*b)*Tan[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b + b*Ta
n[e + f*x]^2])
```

Rubi [A] time = 0.139465, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4146, 413, 385, 217, 206}

$$-\frac{a(3a+5b)\tan(e+fx)}{3b^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{a\tan(e+fx)\sec^2(e+fx)}{3bf(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)*f)
- (a*Sec[e + f*x]^2*Tan[e + f*x])/(3*b*(a + b)*f*(a + b + b*Tan[e + f*x]^2)
^(3/2)) - (a*(3*a + 5*b)*Tan[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b + b*Ta
n[e + f*x]^2])
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+3b+3(a+b)x^2}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3b(a+b)f}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \text{Subst}$$

$$= -\frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \text{Subst}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan}{3b^2(a+b)^2f\sqrt{a+}}$$

Mathematica [C] time = 10.0698, size = 607, normalized size = 4.56

$$\tan(e+fx)\sec^6(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}(a\cos(2e+2fx)+a+2b)^{5/2}\left(-\frac{24b\sin^2(e+fx)\cos^2(e+fx)\left(-\frac{b\tan^2(e+fx)\sec^2(e+fx)\left(-a\sin^2(e+\right.}{(a+b)^2}\right.}{a}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^6*Sqrt[1 - (2*a*Sin[e +
f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*((-24*b*Cos[e + f*x]^2*HypergeometricPFQ[
{2, 2, 2}, {1, 9/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(-((b*Se
c[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(5/2))/
(a + b) - (Sec[e + f*x]^6*(24*b^3*Hypergeometric2F1[2, 2, 9/2, -((b*Tan[e +
 f*x]^2)/(a + b))]*Sin[e + f*x]^6*(4*b^2 + a*b*(8 - 7*Sin[e + f*x]^2) + a^2
```

```
*(4 - 7*Sin[e + f*x]^2 + 3*Sin[e + f*x]^4))*Sqrt[-((b*Sec[e + f*x]^2*(a + b
 - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)] + 35*(a + b)*Cos[e + f*x]^
2*(15*b^2 + 10*a*b*(3 - 2*Sin[e + f*x]^2) + a^2*(15 - 20*Sin[e + f*x]^2 + 8
*Sin[e + f*x]^4))*(-3*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*(a + b -
a*Sin[e + f*x]^2)^2 - Cos[e + f*x]^2*(-3*a^2*Cos[e + f*x]^2 + 2*a*b*(-3 + S
in[e + f*x]^2) - b^2*(3 + Sin[e + f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b
- a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])))/(a + b)^5))/(315*(2*a +
2*b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*
Sqrt[2*a + 2*b - 2*a*Sin[e + f*x]^2]*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2)
*(-((b*Tan[e + f*x]^2)/(a + b)))^(5/2))
```

Maple [C] time = 0.469, size = 3010, normalized size = 22.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/(a+b)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2*sin(f*x+e)*(b+a*c
os(f*x+e)^2)*(6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+12*sin(f*x+e)
*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+co
s(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-
a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),
(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2))*a*b^2-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
```

```
+b)^2)^(1/2))*a^3-6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
2)/(a+b)^2)^(1/2))*a^2*b-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+6*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+12*sin
(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+6*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3-3*2^(1/2
)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-6*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-3*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-3*cos(f*x+e)^3*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-5*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a^2*b+3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3
+5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)*
```

```
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-6*cos(f*x+e)*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b^2+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2
*b+6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2)/(-1+cos(f*x+e))/cos(f*x
+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.46035, size = 1586, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^
4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(b)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^3*b + 5*a^2*b^2)*cos(f*
x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f*cos(f*x
+ e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^2*b^5 + 2*a*
b^6 + b^7)*f), 1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 +
 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-b)*arc
tan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) -
 2*((3*a^3*b + 5*a^2*b^2)*cos(f*x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x
+ e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 +
 2*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*
cos(f*x + e)^2 + (a^2*b^5 + 2*a*b^6 + b^7)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.290 $\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}} dx$

Optimal. Leaf size=79

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (2*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0928341, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4146, 378, 191}

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (2*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
```

```
&& GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] time = 4.55023, size = 74, normalized size = 0.94

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)(a\cos(2(e+fx))+2a+3b)}{6f(a+b)^2\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*(2*a + 3*b + a*Cos[2*(e + f*x)])*Sec[e + f*
x]^4*Tan[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] time = 0.304, size = 76, normalized size = 1.

$$\frac{\sin\left(fx+e\right)\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)\left(2\,a\left(\cos\left(fx+e\right)\right)^2+a+3\,b\right)}{3\,f\,(a+b)^2\left(\cos\left(fx+e\right)\right)^5}\left(\frac{b+a\left(\cos\left(fx+e\right)\right)^2}{\left(\cos\left(fx+e\right)\right)^2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/(a+b)^2*sin(f*x+e)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2+a+3*b)/cos(f*
x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)
```

Maxima [A] time = 1.18301, size = 158, normalized size = 2.

$$\frac{\frac{2\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}(a+b)^2}+\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}(a+b)}-\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}b}+\frac{\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^2+a+b}(a+b)b}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)) - tan(f*x + e)/((b*tan(f*x +
e)^2 + a + b)^(3/2)*b) + tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a +
b)*b))/f
```

Fricas [A] time = 0.842701, size = 313, normalized size = 3.96

$$\frac{\left(2\,a\cos\left(fx+e\right)^3+(a+3\,b)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\,\sin\left(fx+e\right)}{3\left(\left(a^4+2\,a^3b+a^2b^2\right)f\cos\left(fx+e\right)^4+2\left(a^3b+2\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2+\left(a^2b^2+2\,ab^3+b^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*cos(f*x + e)^3 + (a + 3*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^4
 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^
4)*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [B] time = 2.4051, size = 560, normalized size = 7.09

$$-\frac{2\left(\left(\left(\frac{3\left(a^6b^4\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)+2a^5b^5\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)+a^4b^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^7b^4+3a^5b^5+3a^5b^6+a^4b^7}-\frac{2\left(a^6b^4\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)-2\cdots\right.}{\cdots}\right.\right.\right.}{3\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*((3*(a^6*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2*a^5*b^5*sgn(tan(1/2*f
*x + 1/2*e)^2 - 1) + a^4*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x +
 1/2*e)^2/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7) - 2*(a^6*b^4*sgn(tan(
1/2*f*x + 1/2*e)^2 - 1) - 2*a^5*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3*a^4
*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^
4*b^7))*tan(1/2*f*x + 1/2*e)^2 + 3*(a^6*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
 + 2*a^5*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^4*b^6*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7))*tan(1/2*f*x + 1/2
*e)/((a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3/2)*f)
```

3.291 $\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] Tan[e + f*x]/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*
x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0895528, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4146, 192, 191}

$$\frac{2\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```



```
[Out] Tan[e + f*x]/(3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*
x])/(3*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] time = 6.13418, size = 215, normalized size = 3.03

$$\frac{(3a+b)\sec^5(e+fx)\left(\frac{2\sqrt{2}\sin(e+fx)}{(a+b)^2\sqrt{-a\sin^2(e+fx)+a+b}} + \frac{\sqrt{2}\sin(e+fx)}{(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)(a\cos(2e+2fx)+a+2b)^{5/2}}{48af\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{\tan(e+fx)\sec}{8\sqrt{2}af\left(-a\sin^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*a + b)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*((Sqrt[2]*Si
n[e + f*x])/((a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (2*Sqrt[2]*Sin[e +
 f*x])/((a + b)^2*Sqrt[a + b - a*Sin[e + f*x]^2])))/(48*a*f*(a + b*Sec[e +
f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*Tan[e
 + f*x])/(8*Sqrt[2]*a*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x
]^2)^(3/2))
```

Maple [A] time = 0.266, size = 85, normalized size = 1.2

$$\frac{\sin(fx+e)\left(3a(\cos(fx+e))^2+b(\cos(fx+e))^2+2b\right)\left(b+a(\cos(fx+e))^2\right)}{3f(a+b)^2(\cos(fx+e))^5}\left(\frac{b+a(\cos(fx+e))^2}{(\cos(fx+e))^2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/(a+b)^2*sin(f*x+e)*(3*a*cos(f*x+e)^2+b*cos(f*x+e)^2+2*b)*(b+a*cos(f*x
+e)^2)/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)
```

Maxima [A] time = 1.20573, size = 82, normalized size = 1.15

$$\frac{\frac{2\,\tan\left(fx+e\right)}{\sqrt{b\tan\left(fx+e\right)^{2}+a+b}(a+b)^{2}}+\frac{\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}(a+b)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

Fricas [B] time = 0.912419, size = 313, normalized size = 4.41

$$\frac{\left((3\,a+b)\cos\left(fx+e\right)^{3}+2\,b\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\sin\left(fx+e\right)}{3\left(\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{4}+2\left(a^{3}b+2\,a^{2}b^{2}+ab^{3}\right)f\cos\left(fx+e\right)^{2}+\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a + b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^4
 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 + b^
4)*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [B] time = 2.44098, size = 562, normalized size = 7.92

$$-\frac{2\left(\left(\left(\frac{3\left(a^6b^4\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)+2a^5b^5\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)+a^4b^6\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^7b^4+3a^6b^5+3a^5b^6+a^4b^7}-\frac{2\left(3a^6b^4\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)+\right.}{}\right.\right.\right.}{3\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*((3*(a^6*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 2*a^5*b^5*sgn(tan(1/2*f
*x + 1/2*e)^2 - 1) + a^4*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))*tan(1/2*f*x +
 1/2*e)^2/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7) - 2*(3*a^6*b^4*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1) + 2*a^5*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - a^4
*b^6*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^
4*b^7))*tan(1/2*f*x + 1/2*e)^2 + 3*(a^6*b^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
 + 2*a^5*b^5*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + a^4*b^6*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1))/(a^7*b^4 + 3*a^6*b^5 + 3*a^5*b^6 + a^4*b^7))*tan(1/2*f*x + 1/2
*e)/((a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)^(3/2)*f)
```

3.292 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.102312, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \text{Sub}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \text{Sub}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \text{Sub}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\,\text{t}}{3a^2(a+b)^2f\sqrt{a+}}$$

Mathematica [C] time = 6.51743, size = 1927, normalized size = 15.42

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
```

```
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [C] time = 0.4, size = 3016, normalized size = 24.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/(a+b)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2*sin(f*x+e)*(b+a*c
os(f*x+e)^2)*(6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-
a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+12*cos(f*x+e
)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*a*b^2-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^3-6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a^2*b-3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
```

```
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b
)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)+12
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e)
)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2
)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2))*a*b^2*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)-3*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)-6*2^(1/2
)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2*sin(f*x+e)-3*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b
^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-6*cos(f*x+e)^3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b^2+6*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b+4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-5*c
os(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*cos(f*x+e)*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a*b^2+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(f*x+e))/(
(b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32619, size = 2021, normalized size = 16.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

3.293 $\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{6a^3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(a-5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}+\frac{b(3a+5b)\tan(e+fx)}{6a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+$$

```
[Out] ((a - 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) + (b*(3*a + 5*b)*Tan[e + f*x])/(6*a^2*(a + b)*f*(a + b + b*Tan[e +
 f*x]^2)^(3/2)) + (b*(3*a^2 + 22*a*b + 15*b^2)*Tan[e + f*x])/(6*a^3*(a + b)
^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.242584, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{6a^3f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(a-5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{7/2}f}+\frac{b(3a+5b)\tan(e+fx)}{6a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] ((a - 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(7/2)*f) + (Cos[e + f*x]*Sin[e + f*x])/(2*a*f*(a + b + b*Tan[e + f*x]^2
)^(3/2)) + (b*(3*a + 5*b)*Tan[e + f*x])/(6*a^2*(a + b)*f*(a + b + b*Tan[e +
 f*x]^2)^(3/2)) + (b*(3*a^2 + 22*a*b + 15*b^2)*Tan[e + f*x])/(6*a^3*(a + b)
^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
```

```
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-a+b-4bx^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\left(3a^2+\right.}{6a^3(a+b)^2}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\left(3a^2+\right.}{6a^3(a+b)^2}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\left(3a^2+\right.}{6a^3(a+b)^2}$$

$$= \frac{(a-5b)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5}{6a^2(a+b)f\left(a+\right.}$$

Mathematica [C] time = 17.5405, size = 1775, normalized size = 9.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2,
```

5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((15*a*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(2*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e + f*x]^6*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x]))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^6*Sin[e + f*x]*(2*f*(5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - 2*f*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x]) + Sin[e + f*x]^2*(5*a*((21*a*f*AppellF1[5/2, -3, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (18*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5) - 6*(a + b)*((3*a*f*AppellF1[5/2, -2, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (12*f*AppellF1[5/2, -1, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))

Maple [C] time = 0.646, size = 4270, normalized size = 22.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/6/f/(a+b)^2/a^3/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(b+a*c
os(f*x+e)^2)*(15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^4*sin(
f*x+e)-15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4+20*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a*b^3-6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*cos(f*x+e)^2*a^3*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)*a^2*b^2+22*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a*b^3-22
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*cos(f*x+e)^5*a^4+15*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*c
os(f*x+e)*b^4-3*a^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4+9*
2^(1/2)*a^3*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*sin(f*x+e
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*b+27*2^(1/2)*a^2*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*s
in(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*b^2-3*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2
))*a^3*b+9*sin(f*x+e)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*Elli
pticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-
(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2
```

```
*b^2+27*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^3*sin(f*x+e)-
54*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*a*b^3*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*b*sin(f*x+e
)-18*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^
(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2))*a^2*b^2*sin(f*x+e)-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2*b^2+6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^
3*b+30*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2*b^2-30*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*b^4*sin(f*x+e)-3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^4+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-
(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2))*a^4+6*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+3*cos
(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-6*cos(f*x+e)^4*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-20*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a*b^3-30*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
```

```
a^2*b^2+15*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)
))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/
sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^
2)^(1/2))*a*b^3-18*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3*b-54*cos(
f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b^2-30*cos(f*x+e)^2*sin(f*x+e)*2
^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*a*b^3)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/c
os(f*x+e)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 6.16191, size = 2330, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(a^3*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b^
2 - 5*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^4)
*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)
^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b
^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e)) + 8*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + 2*(3*a^4*
b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^2*b^3 + 15*
a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*b^2 + a^
5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), -1/24*(3*(a^3
*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b^2 - 5*a^2*b^3
)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^4)*cos(f*x + e)
^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(
f*x + e)^2)*sin(f*x + e))) - 4*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5
+ 2*(3*a^4*b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^
2*b^3 + 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a
^6*b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.294 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{\left(3a^2-10ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}+\frac{b\left(-15a^2b+9a^3-145ab^2-105b^3\right)\tan(e+fx)}{24a^4f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{b\left(9a^2-18ab-35\right.}{24a^3f(a+b)\left(a+b\,\mathrm{ta}\right.}$$

```
[Out] ((3*a^2 - 10*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) + ((3*a - 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^2 - 18*a*b - 35*b^2)*Tan[e +
 f*x])/(24*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^3 - 15
*a^2*b - 145*a*b^2 - 105*b^3)*Tan[e + f*x])/(24*a^4*(a + b)^2*f*Sqrt[a + b
+ b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.338383, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{\left(3a^2-10ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8a^{9/2}f}+\frac{b\left(-15a^2b+9a^3-145ab^2-105b^3\right)\tan(e+fx)}{24a^4f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{b\left(9a^2-18ab-35\right.}{24a^3f(a+b)\left(a+b\,\mathrm{ta}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] ((3*a^2 - 10*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(8*a^(9/2)*f) + ((3*a - 7*b)*Cos[e + f*x]*Sin[e + f*x])/(8*a
^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos[e + f*x]^3*Sin[e + f*x])/(4*a
*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^2 - 18*a*b - 35*b^2)*Tan[e +
 f*x])/(24*a^3*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(9*a^3 - 15
*a^2*b - 145*a*b^2 - 105*b^3)*Tan[e + f*x])/(24*a^4*(a + b)^2*f*Sqrt[a + b
+ b*Tan[e + f*x]^2])
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3a+b-6bx^2}{(1+x^2)^2(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{4af}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a^2+2ab+7}{(1+x^2)}\right.}{}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\left(9a^2-18ab-\right.}{24a^3(a+b)f\left(a+\right.}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\left(9a^2-18ab-\right.}{24a^3(a+b)f\left(a+\right.}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\left(9a^2-18ab-\right.}{24a^3(a+b)f\left(a+\right.}$$

$$= \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b\left(9a^2-18ab-\right.}{24a^3(a+b)f\left(a+\right.}$$

$$= \frac{\left(3a^2-10ab+35b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} + \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{c}{4af}$$

<u>Mathematica</u> [C] time = 21.0068, size = 1777, normalized size = 6.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(
5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2,
 -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)
*((15*a*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f
*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*
Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1
/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^
9)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4,
 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
 -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*Appe
llF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e
+ f*x]^2)) - (3*Sqrt[2]*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/((a + b - a*Sin
[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[
e + f*x]^8*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (8*f*
AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos
[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(
3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -4, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^8*Sin[e + f*x]
*(2*f*(5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/
2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*S
in[e + f*x])/(3*(a + b)) - (8*f*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2
*(5*a*((21*a*f*AppellF1[5/2, -4, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (24*f*AppellF1[5/2, -3
, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e
+ f*x])/5) - 8*(a + b)*((3*a*f*AppellF1[5/2, -3, 7/2, 7/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (18*f*Appe
llF1[5/2, -2, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
 f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*
```

```
(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
 f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))
```

Maple [C] time = 0.93, size = 5600, normalized size = 21.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 18.1044, size = 2708, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(3*a^4*b^2 - 4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6
 - 4*a^5*b + 18*a^4*b^2 + 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^
```

```
5*b - 4*a^4*b^2 + 18*a^3*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(
-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(6
*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11*a^4*b^2 -
 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^3 - 70*a^2*
b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 105*a*b^5)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^9
 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), -1/96*(3*(3*a^4*b^2 -
4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6 - 4*a^5*b + 18*a^4*b^2
+ 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^5*b - 4*a^4*b^2 + 18*a^3
*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(6
*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11*a^4*b^2 -
 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^3 - 70*a^2*
b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 105*a*b^5)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^9
 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*
cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos \left(fx+e\right)^4}{\left(b \sec \left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.295 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=332

$$\frac{5(a-3b)\left(a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}+\frac{b\left(38a^2b^2-20a^3b+15a^4+420ab^3+315b^4\right)\tan(e+fx)}{48a^5f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{b\left(-25a^2b\right.}{48a^4f(}$$

```
[Out] (5*(a - 3*b)*(a^2 + 7*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(16*a^(11/2)*f) + ((5*a^2 - 10*a*b + 21*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((5*a - 9*b)*Cos[
e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos
[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15
*a^3 - 25*a^2*b + 49*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15*a^4 - 20*a^3*b + 38*a^2*b^2 + 420*a*b
^3 + 315*b^4)*Tan[e + f*x])/(48*a^5*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]
^2])
```

Rubi [A] time = 0.428734, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4146, 414, 527, 12, 377, 203}

$$\frac{5(a-3b)\left(a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16a^{11/2}f}+\frac{b\left(38a^2b^2-20a^3b+15a^4+420ab^3+315b^4\right)\tan(e+fx)}{48a^5f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{b\left(-25a^2b\right.}{48a^4f(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (5*(a - 3*b)*(a^2 + 7*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/(16*a^(11/2)*f) + ((5*a^2 - 10*a*b + 21*b^2)*Cos[e + f*x]*Si
n[e + f*x])/(16*a^3*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((5*a - 9*b)*Cos[
e + f*x]^3*Sin[e + f*x])/(24*a^2*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (Cos
[e + f*x]^5*Sin[e + f*x])/(6*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15
*a^3 - 25*a^2*b + 49*a*b^2 + 105*b^3)*Tan[e + f*x])/(48*a^4*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^(3/2)) + (b*(15*a^4 - 20*a^3*b + 38*a^2*b^2 + 420*a*b
^3 + 315*b^4)*Tan[e + f*x])/(48*a^5*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]
```

```
^2])
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^4\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-5a+b-8bx^2}{\left(1+x^2\right)^3\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{6af}$$

$$= \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3\left(5a^2+\right.}{\left(1+x\right.}\right.}{}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{}{6a}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{}{6a}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{}{6a}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{}{6a}$$

$$= \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{}{6a}$$

$$= \frac{5(a-3b)\left(a^2+7b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} + \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$

Mathematica [C] time = 24.7166, size = 1776, normalized size = 5.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^16*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(
5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2,
 -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)
*((15*a*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]
^2)/(a + b)]*Cos[e + f*x]^11*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e +
f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin
[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[
1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]
^11)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -
5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/
2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*Ap
pellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2)) - (15*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(2*Sqrt[2]*(a + b -
a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*Cos[e + f*x]^10*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) -
 (10*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)]*Cos[e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^
(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x
]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e +
 f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -5,
 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^10*Sin[
e + f*x]*(10*f*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*
x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*Appe
llF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e +
```

```
 f*x]*Sin[e + f*x])/(3*(a + b)) - (10*f*AppellF1[3/2, -4, 5/2, 5/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + 5*Sin[
e + f*x]^2*(a*((21*a*f*AppellF1[5/2, -5, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - 6*f*AppellF1[5
/2, -4, 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*
Sin[e + f*x]) - 2*(a + b)*((3*a*f*AppellF1[5/2, -4, 7/2, 7/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (24*f*A
ppellF1[5/2, -3, 5/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/5))))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*
(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[
e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)^2)))
```

Maple [C] time = 1.471, size = 6934, normalized size = 20.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 49.9991, size = 3035, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b^5 - 35*a*b^6 - 21*b^7
+ (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4 - 21*a^2*b^5)*cos(f*x
+ e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4 - 35*a^2*b^5 - 21*a*b^
6)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b
^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a
*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e)) + 8*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 2*(5*a^
7 + a^6*b - 13*a^5*b^2 - 9*a^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 + 6*a^5*b^2 +
 32*a^4*b^3 + 21*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 15*a^5*b^2 + 31*a^
4*b^3 + 287*a^3*b^4 + 210*a^2*b^5)*cos(f*x + e)^3 + (15*a^5*b^2 - 20*a^4*b^
3 + 38*a^3*b^4 + 420*a^2*b^5 + 315*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*
x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*
a^7*b^3 + a^6*b^4)*f), -1/192*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b
^5 - 35*a*b^6 - 21*b^7 + (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4
 - 21*a^2*b^5)*cos(f*x + e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4
 - 35*a^2*b^5 - 21*a*b^6)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*(a^
7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 2*(5*a^7 + a^6*b - 13*a^5*b^2 - 9*a
^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 + 6*a^5*b^2 + 32*a^4*b^3 + 21*a^3*b^4)*co
s(f*x + e)^5 + 2*(15*a^6*b - 15*a^5*b^2 + 31*a^4*b^3 + 287*a^3*b^4 + 210*a^
2*b^5)*cos(f*x + e)^3 + (15*a^5*b^2 - 20*a^4*b^3 + 38*a^3*b^4 + 420*a^2*b^5
 + 315*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*
b^2 + a^7*b^3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.296 $\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{15a^3d(a+b)^3\sqrt{a+b\tan^2(c+dx)+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tan(c+dx)}{15a^2d(a+b)^2\left(a+b\tan^2(c+dx)+b\right)^{3/2}}-\frac{\quad}{5ad(a}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]]/(a^(7/2)*d) -
 (b*Tan[c + d*x])/(5*a*(a + b)*d*(a + b + b*Tan[c + d*x]^2)^(5/2)) - (b*(9*
a + 5*b)*Tan[c + d*x])/(15*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^(3/2)
) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tan[c + d*x])/(15*a^3*(a + b)^3*d*Sqrt[a
+ b + b*Tan[c + d*x]^2])
```

Rubi [A] time = 0.193807, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 203}

$$-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{15a^3d(a+b)^3\sqrt{a+b\tan^2(c+dx)+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{a^{7/2}d}-\frac{b(9a+5b)\tan(c+dx)}{15a^2d(a+b)^2\left(a+b\tan^2(c+dx)+b\right)^{3/2}}-\frac{\quad}{5ad(a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]]/(a^(7/2)*d) -
 (b*Tan[c + d*x])/(5*a*(a + b)*d*(a + b + b*Tan[c + d*x]^2)^(5/2)) - (b*(9*
a + 5*b)*Tan[c + d*x])/(15*a^2*(a + b)^2*d*(a + b + b*Tan[c + d*x]^2)^(3/2)
) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tan[c + d*x])/(15*a^3*(a + b)^3*d*Sqrt[a
+ b + b*Tan[c + d*x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{7/2}}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{5a+b-4bx^2}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(c+dx)\right)}{5a(a+b)d}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} + \text{Su}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} - \frac{b}{15a}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} - \frac{b}{15a}$$

$$= -\frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^{3/2}} - \frac{b}{15a}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tan(c+dx)}{5a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^{5/2}} - \frac{b(9a+5b)\,\text{t}}{15a^2(a+b)^2d\left(a+b\right.}$$

Mathematica [C] time = 18.7578, size = 1777, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)]*Cos[c + d*x]^6*Sin[c + d*x])/(8*Sqrt[2]*d*(a + b*Sec[c + d*x]^2)^(7
/2)*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2,
 Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2,
```

```
5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2,
-2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)*
((21*a*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^
2)/(a + b)]*Cos[c + d*x]^7*Sin[c + d*x]^2)/(8*Sqrt[2]*(a + b - a*Sin[c + d*
x]^2)^(9/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
 + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*S
in[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x
]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)*AppellF1[1/
2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^7
)/(8*Sqrt[2]*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3,
7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2,
-3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*Appel
lF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c +
 d*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
 + d*x]^2)/(a + b)]*Cos[c + d*x]^5*Sin[c + d*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2,
(a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]
^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin
[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)*Cos
[c + d*x]^6*Sin[c + d*x]*((7*a*d*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2
, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x])/(3*(a + b)) - 2*d*
AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos
[c + d*x]*Sin[c + d*x]))/(8*Sqrt[2]*d*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(
a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c +
d*x]^2)/(a + b)])*Sin[c + d*x]^2)) - (3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2,
 Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^6*Sin[c + d*x]*(2
*d*(7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
 b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x
]^2)/(a + b)])*Cos[c + d*x]*Sin[c + d*x] + 3*(a + b)*((7*a*d*AppellF1[3/2,
-3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[
c + d*x])/(3*(a + b)) - 2*d*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*
Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x]) + Sin[c + d*x]^2*(7*a*(
(27*a*d*AppellF1[5/2, -3, 11/2, 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a
+ b)]*Cos[c + d*x]*Sin[c + d*x])/(5*(a + b)) - (18*d*AppellF1[5/2, -2, 9/2,
 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x]
)/5) - 6*(a + b)*((21*a*d*AppellF1[5/2, -2, 9/2, 7/2, Sin[c + d*x]^2, (a*Si
n[c + d*x]^2)/(a + b)]*Cos[c + d*x]*Sin[c + d*x])/(5*(a + b)) - (12*d*Appel
lF1[5/2, -1, 7/2, 7/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c +
d*x]*Sin[c + d*x])/5))))/(8*Sqrt[2]*d*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(
a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a +
b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(
a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c +
d*x]^2)/(a + b)])*Sin[c + d*x]^2)^2)))
```

Maple [C] time = 0.862, size = 6116, normalized size = 34.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)^2)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 7.40266, size = 2795, normalized size = 15.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 + a^3*b^3
 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*
cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*x + c)^2
)*sqrt(-a)*log(128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 +
32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*
b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^
2 + 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3 -
 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(d
```

```
*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))
 + 8*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*cos(d*x + c)^5 + (75*a^4*b^2 + 9
4*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (33*a^3*b^3 + 40*a^2*b^4 + 15*a*b^
5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c))/
((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8
*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a
^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4
*b^6)*d), -1/60*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 +
 a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a
^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*
x + c)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(a^2 - a*b)*cos(d*x
+ c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)^2 +
 b)/cos(d*x + c)^2)/((2*a^3*cos(d*x + c)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(d*x + c)^2)*sin(d*x + c))) + 4*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*
cos(d*x + c)^5 + (75*a^4*b^2 + 94*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (3
3*a^3*b^3 + 40*a^2*b^4 + 15*a*b^5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b
)/cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*c
os(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^
4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b
^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)^2+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c)^2 + a)^(-7/2), x)
```

3.297 $\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

```
[Out] ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Rubi [A] time = 0.0197271, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4128, 377, 203}

$$\tan^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sec[x]^2],x]
```

```
[Out] ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{2+x^2}}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\\ &= \tan^{-1}\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.025263, size = 37, normalized size = 2.64

$$\frac{\sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)\sqrt{\cos(2x)+3}\sec(x)}{\sqrt{2}\sqrt{\sec^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sec[x]^2],x]
```

```
[Out] (ArcSin[Sin[x]/Sqrt[2]]*Sqrt[3 + Cos[2*x]]*Sec[x])/(Sqrt[2]*Sqrt[1 + Sec[x]
^2])
```

Maple [C] time = 0.227, size = 142, normalized size = 10.1

$$\frac{\left(-\frac{1}{2}+\frac{i}{2}\right)(\sin(x))^2}{\cos(x)(-1+\cos(x))}\sqrt{\frac{i\cos(x)+1-i+\cos(x)}{\cos(x)+1}}\sqrt{-\frac{i\cos(x)-1-i-\cos(x)}{\cos(x)+1}}\left(2\,(-1)^{3/4}\,\text{EllipticPi}\left(\frac{\sqrt[4]{-1}\,(-1+\cos(x))}{\sin(x)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sec(x)^2)^(1/2),x)
```

```
[Out] (-1/2+1/2*I)*sin(x)^2*((I*cos(x)+1-I+cos(x))/(cos(x)+1))^(1/2)*(-(I*cos(x)-
1-I-cos(x))/(cos(x)+1))^(1/2)*(2*(-1)^(3/4)*EllipticPi((-1)^(1/4)*(-1+cos(x
))/sin(x),I,I)-2*(-1)^(1/4)*EllipticPi((-1)^(1/4)*(-1+cos(x))/sin(x),I,I)+2
^(1/2)*EllipticF((1/2+1/2*I)*2^(1/2)*(-1+cos(x))/sin(x),I))/((cos(x)^2+1)/c
os(x)^2)^(1/2)/cos(x)/(-1+cos(x))
```

Maxima [B] time = 1.78543, size = 524, normalized size = 37.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*
sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(
4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(
4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 1/2*arctan2(2*(2*(6*c
os(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*
x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*
x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6*cos(2*x)
 + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin(4*x)*sin(
2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x) + 6*
sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6)
```

Fricas [B] time = 0.509012, size = 177, normalized size = 12.64

$$\frac{1}{2}\,\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos\left(x\right)^3\sin\left(x\right)+\cos\left(x\right)\sin\left(x\right)}{\cos\left(x\right)^4+\cos\left(x\right)^2-1}\right)-\frac{1}{2}\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))/
(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec^2(x) + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sec(x)**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(x)^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sec(x)^2 + 1), x)
```

3.298 $\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=111

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^m\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m}{2};\frac{1}{2},-p;\frac{m+2}{2};\sec^2(e+fx),-\right.}{fm}$$

```
[Out] (AppellF1[m/2, 1/2, -p, (2 + m)/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]
*Cot[e + f*x]*(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p*Sqrt[-Tan[e + f*x
]^2])/(f*m*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

Rubi [F] time = 0.0467519, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p, x]
```

Rubi steps

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$

Mathematica [B] time = 18.4312, size = 2195, normalized size = 19.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(d*Sec[e + f*x])^m*(Sec[e
```

```
+ f*x]^2)^(-1 + m/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a +
b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a +
b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(m/2 + p))/(3*(a
+ b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*
(a + b)*p*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-1
 + m/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 1 - m/2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF
1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1 + m/2 + p)*Appel
lF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*
x]^2)/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 -
 m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m
/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b
)) - (2*(1 - m/2)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan
[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1
[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF
1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a
 + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-1 + m/2 + p)*Tan[e + f*x]
*(2*(2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3
*(a + b)*((2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*(1 -
 m/2)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2*b*p*((-6*b*(
1 - p)*AppellF1[5/2, 1 - m/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (6*(1 - m/2)*Appe
llF1[5/2, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
```

```
))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + (a + b)*(-2 + m)*((6*b*p*AppellF1[5/2
, 2 - m/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[
e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (6*(2 - m/2)*AppellF1[5/2, 3 - m/2,
-p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*Appell
F1[3/2, 2 - m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*
Tan[e + f*x]^2)^2))
```

Maple [F] time = 1.051, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{m}\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\left(d\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \left(d \sec \left(fx+e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

3.299 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p+2,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)\right.\right.}{f}$$

```
[Out] (AppellF1[1/2, 2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))
^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.22342, antiderivative size = 124, normalized size of antiderivative = 1.2, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right)^pF_1\left(\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e +
 f*x]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+\frac{b}{1-x^2}\right)^p}{\left(1-x^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int \left(1-x^2\right.\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int \left(1-x^2\right.\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b-a\sin^2(e+\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right.}{}$$

Mathematica [B] time = 17.3968, size = 1989, normalized size = 19.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]^3*(Sec[e + f*x]^
2)^(1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[
1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*
p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*
x)])^p*(Sec[e + f*x]^2)^(3/2 + p))/(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2
, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*App
ellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*T
an[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(
Sec[e + f*x]^2)^(1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF
1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*
b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(1/2 + p)*AppellF1[1/2, -
1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*
Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*A
ppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2,
 -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
f*x]^2*Tan[e + f*x])/(3*(a + b)) + (AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a
 + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2
, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b +
 a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p)*Tan[e + f*x]*(2*(2*b*p*Ap
pellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
```

```
)] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2
, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(3*(a + b)) + (AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan
[e + f*x]^2*(2*b*p*((-6*b*(1 - p)*AppellF1[5/2, -1/2, 2 - p, 7/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a +
 b)) + (3*AppellF1[5/2, 1/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + (a + b)*((6*b*p*AppellF1[5/
2, 1/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b)) - (3*AppellF1[5/2, 3/2, -p, 7/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/
(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{3}\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

3.300 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p+1,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)+\right.\right.}{f}$$

```
[Out] (AppellF1[1/2, 1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))
^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.17425, antiderivative size = 124, normalized size of antiderivative = 1.2, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right)^pF_1\left(\frac{1}{2}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
(Cos[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e +
 f*x]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+\frac{b}{1-x^2}\right)^p}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int\left(1-x^2\right.\right.}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int\left(1-x^2\right.\right.}{f}$$

$$=\frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b-a\sin^2(e\right.\right.}{}$$

$$=\frac{F_1\left(\frac{1}{2};1+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right.}{}$$

Mathematica [B] time = 16.8257, size = 1995, normalized size = 19.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]*(Sec[e + f*x]^2)^
(-1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1[1/
2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*A
ppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^
p*(Sec[e + f*x]^2)^(1/2 + p))/(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1/2, 1 - p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*AppellF1[3
/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e +
f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e +
f*x]^2)^(-1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2,
1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*Appe
llF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
- (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1/2 + p)*AppellF1[1/2, 1/2, -p,
3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e
+ f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[
1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p
*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)]
)^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 1/2, 1 -
 p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/(3*(a + b)) - (AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appe
llF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, 1/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-1/2 + p)*Tan[e + f*x]*(2*(2*b*p*AppellF1[3/2,
1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)*
```

```
AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
)*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1/2, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e
+ f*x])/(3*(a + b)) - (AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) + Tan[e + f*x]^2*(2
*b*p*((-6*b*(1 - p)*AppellF1[5/2, 1/2, 2 - p, 7/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (3*Appel
lF1[5/2, 3/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*S
ec[e + f*x]^2*Tan[e + f*x])/5) - (a + b)*((6*b*p*AppellF1[5/2, 3/2, 1 - p,
7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e +
 f*x])/(5*(a + b)) - (9*AppellF1[5/2, 5/2, -p, 7/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*Appe
llF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F] time = 0.395, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

3.301 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)+a\right.\right.}{f}$$

```
[Out] (AppellF1[1/2, p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*(Cos
[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2))^p)/
(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.120728, antiderivative size = 122, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right)^pF_1\left(\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*(Cos
[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e + f*x
]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(a+\frac{b}{1-x^2}\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int\left(1-x^2\right)\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int\left(1-x^2\right)\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b-a\sin^2(e+\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a\right.}{f}$$

Mathematica [B] time = 16.7777, size = 1983, normalized size = 19.63

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*(
a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 3/2, -p,
 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/
2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a
+ b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[
e + f*x]^2)^(-1/2 + p))/(-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2
, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*
x]^2)^(-3/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*AppellF1[1/2, 3
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*Appe
llF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-3/2 + p)*AppellF1[1/2, 3/2, -p
, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]^2)/(-3*(a + b)*Appell
F1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2
*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 3/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]))/(-3*(a + b)*A
ppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*AppellF1[1/2, 3/2,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[
2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)*Tan[e + f*x]*(2*(-2*b*p*AppellF
1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3
```

```
*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3/2, 3/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) + Tan[e + f*x]
^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 3/2, 2 - p, 7/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (9
*AppellF1[5/2, 5/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + 3*(a + b)*((6*b*p*AppellF1[5/2, 5/2,
 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2
*Tan[e + f*x])/(5*(a + b)) - 3*AppellF1[5/2, 7/2, -p, 7/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]))))/(-3*(a + b)
*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F] time = 0.512, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

3.302 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p-1,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)+\right.\right.}{f}$$

```
[Out] (AppellF1[1/2, -1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
)^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.161056, antiderivative size = 124, normalized size of antiderivative = 1.2, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right)^pF_1\left(\frac{1}{2}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*(Cos[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e
+ f*x]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p
)
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+\frac{b}{1-x^2}\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int\left(1-x\right.\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int\left(1-x\right.\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b-a\sin^2(e\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};-1+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\cos^2(e+f\right.}{}$$

Mathematica [B] time = 17.1389, size = 1987, normalized size = 19.29

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*(
a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 5/2, -p,
 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/
2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a
+ b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[
e + f*x]^2)^(-3/2 + p))/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2
, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*
x]^2)^(-5/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*AppellF1[1/2, 5
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*Appe
llF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
+ 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-5/2 + p)*AppellF1[1/2, 5/2, -p
, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]^2)/(-3*(a + b)*Appell
F1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2
*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]*((2*b*p*AppellF1[3/2, 5/
2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/(3*(a + b)) - (5*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(-3*(a
+ b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a + b)*AppellF1[1/2
, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b +
a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p)*Tan[e + f*x]*(2*(-2*b*p*A
ppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
```

```
)] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2*b*p*AppellF1[3
/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e
+ f*x]^2*Tan[e + f*x])/(3*(a + b)) - (5*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/3) +
Tan[e + f*x]^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 5/2, 2 - p, 7/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(
a + b)) - 3*AppellF1[5/2, 7/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) + 5*(a + b)*((6*b*p*AppellF1[5
/2, 7/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e
+ f*x]^2*Tan[e + f*x])/(5*(a + b)) - (21*AppellF1[5/2, 9/2, -p, 7/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5)))
)/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F] time = 1.056, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{3} \left(a+b \left(\sec \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \cos \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

3.303 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};p-2,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\left(\sec^2(e+fx)\left(-a\sin^2(e+fx)\right.\right.}{f}$$

```
[Out] (AppellF1[1/2, -2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*(Cos[e + f*x]^2)^p*Sin[e + f*x]*(Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)
)^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.180578, antiderivative size = 124, normalized size of antiderivative = 1.2, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4148, 6722, 1974, 430, 429}

$$\frac{\sin(e+fx)\cos^2(e+fx)^p\left(-a\sin^2(e+fx)+a+b\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\left(a\cos^2(e+fx)+b\right)^{-p}\left(a+b\sec^2(e+fx)\right)^pF_1\left(\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*(Cos[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(a + b - a*Sin[e
+ f*x]^2)^p)/(f*(b + a*Cos[e + f*x]^2)^p*(1 - (a*Sin[e + f*x]^2)/(a + b))^p
)
```

Rule 4148

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
```

```
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+\frac{b}{1-x^2}\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int \left(1-x^2\right.\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\right)\text{Subst}\left(\int \left(1-x^2\right.\right.}{f}$$

$$= \frac{\left(\cos^2(e+fx)^p\left(b+a\cos^2(e+fx)\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(a+b-a\sin^2(e+\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};-2+p,-p;\frac{3}{2};\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\left(b+a\cos^2(e+fx\right.}{}$$

Mathematica [B] time = 17.5867, size = 1997, normalized size = 19.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*Cos[e + f*x]^4*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-7/2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF
1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*
b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2,
 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a +
b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))])*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 +
 p)*(Sec[e + f*x]^2)^(-7/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*
AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-7/2 + p)*Appell
F1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a +
2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*x]^2)/(-3
*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*(a + b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*x]*((2*b*p*A
ppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (7*AppellF1[3/2, 9/2, -p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/3))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (3*(a +
b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)*Tan[e + f*
x]*(2*(-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
```

```
f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] - 3*(a + b)*((2
*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (7*AppellF1[3/2, 9/2, -
p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[
e + f*x])/3) + Tan[e + f*x]^2*(-2*b*p*((-6*b*(1 - p)*AppellF1[5/2, 7/2, 2 -
 p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan
[e + f*x])/(5*(a + b)) - (21*AppellF1[5/2, 9/2, 1 - p, 7/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) + 7*(a + b
)*((6*b*p*AppellF1[5/2, 9/2, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (27*AppellF1[5/2,
11/2, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]
^2*Tan[e + f*x])/5))))/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2,
 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2)^2))
```

Maple [F] time = 0.939, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{5} \left(a+b \left(\sec \left(fx+e\right)\right)^{2}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \cos \left(fx+e\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

3.304 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=216

$$\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] -(((3*a - 2*b*(2 + p))*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b^
2*f*(3 + 2*p)*(5 + 2*p))) + (Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e +
 f*x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*
p + p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Ta
n[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b
*Tan[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.226009, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4146, 416, 388, 246, 245}

$$\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+f}{a+b}\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] -(((3*a - 2*b*(2 + p))*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b^
2*f*(3 + 2*p)*(5 + 2*p))) + (Sec[e + f*x]^2*Tan[e + f*x]*(a + b + b*Tan[e +
 f*x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*
p + p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Ta
n[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b
*Tan[e + f*x]^2)/(a + b))^p)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)^2\left(a+b+bx^2\right)^p\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{\text{Subst}\left(\int\left(a+b+bx\right.\right.}{}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e+fx)\tan}{}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e+fx)\tan}{}$$

$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\sec^2(e+fx)\tan}{}$$

Mathematica [A] time = 2.05395, size = 149, normalized size = 0.69

$$\frac{\tan(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left(3\tan^4(e+fx)\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+10\,\text{t}\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*(15*Hypergeometric2F1[1/2, -p, 3/2,
-((b*Tan[e + f*x]^2)/(a + b))] + 10*Hypergeometric2F1[3/2, -p, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/2, -p, 7/2,
-((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4))/(15*f*(1 + (b*Tan[e + f*x]^
2)/(a + b))^p)
```

Maple [F] time = 0.503, size = 0, normalized size = 0.

$$\int\left(\sec\left(fx+e\right)\right)^6\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}\sec \left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(fx+e\right)^{2}+a\right)^{p}\sec \left(fx+e\right)^{6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

3.305 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=129

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{bf(2p+3)}-\frac{(a-2b(p+1))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hyp}}{bf(2p+3)}$$

```
[Out] (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a - 2
*b*(1 + p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f
*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.0992134, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4146, 388, 246, 245}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{bf(2p+3)}-\frac{(a-2b(p+1))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] (Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a - 2
*b*(1 + p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f
*x]^2)/(a + b))^p)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+b+bx^2\right)^p\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-2b(1+p))\,\text{Subst}\left(\int \left(a+b+\right.\right.}{bf(3+2p)}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{\left((a-2b(1+p))\left(a+b+b\tan^2(e\right.\right.}{}$$

$$= \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-2b(1+p))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan}{a}\right.}{}$$

Mathematica [A] time = 2.26217, size = 126, normalized size = 0.98

$$\frac{\tan(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p\left((2b(p+1)-a)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+\left(a\right.\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*((-a + 2*b*(1 + p))*Hypergeometric2F
1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b + b*Tan[e + f*x]^2)
*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2
)/(a + b))^p)
```

Maple [F] time = 0.426, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{4} \left(a+b \left(\sec \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \sec \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \sec \left(fx+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

3.306 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=72

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.0641613, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 246, 245}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x
]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(a+b+bx^2\right)^p\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^2}{a+b}\right)^p\,dx, x, \tan\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e\right.}{a+b}}{f}$$

Mathematica [A] time = 1.03902, size = 71, normalized size = 0.99

$$\frac{\tan(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\left(a+b\sec^2(e+fx)\right)^p \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b*Sec[
e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^2\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \sec \left(fx+e\right)^{2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \sec \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

3.307 $\int \left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rubi [A] time = 0.0621683, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\,\text{ta}}{}\right.}{f}$$

Mathematica [B] time = 6.26618, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
```

```
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \left(a + b\left(\sec\left(fx + e\right)\right)^{2}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx + e\right)^{2} + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx + e\right)^{2} + a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^{2}\left(e + fx\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.308 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rubi [A] time = 0.0820397, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{\left(1+x^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{\left(1+x^2\right)^2}\,dx, x, \tan(\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+\right.}{f}$$

Mathematica [B] time = 15.8679, size = 1914, normalized size = 23.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-
2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, 2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF
1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(
a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
 f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e +
```

```
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*a*
(a + b)*p*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-2 + p)*
Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/
2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2) + (6*(a + b)*(-2 + p)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)
^(-2 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3,
 -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)
+ (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan
[e + f*x]*((2*b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[
3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*
x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2
, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (
3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*Tan[e +
 f*x]*(4*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p
*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (4*AppellF1[3/2, 3, -p, 5/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
)/3) + 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 2, 2 - p, 7/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b)) - (12*AppellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 2*(a + b)*((6*b*p*App
ellF1[5/2, 3, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*S
ec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (18*AppellF1[5/2, 4, -p, 7/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5
))))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F] time = 0.663, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^{2} \left(a+b \left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec\left(fx+e\right)^{2}+a\right)^{p} \cos\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec\left(fx+e\right)^{2}+a\right)^{p} \cos\left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

3.309 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] (AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rubi [A] time = 0.0808565, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] (AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{\left(1+x^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{\left(1+x^2\right)^3}\,dx,x,\tan(e$$

$$= \frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+f\right.}{f}$$

Mathematica [B] time = 16.4021, size = 1912, normalized size = 23.04

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]^3*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-3 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2,
 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Appel
lF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3
*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e
 + f*x]^2)^(-2 + p))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e
```

```
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*
a*(a + b)*p*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-3 + p
)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[
3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) + (6*(a + b)*(-3 + p)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-3 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2,
4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2
) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p)*T
an[e + f*x]*((2*b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - 2*AppellF1
[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f
*x]^2*Tan[e + f*x]))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (3*
(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
+ b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3 + p)*Tan[e + f
*x]*(4*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*T
an[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*A
ppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - 2*AppellF1[3/2, 4, -p, 5/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]) +
 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 3, 2 - p, 7/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a
+ b)) - (18*AppellF1[5/2, 4, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 3*(a + b)*((6*b*p*AppellF1[
5/2, 4, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
 f*x]^2*Tan[e + f*x])/(5*(a + b)) - (24*AppellF1[5/2, 5, -p, 7/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(
3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(
a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F] time = 0.73, size = 0, normalized size = 0.

$$\int \left(\cos \left(f x+e\right)\right)^{4} \left(a+b \left(\sec \left(f x+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(b \sec \left(f x + e\right)^{2} + a\right)}^{p} \cos \left(f x + e\right)^{4}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(b \sec \left(f x + e\right)^{2} + a\right)}^{p} \cos \left(f x + e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}\cos \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

3.310 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] (AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rubi [A] time = 0.0800884, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4146, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] (AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```


Rule 4146

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{\left(1+x^2\right)^4}\,dx, x, \tan(\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};4,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+\right.}{f}$$

Mathematica [B] time = 17.2577, size = 1914, normalized size = 23.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))]*Cos[e + f*x]^5*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-4 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2,
 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*Appel
lF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4
*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x
]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e
 + f*x]^2)^(-3 + p))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e
```

```
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p,
5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*
a*(a + b)*p*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-4 + p
)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p,
 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[
3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) + (6*(a + b)*(-4 + p)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((
b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^
2)^(-4 + p)*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2,
 -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2,
5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2
) + (3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*T
an[e + f*x]*((2*b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (8*AppellF
1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e +
f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]
^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5,
-p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) -
 (3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-4 + p)*Tan[e
 + f*x]*(4*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b
*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
b))]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (8*AppellF1[3/2, 5, -p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*b*(1 - p)*AppellF1[5/2, 4, 2 - p, 7/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
)/(5*(a + b)) - (24*AppellF1[5/2, 5, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])/5) - 4*(a + b)*((6*b*p*A
ppellF1[5/2, 5, 1 - p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - 6*AppellF1[5/2, 6, -p, 7/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x])))
)/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan
[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^
2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)^2))
```

Maple [F] time = 0.758, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{6} \left(a+b \left(\sec \left(fx+e\right)\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \cos \left(fx+e\right)^{6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(fx+e\right)^{2}+a\right)^{p} \cos \left(fx+e\right)^{6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

3.311 $\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{(a-2b)\sec^4(e+fx)}{4f}-\frac{(2a-b)\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^6(e+fx)}{6f}$$

```
[Out] -((a*Log[Cos[e + f*x]])/f) - ((2*a - b)*Sec[e + f*x]^2)/(2*f) + ((a - 2*b)*
Sec[e + f*x]^4)/(4*f) + (b*Sec[e + f*x]^6)/(6*f)
```

Rubi [A] time = 0.0619932, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 76}

$$\frac{(a-2b)\sec^4(e+fx)}{4f}-\frac{(2a-b)\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) - ((2*a - b)*Sec[e + f*x]^2)/(2*f) + ((a - 2*b)*
Sec[e + f*x]^4)/(4*f) + (b*Sec[e + f*x]^6)/(6*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^2\right)}{x^7}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)^2(b+ax)}{x^4}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^4}+\frac{a-2b}{x^3}+\frac{-2a+b}{x^2}+\frac{a}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{a\log(\cos(e+fx))}{f}-\frac{(2a-b)\sec^2(e+fx)}{2f}+\frac{(a-2b)\sec^4(e+fx)}{4f}+\frac{b\sec^6(e
\end{aligned}
$$

Mathematica [A] time = 0.177252, size = 55, normalized size = 0.76

$$\frac{b\tan^6(e+fx)}{6f}-\frac{a\left(-\tan^4(e+fx)+2\tan^2(e+fx)+4\log(\cos(e+fx))\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

```
[Out] (b*Tan[e + f*x]^6)/(6*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan
[e + f*x]^4))/(4*f)
```

Maple [A] time = 0.051, size = 65, normalized size = 0.9

$$\frac{\left(\tan\left(fx+e\right)\right)^4 a}{4f}-\frac{\left(\tan\left(fx+e\right)\right)^2 a}{2f}-\frac{a\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{b\left(\sin\left(fx+e\right)\right)^6}{6f\left(\cos\left(fx+e\right)\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x)
```

```
[Out] 1/4/f*tan(f*x+e)^4*a-1/2/f*a*tan(f*x+e)^2-a*ln(cos(f*x+e))/f+1/6/f*b*sin(f*
x+e)^6/cos(f*x+e)^6
```

Maxima [A] time = 0.999829, size = 128, normalized size = 1.78

$$-\frac{6\,a\log\left(\sin\left(fx+e\right)^{2}-1\right)-\frac{6\,(2\,a-b)\sin\left(fx+e\right)^{4}-3\,(7\,a-2\,b)\sin\left(fx+e\right)^{2}+9\,a-2\,b}{\sin\left(fx+e\right)^{6}-3\,\sin\left(fx+e\right)^{4}+3\,\sin\left(fx+e\right)^{2}-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*a*log(sin(f*x + e)^2 - 1) - (6*(2*a - b)*sin(f*x + e)^4 - 3*(7*a -
 2*b)*sin(f*x + e)^2 + 9*a - 2*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*si
n(f*x + e)^2 - 1))/f
```

Fricas [A] time = 0.541261, size = 177, normalized size = 2.46

$$-\frac{12\,a\cos\left(fx+e\right)^{6}\log\left(-\cos\left(fx+e\right)\right)+6\,(2\,a-b)\cos\left(fx+e\right)^{4}-3\,(a-2\,b)\cos\left(fx+e\right)^{2}-2\,b}{12\,f\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(2*a - b)*cos(f*x + e)^4
- 3*(a - 2*b)*cos(f*x + e)^2 - 2*b)/(f*cos(f*x + e)^6)
```

Sympy [A] time = 8.28924, size = 116, normalized size = 1.61

$$\begin{cases}\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{4}\left(e+fx\right)}{4f}-\frac{a\tan^{2}\left(e+fx\right)}{2f}+\frac{b\tan^{4}\left(e+fx\right)\sec^{2}\left(e+fx\right)}{6f}-\frac{b\tan^{2}\left(e+fx\right)\sec^{2}\left(e+fx\right)}{6f}+\frac{b\sec^{2}\left(e+fx\right)}{6f} & \text{for}\ f\neq 0\\ x\left(a+b\sec^{2}\left(e\right)\right)\tan^{5}\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**2/(6*f) - b*tan(e +
f*x)**2*sec(e + f*x)**2/(6*f) + b*sec(e + f*x)**2/(6*f), Ne(f, 0)), (x*(a +
 b*sec(e)**2)*tan(e)**5, True))
```

Giac [B] time = 3.22155, size = 408, normalized size = 5.67

$$\frac{6\,a\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+2\right)-6\,a\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-2\right)+\frac{11\,a\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)^{3}+90\,a\left(\frac{\cos\left(fx+e\right)+}{\cos\left(fx+e\right)-}\right.}{\left(\frac{\cos\left(fx\right.}{\cos\left(fx\right.}\right.}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/12*(6*a*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 2) - 6*a*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (
cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2) + (11*a*((cos(f*x + e) + 1)/(cos(
f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^3 + 90*a*((cos(f*x +
 e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 27
6*a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
e) + 1)) + 280*a - 128*b)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 2)^3)/f
```

3.312 $\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx$

Optimal. Leaf size=49

$$\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^4(e+fx)}{4f}$$

```
[Out] (a*Log[Cos[e + f*x]])/f + ((a - b)*Sec[e + f*x]^2)/(2*f) + (b*Sec[e + f*x]^
4)/(4*f)
```

Rubi [A] time = 0.0488434, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 76}

$$\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

```
[Out] (a*Log[Cos[e + f*x]])/f + ((a - b)*Sec[e + f*x]^2)/(2*f) + (b*Sec[e + f*x]^
4)/(4*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^2)}{x^5}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)}{x^3}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a-b}{x^2}-\frac{a}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= \frac{a\log(\cos(e+fx))}{f}+\frac{(a-b)\sec^2(e+fx)}{2f}+\frac{b\sec^4(e+fx)}{4f}
\end{aligned}
$$

Mathematica [A] time = 0.080193, size = 43, normalized size = 0.88

$$
\frac{a\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}+\frac{b\tan^4(e+fx)}{4f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

```
[Out] (b*Tan[e + f*x]^4)/(4*f) + (a*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f)
```

Maple [A] time = 0.048, size = 50, normalized size = 1.

$$
\frac{\left(\tan\left(fx+e\right)\right)^2 a}{2f}+\frac{a\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{b\left(\sin\left(fx+e\right)\right)^4}{4f\left(\cos\left(fx+e\right)\right)^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x)
```

```
[Out] 1/2/f*a*tan(f*x+e)^2+a*ln(cos(f*x+e))/f+1/4/f*b*sin(f*x+e)^4/cos(f*x+e)^4
```

Maxima [A] time = 1.03266, size = 86, normalized size = 1.76

$$\frac{2\,a\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{2\,(a-b)\sin\left(fx+e\right)^2-2\,a+b}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*log(sin(f*x + e)^2 - 1) - (2*(a - b)*sin(f*x + e)^2 - 2*a + b)/(si
n(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.525979, size = 128, normalized size = 2.61

$$\frac{4\,a\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)+2\,(a-b)\cos\left(fx+e\right)^2+b}{4\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*cos(f*x + e)^4*log(-cos(f*x + e)) + 2*(a - b)*cos(f*x + e)^2 + b)/
(f*cos(f*x + e)^4)
```

Sympy [A] time = 2.35166, size = 80, normalized size = 1.63

$$\begin{cases} -\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{4f}-\frac{b\sec^2\left(e+fx\right)}{4f} & \text{for } f\neq 0 \\ x\left(a+b\sec^2\left(e\right)\right)\tan^3\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
tan(e + f*x)**2*sec(e + f*x)**2/(4*f) - b*sec(e + f*x)**2/(4*f), Ne(f, 0)),
 (x*(a + b*sec(e)**2)*tan(e)**3, True))
```

Giac [B] time = 1.83268, size = 343, normalized size = 7.

$$-\frac{2\,a\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+2\right)-2\,a\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-2\right)+\frac{3\,a\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)^{2}+20\,a\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+28\,a-16\,b}{\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+2\right)^{2}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*a*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 2) - 2*a*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (
cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2) + (3*a*((cos(f*x + e) + 1)/(cos(f
*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 20*a*((cos(f*x +
e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 28*a
- 16*b)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 2)^2)/f
```

3.313 $\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx$

Optimal. Leaf size=30

$$\frac{b\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Rubi [A] time = 0.0236449, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4138, 14}

$$\frac{b\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx=-\frac{\text{Subst}\left(\int \frac{b+ax^2}{x^3}\,dx,x,\cos(e+fx)\right)}{f}$$
$$=-\frac{\text{Subst}\left(\int \left(\frac{b}{x^3}+\frac{a}{x}\right)\,dx,x,\cos(e+fx)\right)}{f}$$
$$=-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^2(e+fx)}{2f}$$

Mathematica [A] time = 0.0194704, size = 30, normalized size = 1.

$$\frac{b\sec^2(e+fx)}{2f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

Maple [A] time = 0.018, size = 28, normalized size = 0.9

$$\frac{b\left(\sec\left(fx+e\right)\right)^2}{2f}+\frac{a\ln\left(\sec\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e),x)
```

```
[Out] 1/2*b*sec(f*x+e)^2/f+1/f*a*ln(sec(f*x+e))
```

Maxima [A] time = 0.99568, size = 45, normalized size = 1.5

$$-\frac{a\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{b}{\sin\left(fx+e\right)^2-1}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

Fricas [A] time = 0.532567, size = 93, normalized size = 3.1

$$-\frac{2\,a\cos\left(fx+e\right)^{2}\log\left(-\cos\left(fx+e\right)\right)-b}{2\,f\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cos(f*x + e)^2*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^2)
```

Sympy [A] time = 0.621305, size = 42, normalized size = 1.4

$$\begin{cases} \frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\sec^{2}\left(e+fx\right)}{2f} & \text{for}\ f\neq 0\\ x\left(a+b\sec^{2}\left(e\right)\right)\tan\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**2/(2*f), Ne(f
, 0)), (x*(a + b*sec(e)**2)*tan(e), True))
```

Giac [B] time = 1.42341, size = 275, normalized size = 9.17

$$\frac{a\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+2\right)-a\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-2\right)+\frac{a\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+2\,a-4\,b}{\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 2) - a*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f
*x + e) - 1)/(cos(f*x + e) + 1) - 2) + (a*((cos(f*x + e) + 1)/(cos(f*x + e)
 - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a - 4*b)/((cos(f*x + e)
+ 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/f
```

3.314 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a+b)\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

```
[Out] -((b*Log[Cos[e + f*x]])/f) + ((a + b)*Log[Sin[e + f*x]])/f
```

Rubi [A] time = 0.0467835, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log(\sin(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*Log[Cos[e + f*x]])/f) + ((a + b)*Log[Sin[e + f*x]])/f
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= -\frac{\text{Subst}\left(\int \frac{b+ax^2}{x(1-x^2)}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)x}\,dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{-1+x}+\frac{b}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{b\log(\cos(e+fx))}{f}+\frac{(a+b)\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A] time = 0.0305384, size = 44, normalized size = 1.57

$$\frac{a(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}-\frac{b(\log(\cos(e+fx))-\log(\sin(e+fx)))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*(Log[Cos[e + f*x]] - Log[Sin[e + f*x]]))/f) + (a*(Log[Cos[e + f*x]] +
Log[Tan[e + f*x]]))/f
```

Maple [A] time = 0.046, size = 26, normalized size = 0.9

$$\frac{b\ln\left(\tan\left(fx+e\right)\right)}{f}+\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*b*ln(tan(f*x+e))+a*ln(sin(f*x+e))/f
```

Maxima [A] time = 1.02044, size = 45, normalized size = 1.61

$$-\frac{b\log\left(\sin\left(fx+e\right)^2-1\right)-(a+b)\log\left(\sin\left(fx+e\right)^2\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*log(sin(f*x + e)^2 - 1) - (a + b)*log(sin(f*x + e)^2))/f
```

Fricas [A] time = 0.525149, size = 99, normalized size = 3.54

$$-\frac{b\log\left(\cos\left(fx+e\right)^2\right)-(a+b)\log\left(-\frac{1}{4}\,\cos\left(fx+e\right)^2+\frac{1}{4}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(cos(f*x + e)^2) - (a + b)*log(-1/4*cos(f*x + e)^2 + 1/4))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

<u>Giac</u> [B] time = 1.45604, size = 140, normalized size = 5.

$$-\frac{a \log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right) + b \log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right)}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + 2) + b*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(
f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/f
```

3.315 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=32

$$-\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] -((a + b)*Csc[e + f*x]^2)/(2*f) - (a*Log[Sin[e + f*x]])/f
```

Rubi [A] time = 0.0505279, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 444, 43}

$$-\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + b)*Csc[e + f*x]^2)/(2*f) - (a*Log[Sin[e + f*x]])/f
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= -\frac{\text{Subst}\left(\int \frac{x(b+ax^2)}{(1-x^2)^2}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{b+ax}{(1-x)^2}\,dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int\left(\frac{a+b}{(-1+x)^2}+\frac{a}{-1+x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{(a+b)\csc^2(e+fx)}{2f}-\frac{a\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A] time = 0.164611, size = 52, normalized size = 1.62

$$-\frac{a\left(\cot^2(e+fx)+2\log(\tan(e+fx))+2\log(\cos(e+fx))\right)}{2f}-\frac{b\csc^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(b*Csc[e + f*x]^2)/(2*f) - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Lo
g[Tan[e + f*x]]))/(2*f)
```

Maple [A] time = 0.054, size = 43, normalized size = 1.3

$$-\frac{\left(\cot\left(fx+e\right)\right)^2 a}{2f}-\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{b}{2f\left(\sin\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2/f*a*cot(f*x+e)^2-a*ln(sin(f*x+e))/f-1/2/f*b/sin(f*x+e)^2
```

Maxima [A] time = 0.991948, size = 39, normalized size = 1.22

$$-\frac{a\log\left(\sin\left(fx+e\right)^2\right)+\frac{a+b}{\sin\left(fx+e\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(sin(f*x + e)^2) + (a + b)/sin(f*x + e)^2)/f
```

Fricas [A] time = 0.528583, size = 116, normalized size = 3.62

$$-\frac{2\left(a\cos\left(fx+e\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)-a-b}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(f*x + e)^2 - a)*log(1/2*sin(f*x + e)) - a - b)/(f*cos(f*x +
e)^2 - f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

Giac [B] time = 1.41751, size = 204, normalized size = 6.38

$$\frac{8\,a\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)-4\,a\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+\frac{\left(a+b+\frac{4\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)\left(\cos\left(fx+e\right)+1\right)}{\cos\left(fx+e\right)-1}+\frac{a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*a*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 4*a*log(-(cos(f*
x + e) - 1)/(cos(f*x + e) + 1)) + (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1))*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) + a*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/f
```

3.316 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{(2a+b)\csc^2(e+fx)}{2f}+\frac{a\log(\sin(e+fx))}{f}$$

```
[Out] ((2*a + b)*Csc[e + f*x]^2)/(2*f) - ((a + b)*Csc[e + f*x]^4)/(4*f) + (a*Log[
Sin[e + f*x]])/f
```

Rubi [A] time = 0.0714023, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 77}

$$-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{(2a+b)\csc^2(e+fx)}{2f}+\frac{a\log(\sin(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((2*a + b)*Csc[e + f*x]^2)/(2*f) - ((a + b)*Csc[e + f*x]^4)/(4*f) + (a*Log[
Sin[e + f*x]])/f
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^3(b+ax^2)}{(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\operatorname{Subst}\left(\int \frac{x(b+ax)}{(1-x)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\operatorname{Subst}\left(\int \left(\frac{-a-b}{(-1+x)^3}+\frac{-2a-b}{(-1+x)^2}-\frac{a}{-1+x}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{(2a+b)\csc^2(e+fx)}{2f}-\frac{(a+b)\csc^4(e+fx)}{4f}+\frac{a\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A] time = 0.174829, size = 64, normalized size = 1.25

$$\frac{a\left(-\cot^4(e+fx)+2\cot^2(e+fx)+4\log(\tan(e+fx))+4\log(\cos(e+fx))\right)}{4f}-\frac{b\cot^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(b*Cot[e + f*x]^4)/(4*f) + (a*(2*Cot[e + f*x]^2 - Cot[e + f*x]^4 + 4*Log[C
os[e + f*x]] + 4*Log[Tan[e + f*x]]))/(4*f)
```

Maple [A] time = 0.051, size = 64, normalized size = 1.3

$$-\frac{a\left(\cot\left(fx+e\right)\right)^4}{4\,f}+\frac{\left(\cot\left(fx+e\right)\right)^2 a}{2\,f}+\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{b\left(\cos\left(fx+e\right)\right)^4}{4\,f\left(\sin\left(fx+e\right)\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/4/f*a*cot(f*x+e)^4+1/2/f*a*cot(f*x+e)^2+a*ln(sin(f*x+e))/f-1/4/f*b/sin(f
*x+e)^4*cos(f*x+e)^4
```

Maxima [A] time = 0.9878, size = 66, normalized size = 1.29

$$\frac{2\,a\log\left(\sin\left(fx+e\right)^2\right)+\frac{2\,(2\,a+b)\sin\left(fx+e\right)^2-a-b}{\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*log(sin(f*x + e)^2) + (2*(2*a + b)*sin(f*x + e)^2 - a - b)/sin(f*x
 + e)^4)/f
```

Fricas [A] time = 0.551675, size = 215, normalized size = 4.22

$$-\frac{2\,(2\,a+b)\cos\left(fx+e\right)^2-4\left(a\cos\left(fx+e\right)^4-2\,a\cos\left(fx+e\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)-3\,a-b}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*a + b)*cos(f*x + e)^2 - 4*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2
 + a)*log(1/2*sin(f*x + e)) - 3*a - b)/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)
^2 + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49263, size = 339, normalized size = 6.65

$$-\frac{64\,a\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)-32\,a\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+\frac{\left(a+b+\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}+\frac{12\,\cdots}{\cdots}}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*a*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 32*a*log(-(co
s(f*x + e) - 1)/(cos(f*x + e) + 1)) + (a + b + 12*a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2
 + 12*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*b*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2)/f
```

3.317 $\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx$

Optimal. Leaf size=64

$$\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^7(e+fx)}{7f}$$

```
[Out] -(a*x) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (a*Tan[e + f*x]^5)
/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.0615938, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

```
[Out] -(a*x) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*f) + (a*Tan[e + f*x]^5)
/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^6\left(a+b\left(1+x^2\right)\right)}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a-ax^2+ax^4+bx^6-\frac{a}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}-\frac{a\,\text{Subst}}{}\\
&= -ax+\frac{a\tan(e+fx)}{f}-\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^5(e+fx)}{5f}+\frac{b\tan^7(e+fx)}{7f}
\end{aligned}
$$

Mathematica [A] time = 0.0277987, size = 73, normalized size = 1.14

$$
\frac{a\tan^5(e+fx)}{5f}-\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^7(e+fx)}{7f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3*
f) + (a*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

Maple [A] time = 0.052, size = 61, normalized size = 1.

$$
\frac{1}{f}\left(a\left(\frac{\left(\tan\left(fx+e\right)\right)^5}{5}-\frac{\left(\tan\left(fx+e\right)\right)^3}{3}+\tan\left(fx+e\right)-fx-e\right)+\frac{b\left(\sin\left(fx+e\right)\right)^7}{7\left(\cos\left(fx+e\right)\right)^7}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x)
```

```
[Out] 1/f*(a*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-f*x-e)+1/7*b*sin(f*x+e
)^7/cos(f*x+e)^7)
```

Maxima [A] time = 1.49228, size = 76, normalized size = 1.19

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,a\tan\left(fx+e\right)^{5}-35\,a\tan\left(fx+e\right)^{3}-105\left(fx+e\right)a+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 10
5*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

Fricas [A] time = 0.518405, size = 231, normalized size = 3.61

$$-\frac{105\,afx\cos\left(fx+e\right)^{7}-\left((161\,a-15\,b)\cos\left(fx+e\right)^{6}-(77\,a-45\,b)\cos\left(fx+e\right)^{4}+3\,(7\,a-15\,b)\cos\left(fx+e\right)^{2}+15\,b\right.}{105\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/105*(105*a*f*x*cos(f*x + e)^7 - ((161*a - 15*b)*cos(f*x + e)^6 - (77*a -
 45*b)*cos(f*x + e)^4 + 3*(7*a - 15*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e))
/(f*cos(f*x + e)^7)
```

Sympy [A] time = 7.25981, size = 66, normalized size = 1.03

$$a\left(\begin{cases}-x+\frac{\tan^{5}\left(e+fx\right)}{5f}-\frac{\tan^{3}\left(e+fx\right)}{3f}+\frac{\tan\left(e+fx\right)}{f} & \text{for}\: f\neq 0\\x\tan^{6}\left(e\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}x\tan^{6}\left(e\right)\sec^{2}\left(e\right) & \text{for}\: f=0\\\frac{\tan^{7}\left(e+fx\right)}{7f} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**6,x)
```

```
[Out] a*Piecewise((-x + tan(e + f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e + f
*x)/f, Ne(f, 0)), (x*tan(e)**6, True)) + b*Piecewise((x*tan(e)**6*sec(e)**2
, Eq(f, 0)), (tan(e + f*x)**7/(7*f), True))
```

Giac [A] time = 4.79473, size = 82, normalized size = 1.28

$$\frac{15\,b\tan\left(fx+e\right)^{7}+21\,a\tan\left(fx+e\right)^{5}-35\,a\tan\left(fx+e\right)^{3}-105\left(fx+e\right)a+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 10
5*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

3.318 $\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}+ax+\frac{b\tan^5(e+fx)}{5f}$$

```
[Out] a*x - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^5)/(5
*f)
```

Rubi [A] time = 0.0565195, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$\frac{a\tan^3(e+fx)}{3f}-\frac{a\tan(e+fx)}{f}+ax+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

```
[Out] a*x - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f) + (b*Tan[e + f*x]^5)/(5
*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(-a+ax^2+bx^4+\frac{a}{1+x^2}\right)\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}+\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\text{ta}\right.}{f}$$

$$= ax-\frac{a\tan(e+fx)}{f}+\frac{a\tan^3(e+fx)}{3f}+\frac{b\tan^5(e+fx)}{5f}$$

Mathematica [A] time = 0.0206226, size = 57, normalized size = 1.19

$$\frac{a\tan^3(e+fx)}{3f}+\frac{a\tan^{-1}(\tan(e+fx))}{f}-\frac{a\tan(e+fx)}{f}+\frac{b\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f)
+ (b*Tan[e + f*x]^5)/(5*f)
```

Maple [A] time = 0.048, size = 50, normalized size = 1.

$$\frac{1}{f}\left(a\left(\frac{\left(\tan\left(fx+e\right)\right)^3}{3}-\tan\left(fx+e\right)+fx+e\right)+\frac{b\left(\sin\left(fx+e\right)\right)^5}{5\left(\cos\left(fx+e\right)\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x)
```

```
[Out] 1/f*(a*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e)+1/5*b*sin(f*x+e)^5/cos(f*x+e)^5)
```

Maxima [A] time = 1.47521, size = 61, normalized size = 1.27

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,a\tan\left(fx+e\right)^{3}+15\,\left(fx+e\right)a-15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(f
*x + e))/f
```

Fricas [A] time = 0.495778, size = 177, normalized size = 3.69

$$\frac{15\,afx\cos\left(fx+e\right)^{5}-\left((20\,a-3\,b)\cos\left(fx+e\right)^{4}-(5\,a-6\,b)\cos\left(fx+e\right)^{2}-3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a*f*x*cos(f*x + e)^5 - ((20*a - 3*b)*cos(f*x + e)^4 - (5*a - 6*b)*
cos(f*x + e)^2 - 3*b)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [A] time = 2.9966, size = 54, normalized size = 1.12

$$a\left(\begin{cases} x+\frac{\tan^{3}\left(e+fx\right)}{3f}-\frac{\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\tan^{4}\left(e\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases} x\tan^{4}\left(e\right)\sec^{2}\left(e\right) & \text{for}\ f=0\\ \frac{\tan^{5}\left(e+fx\right)}{5f} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**4,x)
```

```
[Out] a*Piecewise((x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f, Ne(f, 0)), (x*tan(
e)**4, True)) + b*Piecewise((x*tan(e)**4*sec(e)**2, Eq(f, 0)), (tan(e + f*x
)**5/(5*f), True))
```

Giac [A] time = 2.26045, size = 66, normalized size = 1.38

$$\frac{3\,b\tan\left(fx+e\right)^{5}+5\,a\tan\left(fx+e\right)^{3}+15\left(fx+e\right)a-15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(f
*x + e))/f
```

3.319 $\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx$

Optimal. Leaf size=32

$$\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^3(e+fx)}{3f}$$

```
[Out] -(a*x) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0520089, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$\frac{a\tan(e+fx)}{f}-ax+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

```
[Out] -(a*x) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a+bx^2-\frac{a}{1+x^2}\right)\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}-\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -ax+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.0153752, size = 41, normalized size = 1.28

$$-\frac{a\tan^{-1}(\tan(e+fx))}{f}+\frac{a\tan(e+fx)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3*
f)
```

Maple [A] time = 0.04, size = 41, normalized size = 1.3

$$\frac{1}{f}\left(a\left(\tan\left(fx+e\right)-fx-e\right)+\frac{b\left(\sin\left(fx+e\right)\right)^3}{3\left(\cos\left(fx+e\right)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x)
```

```
[Out] 1/f*(a*(tan(f*x+e)-f*x-e)+1/3*b*sin(f*x+e)^3/cos(f*x+e)^3)
```

Maxima [A] time = 1.47493, size = 45, normalized size = 1.41

$$\frac{b\tan\left(fx+e\right)^{3}-3\left(fx+e\right)a+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

Fricas [A] time = 0.486491, size = 130, normalized size = 4.06

$$-\frac{3\,afx\cos\left(fx+e\right)^{3}-\left((3\,a-b)\cos\left(fx+e\right)^{2}+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*f*x*cos(f*x + e)^3 - ((3*a - b)*cos(f*x + e)^2 + b)*sin(f*x + e))
/(f*cos(f*x + e)^3)
```

Sympy [A] time = 1.66331, size = 42, normalized size = 1.31

$$a\left(\begin{cases}-x+\frac{\tan\left(e+fx\right)}{f} & \text{for}\ f\neq 0\\ x\tan^{2}\left(e\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}x\tan^{2}\left(e\right)\sec^{2}\left(e\right) & \text{for}\ f=0\\ \frac{\tan^{3}\left(e+fx\right)}{3f} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**2,x)
```

```
[Out] a*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True)) + b*Piece
wise((x*tan(e)**2*sec(e)**2, Eq(f, 0)), (tan(e + f*x)**3/(3*f), True))
```

Giac [A] time = 1.60415, size = 49, normalized size = 1.53

$$\frac{b\tan\left(fx+e\right)^{3}-3\left(fx+e\right)a+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

3.320 $\int \left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\tan(e+fx)}{f}$$

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rubi [A] time = 0.0126401, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3767, 8}

$$ax+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)\,dx &= ax+b\int \sec^2(e+fx)\,dx\\ &= ax-\frac{b\,\text{Subst}\left(\int 1\,dx,x,-\tan(e+fx)\right)}{f}\\ &= ax+\frac{b\tan(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.0027025, size = 15, normalized size = 1.

$$ax + \frac{b\tan(e + fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[e + f*x]^2,x]
```

```
[Out] a*x + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.015, size = 16, normalized size = 1.1

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(f*x+e)^2,x)
```

```
[Out] a*x+b*tan(f*x+e)/f
```

Maxima [A] time = 0.998696, size = 20, normalized size = 1.33

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

Fricas [B] time = 0.47363, size = 76, normalized size = 5.07

$$\frac{afx\cos\left(fx + e\right) + b\sin\left(fx + e\right)}{f\cos\left(fx + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] (a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)**2,x)
```

```
[Out] Integral(a + b*sec(e + f*x)**2, x)
```

Giac [A] time = 1.26665, size = 22, normalized size = 1.47

$$ax + \frac{b\tan\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] a*x + b*tan(f*x + e)/f
```

3.321 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx$

Optimal. Leaf size=19

$$-\frac{(a+b)\cot(e+fx)}{f}-ax$$

```
[Out] -(a*x) - ((a + b)*Cot[e + f*x])/f
```

Rubi [A] time = 0.0537068, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$-\frac{(a+b)\cot(e+fx)}{f}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(a*x) - ((a + b)*Cot[e + f*x])/f
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{a+b(1+x^2)}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^2}-\frac{a}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f} \\
&= -\frac{(a+b)\cot(e+fx)}{f}-\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f} \\
&= -ax-\frac{(a+b)\cot(e+fx)}{f}
\end{aligned}
$$

Mathematica [C] time = 0.0303523, size = 43, normalized size = 2.26

$$
-\frac{a\cot(e+fx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}-\frac{b\cot(e+fx)}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((b*Cot[e + f*x])/f) - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Ta
n[e + f*x]^2])/f
```

Maple [A] time = 0.043, size = 33, normalized size = 1.7

$$
\frac{a\left(-\cot\left(fx+e\right)-fx-e\right)-b\cot\left(fx+e\right)}{f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-cot(f*x+e)-f*x-e)-b*cot(f*x+e))
```

Maxima [A] time = 1.47532, size = 34, normalized size = 1.79

$$-\frac{(fx+e)a+\frac{a+b}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -((f*x + e)*a + (a + b)/tan(f*x + e))/f
```

Fricas [A] time = 0.482401, size = 85, normalized size = 4.47

$$-\frac{afx\sin(fx+e)+(a+b)\cos(fx+e)}{f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -(a*f*x*sin(f*x + e) + (a + b)*cos(f*x + e))/(f*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2(e+fx)\right)\cot^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

Giac [B] time = 1.28856, size = 77, normalized size = 4.05

$$-\frac{2(fx+e)a-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+\frac{a+b}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)*a - a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e) + (a
+ b)/tan(1/2*f*x + 1/2*e))/f
```

3.322 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=33

$$-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{a\cot(e+fx)}{f}+ax$$

```
[Out] a*x + (a*Cot[e + f*x])/f - ((a + b)*Cot[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0585496, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{a\cot(e+fx)}{f}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] a*x + (a*Cot[e + f*x])/f - ((a + b)*Cot[e + f*x]^3)/(3*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{\text{Subst}\left(\int \frac{a+b\left(1+x^2\right)}{x^4\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int \left(\frac{a+b}{x^4}-\frac{a}{x^2}+\frac{a}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{a\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}+\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=ax+\frac{a\cot(e+fx)}{f}-\frac{(a+b)\cot^3(e+fx)}{3f}$$

Mathematica [C] time = 0.021564, size = 51, normalized size = 1.55

$$-\frac{a\cot^3(e+fx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3f}-\frac{b\cot^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(b*Cot[e + f*x]^3)/(3*f) - (a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -1
/2, -Tan[e + f*x]^2])/(3*f)
```

Maple [A] time = 0.048, size = 48, normalized size = 1.5

$$\frac{1}{f}\left(a\left(-\frac{\left(\cot\left(fx+e\right)\right)^3}{3}+\cot\left(fx+e\right)+fx+e\right)-\frac{b\left(\cos\left(fx+e\right)\right)^3}{3\left(\sin\left(fx+e\right)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-1/3*b/sin(f*x+e)^3*cos(f*x+e)^3
)
```

Maxima [A] time = 1.51366, size = 55, normalized size = 1.67

$$\frac{3\left(fx+e\right)a+\frac{3\,a\tan\left(fx+e\right)^{2}-a-b}{\tan\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a + (3*a*tan(f*x + e)^2 - a - b)/tan(f*x + e)^3)/f
```

Fricas [B] time = 0.478066, size = 185, normalized size = 5.61

$$\frac{(4\,a+b)\cos\left(fx+e\right)^{3}-3\,a\cos\left(fx+e\right)+3\left(afx\cos\left(fx+e\right)^{2}-afx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^{2}-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((4*a + b)*cos(f*x + e)^3 - 3*a*cos(f*x + e) + 3*(a*f*x*cos(f*x + e)^2
- a*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

Giac [B] time = 1.26371, size = 161, normalized size = 4.88

$$\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+24\left(fx+e\right)a-15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\frac{15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{\tan}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*f*x + 1/2*e)^3 + b*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a
- 15*a*tan(1/2*f*x + 1/2*e) - 3*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*x
+ 1/2*e)^2 + 3*b*tan(1/2*f*x + 1/2*e)^2 - a - b)/tan(1/2*f*x + 1/2*e)^3)/f
```

3.323 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

Optimal. Leaf size=51

$$-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-ax$$

```
[Out] -(a*x) - (a*Cot[e + f*x])/f + (a*Cot[e + f*x]^3)/(3*f) - ((a + b)*Cot[e + f
*x]^5)/(5*f)
```

Rubi [A] time = 0.0628051, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4141, 1802, 203}

$$-\frac{(a+b)\cot^5(e+fx)}{5f}+\frac{a\cot^3(e+fx)}{3f}-\frac{a\cot(e+fx)}{f}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(a*x) - (a*Cot[e + f*x])/f + (a*Cot[e + f*x]^3)/(3*f) - ((a + b)*Cot[e + f
*x]^5)/(5*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+b(1+x^2)}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a+b}{x^6}-\frac{a}{x^4}+\frac{a}{x^2}-\frac{a}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a\cot(e+fx)}{f}+\frac{a\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}-\frac{a\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx$$
$$= -ax-\frac{a\cot(e+fx)}{f}+\frac{a\cot^3(e+fx)}{3f}-\frac{(a+b)\cot^5(e+fx)}{5f}$$

Mathematica [C] time = 0.029903, size = 51, normalized size = 1.

$$-\frac{a\cot^5(e+fx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f}-\frac{b\cot^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(b*Cot[e + f*x]^5)/(5*f) - (a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -3
/2, -Tan[e + f*x]^2])/(5*f)
```

Maple [A] time = 0.051, size = 63, normalized size = 1.2

$$\frac{1}{f}\left(a\left(-\frac{\left(\cot\left(fx+e\right)\right)^5}{5}+\frac{\left(\cot\left(fx+e\right)\right)^3}{3}-\cot\left(fx+e\right)-fx-e\right)-\frac{b\left(\cos\left(fx+e\right)\right)^5}{5\left(\sin\left(fx+e\right)\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f*(a*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-1/5*b/sin(f*x+
e)^5*cos(f*x+e)^5)
```

Maxima [A] time = 1.56723, size = 70, normalized size = 1.37

$$-\frac{15\left(fx+e\right)a+\frac{15\,a\tan\left(fx+e\right)^4-5\,a\tan\left(fx+e\right)^2+3\,a+3\,b}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*a + (15*a*tan(f*x + e)^4 - 5*a*tan(f*x + e)^2 + 3*a + 3
*b)/tan(f*x + e)^5)/f
```

Fricas [B] time = 0.491892, size = 286, normalized size = 5.61

$$-\frac{(23\,a+3\,b)\cos\left(fx+e\right)^5-35\,a\cos\left(fx+e\right)^3+15\,a\cos\left(fx+e\right)+15\left(afx\cos\left(fx+e\right)^4-2\,afx\cos\left(fx+e\right)^2+afx\right)}{15\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/15*((23*a + 3*b)*cos(f*x + e)^5 - 35*a*cos(f*x + e)^3 + 15*a*cos(f*x + e
) + 15*(a*f*x*cos(f*x + e)^4 - 2*a*f*x*cos(f*x + e)^2 + a*f*x)*sin(f*x + e)
)/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46628, size = 246, normalized size = 4.82

$$\frac{3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-15\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-480\,\left(fx+e\right)a+330\,a\,\mathrm{t}}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 + 3*b*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(1
/2*f*x + 1/2*e)^3 - 15*b*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a + 330*a*t
an(1/2*f*x + 1/2*e) + 30*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1/2*
e)^4 + 30*b*tan(1/2*f*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 - 15*b*tan
(1/2*f*x + 1/2*e)^2 + 3*a + 3*b)/tan(1/2*f*x + 1/2*e)^5)/f
```

3.324 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan^5(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{\left(a^2-4ab+b^2\right)\sec^4(e+fx)}{4f}-\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(a-b)\sec^6(e+fx)}{3f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{b^2\sec^8(e+fx)}{8f}$$

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) - (a*(a - b)*Sec[e + f*x]^2)/f + ((a^2 - 4*a*b
 + b^2)*Sec[e + f*x]^4)/(4*f) + ((a - b)*b*Sec[e + f*x]^6)/(3*f) + (b^2*Sec
[e + f*x]^8)/(8*f)
```

Rubi [A] time = 0.10066, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{\left(a^2-4ab+b^2\right)\sec^4(e+fx)}{4f}-\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(a-b)\sec^6(e+fx)}{3f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{b^2\sec^8(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```


```
[Out] -((a^2*Log[Cos[e + f*x]])/f) - (a*(a - b)*Sec[e + f*x]^2)/f + ((a^2 - 4*a*b
 + b^2)*Sec[e + f*x]^4)/(4*f) + ((a - b)*b*Sec[e + f*x]^6)/(3*f) + (b^2*Sec
[e + f*x]^8)/(8*f)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^2\right)^2}{x^9}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)^2(b+ax)^2}{x^5}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{b^2}{x^5}+\frac{2(a-b)b}{x^4}+\frac{a^2-4ab+b^2}{x^3}-\frac{2a(a-b)}{x^2}+\frac{a^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{a^2\log(\cos(e+fx))}{f}-\frac{a(a-b)\sec^2(e+fx)}{f}+\frac{\left(a^2-4ab+b^2\right)\sec^4(e+fx)}{4f}
\end{aligned}
$$

Mathematica [A] time = 0.48123, size = 126, normalized size = 1.26

$$
-\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(-6\left(a^2-4ab+b^2\right)\sec^4(e+fx)+24a^2\log(\cos(e+fx))-8b(a-b)\sec^6(e+fx)+\right.}{6f(a\cos(2e+2fx)+a+2b)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

```
[Out] -(Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(24*a^2*Log[Cos[e + f*x]] + 24*a*
(a - b)*Sec[e + f*x]^2 - 6*(a^2 - 4*a*b + b^2)*Sec[e + f*x]^4 - 8*(a - b)*b
*Sec[e + f*x]^6 - 3*b^2*Sec[e + f*x]^8))/(6*f*(a + 2*b + a*Cos[2*e + 2*f*x]
)^2)
```

Maple [A] time = 0.061, size = 120, normalized size = 1.2

$$
\frac{\left(\tan\left(fx+e\right)\right)^4 a^2}{4f}-\frac{a^2\left(\tan\left(fx+e\right)\right)^2}{2f}-\frac{a^2\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{ab\left(\sin\left(fx+e\right)\right)^6}{3f\left(\cos\left(fx+e\right)\right)^6}+\frac{b^2\left(\sin\left(fx+e\right)\right)^6}{8f\left(\cos\left(fx+e\right)\right)^8}+\frac{b^2\left(\sin\left(\right.\right.}{24f\left(\cos\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x)
```

```
[Out] 1/4/f*tan(f*x+e)^4*a^2-1/2/f*a^2*tan(f*x+e)^2-a^2*ln(cos(f*x+e))/f+1/3/f*a*
b*sin(f*x+e)^6/cos(f*x+e)^6+1/8/f*b^2*sin(f*x+e)^6/cos(f*x+e)^8+1/24/f*b^2*
sin(f*x+e)^6/cos(f*x+e)^6
```

Maxima [A] time = 1.05593, size = 198, normalized size = 1.98

$$-\frac{12\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{24\,(a^2-ab)\sin(fx+e)^6-6\,(11\,a^2-8\,ab-b^2)\sin(fx+e)^4+4\,(15\,a^2-8\,ab-b^2)\sin(fx+e)^2-18\,a^2+8\,ab+b^2}{\sin(fx+e)^8-4\,\sin(fx+e)^6+6\,\sin(fx+e)^4-4\,\sin(fx+e)^2+1}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*a^2*log(sin(f*x + e)^2 - 1) - (24*(a^2 - a*b)*sin(f*x + e)^6 - 6*
(11*a^2 - 8*a*b - b^2)*sin(f*x + e)^4 + 4*(15*a^2 - 8*a*b - b^2)*sin(f*x +
e)^2 - 18*a^2 + 8*a*b + b^2)/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x
 + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.560644, size = 242, normalized size = 2.42

$$-\frac{24\,a^2\cos\left(fx+e\right)^8\log\left(-\cos\left(fx+e\right)\right)+24\left(a^2-ab\right)\cos\left(fx+e\right)^6-6\left(a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^4-8\left(ab-b^2\right)\cos}{24\,f\cos\left(fx+e\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(24*a^2*cos(f*x + e)^8*log(-cos(f*x + e)) + 24*(a^2 - a*b)*cos(f*x +
e)^6 - 6*(a^2 - 4*a*b + b^2)*cos(f*x + e)^4 - 8*(a*b - b^2)*cos(f*x + e)^2
- 3*b^2)/(f*cos(f*x + e)^8)
```

Sympy [A] time = 21.8335, size = 190, normalized size = 1.9

$$\begin{cases} \frac{a^2 \log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{a^2 \tan^4\left(e+fx\right)}{4f} - \frac{a^2 \tan^2\left(e+fx\right)}{2f} + \frac{ab \tan^4\left(e+fx\right) \sec^2\left(e+fx\right)}{3f} - \frac{ab \tan^2\left(e+fx\right) \sec^2\left(e+fx\right)}{3f} + \frac{ab \sec^2\left(e+fx\right)}{3f} + \frac{b^2 \tan \\ x\left(a+b\sec^2\left(e\right)\right)^2 \tan^5\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f)
 - a**2*tan(e + f*x)**2/(2*f) + a*b*tan(e + f*x)**4*sec(e + f*x)**2/(3*f) -
 a*b*tan(e + f*x)**2*sec(e + f*x)**2/(3*f) + a*b*sec(e + f*x)**2/(3*f) + b*
*2*tan(e + f*x)**4*sec(e + f*x)**4/(8*f) - b**2*tan(e + f*x)**2*sec(e + f*x
)**4/(12*f) + b**2*sec(e + f*x)**4/(24*f), Ne(f, 0)), (x*(a + b*sec(e)**2)*
*2*tan(e)**5, True))
```

Giac [B] time = 3.19647, size = 628, normalized size = 6.28

$$12\,a^2 \log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1} - \frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1} + 2\right) - 12\,a^2 \log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1} - \frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1} - 2\right) + \frac{25\,a^2\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1} + \frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)^4 + 248\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/24*(12*a^2*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 2) - 12*a^2*log(-(cos(f*x + e) + 1)/(cos(f*x + e) -
1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2) + (25*a^2*((cos(f*x + e) +
1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^4 + 248*a^2*
((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) +
 1))^3 + 984*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1
)/(cos(f*x + e) + 1))^2 + 1760*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) +
 (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 512*a*b*((cos(f*x + e) + 1)/(cos(
f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 256*b^2*((cos(f*x
+ e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 116
8*a^2 - 1024*a*b + 256*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 2)^4)/f
```

3.325 $\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(2a-b)\sec^4(e+fx)}{4f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{b^2\sec^6(e+fx)}{6f}$$

```
[Out] (a^2*Log[Cos[e + f*x]])/f + (a*(a - 2*b)*Sec[e + f*x]^2)/(2*f) + ((2*a - b)
*b*Sec[e + f*x]^4)/(4*f) + (b^2*Sec[e + f*x]^6)/(6*f)
```

Rubi [A] time = 0.0840397, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 76}

$$\frac{a^2\log(\cos(e+fx))}{f}+\frac{b(2a-b)\sec^4(e+fx)}{4f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{b^2\sec^6(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

```
[Out] (a^2*Log[Cos[e + f*x]])/f + (a*(a - 2*b)*Sec[e + f*x]^2)/(2*f) + ((2*a - b)
*b*Sec[e + f*x]^4)/(4*f) + (b^2*Sec[e + f*x]^6)/(6*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(1-x^2)\left(b+ax^2\right)^2}{x^7}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)(b+ax)^2}{x^4}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^4}+\frac{(2a-b)b}{x^3}+\frac{a(a-2b)}{x^2}-\frac{a^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{a^2\log(\cos(e+fx))}{f}+\frac{a(a-2b)\sec^2(e+fx)}{2f}+\frac{(2a-b)b\sec^4(e+fx)}{4f}+\frac{b^2 \text{s}}{}\end{aligned}$$

Mathematica [A] time = 0.277409, size = 107, normalized size = 1.39

$$\frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12a^2\log(\cos(e+fx))+3b(2a-b)\sec^4(e+fx)+6a(a-2b)\sec^2(e+fx)+2b^2\sec^{}\right.}{3f(a\cos(2e+2fx)+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

```
[Out] (Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a^2*Log[Cos[e + f*x]] + 6*a*(a
 - 2*b)*Sec[e + f*x]^2 + 3*(2*a - b)*b*Sec[e + f*x]^4 + 2*b^2*Sec[e + f*x]^
6))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2)
```

Maple [A] time = 0.056, size = 103, normalized size = 1.3

$$\frac{a^2\left(\tan\left(fx+e\right)\right)^2}{2f}+\frac{a^2\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{ab\left(\sin\left(fx+e\right)\right)^4}{2f\left(\cos\left(fx+e\right)\right)^4}+\frac{b^2\left(\sin\left(fx+e\right)\right)^4}{6f\left(\cos\left(fx+e\right)\right)^6}+\frac{b^2\left(\sin\left(fx+e\right)\right)^4}{12f\left(\cos\left(fx+e\right)\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x)
```

```
[Out] 1/2/f*a^2*tan(f*x+e)^2+a^2*ln(cos(f*x+e))/f+1/2/f*a*b*sin(f*x+e)^4/cos(f*x+
e)^4+1/6/f*b^2*sin(f*x+e)^4/cos(f*x+e)^6+1/12/f*b^2*sin(f*x+e)^4/cos(f*x+e)
^4
```

Maxima [A] time = 1.00919, size = 154, normalized size = 2.

$$\frac{6\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{6\left(a^2-2\,ab\right)\sin\left(fx+e\right)^4-3\left(4\,a^2-6\,ab-b^2\right)\sin\left(fx+e\right)^2+6\,a^2-6\,ab-b^2}{\sin\left(fx+e\right)^6-3\,\sin\left(fx+e\right)^4+3\,\sin\left(fx+e\right)^2-1}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(6*a^2*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 2*a*b)*sin(f*x + e)^4 - 3*(
4*a^2 - 6*a*b - b^2)*sin(f*x + e)^2 + 6*a^2 - 6*a*b - b^2)/(sin(f*x + e)^6
- 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/f
```

Fricas [A] time = 0.541965, size = 192, normalized size = 2.49

$$\frac{12\,a^2\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\left(a^2-2\,ab\right)\cos\left(fx+e\right)^4+3\left(2\,ab-b^2\right)\cos\left(fx+e\right)^2+2\,b^2}{12\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^2*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(a^2 - 2*a*b)*cos(f*x +
e)^4 + 3*(2*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)/(f*cos(f*x + e)^6)
```

Sympy [A] time = 7.92297, size = 128, normalized size = 1.66

$$\begin{cases} -\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{a^2\tan^2\left(e+fx\right)}{2f} + \frac{ab\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{2f} - \frac{ab\sec^2\left(e+fx\right)}{2f} + \frac{b^2\tan^2\left(e+fx\right)\sec^4\left(e+fx\right)}{6f} - \frac{b^2\sec^4\left(e+fx\right)}{12f} & \text{for } f \\ x\left(a+b\sec^2\left(e\right)\right)^2\tan^3\left(e\right) & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*f
) + a*b*tan(e + f*x)**2*sec(e + f*x)**2/(2*f) - a*b*sec(e + f*x)**2/(2*f) +
 b**2*tan(e + f*x)**2*sec(e + f*x)**4/(6*f) - b**2*sec(e + f*x)**4/(12*f),
Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e)**3, True))
```

Giac [B] time = 1.85568, size = 560, normalized size = 7.27

$$-\frac{6\,a^2\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1} - \frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1} + 2\right) - 6\,a^2\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1} - \frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1} - 2\right) + \frac{11\,a^2\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1} + \frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)^3 + 90\,a^2\left(\frac{c}{c}\right.}{}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/12*(6*a^2*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 2) - 6*a^2*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1
) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2) + (11*a^2*((cos(f*x + e) + 1
)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^3 + 90*a^2*((
cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1
))^2 + 228*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/
(cos(f*x + e) + 1)) - 96*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(
f*x + e) - 1)/(cos(f*x + e) + 1)) - 48*b^2*((cos(f*x + e) + 1)/(cos(f*x + e
) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 184*a^2 - 192*a*b + 32*b^
2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 2)^3)/f
```

3.326 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan(e+fx)\,dx$

Optimal. Leaf size=48

$$-\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) + (a*b*Sec[e + f*x]^2)/f + (b^2*Sec[e + f*x]^4
)/(4*f)
```

Rubi [A] time = 0.0419613, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$-\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

```
[Out] -((a^2*Log[Cos[e + f*x]])/f) + (a*b*Sec[e + f*x]^2)/f + (b^2*Sec[e + f*x]^4
)/(4*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^2\tan(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^5}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^3}\,dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2ab}{x^2}+\frac{a^2}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\
&= -\frac{a^2\log(\cos(e+fx))}{f}+\frac{ab\sec^2(e+fx)}{f}+\frac{b^2\sec^4(e+fx)}{4f}
\end{aligned}$$

Mathematica [A] time = 0.122499, size = 82, normalized size = 1.71

$$-\frac{\sec^4(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(4a^2\cos^4(e+fx)\log(\cos(e+fx))-4ab\cos^2(e+fx)-b^2\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

```
[Out] -(((b + a*Cos[e + f*x]^2)^2*(-b^2 - 4*a*b*Cos[e + f*x]^2 + 4*a^2*Cos[e + f*
x]^4*Log[Cos[e + f*x]])*Sec[e + f*x]^4)/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2
))
```

Maple [A] time = 0.022, size = 46, normalized size = 1.

$$\frac{b^2\left(\sec\left(fx+e\right)\right)^4}{4f}+\frac{ab\left(\sec\left(fx+e\right)\right)^2}{f}+\frac{a^2\ln\left(\sec\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x)
```

```
[Out] 1/4*b^2*sec(f*x+e)^4/f+a*b*sec(f*x+e)^2/f+1/f*a^2*ln(sec(f*x+e))
```

Maxima [A] time = 0.981485, size = 90, normalized size = 1.88

$$-\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{4\,ab\sin\left(fx+e\right)^2-4\,ab-b^2}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*log(sin(f*x + e)^2 - 1) + (4*a*b*sin(f*x + e)^2 - 4*a*b - b^2)/
(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

Fricas [A] time = 0.530731, size = 130, normalized size = 2.71

$$-\frac{4\,a^2\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)-4\,ab\cos\left(fx+e\right)^2-b^2}{4\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^2*cos(f*x + e)^4*log(-cos(f*x + e)) - 4*a*b*cos(f*x + e)^2 - b^2)
/(f*cos(f*x + e)^4)
```

Sympy [A] time = 2.19811, size = 61, normalized size = 1.27

$$\begin{cases}\frac{a^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{ab\sec^2\left(e+fx\right)}{f}+\frac{b^2\sec^4\left(e+fx\right)}{4f} & \text{for}\ f\neq 0\\ x\left(a+b\sec^2\left(e\right)\right)^2\tan\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e),x)
```

```
[Out] Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a*b*sec(e + f*x)**2/f + b*
*2*sec(e + f*x)**4/(4*f), Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e), True))
```

Giac [B] time = 1.36989, size = 486, normalized size = 10.12

$$\frac{2\,a^2\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+2\right)-2\,a^2\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-2\right)+\frac{3\,a^2\left(\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}+\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)^2+12\,a^2\left(\frac{\cos(}{\cos(}\right.}{}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/4*(2*a^2*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 2) - 2*a^2*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1)
- (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2) + (3*a^2*((cos(f*x + e) + 1)/(
cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 12*a^2*((cos
(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))
- 16*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f
*x + e) + 1)) - 8*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e
) - 1)/(cos(f*x + e) + 1)) + 12*a^2 - 32*a*b)/((cos(f*x + e) + 1)/(cos(f*x
+ e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^2)/f
```

3.327 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{(a+b)^2\log(\sin(e+fx))}{f}-\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}$$

```
[Out] -((b*(2*a + b)*Log[Cos[e + f*x]])/f) + ((a + b)^2*Log[Sin[e + f*x]])/f + (b
^2*Sec[e + f*x]^2)/(2*f)
```

Rubi [A] time = 0.0738816, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$\frac{(a+b)^2\log(\sin(e+fx))}{f}-\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((b*(2*a + b)*Log[Cos[e + f*x]])/f) + ((a + b)^2*Log[Sin[e + f*x]])/f + (b
^2*Sec[e + f*x]^2)/(2*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x^3\left(1-x^2\right)}\,dx,x,\cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)x^2}\,dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x}+\frac{b^2}{x^2}+\frac{b(2a+b)}{x}\right)dx,x,\cos^2(e+fx)\right)}{2f}\\ &= -\frac{b(2a+b)\log(\cos(e+fx))}{f}+\frac{(a+b)^2\log(\sin(e+fx))}{f}+\frac{b^2\sec^2(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.22654, size = 84, normalized size = 1.58

$$\frac{2(a\cos(e+fx)+b\sec(e+fx))^2\left(2\cos^2(e+fx)\left((a+b)^2\log(\sin(e+fx))-b(2a+b)\log(\cos(e+fx))\right)+b^2\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(b^2 + 2*Cos[e + f*x]^2*(-(b*(2*a + b)*Log[Cos[e + f*x]]) + (a + b)^2*Lo
g[Sin[e + f*x]]))*(a*Cos[e + f*x] + b*Sec[e + f*x])^2)/(f*(a + 2*b + a*Cos[
2*(e + f*x)])^2)
```

Maple [A] time = 0.057, size = 60, normalized size = 1.1

$$\frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f}+2\,\frac{ab\ln\left(\tan\left(fx+e\right)\right)}{f}+\frac{b^2}{2\,f\left(\cos\left(fx+e\right)\right)^2}+\frac{b^2\ln\left(\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] a^2*ln(sin(f*x+e))/f+2/f*a*b*ln(tan(f*x+e))+1/2/f*b^2/cos(f*x+e)^2+1/f*b^2*
ln(tan(f*x+e))
```

Maxima [A] time = 0.984089, size = 86, normalized size = 1.62

$$-\frac{(2\,ab+b^2)\log\left(\sin\left(fx+e\right)^2-1\right)-(a^2+2\,ab+b^2)\log\left(\sin\left(fx+e\right)^2\right)+\frac{b^2}{\sin\left(fx+e\right)^2-1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (a^2 + 2*a*b + b^2)*log(sin(f
*x + e)^2) + b^2/(sin(f*x + e)^2 - 1))/f
```

Fricas [A] time = 0.535818, size = 203, normalized size = 3.83

$$-\frac{(2\,ab+b^2)\cos\left(fx+e\right)^2\log\left(\cos\left(fx+e\right)^2\right)-(a^2+2\,ab+b^2)\cos\left(fx+e\right)^2\log\left(-\frac{1}{4}\cos\left(fx+e\right)^2+\frac{1}{4}\right)-b^2}{2\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((2*a*b + b^2)*cos(f*x + e)^2*log(cos(f*x + e)^2) - (a^2 + 2*a*b + b^2
)*cos(f*x + e)^2*log(-1/4*cos(f*x + e)^2 + 1/4) - b^2)/(f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x), x)
```

Giac [B] time = 1.39487, size = 358, normalized size = 6.75

$$-\frac{a^2\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)+\left(2\,ab+b^2\right)\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right)-\frac{2\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+b^2\left(\cdots\right.}{\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\cdots}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 2) + (2*a*b + b^2)*log(-(cos(f*x + e) + 1)/(cos(f*x + e
) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2) - (2*a*b*((cos(f*x + e)
 + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + b^2*((c
os(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)
) + 4*a*b - 2*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 2))/f
```

3.328 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=57

$$-\frac{\left(a^2-b^2\right)\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}$$

```
[Out] -((a + b)^2*Csc[e + f*x]^2)/(2*f) - (b^2*Log[Cos[e + f*x]])/f - ((a^2 - b^2
)*Log[Sin[e + f*x]])/f
```

Rubi [A] time = 0.0805385, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$-\frac{\left(a^2-b^2\right)\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^2(e+fx)}{2f}-\frac{b^2\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + b)^2*Csc[e + f*x]^2)/(2*f) - (b^2*Log[Cos[e + f*x]])/f - ((a^2 - b^2
)*Log[Sin[e + f*x]])/f
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\text{Subst}\left(\int \frac{\left(b+ax^2\right)^2}{x\left(1-x^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^2x}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{(-1+x)^2}+\frac{a^2-b^2}{-1+x}+\frac{b^2}{x}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+b)^2\csc^2(e+fx)}{2f} - \frac{b^2\log(\cos(e+fx))}{f} - \frac{\left(a^2-b^2\right)\log(\sin(e+fx))}{f}$$

Mathematica [A] time = 0.181189, size = 81, normalized size = 1.42

$$-\frac{2\left(a\cos^2(e+fx)+b\right)^2\left(2\left(a^2-b^2\right)\log(\sin(e+fx))+(a+b)^2\csc^2(e+fx)+2b^2\log(\cos(e+fx))\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-2*(b + a*Cos[e + f*x]^2)^2*((a + b)^2*Csc[e + f*x]^2 + 2*b^2*Log[Cos[e +
f*x]] + 2*(a^2 - b^2)*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])
^2)
```

Maple [A] time = 0.063, size = 78, normalized size = 1.4

$$-\frac{a^2\left(\cot\left(fx+e\right)\right)^2}{2f} - \frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f} - \frac{ab}{f\left(\sin\left(fx+e\right)\right)^2} - \frac{b^2}{2f\left(\sin\left(fx+e\right)\right)^2} + \frac{b^2\ln\left(\tan\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f*a^2*cot(f*x+e)^2-a^2*ln(sin(f*x+e))/f-1/f*a*b/sin(f*x+e)^2-1/2/f*b^2
/sin(f*x+e)^2+1/f*b^2*ln(tan(f*x+e))
```

Maxima [A] time = 0.983106, size = 81, normalized size = 1.42

$$-\frac{b^2 \log\left(\sin\left(fx+e\right)^2-1\right)+\left(a^2-b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a^2+2\,ab+b^2}{\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - b^2)*log(sin(f*x + e)^2) + (a^2
+ 2*a*b + b^2)/sin(f*x + e)^2)/f
```

Fricas [A] time = 0.527658, size = 231, normalized size = 4.05

$$\frac{a^2+2\,ab+b^2-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(\cos\left(fx+e\right)^2\right)-\left(\left(a^2-b^2\right)\cos\left(fx+e\right)^2-a^2+b^2\right)\log\left(-\frac{1}{4}\cos\left(fx+e\right)^2+\right.}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2 + 2*a*b + b^2 - (b^2*cos(f*x + e)^2 - b^2)*log(cos(f*x + e)^2) - (
(a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2)*log(-1/4*cos(f*x + e)^2 + 1/4))/(f*
cos(f*x + e)^2 - f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40244, size = 339, normalized size = 5.95

$$\frac{a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+4\,a^2\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f
*x + e) + 1)) + 2*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e
) - 1)/(cos(f*x + e) + 1)) + b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (
cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 4*a^2*log(-(cos(f*x + e) + 1)/(cos(
f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2) - 4*b^2*log(-(co
s(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1)
- 2))/f
```

3.329 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=51

$$\frac{a^2\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a(a+b)\csc^2(e+fx)}{f}$$

```
[Out] (a*(a + b)*Csc[e + f*x]^2)/f - ((a + b)^2*Csc[e + f*x]^4)/(4*f) + (a^2*Log[
Sin[e + f*x]])/f
```

Rubi [A] time = 0.0865154, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 444, 43}

$$\frac{a^2\log(\sin(e+fx))}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a(a+b)\csc^2(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a*(a + b)*Csc[e + f*x]^2)/f - ((a + b)^2*Csc[e + f*x]^4)/(4*f) + (a^2*Log[
Sin[e + f*x]])/f
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \frac{x\left(b+ax^2\right)^2}{\left(1-x^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{(1-x)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{(-1+x)^3}-\frac{2a(a+b)}{(-1+x)^2}-\frac{a^2}{-1+x}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{a(a+b)\csc^2(e+fx)}{f}-\frac{(a+b)^2\csc^4(e+fx)}{4f}+\frac{a^2\log(\sin(e+fx))}{f}\end{aligned}$$

Mathematica [A] time = 0.260646, size = 77, normalized size = 1.51

$$-\frac{\left(a\cos^2(e+fx)+b\right)^2\left(-4a^2\log(\sin(e+fx))+(a+b)^2\csc^4(e+fx)-4a(a+b)\csc^2(e+fx)\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(((b + a*Cos[e + f*x]^2)^2*(-4*a*(a + b)*Csc[e + f*x]^2 + (a + b)^2*Csc[e
+ f*x]^4 - 4*a^2*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2))
```

Maple [A] time = 0.06, size = 87, normalized size = 1.7

$$-\frac{a^2\left(\cot\left(fx+e\right)\right)^4}{4f}+\frac{a^2\left(\cot\left(fx+e\right)\right)^2}{2f}+\frac{a^2\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{ab\left(\cos\left(fx+e\right)\right)^4}{2f\left(\sin\left(fx+e\right)\right)^4}-\frac{b^2}{4f\left(\sin\left(fx+e\right)\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/4/f*a^2*cot(f*x+e)^4+1/2/f*a^2*cot(f*x+e)^2+a^2*ln(sin(f*x+e))/f-1/2/f*a
*b/sin(f*x+e)^4*cos(f*x+e)^4-1/4/f*b^2/sin(f*x+e)^4
```

Maxima [A] time = 1.06555, size = 82, normalized size = 1.61

$$\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2\right)+\frac{4\,\left(a^2+ab\right)\sin\left(fx+e\right)^2-a^2-2\,ab-b^2}{\sin\left(fx+e\right)^4}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*log(sin(f*x + e)^2) + (4*(a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*
b - b^2)/sin(f*x + e)^4)/f
```

Fricas [A] time = 0.520697, size = 242, normalized size = 4.75

$$-\frac{4\left(a^2+ab\right)\cos\left(fx+e\right)^2-3\,a^2-2\,ab+b^2-4\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - 3*a^2 - 2*a*b + b^2 - 4*(a^2*cos(f*x +
 e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*log(1/2*sin(f*x + e)))/(f*cos(f*x + e)^
4 - 2*f*cos(f*x + e)^2 + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39903, size = 474, normalized size = 9.29

$$-\frac{64\,a^2\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)-32\,a^2\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+\frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(f}{(\cos(fx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/64*(64*a^2*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 32*a^2*log(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 12*a^2*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b^2*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1
)^2/(cos(f*x + e) + 1)^2 + (a^2 + 2*a*b + b^2 + 12*a^2*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/(cos(f*x +
 e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2)/f
```

3.330 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan^6(e+fx)\,dx$

Optimal. Leaf size=95

$$\frac{a^2\tan^5(e+fx)}{5f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e +
 f*x]^5)/(5*f) + (b*(2*a + b)*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/
(9*f)
```

Rubi [A] time = 0.107516, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 203}

$$\frac{a^2\tan^5(e+fx)}{5f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^7(e+fx)}{7f}+\frac{b^2\tan^9(e+fx)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```


```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f - (a^2*Tan[e + f*x]^3)/(3*f) + (a^2*Tan[e +
 f*x]^5)/(5*f) + (b*(2*a + b)*Tan[e + f*x]^7)/(7*f) + (b^2*Tan[e + f*x]^9)/
(9*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b\left(1+x^2\right)\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(a^2-a^2x^2+a^2x^4+b(2a+b)x^6+b^2x^8-\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right.}{f}$$

$$= \frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+b)\tan^7(e+f}{7f}$$

$$= -a^2x+\frac{a^2\tan(e+fx)}{f}-\frac{a^2\tan^3(e+fx)}{3f}+\frac{a^2\tan^5(e+fx)}{5f}+\frac{b(2a+b)\tan}{7f}$$

Mathematica [B] time = 2.1231, size = 275, normalized size = 2.89

$$-\frac{4\sec^9(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(\left(231a^2-270ab+5b^2\right)\tan(e)\cos^7(e+fx)-3\left(21a^2-90ab+25b^2\right)\tan(e)\cos^5(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^9*(315*a^2*f*x*Cos[e + f*x]^9 - 3
5*b^2*Sec[e]*Sin[f*x] - 5*(18*a - 19*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] -
3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^4*Sec[e]*Sin[f*x] + (231*a^2 - 27
0*a*b + 5*b^2)*Cos[e + f*x]^6*Sec[e]*Sin[f*x] - (483*a^2 - 90*a*b - 10*b^2)
*Cos[e + f*x]^8*Sec[e]*Sin[f*x] - 35*b^2*Cos[e + f*x]*Tan[e] - 5*(18*a - 19
*b)*b*Cos[e + f*x]^3*Tan[e] - 3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^5*T
an[e] + (231*a^2 - 270*a*b + 5*b^2)*Cos[e + f*x]^7*Tan[e]))/(315*f*(a + 2*b
 + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.07, size = 105, normalized size = 1.1

$$\frac{1}{f}\left(a^{2}\left(\frac{\left(\tan\left(fx+e\right)\right)^{5}}{5}-\frac{\left(\tan\left(fx+e\right)\right)^{3}}{3}+\tan\left(fx+e\right)-fx-e\right)+\frac{2\,ab\left(\sin\left(fx+e\right)\right)^{7}}{7\left(\cos\left(fx+e\right)\right)^{7}}+b^{2}\left(\frac{\left(\sin\left(fx+e\right)\right)^{7}}{9\left(\cos\left(fx+e\right)\right)^{9}}+\frac{2}{63}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x)
```

```
[Out] 1/f*(a^2*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-f*x-e)+2/7*a*b*sin(f
*x+e)^7/cos(f*x+e)^7+b^2*(1/9*sin(f*x+e)^7/cos(f*x+e)^9+2/63*sin(f*x+e)^7/c
os(f*x+e)^7))
```

Maxima [A] time = 1.59323, size = 113, normalized size = 1.19

$$\frac{35\,b^{2}\tan\left(fx+e\right)^{9}+45\left(2\,ab+b^{2}\right)\tan\left(fx+e\right)^{7}+63\,a^{2}\tan\left(fx+e\right)^{5}-105\,a^{2}\tan\left(fx+e\right)^{3}-315\left(fx+e\right)a^{2}+315\,a^{2}}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b + b^2)*tan(f*x + e)^7 + 63*a^2*tan
(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 315*a^2*tan(f*x
+ e))/f
```

Fricas [A] time = 0.548164, size = 342, normalized size = 3.6

$$-\frac{315\,a^{2}fx\cos\left(fx+e\right)^{9}-\left(\left(483\,a^{2}-90\,ab-10\,b^{2}\right)\cos\left(fx+e\right)^{8}-\left(231\,a^{2}-270\,ab+5\,b^{2}\right)\cos\left(fx+e\right)^{6}+3\left(21\,a^{2}-90\right.\right.}{315\,f\cos\left(fx+e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/315*(315*a^2*f*x*cos(f*x + e)^9 - ((483*a^2 - 90*a*b - 10*b^2)*cos(f*x +
 e)^8 - (231*a^2 - 270*a*b + 5*b^2)*cos(f*x + e)^6 + 3*(21*a^2 - 90*a*b + 2
```

```
5*b^2)*cos(f*x + e)^4 + 5*(18*a*b - 19*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*
x + e))/(f*cos(f*x + e)^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec^2\left(e+fx\right)\right)^2\tan^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**6,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**6, x)
```

Giac [A] time = 4.32276, size = 132, normalized size = 1.39

$$\frac{35\,b^2\tan\left(fx+e\right)^9+90\,ab\tan\left(fx+e\right)^7+45\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5-105\,a^2\tan\left(fx+e\right)^3-315\,\left(fx\right.}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 45*b^2*tan(f*x + e)^
7 + 63*a^2*tan(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 31
5*a^2*tan(f*x + e))/f
```

3.331 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan^4(e+fx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tan^3(e+fx)}{3f}-\frac{a^2\tan(e+fx)}{f}+a^2x+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

```
[Out] a^2*x - (a^2*Tan[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) + (b*(2*a + b)*Ta
n[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.097092, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 203}

$$\frac{a^2\tan^3(e+fx)}{3f}-\frac{a^2\tan(e+fx)}{f}+a^2x+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

```
[Out] a^2*x - (a^2*Tan[e + f*x])/f + (a^2*Tan[e + f*x]^3)/(3*f) + (b*(2*a + b)*Ta
n[e + f*x]^5)/(5*f) + (b^2*Tan[e + f*x]^7)/(7*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(-a^2+a^2x^2+b(2a+b)x^4+b^2x^6+\frac{a^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+}{7f}$$

$$= a^2x-\frac{a^2\tan(e+fx)}{f}+\frac{a^2\tan^3(e+fx)}{3f}+\frac{b(2a+b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7}{7}$$

Mathematica [B] time = 1.14626, size = 395, normalized size = 5.13

$$\frac{\sec(e)\sec^7(e+fx)\left(4480a^2\sin(2e+fx)-3780a^2\sin(2e+3fx)+2100a^2\sin(4e+3fx)-1540a^2\sin(4e+5fx)+420\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

```
[Out] (Sec[e]*Sec[e + f*x]^7*(3675*a^2*f*x*Cos[f*x] + 3675*a^2*f*x*Cos[2*e + f*x]
 + 2205*a^2*f*x*Cos[2*e + 3*f*x] + 2205*a^2*f*x*Cos[4*e + 3*f*x] + 735*a^2*
f*x*Cos[4*e + 5*f*x] + 735*a^2*f*x*Cos[6*e + 5*f*x] + 105*a^2*f*x*Cos[6*e +
 7*f*x] + 105*a^2*f*x*Cos[8*e + 7*f*x] - 5320*a^2*Sin[f*x] + 1680*a*b*Sin[f
*x] + 840*b^2*Sin[f*x] + 4480*a^2*Sin[2*e + f*x] - 1260*a*b*Sin[2*e + f*x]
+ 420*b^2*Sin[2*e + f*x] - 3780*a^2*Sin[2*e + 3*f*x] + 924*a*b*Sin[2*e + 3*
f*x] - 168*b^2*Sin[2*e + 3*f*x] + 2100*a^2*Sin[4*e + 3*f*x] - 840*a*b*Sin[4
*e + 3*f*x] - 420*b^2*Sin[4*e + 3*f*x] - 1540*a^2*Sin[4*e + 5*f*x] + 168*a*
b*Sin[4*e + 5*f*x] + 84*b^2*Sin[4*e + 5*f*x] + 420*a^2*Sin[6*e + 5*f*x] - 4
20*a*b*Sin[6*e + 5*f*x] - 280*a^2*Sin[6*e + 7*f*x] + 84*a*b*Sin[6*e + 7*f*x
] + 12*b^2*Sin[6*e + 7*f*x]))/(13440*f)
```

Maple [A] time = 0.054, size = 94, normalized size = 1.2

$$\frac{1}{f}\left(a^2\left(\frac{\left(\tan\left(fx+e\right)\right)^3}{3}-\tan\left(fx+e\right)+fx+e\right)+\frac{2ab\left(\sin\left(fx+e\right)\right)^5}{5\left(\cos\left(fx+e\right)\right)^5}+b^2\left(\frac{\left(\sin\left(fx+e\right)\right)^5}{7\left(\cos\left(fx+e\right)\right)^7}+\frac{2\left(\sin\left(fx+e\right)\right)^5}{35\left(\cos\left(fx+e\right)\right)^5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x)
```

```
[Out] 1/f*(a^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e)+2/5*a*b*sin(f*x+e)^5/cos(f*x+e
)^5+b^2*(1/7*sin(f*x+e)^5/cos(f*x+e)^7+2/35*sin(f*x+e)^5/cos(f*x+e)^5))
```

Maxima [A] time = 1.46163, size = 96, normalized size = 1.25

$$\frac{15\,b^2\tan\left(fx+e\right)^7+21\left(2\,ab+b^2\right)\tan\left(fx+e\right)^5+35\,a^2\tan\left(fx+e\right)^3+105\left(fx+e\right)a^2-105\,a^2\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + b^2)*tan(f*x + e)^5 + 35*a^2*tan
(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

Fricas [A] time = 0.520851, size = 273, normalized size = 3.55

$$\frac{105\,a^2fx\cos\left(fx+e\right)^7-\left(2\left(70\,a^2-21\,ab-3\,b^2\right)\cos\left(fx+e\right)^6-\left(35\,a^2-84\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-6\left(7\,ab-4\,b^2\right)\text{co}}{105\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^2*f*x*cos(f*x + e)^7 - (2*(70*a^2 - 21*a*b - 3*b^2)*cos(f*x +
e)^6 - (35*a^2 - 84*a*b + 3*b^2)*cos(f*x + e)^4 - 6*(7*a*b - 4*b^2)*cos(f*x
 + e)^2 - 15*b^2)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^2 \tan^4\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**4, x)
```

Giac [A] time = 2.46368, size = 113, normalized size = 1.47

$$\frac{15\,b^2\tan\left(fx+e\right)^7 + 42\,ab\tan\left(fx+e\right)^5 + 21\,b^2\tan\left(fx+e\right)^5 + 35\,a^2\tan\left(fx+e\right)^3 + 105\left(fx+e\right)a^2 - 105\,a^2\tan\left(fx\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)^
5 + 35*a^2*tan(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

3.332 $\int \left(a+b\sec^2(e+fx)\right)^2 \tan^2(e+fx)\,dx$

Optimal. Leaf size=59

$$\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f + (b*(2*a + b)*Tan[e + f*x]^3)/(3*f) + (b^2
*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0926242, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 203}

$$\frac{a^2\tan(e+fx)}{f}-a^2x+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

```
[Out] -(a^2*x) + (a^2*Tan[e + f*x])/f + (b*(2*a + b)*Tan[e + f*x]^3)/(3*f) + (b^2
*Tan[e + f*x]^5)/(5*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int \left(a^2+b(2a+b)x^2+b^2x^4-\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}-\frac{a^2\,\text{Subst}\left(\int \frac{}{1}\right.}{}$$
$$=-a^2x+\frac{a^2\tan(e+fx)}{f}+\frac{b(2a+b)\tan^3(e+fx)}{3f}+\frac{b^2\tan^5(e+fx)}{5f}$$

Mathematica [B] time = 0.816515, size = 281, normalized size = 4.76

$$-\frac{\sec(e)\sec^5(e+fx)\left(120a^2\sin(2e+fx)-120a^2\sin(2e+3fx)+30a^2\sin(4e+3fx)-30a^2\sin(4e+5fx)+150a^2fx\,c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

```
[Out] -(Sec[e]*Sec[e + f*x]^5*(150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e + f*x]
+ 75*a^2*f*x*Cos[2*e + 3*f*x] + 75*a^2*f*x*Cos[4*e + 3*f*x] + 15*a^2*f*x*Co
s[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*Sin[f*x] + 80*a*b*Si
n[f*x] - 20*b^2*Sin[f*x] + 120*a^2*Sin[2*e + f*x] - 120*a*b*Sin[2*e + f*x]
- 60*b^2*Sin[2*e + f*x] - 120*a^2*Sin[2*e + 3*f*x] + 40*a*b*Sin[2*e + 3*f*x
] + 20*b^2*Sin[2*e + 3*f*x] + 30*a^2*Sin[4*e + 3*f*x] - 60*a*b*Sin[4*e + 3*
f*x] - 30*a^2*Sin[4*e + 5*f*x] + 20*a*b*Sin[4*e + 5*f*x] + 4*b^2*Sin[4*e +
5*f*x]))/(480*f)
```

Maple [A] time = 0.05, size = 85, normalized size = 1.4

$$\frac{1}{f}\left(a^2\left(\tan\left(fx+e\right)-fx-e\right)+\frac{2\left(\sin\left(fx+e\right)\right)^3ab}{3\left(\cos\left(fx+e\right)\right)^3}+b^2\left(\frac{\left(\sin\left(fx+e\right)\right)^3}{5\left(\cos\left(fx+e\right)\right)^5}+\frac{2\left(\sin\left(fx+e\right)\right)^3}{15\left(\cos\left(fx+e\right)\right)^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x)
```

```
[Out] 1/f*(a^2*(tan(f*x+e)-f*x-e)+2/3*a*b*sin(f*x+e)^3/cos(f*x+e)^3+b^2*(1/5*sin(
f*x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3))
```

Maxima [A] time = 1.49729, size = 78, normalized size = 1.32

$$\frac{3\,b^{2}\tan\left(fx+e\right)^{5}+5\left(2\,ab+b^{2}\right)\tan\left(fx+e\right)^{3}-15\left(fx+e\right)a^{2}+15\,a^{2}\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b + b^2)*tan(f*x + e)^3 - 15*(f*x + e)*
a^2 + 15*a^2*tan(f*x + e))/f
```

Fricas [A] time = 0.504371, size = 205, normalized size = 3.47

$$-\frac{15\,a^{2}fx\cos\left(fx+e\right)^{5}-\left(\left(15\,a^{2}-10\,ab-2\,b^{2}\right)\cos\left(fx+e\right)^{4}+\left(10\,ab-b^{2}\right)\cos\left(fx+e\right)^{2}+3\,b^{2}\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(15*a^2*f*x*cos(f*x + e)^5 - ((15*a^2 - 10*a*b - 2*b^2)*cos(f*x + e)^
4 + (10*a*b - b^2)*cos(f*x + e)^2 + 3*b^2)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{2}\left(e+fx\right)\right)^{2}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**2, x)
```

Giac [A] time = 1.48211, size = 95, normalized size = 1.61

$$\frac{3\,b^2\tan\left(fx+e\right)^5+10\,ab\tan\left(fx+e\right)^3+5\,b^2\tan\left(fx+e\right)^3-15\left(fx+e\right)a^2+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 5*b^2*tan(f*x + e)^3 -
 15*(f*x + e)*a^2 + 15*a^2*tan(f*x + e))/f
```

3.333 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=40

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0289237, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 390, 203}

$$a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + (b*(2*a + b)*Tan[e + f*x])/f + (b^2*Tan[e + f*x]^3)/(3*f)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+b+bx^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\operatorname{Subst}\left(\int \left(b(2a+b)+b^2x^2+\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= a^2x+\frac{b(2a+b)\tan(e+fx)}{f}+\frac{b^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [B] time = 0.373289, size = 106, normalized size = 2.65

$$\frac{4\sec^3(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(3a^2fx\cos^3(e+fx)+2b(3a+b)\sec(e)\sin(fx)\cos^2(e+fx)+b^2\tan(e)\cos(e+fx\right.}{3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(3*a^2*f*x*Cos[e + f*x]^3 + b^2*
Sec[e]*Sin[f*x] + 2*b*(3*a + b)*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + b^2*Cos[e
+ f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.031, size = 48, normalized size = 1.2

$$\frac{1}{f}\left(a^2\left(fx+e\right)+2\,ab\tan\left(fx+e\right)-b^2\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(f*x+e)+2*a*b*tan(f*x+e)-b^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [A] time = 1.00205, size = 59, normalized size = 1.48

$$a^2x + \frac{\left(\tan\left(fx+e\right)^3 + 3\,\tan\left(fx+e\right)\right)b^2}{3\,f} + \frac{2\,ab\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

Fricas [A] time = 0.482157, size = 142, normalized size = 3.55

$$\frac{3\,a^2fx\cos\left(fx+e\right)^3 + \left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2 + b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin(
f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.27813, size = 72, normalized size = 1.8

$$\frac{b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)a^2+6\,ab\tan\left(fx+e\right)+3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan(
f*x + e))/f
```

3.334 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=36

$$a^2(-x)-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

```
[Out] -(a^2*x) - ((a + b)^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rubi [A] time = 0.0803038, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 203}

$$a^2(-x)-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*x) - ((a + b)^2*Cot[e + f*x])/f + (b^2*Tan[e + f*x])/f
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^2}{x^2\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{(a+b)^2}{x^2}-\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -a^2x-\frac{(a+b)^2\cot(e+fx)}{f}+\frac{b^2\tan(e+fx)}{f}$$

Mathematica [B] time = 0.69248, size = 82, normalized size = 2.28

$$-\frac{4\sec(e+fx)\left(a\cos^2(e+fx)+b\right)^2\left(a^2fx\cos(e+fx)-\sin(fx)\left((a+b)^2\csc(e)\cot(e+fx)+b^2\sec(e)\right)\right)}{f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]*(a^2*f*x*Cos[e + f*x] - ((a + b)^
2*Cot[e + f*x]*Csc[e] + b^2*Sec[e])*Sin[f*x]))/(f*(a + 2*b + a*Cos[2*(e + f
*x)])^2)
```

Maple [A] time = 0.049, size = 66, normalized size = 1.8

$$\frac{1}{f}\left(a^2\left(-\cot\left(fx+e\right)-fx-e\right)-2\,ab\cot\left(fx+e\right)+b^2\left(\frac{1}{\sin\left(fx+e\right)\cos\left(fx+e\right)}-2\,\cot\left(fx+e\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-cot(f*x+e)-f*x-e)-2*a*b*cot(f*x+e)+b^2*(1/sin(f*x+e)/cos(f*x+e)-
2*cot(f*x+e)))
```

Maxima [A] time = 1.47566, size = 62, normalized size = 1.72

$$-\frac{\left(fx+e\right)a^2-b^2\tan\left(fx+e\right)+\frac{a^2+2\,ab+b^2}{\tan\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

Fricas [A] time = 0.488268, size = 153, normalized size = 4.25

$$-\frac{a^2fx\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(a^2+2\,ab+2\,b^2\right)\cos\left(fx+e\right)^2-b^2}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*f*x*cos(f*x + e)*sin(f*x + e) + (a^2 + 2*a*b + 2*b^2)*cos(f*x + e)^2
- b^2)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^2\left(e+fx\right)\right)^2\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**2, x)
```

Giac [A] time = 1.29031, size = 66, normalized size = 1.83

$$-\frac{(fx+e)a^2 - b^2\tan(fx+e) + \frac{a^2+2ab+b^2}{\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

3.335 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=45

$$\frac{\left(a^2-b^2\right)\cot(e+fx)}{f}+a^2x-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

```
[Out] a^2*x + ((a^2 - b^2)*Cot[e + f*x])/f - ((a + b)^2*Cot[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0868653, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 203}

$$\frac{\left(a^2-b^2\right)\cot(e+fx)}{f}+a^2x-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] a^2*x + ((a^2 - b^2)*Cot[e + f*x])/f - ((a + b)^2*Cot[e + f*x]^3)/(3*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b\left(1+x^2\right)\right)^2}{x^4\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{(a+b)^2}{x^4}+\frac{-a^2+b^2}{x^2}+\frac{a^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\left(a^2-b^2\right)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}+\frac{a^2\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$=a^2x+\frac{\left(a^2-b^2\right)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

Mathematica [B] time = 0.830663, size = 160, normalized size = 3.56

$$\frac{\csc(e)\csc^3(e+fx)\left(-12a^2\sin(2e+fx)+8a^2\sin(2e+3fx)-9a^2fx\cos(2e+fx)-3a^2fx\cos(2e+3fx)+3a^2fx\cos(4\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^3*(9*a^2*f*x*Cos[f*x] - 9*a^2*f*x*Cos[2*e + f*x] - 3*a
^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*x] + 12
*b^2*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 12*a*b*Sin[2*e + f*x] + 8*a^2*Sin[2
*e + 3*f*x] + 4*a*b*Sin[2*e + 3*f*x] - 4*b^2*Sin[2*e + 3*f*x]))/(24*f)
```

Maple [A] time = 0.053, size = 73, normalized size = 1.6

$$\frac{1}{f}\left(a^2\left(-\frac{\left(\cot\left(fx+e\right)\right)^3}{3}+\cot\left(fx+e\right)+fx+e\right)-\frac{2\,ab\left(\cos\left(fx+e\right)\right)^3}{3\left(\sin\left(fx+e\right)\right)^3}+b^2\left(-\frac{2}{3}-\frac{\left(\csc\left(fx+e\right)\right)^2}{3}\right)\cot\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-2/3*a*b/sin(f*x+e)^3*cos(f*x+
e)^3+b^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

Maxima [A] time = 1.51048, size = 80, normalized size = 1.78

$$\frac{3\left(fx+e\right)a^2+\frac{3\left(a^2-b^2\right)\tan\left(fx+e\right)^2-a^2-2\,ab-b^2}{\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(f*x + e)*a^2 + (3*(a^2 - b^2)*tan(f*x + e)^2 - a^2 - 2*a*b - b^2)/t
an(f*x + e)^3)/f
```

Fricas [B] time = 0.492148, size = 220, normalized size = 4.89

$$\frac{2\left(2\,a^2+ab-b^2\right)\cos\left(fx+e\right)^3-3\left(a^2-b^2\right)\cos\left(fx+e\right)+3\left(a^2fx\cos\left(fx+e\right)^2-a^2fx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(2*a^2 + a*b - b^2)*cos(f*x + e)^3 - 3*(a^2 - b^2)*cos(f*x + e) + 3*
(a^2*f*x*cos(f*x + e)^2 - a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*si
n(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35105, size = 252, normalized size = 5.6

$$\frac{a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2ab\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+24\left(fx+e\right)a^2-15a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6ab\tan}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 + 2*a*b*tan(1/2*f*x + 1/2*e)^3 + b^2*tan(1
/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a^2 - 15*a^2*tan(1/2*f*x + 1/2*e) - 6*a*b*
tan(1/2*f*x + 1/2*e) + 9*b^2*tan(1/2*f*x + 1/2*e) + (15*a^2*tan(1/2*f*x + 1
/2*e)^2 + 6*a*b*tan(1/2*f*x + 1/2*e)^2 - 9*b^2*tan(1/2*f*x + 1/2*e)^2 - a^2
 - 2*a*b - b^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

3.336 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=65

$$\frac{\left(a^2-b^2\right)\cot^3(e+fx)}{3f}-\frac{a^2\cot(e+fx)}{f}-a^2x-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

```
[Out] -(a^2*x) - (a^2*Cot[e + f*x])/f + ((a^2 - b^2)*Cot[e + f*x]^3)/(3*f) - ((a
+ b)^2*Cot[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0933106, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4141, 1802, 203}

$$\frac{\left(a^2-b^2\right)\cot^3(e+fx)}{3f}-\frac{a^2\cot(e+fx)}{f}-a^2x-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a^2*x) - (a^2*Cot[e + f*x])/f + ((a^2 - b^2)*Cot[e + f*x]^3)/(3*f) - ((a
+ b)^2*Cot[e + f*x]^5)/(5*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^2}{x^6\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{x^6}+\frac{-a^2+b^2}{x^4}+\frac{a^2}{x^2}-\frac{a^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{a^2\cot(e+fx)}{f}+\frac{\left(a^2-b^2\right)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}-\frac{a^2\,\text{Subs}}{}$$
$$= -a^2x-\frac{a^2\cot(e+fx)}{f}+\frac{\left(a^2-b^2\right)\cot^3(e+fx)}{3f}-\frac{(a+b)^2\cot^5(e+fx)}{5f}$$

Mathematica [B] time = 1.06115, size = 256, normalized size = 3.94

$$\frac{\csc(e)\csc^5(e+fx)\left(180a^2\sin(2e+fx)-140a^2\sin(2e+3fx)-90a^2\sin(4e+3fx)+46a^2\sin(4e+5fx)+150a^2fx\,\text{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^5*(-150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e + f*x]
+ 75*a^2*f*x*Cos[2*e + 3*f*x] - 75*a^2*f*x*Cos[4*e + 3*f*x] - 15*a^2*f*x*Co
s[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] + 280*a^2*Sin[f*x] + 120*a*b*S
in[f*x] + 20*b^2*Sin[f*x] + 180*a^2*Sin[2*e + f*x] - 60*b^2*Sin[2*e + f*x]
- 140*a^2*Sin[2*e + 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] - 90*a^2*Sin[4*e + 3*f
*x] - 60*a*b*Sin[4*e + 3*f*x] + 46*a^2*Sin[4*e + 5*f*x] + 12*a*b*Sin[4*e +
5*f*x] - 4*b^2*Sin[4*e + 5*f*x]))/(480*f)
```

Maple [A] time = 0.059, size = 107, normalized size = 1.7

$$\frac{1}{f}\left(a^2\left(-\frac{\left(\cot\left(fx+e\right)\right)^5}{5}+\frac{\left(\cot\left(fx+e\right)\right)^3}{3}-\cot\left(fx+e\right)-fx-e\right)-\frac{2\,ab\left(\cos\left(fx+e\right)\right)^5}{5\left(\sin\left(fx+e\right)\right)^5}+b^2\left(-\frac{\left(\cos\left(fx+e\right)\right)^3}{5\left(\sin\left(fx+e\right)\right)^5}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-2/5*a*b/sin(
f*x+e)^5*cos(f*x+e)^5+b^2*(-1/5/sin(f*x+e)^5*cos(f*x+e)^3-2/15/sin(f*x+e)^3
*cos(f*x+e)^3))
```

Maxima [A] time = 1.47225, size = 97, normalized size = 1.49

$$-\frac{15\left(fx+e\right)a^2+\frac{15\,a^2\tan\left(fx+e\right)^4-5\left(a^2-b^2\right)\tan\left(fx+e\right)^2+3\,a^2+6\,ab+3\,b^2}{\tan\left(fx+e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(15*(f*x + e)*a^2 + (15*a^2*tan(f*x + e)^4 - 5*(a^2 - b^2)*tan(f*x +
e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

Fricas [B] time = 0.499277, size = 328, normalized size = 5.05

$$-\frac{\left(23\,a^2+6\,ab-2\,b^2\right)\cos\left(fx+e\right)^5-5\left(7\,a^2-b^2\right)\cos\left(fx+e\right)^3+15\,a^2\cos\left(fx+e\right)+15\left(a^2fx\cos\left(fx+e\right)^4-2\,a^2fx\,c\right.}{15\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/15*((23*a^2 + 6*a*b - 2*b^2)*cos(f*x + e)^5 - 5*(7*a^2 - b^2)*cos(f*x +
e)^3 + 15*a^2*cos(f*x + e) + 15*(a^2*f*x*cos(f*x + e)^4 - 2*a^2*f*x*cos(f*x
 + e)^2 + a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 +
f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50462, size = 392, normalized size = 6.03

$$3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-30\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*tan(1/2*f*x + 1/2*e)^5 + 3*b^2*
tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 - 30*a*b*tan(1/2*f*x
 + 1/2*e)^3 + 5*b^2*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a^2 + 330*a^2*ta
n(1/2*f*x + 1/2*e) + 60*a*b*tan(1/2*f*x + 1/2*e) - 30*b^2*tan(1/2*f*x + 1/2
*e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 + 60*a*b*tan(1/2*f*x + 1/2*e)^4 - 30*
b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 - 30*a*b*tan(1/2
*f*x + 1/2*e)^2 + 5*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan
(1/2*f*x + 1/2*e)^5)/f
```

3.337 $\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=69

$$-\frac{(a+b)^2\log\left(a\cos^2(e+fx)+b\right)}{2ab^2f}+\frac{(a+2b)\log(\cos(e+fx))}{b^2f}+\frac{\sec^2(e+fx)}{2bf}$$

```
[Out] ((a + 2*b)*Log[Cos[e + f*x]])/(b^2*f) - ((a + b)^2*Log[b + a*Cos[e + f*x]^2
])/(2*a*b^2*f) + Sec[e + f*x]^2/(2*b*f)
```

Rubi [A] time = 0.099578, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$-\frac{(a+b)^2\log\left(a\cos^2(e+fx)+b\right)}{2ab^2f}+\frac{(a+2b)\log(\cos(e+fx))}{b^2f}+\frac{\sec^2(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b)*Log[Cos[e + f*x]])/(b^2*f) - ((a + b)^2*Log[b + a*Cos[e + f*x]^2
])/(2*a*b^2*f) + Sec[e + f*x]^2/(2*b*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-a-2b}{b^2x}+\frac{(a+b)^2}{b^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{(a+2b)\log(\cos(e+fx))}{b^2f}-\frac{(a+b)^2\log\left(b+a\cos^2(e+fx)\right)}{2ab^2f}+\frac{\sec^2(e+fx)}{2bf}\end{aligned}$$

Mathematica [A] time = 0.293114, size = 99, normalized size = 1.43

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(ab\sec^2(e+fx)+(a+b)^2\left(-\log\left(-a\sin^2(e+fx)+a+b\right)\right)+2a(a+2b)\log(c$$
$$4ab^2f\left(a+b\sec^2(e+fx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(2*a*(a + 2*b)*Log[Cos[e + f
*x]] - (a + b)^2*Log[a + b - a*Sin[e + f*x]^2] + a*b*Sec[e + f*x]^2))/(4*a*
b^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [A] time = 0.061, size = 112, normalized size = 1.6

$$-\frac{a\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2fb^2}-\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{fb}-\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2af}+\frac{\ln\left(\cos\left(fx+e\right)\right)a}{fb^2}+2\frac{\ln\left(co\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2/f/b^2*a*ln(b+a*cos(f*x+e)^2)-1/f/b*ln(b+a*cos(f*x+e)^2)-1/2*ln(b+a*cos
(f*x+e)^2)/a/f+1/f/b^2*ln(cos(f*x+e))*a+2*ln(cos(f*x+e))/b/f+1/2/f/b/cos(f*
x+e)^2
```

Maxima [A] time = 1.01583, size = 109, normalized size = 1.58

$$\frac{\frac{(a+2b)\log\left(\sin\left(fx+e\right)^2-1\right)}{b^2}-\frac{1}{b\sin\left(fx+e\right)^2-b}-\frac{\left(a^2+2ab+b^2\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{ab^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + 2*b)*log(sin(f*x + e)^2 - 1)/b^2 - 1/(b*sin(f*x + e)^2 - b) - (a^
2 + 2*a*b + b^2)*log(a*sin(f*x + e)^2 - a - b)/(a*b^2))/f
```

Fricas [A] time = 0.698029, size = 205, normalized size = 2.97

$$-\frac{\left(a^2+2ab+b^2\right)\cos\left(fx+e\right)^2\log\left(a\cos\left(fx+e\right)^2+b\right)-2\left(a^2+2ab\right)\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right)\right)-ab}{2ab^2f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2*log(a*cos(f*x + e)^2 + b) - 2*(a^2
 + 2*a*b)*cos(f*x + e)^2*log(-cos(f*x + e)) - a*b)/(a*b^2*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

Giac [B] time = 3.14862, size = 539, normalized size = 7.81

$$-\frac{\frac{(a^3+3a^2b+3ab^2+b^3)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+2b\right|\right)}{a^2b^2+ab^3}-\frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)}{a}-\frac{(a+2b)\log\left(-\cdots\right.}{\cdots}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f
*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) +
1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b)
)/(a^2*b^2 + a*b^3) - log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 2)/a - (a + 2*b)*log(-(cos(f*x + e) + 1)/(c
os(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)/b^2 + (a*((co
s(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))
 + 2*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x
 + e) + 1)) + 2*a + 8*b)/(b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)))/f
```

3.338 $\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)}{2abf}-\frac{\log(\cos(e+fx))}{bf}$$

```
[Out] -(Log[Cos[e + f*x]]/(b*f)) + ((a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Rubi [A] time = 0.0772854, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 72}

$$\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)}{2abf}-\frac{\log(\cos(e+fx))}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(Log[Cos[e + f*x]]/(b*f)) + ((a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{x(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{x(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx}+\frac{-a-b}{b(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\log(\cos(e+fx))}{bf}+\frac{(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}$$

Mathematica [A] time = 0.114396, size = 41, normalized size = 0.91

$$\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)-2a\log(\cos(e+fx))}{2abf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (-2*a*Log[Cos[e + f*x]] + (a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

Maple [A] time = 0.058, size = 59, normalized size = 1.3

$$\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2fb}+\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2af}-\frac{\ln\left(\cos\left(fx+e\right)\right)}{fb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2/f/b*ln(b+a*cos(f*x+e)^2)+1/2*ln(b+a*cos(f*x+e)^2)/a/f-ln(cos(f*x+e))/b/
f
```

Maxima [A] time = 1.02529, size = 68, normalized size = 1.51

$$\frac{\frac{(a+b)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{ab}-\frac{\log\left(\sin\left(fx+e\right)^2-1\right)}{b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*log(a*sin(f*x + e)^2 - a - b)/(a*b) - log(sin(f*x + e)^2 - 1)/
b)/f
```

Fricas [A] time = 0.596589, size = 100, normalized size = 2.22

$$\frac{(a+b)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\,a\log\left(-\cos\left(fx+e\right)\right)}{2\,abf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*log(a*cos(f*x + e)^2 + b) - 2*a*log(-cos(f*x + e)))/(a*b*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Giac [B] time = 1.66071, size = 319, normalized size = 7.09

$$\frac{\frac{(a^2+2\,ab+b^2)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2\,a+2\,b\right|\right)}{a^2b+ab^2}-\frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)}{a}-\frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos}{\cos}\right.}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1)
+ (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) + 1)/(cos(f*x +
 e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b))/(a^2*b + a*
b^2) - log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 2)/a - log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f
*x + e) - 1)/(cos(f*x + e) + 1) - 2)/b)/f
```

3.339 $\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2af}$$

```
[Out] -Log[b + a*Cos[e + f*x]^2]/(2*a*f)
```

Rubi [A] time = 0.0308976, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 260}

$$-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -Log[b + a*Cos[e + f*x]^2]/(2*a*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x}{b+ax^2}\,dx, x, \cos(e+fx)\right)}{f} = -\frac{\log\left(b+a\cos^2(e+fx)\right)}{2af}$$

Mathematica [A] time = 0.173116, size = 26, normalized size = 1.13

$$-\frac{\log(a\cos(2(e+fx))+a+2b)}{2af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -Log[a + 2*b + a*Cos[2*(e + f*x)]]/(2*a*f)
```

Maple [A] time = 0.025, size = 37, normalized size = 1.6

$$-\frac{\ln\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)}{2fa}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2/f/a*ln(a+b*sec(f*x+e)^2)+1/f/a*ln(sec(f*x+e))
```

Maxima [A] time = 1.08854, size = 35, normalized size = 1.52

$$-\frac{\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(a*sin(f*x + e)^2 - a - b)/(a*f)
```

Fricas [A] time = 0.508049, size = 51, normalized size = 2.22

$$-\frac{\log\left(a\cos\left(fx+e\right)^2+b\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -1/2*log(a*cos(f*x + e)^2 + b)/(a*f)
```

Sympy [A] time = 15.8197, size = 128, normalized size = 5.57

$$\begin{cases} \frac{\tilde{\infty}x\tan(e)}{\sec^2(e)} & \text{for } a=0 \wedge b=0 \wedge f=0 \\ \frac{\log\left(\tan^2(e+fx)+1\right)}{2af} & \text{for } b=0 \\ \frac{x\tan(e)}{a+b\sec^2(e)} & \text{for } f=0 \\ -\frac{1}{2bf\sec^2(e+fx)} & \text{for } a=0 \\ -\frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sec(e+fx)\right)}{2af} - \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sec(e+fx)\right)}{2af} + \frac{\log\left(\tan^2(e+fx)+1\right)}{2af} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Piecewise((zoo*x*tan(e)/sec(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(ta
n(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0)), (x*tan(e)/(a + b*sec(e)**2), Eq(f, 0
)), (-1/(2*b*f*sec(e + f*x)**2), Eq(a, 0)), (-log(-I*sqrt(a)*sqrt(1/b) + se
c(e + f*x))/(2*a*f) - log(I*sqrt(a)*sqrt(1/b) + sec(e + f*x))/(2*a*f) + log
(tan(e + f*x)**2 + 1)/(2*a*f), True))
```

Giac [B] time = 1.39131, size = 186, normalized size = 8.09

$$-\frac{\frac{\log\left(a+b+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)}{a}-\frac{2\,\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)}{a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/2*(log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a - 2*log(-(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 1)/a)/f
```

3.340 $\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log(\sin(e+fx))}{f(a+b)}+\frac{b\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)}$$

```
[Out] (b*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)*f) + Log[Sin[e + f*x]]/((a + b)*
f)
```

Rubi [A] time = 0.0803005, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 72}

$$\frac{\log(\sin(e+fx))}{f(a+b)}+\frac{b\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (b*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)*f) + Log[Sin[e + f*x]]/((a + b)*
f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(b+ax^2)}\,dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{(-a-b)(-1+x)} - \frac{b}{(a+b)(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{b\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)f} + \frac{\log(\sin(e+fx))}{(a+b)f}
\end{aligned}$$

Mathematica [A] time = 0.105223, size = 43, normalized size = 0.93

$$\frac{b\log\left(-a\sin^2(e+fx)+a+b\right)+2a\log(\sin(e+fx))}{2a^2f+2abf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] (2*a*Log[Sin[e + f*x]] + b*Log[a + b - a*Sin[e + f*x]^2])/(2*a^2*f + 2*a*b*
f)
```

Maple [A] time = 0.078, size = 73, normalized size = 1.6

$$\frac{b\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2\,(a+b)\,af} + \frac{\ln\left(1+\cos\left(fx+e\right)\right)}{f\,(2\,a+2\,b)} + \frac{\ln\left(-1+\cos\left(fx+e\right)\right)}{f\,(2\,a+2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2*b*ln(b+a*cos(f*x+e)^2)/a/(a+b)/f+1/f/(2*a+2*b)*ln(1+cos(f*x+e))+1/f/(2*
a+2*b)*ln(-1+cos(f*x+e))
```

Maxima [A] time = 0.980614, size = 68, normalized size = 1.48

$$\frac{\frac{b\log\left(a\sin\left(fx+e\right)^{2}-a-b\right)}{a^{2}+ab}+\frac{\log\left(\sin\left(fx+e\right)^{2}\right)}{a+b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(a*sin(f*x + e)^2 - a - b)/(a^2 + a*b) + log(sin(f*x + e)^2)/(a +
 b))/f
```

Fricas [A] time = 0.663496, size = 107, normalized size = 2.33

$$\frac{b\log\left(a\cos\left(fx+e\right)^{2}+b\right)+2\,a\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{2\left(a^{2}+ab\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(a*cos(f*x + e)^2 + b) + 2*a*log(1/2*sin(f*x + e)))/((a^2 + a*b)*
f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(e+fx\right)}{a+b\sec^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

Giac [B] time = 1.36048, size = 235, normalized size = 5.11

$$\frac{\frac{b \log\left(a+b+\frac{2 a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2 b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)}{a^{2}+ab}-\frac{2 \log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)}{a}+\frac{\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a+b}}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^2 + a*b) - 2*log(-(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 1)/a + log(-(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1))/(a + b))/f
```

3.341 $\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{b^2\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^2}-\frac{\csc^2(e+fx)}{2f(a+b)}-\frac{(a+2b)\log(\sin(e+fx))}{f(a+b)^2}$$

```
[Out] -Csc[e + f*x]^2/(2*(a + b)*f) - (b^2*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b
)^2*f) - ((a + 2*b)*Log[Sin[e + f*x]])/((a + b)^2*f)
```

Rubi [A] time = 0.110046, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$-\frac{b^2\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^2}-\frac{\csc^2(e+fx)}{2f(a+b)}-\frac{(a+2b)\log(\sin(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -Csc[e + f*x]^2/(2*(a + b)*f) - (b^2*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b
)^2*f) - ((a + 2*b)*Log[Sin[e + f*x]])/((a + b)^2*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{\left(1-x^2\right)^2\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^2(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2}+\frac{a+2b}{(a+b)^2(-1+x)}+\frac{b^2}{(a+b)^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\csc^2(e+fx)}{2(a+b)f}-\frac{b^2\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^2f}-\frac{(a+2b)\log(\sin(e+fx))}{(a+b)^2f}$$

Mathematica [A] time = 0.234977, size = 100, normalized size = 1.35

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(b^2\log\left(-a\sin^2(e+fx)+a+b\right)+a(a+b)\csc^2(e+fx)+2a(a+2b)\log(\sin(}{4af(a+b)^2\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*(a*(a + b)*Csc[e + f*x]^2 + 2*a*(a + 2*b)*
Log[Sin[e + f*x]] + b^2*Log[a + b - a*Sin[e + f*x]^2])*Sec[e + f*x]^2)/(4*a
*(a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

Maple [B] time = 0.087, size = 158, normalized size = 2.1

$$-\frac{b^2\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2a(a+b)^2f}-\frac{1}{f(4a+4b)\left(1+\cos\left(fx+e\right)\right)}-\frac{\ln\left(1+\cos\left(fx+e\right)\right)a}{2f(a+b)^2}-\frac{\ln\left(1+\cos\left(fx+e\right)\right)b}{f(a+b)^2}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/2*b^2*ln(b+a*cos(f*x+e)^2)/a/(a+b)^2/f-1/f/(4*a+4*b)/(1+cos(f*x+e))-1/2/
f/(a+b)^2*ln(1+cos(f*x+e))*a-1/f/(a+b)^2*ln(1+cos(f*x+e))*b+1/f/(4*a+4*b)/(
-1+cos(f*x+e))-1/2/f/(a+b)^2*ln(-1+cos(f*x+e))*a-1/f/(a+b)^2*ln(-1+cos(f*x+
e))*b
```

Maxima [A] time = 1.00575, size = 117, normalized size = 1.58

$$-\frac{\frac{b^2 \log\left(a \sin\left(fx+e\right)^2 - a - b\right)}{a^3+2\,a^2b+ab^2} + \frac{(a+2\,b)\log\left(\sin\left(fx+e\right)^2\right)}{a^2+2\,ab+b^2} + \frac{1}{(a+b)\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*log(a*sin(f*x + e)^2 - a - b)/(a^3 + 2*a^2*b + a*b^2) + (a + 2*b)
*log(sin(f*x + e)^2)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(f*x + e)^2))/f
```

Fricas [A] time = 0.867789, size = 289, normalized size = 3.91

$$\frac{a^2 + ab - \left(b^2 \cos\left(fx+e\right)^2 - b^2\right)\log\left(a\cos\left(fx+e\right)^2 + b\right) - 2\left(\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2 - a^2 - 2\,ab\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{2\left(\left(a^3+2\,a^2b+ab^2\right)f\cos\left(fx+e\right)^2 - \left(a^3+2\,a^2b+ab^2\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2 + a*b - (b^2*cos(f*x + e)^2 - b^2)*log(a*cos(f*x + e)^2 + b) - 2*(
(a^2 + 2*a*b)*cos(f*x + e)^2 - a^2 - 2*a*b)*log(1/2*sin(f*x + e)))/((a^3 +
2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

Giac [B] time = 1.40536, size = 419, normalized size = 5.66

$$-\frac{\frac{4\,b^2\log\left(a+b+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{b\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)}{a^3+2\,a^2b+ab^2}+\frac{4\,(a+2\,b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)}{a^2+2\,ab+b^2}-\frac{\left(a+b+\frac{4\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{8\,b\left(\cos\left(fx+e\right.\right.}{\cos\left(fx+e\right)+}\right.}{\left(a^2+2\,ab+b^2\right)\left(\cos\left(fx-\right.\right.}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^3 + 2*a^2*b + a*b^2
) + 4*(a + 2*b)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^2 + 2*a*b +
b^2) - (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/((a^2 + 2*a*b + b^2)*(cos(f*
x + e) - 1)) - 8*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/a - (cos(f
*x + e) - 1)/((a + b)*(cos(f*x + e) + 1)))/f
```

3.342 $\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{f(a+b)^3}+\frac{b^3\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^3}-\frac{\csc^4(e+fx)}{4f(a+b)}+\frac{(2a+3b)\csc^2(e+fx)}{2f(a+b)^2}$$

```
[Out] ((2*a + 3*b)*Csc[e + f*x]^2)/(2*(a + b)^2*f) - Csc[e + f*x]^4/(4*(a + b)*f)
 + (b^3*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)^3*f) + ((a^2 + 3*a*b + 3*b^
2)*Log[Sin[e + f*x]])/((a + b)^3*f)
```

Rubi [A] time = 0.148455, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{f(a+b)^3}+\frac{b^3\log\left(a\cos^2(e+fx)+b\right)}{2af(a+b)^3}-\frac{\csc^4(e+fx)}{4f(a+b)}+\frac{(2a+3b)\csc^2(e+fx)}{2f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```


```
[Out] ((2*a + 3*b)*Csc[e + f*x]^2)/(2*(a + b)^2*f) - Csc[e + f*x]^4/(4*(a + b)*f)
 + (b^3*Log[b + a*Cos[e + f*x]^2])/(2*a*(a + b)^3*f) + ((a^2 + 3*a*b + 3*b^
2)*Log[Sin[e + f*x]])/((a + b)^3*f)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^7}{\left(1-x^2\right)^3\left(b+ax^2\right)}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^3(b+ax)}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)^3}+\frac{-2a-3b}{(a+b)^2(-1+x)^2}+\frac{-a^2-3ab-3b^2}{(a+b)^3(-1+x)}-\frac{b^3}{(a+b)^3(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{(2a+3b)\csc^2(e+fx)}{2(a+b)^2f}-\frac{\csc^4(e+fx)}{4(a+b)f}+\frac{b^3\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^3f}+\frac{\left(a^2+3ab+3b^2\right)\log}{(a+b)^3f}
\end{aligned}$$

Mathematica [A] time = 0.653677, size = 138, normalized size = 1.28

$$\frac{\sec^2(e+fx)(a\cos(2e+2fx)+a+2b)\left(\frac{4\left(a^2+3ab+3b^2\right)\log(\sin(e+fx))}{(a+b)^3}+\frac{2b^3\log\left(-a\sin^2(e+fx)+a+b\right)}{a(a+b)^3}-\frac{\csc^4(e+fx)}{a+b}+\frac{2(2a+3b)\csc^2(e+fx}{(a+b)^2}\right.}{8f\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])*((2*(2*a + 3*b)*Csc[e + f*x]^2)/(a + b)^2 -
 Csc[e + f*x]^4/(a + b) + (4*(a^2 + 3*a*b + 3*b^2)*Log[Sin[e + f*x]])/(a +
b)^3 + (2*b^3*Log[a + b - a*Sin[e + f*x]^2])/(a*(a + b)^3))*Sec[e + f*x]^2)
/(8*f*(a + b*Sec[e + f*x]^2))
```

Maple [B] time = 0.089, size = 293, normalized size = 2.7

$$\frac{b^3 \ln\left(b + a\left(\cos\left(fx+e\right)\right)^2\right)}{2a\left(a+b\right)^3 f} - \frac{1}{2f\left(8a+8b\right)\left(1+\cos\left(fx+e\right)\right)^2} + \frac{7a}{16f\left(a+b\right)^2\left(1+\cos\left(fx+e\right)\right)} + \frac{11b}{16f\left(a+b\right)^2\left(1+\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/2*b^3*ln(b+a*cos(f*x+e)^2)/a/(a+b)^3/f-1/2/f/(8*a+8*b)/(1+cos(f*x+e))^2+7
/16/f/(a+b)^2/(1+cos(f*x+e))*a+11/16/f/(a+b)^2/(1+cos(f*x+e))*b+1/2/f/(a+b)
^3*ln(1+cos(f*x+e))*a^2+3/2/f/(a+b)^3*ln(1+cos(f*x+e))*a*b+3/2/f/(a+b)^3*ln
(1+cos(f*x+e))*b^2-1/2/f/(8*a+8*b)/(-1+cos(f*x+e))^2-7/16/f/(a+b)^2/(-1+cos
(f*x+e))*a-11/16/f/(a+b)^2/(-1+cos(f*x+e))*b+1/2/f/(a+b)^3*ln(-1+cos(f*x+e)
)*a^2+3/2/f/(a+b)^3*ln(-1+cos(f*x+e))*a*b+3/2/f/(a+b)^3*ln(-1+cos(f*x+e))*b
^2
```

Maxima [A] time = 0.994865, size = 196, normalized size = 1.81

$$\frac{\frac{2b^3\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^4+3a^3b+3a^2b^2+ab^3} + \frac{2\left(a^2+3ab+3b^2\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(2a+3b\right)\sin\left(fx+e\right)^2-a-b}{\left(a^2+2ab+b^2\right)\sin\left(fx+e\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^3*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
) + 2*(a^2 + 3*a*b + 3*b^2)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3) + (2*(2*a + 3*b)*sin(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*sin(f*x
+ e)^4))/f
```

Fricas [B] time = 1.32138, size = 610, normalized size = 5.65

$$\frac{3a^3+8a^2b+5ab^2-2\left(2a^3+5a^2b+3ab^2\right)\cos\left(fx+e\right)^2+2\left(b^3\cos\left(fx+e\right)^4-2b^3\cos\left(fx+e\right)^2+b^3\right)\log\left(a\cos\left(fx\right.\right.}{4\left(\left(a^4+3a^3b+3a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4-2\left(a^4+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^3 + 8*a^2*b + 5*a*b^2 - 2*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)
^2 + 2*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*log(a*cos(f*x + e)
^2 + b) + 4*((a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a
*b^2 - 2*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/(
(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42843, size = 733, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/64*(32*b^3*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3) + 32*(a^2 + 3*a*b + 3*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (12*a*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 20*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
)/(a^2 + 2*a*b + b^2) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 32*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 20*b^2*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) - 1)^2/(cos(f*x +
```

```
e) + 1)^2 + 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 144*b^2*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/((a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*(cos(f*x + e) - 1)^2) - 64*log(-(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 1)/a)/f
```

3.343 $\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=83

$$\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f}-\frac{(a+2b)\tan(e+fx)}{b^2f}-\frac{x}{a}+\frac{\tan^3(e+fx)}{3bf}$$

```
[Out] -(x/a) + ((a + b)^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(5
/2)*f) - ((a + 2*b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```

Rubi [A] time = 0.272673, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 479, 582, 522, 203, 205}

$$\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f}-\frac{(a+2b)\tan(e+fx)}{b^2f}-\frac{x}{a}+\frac{\tan^3(e+fx)}{3bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```


```
[Out] -(x/a) + ((a + b)^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(5
/2)*f) - ((a + 2*b)*Tan[e + f*x])/(b^2*f) + Tan[e + f*x]^3/(3*b*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+3(a+2b)x^2)}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3bf}$$

$$= -\frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} + \frac{\text{Subst}\left(\int \frac{3(a+b)(a+2b)+3(a^2+3ab+3b^2)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+\right.}{3b^2f}$$

$$= -\frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{(a+b)^3\,\text{Subs}}{}$$

$$= -\frac{x}{a} + \frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f} - \frac{(a+2b)\tan(e+fx)}{b^2f} + \frac{\tan^3(e+fx)}{3bf}$$

Mathematica [C] time = 3.00328, size = 229, normalized size = 2.76

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\frac{(3a+7b)\sec(e)\sin(fx)\sec(e+fx)}{b^2f} - \frac{3(a+b)^{5/2}(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec(fx)(a\sin(2}{2\sqrt{a+b}\sqrt{b(\cos(e)-i}}\right.}{ab^2f\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{6\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-3*x)/a - (3*(a + b)^(5/2)
*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*
e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin
[2*e]))/(a*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - ((3*a + 7*b)*Sec[e]*Sec[e
 + f*x]*Sin[f*x])/(b^2*f) + (Sec[e]*Sec[e + f*x]^3*Sin[f*x])/(b*f) + (Sec[e
 + f*x]^2*Tan[e])/(b*f)))/(6*(a + b*Sec[e + f*x]^2))
```

Maple [B] time = 0.085, size = 186, normalized size = 2.2

$$\frac{\left(\tan\left(fx+e\right)\right)^{3}}{3\,fb}-\frac{\tan\left(fx+e\right)a}{fb^{2}}-2\,\frac{\tan\left(fx+e\right)}{fb}-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}+\frac{a^{2}}{fb^{2}}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{\left(a+b\right)b}}\right)\frac{1}{\sqrt{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/3*tan(f*x+e)^3/b/f-1/f/b^2*tan(f*x+e)*a-2*tan(f*x+e)/b/f-1/f/a*arctan(tan
(f*x+e))+1/f*a^2/b^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3
/f*a/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/f/((a+b)*b)^(
1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f*b/a/((a+b)*b)^(1/2)*arctan(ta
n(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.595782, size = 890, normalized size = 10.72

$$\left[-\frac{12\,b^{2}fx\cos\left(fx+e\right)^{3}-3\,\left(a^{2}+2\,ab+b^{2}\right)\sqrt{-\frac{a+b}{b}}\cos\left(fx+e\right)^{3}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-2\,\left(3\,ab+4\,b^{2}\right)\cos\left(fx+e\right)^{2}-4\,\left(\left(ab+2\,b^{2}\ldots\right.\right.}{a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(f\ldots\right.}\right.}{12\,ab^{2}f\cos\left(fx+e\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(12*b^2*f*x*cos(f*x + e)^3 - 3*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/b)*
cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
```

```
)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt
(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2
 + b^2)) + 4*((3*a^2 + 7*a*b)*cos(f*x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*
cos(f*x + e)^3), -1/6*(6*b^2*f*x*cos(f*x + e)^3 + 3*(a^2 + 2*a*b + b^2)*sqr
t((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a
+ b)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((3*a^2 + 7*a*b)*cos(f*
x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 4.81404, size = 178, normalized size = 2.14

$$-\frac{\frac{3\left(fx+e\right)}{a}-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}ab^2}-\frac{b^2\tan\left(fx+e\right)^3-3ab\tan\left(fx+e\right)-6b^2\tan\left(fx+e\right)}{b^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(f*x + e)/a - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((f*x + e)
/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2
)*a*b^2) - (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x + e) - 6*b^2*tan(f*x + e))/b
^3)/f
```

3.344 $\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=59

$$-\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f}+\frac{x}{a}+\frac{\tan(e+fx)}{bf}$$

```
[Out] x/a - ((a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(3/2)
*f) + Tan[e + f*x]/(b*f)
```

Rubi [A] time = 0.167438, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 479, 522, 203, 205}

$$-\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f}+\frac{x}{a}+\frac{\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] x/a - ((a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*b^(3/2)
*f) + Tan[e + f*x]/(b*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\tan(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{a+b+(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{bf}$$
$$= \frac{\tan(e+fx)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} - \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+f$$
$$= \frac{x}{a} - \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

Mathematica [C] time = 1.14793, size = 206, normalized size = 3.49

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sqrt{b(\sin(e)+i\cos(e))^4}(a\sec(e)\sin(fx)\sec(e+fx)+bfx)+(a+b)^2(\cos(}{2abf\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((a + b)^2*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(b*f*x + a*Sec[e]*Sec[e + f*x]*Sin[f*x])))
/(2*a*b*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [B] time = 0.074, size = 121, normalized size = 2.1

$$\frac{\tan(fx+e)}{fb} + \frac{\arctan(\tan(fx+e))}{fa} - \frac{a}{fb}\arctan\left(\tan(fx+e)\,b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}} - 2\,\frac{1}{f\sqrt{(a+b)\,b}}\arctan\left(\frac{\tan}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] tan(f*x+e)/b/f+1/f/a*arctan(tan(f*x+e))-1/f*a/b/((a+b)*b)^(1/2)*arctan(tan(
f*x+e)*b/((a+b)*b)^(1/2))-2/f/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)
^(1/2))-1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.573785, size = 720, normalized size = 12.2

$$\left[\frac{4\,bfx\cos\left(fx+e\right)+(a+b)\sqrt{-\frac{a+b}{b}}\cos\left(fx+e\right)\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(ab+2\,b^2\right)\cos\left(fx+e\right)^3-b^2\cos\right.}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right.}{4\,abf\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*f*x*cos(f*x + e) + (a + b)*sqrt(-(a + b)/b)*cos(f*x + e)*log(((a^
2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((
a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*a*sin(f*x
+ e))/(a*b*f*cos(f*x + e)), 1/2*(2*b*f*x*cos(f*x + e) + (a + b)*sqrt((a + b
)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos
(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*a*sin(f*x + e))/(a*b*f*cos(f*x +
e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 2.23345, size = 123, normalized size = 2.08

$$\frac{\frac{fx+e}{a}+\frac{\tan(fx+e)}{b}-\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2+2\,ab+b^2)}{\sqrt{ab+b^2}ab}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((f*x + e)/a + tan(f*x + e)/b - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arct
an(b*tan(f*x + e)/sqrt(a*b + b^2)))*(a^2 + 2*a*b + b^2)/(sqrt(a*b + b^2)*a*
b))/f
```

3.345 $\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}f}-\frac{x}{a}$$

```
[Out] -(x/a) + (Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b
]*f)
```

Rubi [A] time = 0.126275, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4141, 1975, 481, 203, 205}

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}f}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -(x/a) + (Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b
]*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{af}\\
&= -\frac{x}{a} + \frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}f}
\end{aligned}$$

Mathematica [C] time = 0.309408, size = 184, normalized size = 4.

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(fx\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}+(a+b)(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin}{}\right.\right.}{2af\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Co
s[e] - I*Sin[e])^4] + (a + b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((
a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x
]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.081, size = 75, normalized size = 1.6

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}+\frac{1}{f}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}+\frac{b}{fa}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/a*arctan(tan(f*x+e))+1/f/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)
^(1/2))+1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.553306, size = 533, normalized size = 11.59

$$\left[-\frac{4\,fx-\sqrt{-\frac{a+b}{b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2-4\left(\left(ab+2\,b^2\right)\cos\left(fx+e\right)^3-b^2\cos\left(fx+e\right)\right)\sqrt{-\frac{a+b}{b}}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)}{4\,af},-\frac{2\,fx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*f*x - sqrt(-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*c
os(f*x + e))*sqrt(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/(a*f), -1/2*(2*f*x + sqrt((a + b)/b)*arctan(1/2*
((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*
x + e))))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.48892, size = 93, normalized size = 2.02

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)(a+b)}{\sqrt{ab+b^2}a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))*(a + b)/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

3.346 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rubi [A] time = 0.0454022, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4127, 3181, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*f
)
```

Rule 4127

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Dist[b/a, Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x
] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sec^2(e+fx)}\,dx &= \frac{x}{a}-\frac{b\int \frac{1}{b+a\cos^2(e+fx)}\,dx}{a}\\ &= \frac{x}{a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b+(a+b)x^2}\,dx,x,\cot(e+fx)\right)}{af}\\ &= \frac{x}{a}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}\end{aligned}$$

Mathematica [C] time = 0.259887, size = 182, normalized size = 4.04

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(fx\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}+b(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec}{2\sqrt{a+b}}\right.\right.}{2af\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Cos
[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sq
rt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.069, size = 48, normalized size = 1.1

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}-\frac{b}{fa}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/a*arctan(tan(f*x+e))-1/f*b/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)
*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.566201, size = 544, normalized size = 12.09

$$\left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2+4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{b}{a+b}}\sin\left(fx+e\right)+b^2}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right)}{4\,af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f
*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a + b
))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e
)*sin(f*x + e))))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(1/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.38933, size = 92, normalized size = 2.04

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a}-\frac{fx+e}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

3.347 $\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=62

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}-\frac{x}{a}$$

```
[Out] -(x/a) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(3
/2)*f) - Cot[e + f*x]/((a + b)*f)
```

Rubi [A] time = 0.173957, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 480, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```


```
[Out] -(x/a) + (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(3
/2)*f) - Cot[e + f*x]/((a + b)*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b(1+x^2))}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f} + \frac{\text{Subst}\left(\int \frac{-a-2b-bx^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$

$$= -\frac{\cot(e+fx)}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+b+bx^2}\,dx, x, \tan(e+fx)\right)}{a(a+b)f}$$

$$= -\frac{x}{a} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}$$

Mathematica [C] time = 1.35378, size = 204, normalized size = 3.29

$$-\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(b^2(\cos(2e)-i\sin(2e))\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec(fx)(a\sin(2e+fx)-(a+2b)\sin(fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)+\ldots}{2af(a+b)^{3/2}\sqrt{b(\cos(e)-i\sin(e))^4}\left(a+b\sec^2(e+\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b^2*ArcTan[(Sec[f*x]*(Cos[
2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Sq
rt[b*(Cos[e] - I*Sin[e])^4]*((a + b)*f*x - a*Csc[e]*Csc[e + f*x]*Sin[f*x]))
)/(2*a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4]
)
```

Maple [A] time = 0.086, size = 73, normalized size = 1.2

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa} - \frac{1}{f\left(a+b\right)\tan\left(fx+e\right)} + \frac{b^2}{f\left(a+b\right)a}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{\left(a+b\right)b}}\right)\frac{1}{\sqrt{\left(a+b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/a*arctan(tan(f*x+e))-1/f/(a+b)/tan(f*x+e)+1/f/(a+b)*b^2/a/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.57375, size = 757, normalized size = 12.21

$$\left[-\frac{4\,(a+b)fx\sin\left(fx+e\right)-b\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-2\left(3\,ab+4\,b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^3-\left(ab+b^2\right)\cos\left(fx+\right.\right.}{a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2}\right.}{4\left(a^2+ab\right)f\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a + b)*f*x*sin(f*x + e) - b*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b
+ 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*
x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x +
e) + 4*a*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), -1/2*(2*(a + b)*f*x*si
n(f*x + e) + b*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*a*cos(f*x + e
))/((a^2 + a*b)*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.39033, size = 123, normalized size = 1.98

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{(a^2+ab)\sqrt{ab+b^2}}-\frac{fx+e}{a}-\frac{1}{(a+b)\tan(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] ((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))*b^2/((a^2 + a*b)*sqrt(a*b + b^2)) - (f*x + e)/a - 1/((a + b)*tan(f*x +
 e)))/f
```

3.348 $\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}+\frac{(a+2b)\cot(e+fx)}{f(a+b)^2}+\frac{x}{a}$$

```
[Out] x/a - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(5/2)
*f) + ((a + 2*b)*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rubi [A] time = 0.248174, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 480, 583, 522, 203, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{5/2}}-\frac{\cot^3(e+fx)}{3f(a+b)}+\frac{(a+2b)\cot(e+fx)}{f(a+b)^2}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] x/a - (b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(5/2)
*f) + ((a + 2*b)*Cot[e + f*x])/((a + b)^2*f) - Cot[e + f*x]^3/(3*(a + b)*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{-3(a+2b)-3bx^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f} - \frac{\text{Subst}\left(\int \frac{-3(a^2+3ab+3b^2)-3b(a+2b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{3(a+b)^2 f}$$

$$= \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{af} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b+\cdots}\right.}{a(a\cdots}$$

$$= \frac{x}{a} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}f} + \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f}$$

Mathematica [C] time = 3.70735, size = 390, normalized size = 4.53

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{1}{8}\sqrt{a+b}\csc(e)\sqrt{b(\cos(e)-i\sin(e))^4}\csc^3(e+fx)\left(-12a^2\sin(2e+fx)+8a^2\sin\cdots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*b^3*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a + b]*
Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(9*(a + b)^2*f*x*Cos[f*
x] - 9*(a + b)^2*f*x*Cos[2*e + f*x] - 3*a^2*f*x*Cos[2*e + 3*f*x] - 6*a*b*f*
x*Cos[2*e + 3*f*x] - 3*b^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x
] + 6*a*b*f*x*Cos[4*e + 3*f*x] + 3*b^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*
x] - 24*a*b*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 18*a*b*Sin[2*e + f*x] + 8*a^
2*Sin[2*e + 3*f*x] + 14*a*b*Sin[2*e + 3*f*x]))/8))/(6*a*(a + b)^(5/2)*f*(a
+ b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

Maple [A] time = 0.099, size = 110, normalized size = 1.3

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}-\frac{1}{3f\left(a+b\right)\left(\tan\left(fx+e\right)\right)^{3}}+\frac{a}{f\left(a+b\right)^{2}\tan\left(fx+e\right)}+2\frac{b}{f\left(a+b\right)^{2}\tan\left(fx+e\right)}-\frac{b^{3}}{f\left(a+b\right)^{2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x)
```

```
[Out] 1/f/a*arctan(tan(f*x+e))-1/3/f/(a+b)/tan(f*x+e)^3+1/f/(a+b)^2/tan(f*x+e)*a+
2/f/(a+b)^2/tan(f*x+e)*b-1/f/(a+b)^2*b^3/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e
)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.621233, size = 1261, normalized size = 14.66

$$\left[\frac{4\left(4a^{2}+7ab\right)\cos\left(fx+e\right)^{3}+3\left(b^{2}\cos\left(fx+e\right)^{2}-b^{2}\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^{2}+8ab+8b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(3ab+4b^{2}\right)\cos\left(fx+e\right)^{2}+4\left(\left(a^{2}+3ab\right.\right.}{a^{2}\cos\left(fx+e\right)^{4}+2ab}\right.}{12\left(\left(a^{3}+2a^{2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)^2 - b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
```

```
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(a^2 + 2*a*b)*cos(f*x + e) + 12*((
a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f*x)*sin(f*x +
e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
*sin(f*x + e)), 1/6*(2*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)
^2 - b^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/
(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a^2 + 2*a*b)*cos(
f*x + e) + 6*((a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*
f*x)*sin(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^
2*b + a*b^2)*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

Giac [A] time = 1.43611, size = 189, normalized size = 2.2

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)b^3}{\left(a^3+2\,a^2b+ab^2\right)\sqrt{ab+b^2}}-\frac{3\left(fx+e\right)}{a}-\frac{3\,a\tan\left(fx+e\right)^2+6\,b\tan\left(fx+e\right)^2-a-b}{\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^3/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b + b^2)) - 3*(f*x + e)/a -
 (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*tan
(f*x + e)^3))/f
```

3.349 $\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=120

$$-\frac{(a^2+3ab+3b^2)\cot(e+fx)}{f(a+b)^3}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{7/2}}-\frac{\cot^5(e+fx)}{5f(a+b)}+\frac{(a+2b)\cot^3(e+fx)}{3f(a+b)^2}-\frac{x}{a}$$

```
[Out] -(x/a) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(7
/2)*f) - ((a^2 + 3*a*b + 3*b^2)*Cot[e + f*x])/((a + b)^3*f) + ((a + 2*b)*Co
t[e + f*x]^3)/(3*(a + b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```

Rubi [A] time = 0.343942, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 480, 583, 522, 203, 205}

$$-\frac{(a^2+3ab+3b^2)\cot(e+fx)}{f(a+b)^3}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{af(a+b)^{7/2}}-\frac{\cot^5(e+fx)}{5f(a+b)}+\frac{(a+2b)\cot^3(e+fx)}{3f(a+b)^2}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```


```
[Out] -(x/a) + (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*(a + b)^(7
/2)*f) - ((a^2 + 3*a*b + 3*b^2)*Cot[e + f*x])/((a + b)^3*f) + ((a + 2*b)*Co
t[e + f*x]^3)/(3*(a + b)^2*f) - Cot[e + f*x]^5/(5*(a + b)*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-5(a+2b)-5bx^2}{x^4(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} - \frac{\text{Subst}\left(\int \frac{-15(a^2+3ab+3b^2)-15b(a+2b)x^2}{x^2(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+f\right.}{15(a+b)^2f}$$

$$= -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-15(a+2b}{}\right.}{}$$

$$= -\frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\cot^5(e+fx)}{5(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{af}$$

$$= -\frac{x}{a} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{7/2}f} - \frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2f} - \frac{\text{co}}{5}$$

Mathematica [C] time = 2.9304, size = 671, normalized size = 5.59

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(\csc(e)\csc^5(e+fx)\left(540a^2b\sin(2e+fx)-420a^2b\sin(2e+3fx)-240a^2b\sin(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-480*b^4*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt
[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + Csc[e]*Csc[e + f*x]^5*(-150*(a + b)^3*f
*x*Cos[f*x] + 150*(a + b)^3*f*x*Cos[2*e + f*x] + 75*a^3*f*x*Cos[2*e + 3*f*x
] + 225*a^2*b*f*x*Cos[2*e + 3*f*x] + 225*a*b^2*f*x*Cos[2*e + 3*f*x] + 75*b^
```

```
3*f*x*Cos[2*e + 3*f*x] - 75*a^3*f*x*Cos[4*e + 3*f*x] - 225*a^2*b*f*x*Cos[4*
e + 3*f*x] - 225*a*b^2*f*x*Cos[4*e + 3*f*x] - 75*b^3*f*x*Cos[4*e + 3*f*x] -
 15*a^3*f*x*Cos[4*e + 5*f*x] - 45*a^2*b*f*x*Cos[4*e + 5*f*x] - 45*a*b^2*f*x
*Cos[4*e + 5*f*x] - 15*b^3*f*x*Cos[4*e + 5*f*x] + 15*a^3*f*x*Cos[6*e + 5*f*
x] + 45*a^2*b*f*x*Cos[6*e + 5*f*x] + 45*a*b^2*f*x*Cos[6*e + 5*f*x] + 15*b^3
*f*x*Cos[6*e + 5*f*x] + 280*a^3*Sin[f*x] + 780*a^2*b*Sin[f*x] + 680*a*b^2*S
in[f*x] + 180*a^3*Sin[2*e + f*x] + 540*a^2*b*Sin[2*e + f*x] + 480*a*b^2*Sin
[2*e + f*x] - 140*a^3*Sin[2*e + 3*f*x] - 420*a^2*b*Sin[2*e + 3*f*x] - 400*a
*b^2*Sin[2*e + 3*f*x] - 90*a^3*Sin[4*e + 3*f*x] - 240*a^2*b*Sin[4*e + 3*f*x
] - 180*a*b^2*Sin[4*e + 3*f*x] + 46*a^3*Sin[4*e + 5*f*x] + 132*a^2*b*Sin[4*
e + 5*f*x] + 116*a*b^2*Sin[4*e + 5*f*x])))/(960*a*(a + b)^3*f*(a + b*Sec[e
+ f*x]^2))
```

Maple [A] time = 0.112, size = 173, normalized size = 1.4

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa}-\frac{1}{5f\left(a+b\right)\left(\tan\left(fx+e\right)\right)^{5}}+\frac{a}{3f\left(a+b\right)^{2}\left(\tan\left(fx+e\right)\right)^{3}}+\frac{2b}{3f\left(a+b\right)^{2}\left(\tan\left(fx+e\right)\right)^{3}}-\frac{}{f\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x)
```

```
[Out] -1/f/a*arctan(tan(f*x+e))-1/5/f/(a+b)/tan(f*x+e)^5+1/3/f/(a+b)^2/tan(f*x+e)
^3*a+2/3/f/(a+b)^2/tan(f*x+e)^3*b-1/f/(a+b)^3/tan(f*x+e)*a^2-3/f/(a+b)^3/ta
n(f*x+e)*a*b-3/f/(a+b)^3/tan(f*x+e)*b^2+1/f/(a+b)^3*b^4/a/((a+b)*b)^(1/2)*a
rctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.683559, size = 1972, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(23*a^3 + 66*a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 20*(7*a^3 + 21*a^
2*b + 20*a*b^2)*cos(f*x + e)^3 - 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e
)^2 + b^3)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(
3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (
a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x
+ e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(a^3 + 3*a^2*b + 3*
a*b^2)*cos(f*x + e) + 60*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^
4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*
cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), -1/30*(2*(23*a^3 + 66*
a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 10*(7*a^3 + 21*a^2*b + 20*a*b^2)*cos(f*x
 + e)^3 + 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*sqrt(b/(a +
b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x +
e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)
+ 30*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x)
*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.47196, size = 300, normalized size = 2.5

$$\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab+b^2}}-\frac{15(fx+e)}{a}-\frac{15a^2\tan(fx+e)^4+45ab\tan(fx+e)^4+45b^2\tan(fx+e)^4-5a^2\tan(fx+e)^2-15ab\tan(fx+e)^2-}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)^5}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^4/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b + b^2)) - 15
*(f*x + e)/a - (15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 45*b^2*tan(
f*x + e)^4 - 5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 - 10*b^2*tan(f*x
+ e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x +
e)^5))/f
```

3.350 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=77

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)}{2f}-\frac{(a+b)^2}{2a^2bf\left(a\cos^2(e+fx)+b\right)}-\frac{\log(\cos(e+fx))}{b^2f}$$

```
[Out] -(a + b)^2/(2*a^2*b*f*(b + a*Cos[e + f*x]^2)) - Log[Cos[e + f*x]]/(b^2*f) -
 ((a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.105814, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)}{2f}-\frac{(a+b)^2}{2a^2bf\left(a\cos^2(e+fx)+b\right)}-\frac{\log(\cos(e+fx))}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(a + b)^2/(2*a^2*b*f*(b + a*Cos[e + f*x]^2)) - Log[Cos[e + f*x]]/(b^2*f) -
 ((a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x]^2])/(2*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x} - \frac{(a+b)^2}{ab(b+ax)^2} + \frac{-a^2+b^2}{ab^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)^2}{2a^2bf\left(b+a\cos^2(e+fx)\right)} - \frac{\log(\cos(e+fx))}{b^2f} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)}{2f}$$

Mathematica [A] time = 0.45452, size = 109, normalized size = 1.42

$$-\frac{\sec^4(e+fx)(a\cos(2e+2fx)+a+2b)^2\left(\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)+\frac{(a+b)^2}{a^2b\left(a\cos^2(e+fx)+b\right)}+\frac{2\log(\cos(e+fx))}{b^2}\right)}{8f\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + 2*b + a*Cos[2*e + 2*f*x])^2*((a + b)^2/(a^2*b*(b + a*Cos[e + f*x]^2)
) + (2*Log[Cos[e + f*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x]^2]
)*Sec[e + f*x]^4)/(8*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.087, size = 126, normalized size = 1.6

$$\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}{2\,f b^{2}}-\frac{1}{2\,f b\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}-\frac{1}{f a\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}-\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}{2\,a^{2}f}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f/b^2*ln(b+a*cos(f*x+e)^2)-1/2/f/b/(b+a*cos(f*x+e)^2)-1/f/a/(b+a*cos(f*
x+e)^2)-1/2*ln(b+a*cos(f*x+e)^2)/a^2/f-1/2*b/a^2/f/(b+a*cos(f*x+e)^2)-ln(co
s(f*x+e))/b^2/f
```

Maxima [A] time = 1.01365, size = 132, normalized size = 1.71

$$\frac{\frac{a^{2}+2\,ab+b^{2}}{a^{3}b\sin\left(fx+e\right)^{2}-a^{3}b-a^{2}b^{2}}-\frac{\log\left(\sin\left(fx+e\right)^{2}-1\right)}{b^{2}}+\frac{\left(a^{2}-b^{2}\right)\log\left(a\sin\left(fx+e\right)^{2}-a-b\right)}{a^{2}b^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2)/(a^3*b*sin(f*x + e)^2 - a^3*b - a^2*b^2) - log(sin
(f*x + e)^2 - 1)/b^2 + (a^2 - b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^2*b^2))
/f
```

Fricas [A] time = 0.743636, size = 262, normalized size = 3.4

$$-\frac{a^{2}b+2\,ab^{2}+b^{3}-\left(a^{2}b-b^{3}+\left(a^{3}-ab^{2}\right)\cos\left(fx+e\right)^{2}\right)\log\left(a\cos\left(fx+e\right)^{2}+b\right)+2\left(a^{3}\cos\left(fx+e\right)^{2}+a^{2}b\right)\log\left(-\cdots\right)}{2\left(a^{3}b^{2}f\cos\left(fx+e\right)^{2}+a^{2}b^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b + 2*a*b^2 + b^3 - (a^2*b - b^3 + (a^3 - a*b^2)*cos(f*x + e)^2)*
log(a*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(-cos(f*x + e
)))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [B] time = 3.30719, size = 770, normalized size = 10.

$$\frac{\left(a^3+a^2b-ab^2-b^3\right)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+2b\right|\right)}{a^3b^2+a^2b^3}+\frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)}{a^2}-\frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\cdots\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 + a^2*b - a*b^2 - b^3)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x +
e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) + 1)/(c
os(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b))/(a^
3*b^2 + a^2*b^3) + log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 2)/a^2 - log(-(cos(f*x + e) + 1)/(cos(f*x + e)
 - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)/b^2 - (a^3*((cos(f*x + e
) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + a^2*b*
((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) +
 1)) - a*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(c
os(f*x + e) + 1)) - b^3*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x +
 e) - 1)/(cos(f*x + e) + 1)) + 2*a^3 - 6*a^2*b - 6*a*b^2 + 2*b^3)/((a*((cos
(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))
```

```
+ b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
e) + 1)) + 2*a - 2*b)*a^2*b^2))/f
```

3.351 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=51

$$\frac{a+b}{2a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

```
[Out] (a + b)/(2*a^2*f*(b + a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^2
*f)
```

Rubi [A] time = 0.0823833, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 444, 43}

$$\frac{a+b}{2a^2f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (a + b)/(2*a^2*f*(b + a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^2
*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{x\left(1-x^2\right)}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{a+b}{a(b+ax)^2}-\frac{1}{a(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= \frac{a+b}{2a^2f\left(b+a\cos^2(e+fx)\right)}+\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}
\end{aligned}$$

Mathematica [A] time = 0.731966, size = 81, normalized size = 1.59

$$\frac{(a+2b)\log(a\cos(2(e+fx))+a+2b)+a\cos(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+2(a+b)}{2a^2f(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] (2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x
)]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(2*a^2*f*(a + 2*b + a*Cos[2*(e + f*x)
]))
```

Maple [A] time = 0.079, size = 68, normalized size = 1.3

$$\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2a^2f}+\frac{1}{2fa\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}+\frac{b}{2a^2f\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2*ln(b+a*cos(f*x+e)^2)/a^2/f+1/2/f/a/(b+a*cos(f*x+e)^2)+1/2*b/a^2/f/(b+a*
cos(f*x+e)^2)
```

Maxima [A] time = 0.994416, size = 80, normalized size = 1.57

$$-\frac{\frac{a+b}{a^3\sin\left(fx+e\right)^2-a^3-a^2b}-\frac{\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a + b)/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a
 - b)/a^2)/f
```

Fricas [A] time = 0.524043, size = 131, normalized size = 2.57

$$\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+a+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + a + b)/(a^3*f*cos(f
*x + e)^2 + a^2*b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.86911, size = 458, normalized size = 8.98

$$\frac{\frac{\log\left(a+b+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)}{a^{2}}-\frac{2\,\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)}{a^{2}}-\frac{a+b+\frac{6\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a(\cos}{(\cos}}{\left(a+b+\frac{2\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{a(\cos}{(\cos(}\right.}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 - 2*log(-(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + 1)/a^2 - (a + b + 6*a*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((a
+ b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^2))/f
```

3.352 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{b}{2a^2f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

```
[Out] -b/(2*a^2*f*(b + a*Cos[e + f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^2*f)
```

Rubi [A] time = 0.0556289, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$-\frac{b}{2a^2f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -b/(2*a^2*f*(b + a*Cos[e + f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^2*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^3}{\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{x}{(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{b}{a(b+ax)^2}+\frac{1}{a(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{b}{2a^2f\left(b+a\cos^2(e+fx)\right)}-\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}
\end{aligned}$$

Mathematica [A] time = 0.505819, size = 79, normalized size = 1.61

$$-\frac{(a+2b)\log(a\cos(2(e+fx))+a+2b)+a\cos(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+2b}{2a^2f(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x)]*Lo
g[a + 2*b + a*Cos[2*(e + f*x)]])/(2*a^2*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Maple [A] time = 0.036, size = 59, normalized size = 1.2

$$-\frac{\ln\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)}{2\,fa^2}+\frac{1}{2\,fa\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{fa^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/2/f/a^2*ln(a+b*sec(f*x+e)^2)+1/2/f/a/(a+b*sec(f*x+e)^2)+1/f/a^2*ln(sec(f
*x+e))
```

Maxima [A] time = 0.993598, size = 77, normalized size = 1.57

$$\frac{\frac{b}{a^3 \sin\left(fx+e\right)^2 - a^3 - a^2 b} - \frac{\log\left(a \sin\left(fx+e\right)^2 - a - b\right)}{a^2}}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a - b)/a
^2)/f
```

Fricas [A] time = 0.524192, size = 127, normalized size = 2.59

$$-\frac{\left(a \cos\left(fx+e\right)^2 + b\right) \log\left(a \cos\left(fx+e\right)^2 + b\right) + b}{2 \left(a^3 f \cos\left(fx+e\right)^2 + a^2 b f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + b)/(a^3*f*cos(f*x
+ e)^2 + a^2*b*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36198, size = 558, normalized size = 11.39

$$\frac{\frac{a^2+2ab+b^2+\frac{2a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{2ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{(a^3+a^2b)\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)} - \frac{\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(}{c}\right.}{}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*a*b + b^2 + 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*a
*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2)/((a^3 + a^2*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
- 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - log(a + b
+ 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 + 2*log(-(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 1)/a^2)/f
```

3.353 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{b^2}{2a^2f(a+b)\left(a\cos^2(e+fx)+b\right)}+\frac{b(2a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^2}+\frac{\log(\sin(e+fx))}{f(a+b)^2}$$

```
[Out] b^2/(2*a^2*(a + b)*f*(b + a*Cos[e + f*x]^2)) + (b*(2*a + b)*Log[b + a*Cos[e
 + f*x]^2])/(2*a^2*(a + b)^2*f) + Log[Sin[e + f*x]]/((a + b)^2*f)
```

Rubi [A] time = 0.113657, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$\frac{b^2}{2a^2f(a+b)\left(a\cos^2(e+fx)+b\right)}+\frac{b(2a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^2}+\frac{\log(\sin(e+fx))}{f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] b^2/(2*a^2*(a + b)*f*(b + a*Cos[e + f*x]^2)) + (b*(2*a + b)*Log[b + a*Cos[e
 + f*x]^2])/(2*a^2*(a + b)^2*f) + Log[Sin[e + f*x]]/((a + b)^2*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)}+\frac{b^2}{a(a+b)(b+ax)^2}-\frac{b(2a+b)}{a(a+b)^2(b+ax)}\right) dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{b^2}{2a^2(a+b)f\left(b+a\cos^2(e+fx)\right)}+\frac{b(2a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^2f}+\frac{\log(\sin(e+fx))}{(a+b)^2f}$$

Mathematica [A] time = 0.361978, size = 112, normalized size = 1.35

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(\frac{b^2(a+b)}{a^2\left(-a\sin^2(e+fx)+a+b\right)}+\frac{b(2a+b)\log\left(-a\sin^2(e+fx)+a+b\right)}{a^2}+2\log(\sin(e+fx))\right)}{8f(a+b)^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(2*Log[Sin[e + f*x]] + (b*
(2*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (b^2*(a + b))/(a^2*(a + b -
a*Sin[e + f*x]^2))))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.092, size = 155, normalized size = 1.9

$$\frac{b^2\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2f\left(a+b\right)^2a^2}+\frac{b\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{f\left(a+b\right)^2a}+\frac{b^2}{2f\left(a+b\right)^2a\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}+\frac{b^3}{2f\left(a+b\right)^2a^2\left(b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/2/f*b^2/(a+b)^2/a^2*ln(b+a*cos(f*x+e)^2)+1/f*b/(a+b)^2/a*ln(b+a*cos(f*x+e
)^2)+1/2/f*b^2/(a+b)^2/a/(b+a*cos(f*x+e)^2)+1/2*b^3/a^2/(a+b)^2/f/(b+a*cos(
f*x+e)^2)+1/2/f/(a+b)^2*ln(1+cos(f*x+e))+1/2/f/(a+b)^2*ln(-1+cos(f*x+e))
```

Maxima [A] time = 1.00624, size = 158, normalized size = 1.9

$$\frac{\frac{b^2}{a^4+2a^3b+a^2b^2-\left(a^4+a^3b\right)\sin\left(fx+e\right)^2}+\frac{\left(2ab+b^2\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^4+2a^3b+a^2b^2}+\frac{\log\left(\sin\left(fx+e\right)^2\right)}{a^2+2ab+b^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x + e)^2) + (2*a*b
+ b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 2*a^3*b + a^2*b^2) + log(sin(f*
x + e)^2)/(a^2 + 2*a*b + b^2))/f
```

Fricas [A] time = 0.995817, size = 313, normalized size = 3.77

$$\frac{ab^2+b^3+\left(2ab^2+b^3+\left(2a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\left(a^3\cos\left(fx+e\right)^2+a^2b\right)\log\left(\frac{1}{2}\sin\left(f\right.\right.}{2\left(\left(a^5+2a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^4b+2a^3b^2+a^2b^3\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2 + b^3 + (2*a*b^2 + b^3 + (2*a^2*b + a*b^2)*cos(f*x + e)^2)*log(a
*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(1/2*sin(f*x + e))
)/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^
3)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.07845, size = 567, normalized size = 6.83

$$\frac{\frac{(2\,ab+b^2)\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4+2\,a^3b+a^2b^2}+\frac{\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)}{a^2+2\,ab+b^2}-\frac{2\,ab+b^2+\frac{4\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b^2(\cos(fx+e}{\cos(fx+e)+}}{(a^3+a^2b)\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(f}{\cos(fx}\right.}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a*b + b^2)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 2*a^3*b
+ a^2*b^2) + log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2
) - (2*a*b + b^2 + 4*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e
) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((a^3 + a^2*b)*(a
 + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - 2*log(-(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 1)/a^2)/f
```

3.354 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=111

$$-\frac{b^3}{2a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}-\frac{b^2(3a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^3}-\frac{\csc^2(e+fx)}{2f(a+b)^2}-\frac{(a+3b)\log(\sin(e+fx))}{f(a+b)^3}$$

```
[Out] -b^3/(2*a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a + b)
^2*f) - (b^2*(3*a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a^2*(a + b)^3*f) - ((a
 + 3*b)*Log[Sin[e + f*x]])/((a + b)^3*f)
```

Rubi [A] time = 0.157109, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$-\frac{b^3}{2a^2f(a+b)^2\left(a\cos^2(e+fx)+b\right)}-\frac{b^2(3a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^3}-\frac{\csc^2(e+fx)}{2f(a+b)^2}-\frac{(a+3b)\log(\sin(e+fx))}{f(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] -b^3/(2*a^2*(a + b)^2*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a + b)
^2*f) - (b^2*(3*a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a^2*(a + b)^3*f) - ((a
 + 3*b)*Log[Sin[e + f*x]])/((a + b)^3*f)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^7}{\left(1-x^2\right)^2\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^2(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^2(-1+x)^2}+\frac{a+3b}{(a+b)^3(-1+x)}-\frac{b^3}{a(a+b)^2(b+ax)^2}+\frac{b^2(3a+b)}{a(a+b)^3(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{b^3}{2a^2(a+b)^2f\left(b+a\cos^2(e+fx)\right)}-\frac{\csc^2(e+fx)}{2(a+b)^2f}-\frac{b^2(3a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^3f}
\end{aligned}
$$

Mathematica [A] time = 1.33795, size = 130, normalized size = 1.17

$$
-\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(\frac{b^2\left(\frac{2b(a+b)}{a\cos(2(e+fx))+a+2b}+(3a+b)\log\left(-a\sin^2(e+fx)+a+b\right)\right)}{a^2}+(a+b)\csc^2(e+fx)+2(a+3\right.}{8f(a+b)^3\left(a+b\sec^2(e+fx)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])^2*((a + b)*Csc[e + f*x]^2 + 2*(a + 3*b)*Lo
g[Sin[e + f*x]] + (b^2*((2*b*(a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*a
 + b)*Log[a + b - a*Sin[e + f*x]^2]))/a^2)*Sec[e + f*x]^4)/(8*(a + b)^3*f*(
a + b*Sec[e + f*x]^2)^2)
```

Maple [B] time = 0.109, size = 240, normalized size = 2.2

$$-\frac{3 b^{2} \ln \left(b+a \left(\cos \left(f x+e\right)\right)^{2}\right)}{2 f \left(a+b\right)^{3} a}-\frac{b^{3} \ln \left(b+a \left(\cos \left(f x+e\right)\right)^{2}\right)}{2 f \left(a+b\right)^{3} a^{2}}-\frac{b^{3}}{2 f \left(a+b\right)^{3} a \left(b+a \left(\cos \left(f x+e\right)\right)^{2}\right)}-\frac{}{2 f \left(a+b\right)^{3} a^{2} \left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -3/2/f*b^2/(a+b)^3/a*ln(b+a*cos(f*x+e)^2)-1/2/f*b^3/(a+b)^3/a^2*ln(b+a*cos(
f*x+e)^2)-1/2/f*b^3/(a+b)^3/a/(b+a*cos(f*x+e)^2)-1/2*b^4/a^2/(a+b)^3/f/(b+a
*cos(f*x+e)^2)-1/4/f/(a+b)^2/(1+cos(f*x+e))-1/2/f/(a+b)^3*ln(1+cos(f*x+e))*
a-3/2/f/(a+b)^3*ln(1+cos(f*x+e))*b+1/4/f/(a+b)^2/(-1+cos(f*x+e))-1/2/f/(a+b
)^3*ln(-1+cos(f*x+e))*a-3/2/f/(a+b)^3*ln(-1+cos(f*x+e))*b
```

Maxima [A] time = 1.02126, size = 259, normalized size = 2.33

$$-\frac{\frac{\left(3 a b^{2}+b^{3}\right) \log \left(a \sin \left(f x+e\right)^{2}-a-b\right)}{a^{5}+3 a^{4} b+3 a^{3} b^{2}+a^{2} b^{3}}+\frac{\left(a+3 b\right) \log \left(\sin \left(f x+e\right)^{2}\right)}{a^{3}+3 a^{2} b+3 a b^{2}+b^{3}}-\frac{a^{3}+a^{2} b-\left(a^{3}-b^{3}\right) \sin \left(f x+e\right)^{2}}{\left(a^{5}+2 a^{4} b+a^{3} b^{2}\right) \sin \left(f x+e\right)^{4}-\left(a^{5}+3 a^{4} b+3 a^{3} b^{2}+a^{2} b^{3}\right) \sin \left(f x+e\right)^{2}}}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^5 + 3*a^4*b + 3*a^3*
b^2 + a^2*b^3) + (a + 3*b)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 + b
^3) - (a^3 + a^2*b - (a^3 - b^3)*sin(f*x + e)^2)/((a^5 + 2*a^4*b + a^3*b^2)
*sin(f*x + e)^4 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

Fricas [B] time = 1.53509, size = 664, normalized size = 5.98

$$\frac{a^{3} b+a^{2} b^{2}+a b^{3}+b^{4}+\left(a^{4}+a^{3} b-a b^{3}-b^{4}\right) \cos \left(f x+e\right)^{2}-\left(\left(3 a^{2} b^{2}+a b^{3}\right) \cos \left(f x+e\right)^{4}-3 a b^{3}-b^{4}-\left(3 a^{2} b^{2}-2 a b^{3}\right.\right.}{2 \left(\left(a^{6}+3 a^{5} b+3 a^{4} b^{2}+a^{3} b^{3}\right) f \cos \left(f x+e\right)^{4}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*b + a^2*b^2 + a*b^3 + b^4 + (a^4 + a^3*b - a*b^3 - b^4)*cos(f*x +
e)^2 - ((3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - 3*a*b^3 - b^4 - (3*a^2*b^2 - 2
*a*b^3 - b^4)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 2*((a^4 + 3*a^3*b
)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - (a^4 + 2*a^3*b - 3*a^2*b^2)*cos(f*x
+ e)^2)*log(1/2*sin(f*x + e)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos
(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5
*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.94323, size = 1165, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(3*a*b^2 + b^3)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3) + 12*(a + 3*b)*log(-(cos(f*x + e) - 1)/(cos(f
*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^4 + 6*a^3*b + 3*a^2*b^
2 + 10*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a^3*b*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 30*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
 32*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b^4*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 11*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 22*
a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^2*(cos(f*x + e)
- 1)^2/(cos(f*x + e) + 1)^2 + 16*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
```

```
1)^2 - 16*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 4*a^4*(cos(f*x +
 e) - 1)^3/(cos(f*x + e) + 1)^3 + 16*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x +
e) + 1)^3 + 36*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 32*a*b^3
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*b^4*(cos(f*x + e) - 1)^3/(co
s(f*x + e) + 1)^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x + e)
 - 1)^3/(cos(f*x + e) + 1)^3)) - 3*(cos(f*x + e) - 1)/((a^2 + 2*a*b + b^2)*
(cos(f*x + e) + 1)) - 24*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/a^
2)/f
```

3.355 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=140

$$\frac{b^4}{2a^2f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2+4ab+6b^2\right)\log(\sin(e+fx))}{f(a+b)^4}+\frac{b^3(4a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^4}-\frac{\csc^4(e+}{4f(a+}$$

```
[Out] b^4/(2*a^2*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) + ((a + 2*b)*Csc[e + f*x]^2)
/((a + b)^3*f) - Csc[e + f*x]^4/(4*(a + b)^2*f) + (b^3*(4*a + b)*Log[b + a*
Cos[e + f*x]^2])/(2*a^2*(a + b)^4*f) + ((a^2 + 4*a*b + 6*b^2)*Log[Sin[e + f
*x]])/((a + b)^4*f)
```

Rubi [A] time = 0.197253, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{b^4}{2a^2f(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2+4ab+6b^2\right)\log(\sin(e+fx))}{f(a+b)^4}+\frac{b^3(4a+b)\log\left(a\cos^2(e+fx)+b\right)}{2a^2f(a+b)^4}-\frac{\csc^4(e+}{4f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] b^4/(2*a^2*(a + b)^3*f*(b + a*Cos[e + f*x]^2)) + ((a + 2*b)*Csc[e + f*x]^2)
/((a + b)^3*f) - Csc[e + f*x]^4/(4*(a + b)^2*f) + (b^3*(4*a + b)*Log[b + a*
Cos[e + f*x]^2])/(2*a^2*(a + b)^4*f) + ((a^2 + 4*a*b + 6*b^2)*Log[Sin[e + f
*x]])/((a + b)^4*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^9}{\left(1-x^2\right)^3\left(b+ax^2\right)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^3(b+ax)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)^3} - \frac{2(a+2b)}{(a+b)^3(-1+x)^2} + \frac{-a^2-4ab-6b^2}{(a+b)^4(-1+x)} + \frac{b^4}{a(a+b)^3(b+ax)^2} - \frac{b^3(4a+b)}{a(a+b)^4(b+ax)}\right) dx, x,\right.}{2f}$$

$$= \frac{b^4}{2a^2(a+b)^3 f\left(b+a\cos^2(e+fx)\right)} + \frac{(a+2b)\csc^2(e+fx)}{(a+b)^3 f} - \frac{\csc^4(e+fx)}{4(a+b)^2 f} + \frac{b^3(4a+b)\log}{2a^2}$$

Mathematica [A] time = 2.00365, size = 162, normalized size = 1.16

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(\frac{2b^4(a+b)}{a^2\left(-a\sin^2(e+fx)+a+b\right)} + \frac{2b^3(4a+b)\log\left(-a\sin^2(e+fx)+a+b\right)}{a^2} + 4\left(a^2+4ab+6b^2\right)\log(\sin(\right.}{16f(a+b)^4\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(4*(a + b)*(a + 2*b)*Csc[e
 + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 4*a*b + 6*b^2)*Log[Sin[e +
f*x]] + (2*b^3*(4*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (2*b^4*(a + b
))/(a^2*(a + b - a*Sin[e + f*x]^2))))/(16*(a + b)^4*f*(a + b*Sec[e + f*x]^2
)^2)
```

Maple [B] time = 0.126, size = 374, normalized size = 2.7

$$2\,\frac{b^3\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{f\left(a+b\right)^4 a}+\frac{b^4\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2\,f\left(a+b\right)^4 a^2}+\frac{b^4}{2\,f\left(a+b\right)^4 a\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}+\frac{}{2\,f\left(a+b\right)^4 a^2 \left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 2/f*b^3/(a+b)^4/a*ln(b+a*cos(f*x+e)^2)+1/2/f*b^4/(a+b)^4/a^2*ln(b+a*cos(f*x
+e)^2)+1/2/f*b^4/(a+b)^4/a/(b+a*cos(f*x+e)^2)+1/2/f*b^5/(a+b)^4/a^2/(b+a*co
s(f*x+e)^2)-1/16/f/(a+b)^2/(1+cos(f*x+e))^2+7/16/f/(a+b)^3/(1+cos(f*x+e))*a
+15/16/f/(a+b)^3/(1+cos(f*x+e))*b+1/2/f/(a+b)^4*ln(1+cos(f*x+e))*a^2+2/f/(a
+b)^4*ln(1+cos(f*x+e))*a*b+3/f/(a+b)^4*ln(1+cos(f*x+e))*b^2-1/16/f/(a+b)^2/
(-1+cos(f*x+e))^2-7/16/f/(a+b)^3/(-1+cos(f*x+e))*a-15/16/f/(a+b)^3/(-1+cos(
f*x+e))*b+1/2/f/(a+b)^4*ln(-1+cos(f*x+e))*a^2+2/f/(a+b)^4*ln(-1+cos(f*x+e))
*a*b+3/f/(a+b)^4*ln(-1+cos(f*x+e))*b^2
```

Maxima [B] time = 1.01132, size = 377, normalized size = 2.69

$$\frac{\frac{2\left(4\,ab^3+b^4\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4}+\frac{2\left(a^2+4\,ab+6\,b^2\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(2\,a^4+4\,a^3b-b^4\right)\sin\left(fx+e\right)^4+a^4+2\,a^3b+a^2b^2-\left(5\,a^4+13\,a^3b+8\,a^2b^2\right)\sin\left(fx+\right.}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sin\left(fx+e\right)^6-\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4\right)\sin\left(fx+\right.}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4) + 2*(a^2 + 4*a*b + 6*b^2)*log(sin(f*x + e)^2)/(
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*(2*a^4 + 4*a^3*b - b^4)*sin
(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - (5*a^4 + 13*a^3*b + 8*a^2*b^2)*sin(
f*x + e)^2)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*x + e)^6 - (a^6 +
4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sin(f*x + e)^4))/f
```

Fricas [B] time = 2.69687, size = 1196, normalized size = 8.54

$$\frac{3\,a^4b + 10\,a^3b^2 + 7\,a^2b^3 + 2\,ab^4 + 2\,b^5 - 2\left(2\,a^5 + 6\,a^4b + 4\,a^3b^2 - ab^4 - b^5\right)\cos\left(fx + e\right)^4 + \left(3\,a^5 + 6\,a^4b - 5\,a^3b^2 - 8\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^4*b + 10*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 2*b^5 - 2*(2*a^5 + 6*a^4*
b + 4*a^3*b^2 - a*b^4 - b^5)*cos(f*x + e)^4 + (3*a^5 + 6*a^4*b - 5*a^3*b^2
- 8*a^2*b^3 - 4*a*b^4 - 4*b^5)*cos(f*x + e)^2 + 2*((4*a^2*b^3 + a*b^4)*cos(
f*x + e)^6 + 4*a*b^4 + b^5 - (8*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (
4*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*
((a^5 + 4*a^4*b + 6*a^3*b^2)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3
 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 - 6*a^2*b^3)*cos(f*x + e)^4 + (a^5 + 2*a^4*
b - 2*a^3*b^2 - 12*a^2*b^3)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^7 +
4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^
6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7
+ 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2
 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57041, size = 1305, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")

[Out] 1/64*(32*(4*a*b^3 + b^4)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 4*
a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4) + 32*(a^2 + 4*a*b + 6*b^2)*log(-(c
os(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
 b^4) - (12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 40*a*b*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 28*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2*a*b*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 40*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) +
 1) + 28*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 192*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + 288*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)
^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1)^2) - 32*
(4*a^2*b^3 + 5*a*b^4 + b^5 + 8*a^2*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) - 2*a*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^5*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 4*a^2*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ 5*a*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^5*(cos(f*x + e) - 1
)^2/(cos(f*x + e) + 1)^2)/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4
)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - 64*log(-(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 1)/a^2)/f
```

3.356 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{(3a-2b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f}-\frac{x}{a^2}+\frac{(3a+b)\tan(e+fx)}{2ab^2f}-\frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] -(x/a^2) - ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b]])/(2*a^2*b^(5/2)*f) + ((3*a + b)*Tan[e + f*x])/(2*a*b^2*f) - ((a + b)*
Tan[e + f*x]^3)/(2*a*b*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.266504, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 582, 522, 203, 205}

$$-\frac{(3a-2b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f}-\frac{x}{a^2}+\frac{(3a+b)\tan(e+fx)}{2ab^2f}-\frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) - ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b]])/(2*a^2*b^(5/2)*f) + ((3*a + b)*Tan[e + f*x])/(2*a*b^2*f) - ((a + b)*
Tan[e + f*x]^3)/(2*a*b*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(3a+b)x^2\right)}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2abf}$$

$$= \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{(a+b)(3a+b)+\left(3a^2+4ab-b^2\right)x^2}{(1+x^2)\left(a+b+bx^2\right)}\right.}{2ab^2f}$$

$$= \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f}$$

$$= -\frac{x}{a^2} - \frac{(3a-2b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f} + \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e}{2abf\left(a+b+b\tan\right.}$$

Mathematica [C] time = 4.8728, size = 286, normalized size = 2.4

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\frac{(a+b)^2((a+2b)\sin(2e)-a\sin(2fx))}{a^2b^2f(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + \frac{(3a-2b)(a+b)^{3/2}(\cos(2e)-i\sin(2e))(a\cos(2(e+fx))+a+2b)\tan^{-1}}{a^2b^2f\sqrt{b(\cos(e)-i\sin}}\right.}{8\left(a+b\sec^2(e+fx)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(e
 + f*x)]))/a^2 + ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*
Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[
2*e]))/(a^2*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos[2*(e
 + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x])/(b^2*f) - ((a + b)^2*((a + 2*b)*Sin
[2*e] - a*Sin[2*f*x]))/(a^2*b^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8
```

```
*(a + b*Sec[e + f*x]^2)^2)
```

Maple [B] time = 0.09, size = 242, normalized size = 2.

$$\frac{\tan \left(f x + e\right)}{b^{2} f} - \frac{\arctan \left(\tan \left(f x + e\right)\right)}{f a^{2}} + \frac{a \tan \left(f x + e\right)}{2 b^{2} f \left(a + b + b \left(\tan \left(f x + e\right)\right)^{2}\right)} + \frac{\tan \left(f x + e\right)}{f b \left(a + b + b \left(\tan \left(f x + e\right)\right)^{2}\right)} + \frac{}{2 a f \left(a \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] tan(f*x+e)/b^2/f-1/f/a^2*arctan(tan(f*x+e))+1/2/f*a/b^2*tan(f*x+e)/(a+b+b*t
an(f*x+e)^2)+1/f/b*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*tan(f*x+e)/a/f/(a+b+
b*tan(f*x+e)^2)-3/2/f*a/b^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(
1/2))-2/f/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2/f/a/((
a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^2*b/((a+b)*b)^(1/2
)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.637105, size = 1197, normalized size = 10.06

$$\left[-\frac{8 a b^{2} f x \cos \left(f x + e\right)^{3} + 8 b^{3} f x \cos \left(f x + e\right) + \left(\left(3 a^{3} + a^{2} b - 2 a b^{2}\right) \cos \left(f x + e\right)^{3} + \left(3 a^{2} b + a b^{2} - 2 b^{3}\right) \cos \left(f x + e\right)\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b^2*f*x*cos(f*x + e)^3 + 8*b^3*f*x*cos(f*x + e) + ((3*a^3 + a^2*
b - 2*a*b^2)*cos(f*x + e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt(
-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*c
os(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(
a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
b^2)) - 4*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*x + e)
)/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e)), -1/4*(4*a*b^2*f*x*co
s(f*x + e)^3 + 4*b^3*f*x*cos(f*x + e) - ((3*a^3 + a^2*b - 2*a*b^2)*cos(f*x
+ e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a + b)/b)*arctan(1/2
*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f
*x + e))) - 2*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*x
+ e))/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 3.98214, size = 220, normalized size = 1.85

$$-\frac{\frac{2\left(fx+e\right)}{a^2}-\frac{2\tan\left(fx+e\right)}{b^2}+\frac{\left(3a^3+4a^2b-ab^2-2b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^2b^2}-\frac{a^2\tan\left(fx+e\right)+2ab\tan\left(fx+e\right)+b^2\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)ab^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(f*x + e)/a^2 - 2*tan(f*x + e)/b^2 + (3*a^3 + 4*a^2*b - a*b^2 - 2*b
^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
```

```
b^2)))/(sqrt(a*b + b^2)*a^2*b^2) - (a^2*tan(f*x + e) + 2*a*b*tan(f*x + e)
+ b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b^2))/f
```

3.357 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{(a-2b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f}+\frac{x}{a^2}-\frac{(a+b)\tan(e+fx)}{2abf\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(2*a^2*b^(3/2)*f) - ((a + b)*Tan[e + f*x])/(2*a*b*f*(a + b + b*Tan[e + f*x]
^2))
```

Rubi [A] time = 0.176449, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 470, 522, 203, 205}

$$\frac{(a-2b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f}+\frac{x}{a^2}-\frac{(a+b)\tan(e+fx)}{2abf\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] x/a^2 + ((a - 2*b)*Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(2*a^2*b^(3/2)*f) - ((a + b)*Tan[e + f*x])/(2*a*b*f*(a + b + b*Tan[e + f*x]
^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{((a-2b)(a+b))\,\text{Su}}{}$$

$$= \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 2.98309, size = 249, normalized size = 2.77

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{\left(-a^2+ab+2b^2\right)(\cos(2e)-i\sin(2e))(a\cos(2(e+fx))+a+2b)\tan^{-1}\left(\frac{(\cos(2e)-i\sin(2e))\sec(fx)(a\sin(2e+fx)-(a+2b)s}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{bf\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + ((-a^2 + a*b + 2*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-
((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(b*Sq
rt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + ((a + b)*((a + 2*b)*Sin[2*e] -
 a*Sin[2*f*x]))/(b*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*S
ec[e + f*x]^2)^2)
```

Maple [B] time = 0.096, size = 168, normalized size = 1.9

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^{2}}-\frac{\tan\left(fx+e\right)}{2fb\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)}+\frac{1}{2fb}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)b}}\right)\frac{1}{\sqrt{(a+b)b}}-\frac{1}{2fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f/a^2*arctan(tan(f*x+e))-1/2/f/b*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2/f/b/
((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/2/f/a/((a+b)*b)^(1/2
)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)
^2)-1/f/a^2*b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.609202, size = 933, normalized size = 10.37

$$\left[\frac{8abfx\cos\left(fx+e\right)^{2}+8b^{2}fx-4\left(a^{2}+ab\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(\left(a^{2}-2ab\right)\cos\left(fx+e\right)^{2}+ab-2b^{2}\right)\sqrt{-\frac{a+b}{b}}\,\mathrm{l}}{8\left(a^{3}bf\cos\left(fx+e\right)^{2}+a^{2}b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*f*x*cos(f*x + e)^2 + 8*b^2*f*x - 4*(a^2 + a*b)*cos(f*x + e)*sin
(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(-(a + b)/b)*l
```

```
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^3*b
*f*cos(f*x + e)^2 + a^2*b^2*f), 1/4*(4*a*b*f*x*cos(f*x + e)^2 + 4*b^2*f*x -
 2*(a^2 + a*b)*cos(f*x + e)*sin(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 +
a*b - 2*b^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt
((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))))/(a^3*b*f*cos(f*x + e)^2 +
 a^2*b^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 2.25342, size = 170, normalized size = 1.89

$$\frac{\frac{2(fx+e)}{a^2}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2-ab-2b^2)}{\sqrt{ab+b^2}\,a^2b}-\frac{a\tan(fx+e)+b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)ab}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(f*x + e)/a^2 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))*(a^2 - a*b - 2*b^2)/(sqrt(a*b + b^2)*a^2*b) - (a
*tan(f*x + e) + b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b))/f
```

3.358 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=85

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}f\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] -(x/a^2) + ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^2*Sq
rt[b]*Sqrt[a + b]*f) + Tan[e + f*x]/(2*a*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.15566, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4141, 1975, 471, 522, 203, 205}

$$\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}f\sqrt{a+b}}-\frac{x}{a^2}+\frac{\tan(e+fx)}{2af\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*a^2*Sq
rt[b]*Sqrt[a + b]*f) + Tan[e + f*x]/(2*a*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2af}$$

$$= \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{}{a\cdots}\right.}{}$$

$$= -\frac{x}{a^2} + \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}f} + \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 7.70941, size = 346, normalized size = 4.07

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(\frac{\frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a\cos(2(e+fx))+a+2b)}}{b^{3/2}f} - \frac{\frac{(a^2+8ab+8b^2)((a+2b)\sin(2e)-a\sin(2fx))}{bf(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))(a\cos(2(e+fx)\cdots}}{\cdots}\right.}{64\left(a+b\sec^2(e+fx)\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(-((16*x + ((-a^3 + 6*a^2*
b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b
)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])
^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a +
b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))/a
^2) + (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)
 - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(
```

```
b^(3/2)*f)))/(64*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.096, size = 108, normalized size = 1.3

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^{2}}+\frac{\tan\left(fx+e\right)}{2\,af\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)}+\frac{1}{2\,af}\arctan\left(\tan\left(fx+e\right)b\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}+\frac{b}{fa^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/f/a^2*arctan(tan(f*x+e))+1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+1/2/f/a
/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/f/a^2*b/((a+b)*b)^(
1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.597478, size = 1052, normalized size = 12.38

$$\left[-\frac{8\left(a^{2}b+ab^{2}\right)fx\cos\left(fx+e\right)^{2}+8\left(ab^{2}+b^{3}\right)fx-4\left(a^{2}b+ab^{2}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(\left(a^{2}+2\,ab\right)\cos\left(fx+e\right)^{2}+\right.}{8\left(\left(a^{4}b+a^{3}b^{2}\right)f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 8*(a*b^2 + b^3)*f*x - 4*(a^2*
b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b
+ 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))
*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^3*b^2 + a^2*b^3)*f)
, -1/4*(4*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b^2 + b^3)*f*x - 2*(a^2
*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b
 + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a
*b + b^2)*cos(f*x + e)*sin(f*x + e))))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2
+ (a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

Giac [A] time = 1.58837, size = 134, normalized size = 1.58

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2b)}{\sqrt{ab+b^2}\,a^2}-\frac{2(fx+e)}{a^2}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(a + 2*b)/(sqrt(a*b + b^2)*a^2) - 2*(f*x + e)/a^2 + tan(f*x + e)/(
(b*tan(f*x + e)^2 + a + b)*a))/f
```

3.359 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2} dx$

Optimal. Leaf size=92

$$-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rubi [A] time = 0.0854052, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4128, 414, 522, 203, 205}

$$-\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{3/2}}+\frac{x}{a^2}-\frac{b\tan(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] x/a^2 - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(3/2)*f) - (b*Tan[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e +
f*x]^2))
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a+b-bx^2}{(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e+fx)\right)}{a^2f} - \frac{(b(3a+2b)}{}$$

$$= \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

Mathematica [C] time = 1.95369, size = 240, normalized size = 2.61

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(2x(a\cos(2(e+fx))+a+2b)+\frac{b((a+2b)\sin(2e)-a\sin(2fx))}{f(a+b)(\cos(e)-\sin(e))(\sin(e)+\cos(e))}+\frac{b(3a+2b)(\cos(2e)-i\sin}{}\right.}{8a^2\left(a+b\sec^2(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e +
 f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/
2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]
))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Sec[e +
f*x]^2)^2)
```

Maple [A] time = 0.08, size = 127, normalized size = 1.4

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^2}-\frac{b\tan\left(fx+e\right)}{2\left(a+b\right)af\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)}-\frac{3b}{2\left(a+b\right)af}\arctan\left(b\tan\left(fx+e\right)\frac{1}{\sqrt{(a+b)b}}\right)\frac{}{\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f/a^2*arctan(tan(f*x+e))-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)-
3/2/f*b/a/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f*b^
2/a^2/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.621909, size = 1027, normalized size = 11.16

$$\left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2-4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right)+8\left(ab+b^2\right)fx+\left(\left(3\,a^2+2\,ab\right)\cos\left(fx+e\right)^2+3\,ab+2\,b\right.}{8\left(\left(a^4+a^3b\right)f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt(-
b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*co
s(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*
x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f
), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*x + e)
+ 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b
^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

Giac [A] time = 1.26367, size = 161, normalized size = 1.75

$$-\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+2\,b^2)}{(a^3+a^2b)\sqrt{ab+b^2}}+\frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)}-\frac{2\,(fx+e)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e)/
((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

3.360 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=121

$$\frac{b^{3/2}(5a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{5/2}}-\frac{x}{a^2}-\frac{(2a-b)\cot(e+fx)}{2af(a+b)^2}-\frac{b\cot(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] -(x/a^2) + (b^(3/2)*(5*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(5/2)*f) - ((2*a - b)*Cot[e + f*x])/(2*a*(a + b)^2*f) - (b*
Cot[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.253772, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 203, 205}

$$\frac{b^{3/2}(5a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{5/2}}-\frac{x}{a^2}-\frac{(2a-b)\cot(e+fx)}{2af(a+b)^2}-\frac{b\cot(e+fx)}{2af(a+b)\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] -(x/a^2) + (b^(3/2)*(5*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(5/2)*f) - ((2*a - b)*Cot[e + f*x])/(2*a*(a + b)^2*f) - (b*
Cot[e + f*x])/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-3bx^2}{x^2(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{2a^2+6ab+b^2+(2a-b)}{(1+x^2)\left(a+b+bx^2\right)}\right.}{2a(a+}$$

$$= -\frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(\right.}{a^2f}$$

$$= -\frac{x}{a^2} + \frac{b^{3/2}(5a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}f} - \frac{(2a-b)\cot(e+fx)}{2a(a+b)^2f} - \frac{b\cot(e+f}{2a(a+b)f\left(a+b+b\,\mathrm{t}\right.}$$

Mathematica [C] time = 3.92856, size = 288, normalized size = 2.38

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b^2(a\sin(2fx)-(a+2b)\sin(2e))}{a^2f(a+b)^2(\cos(e)-\sin(e))(\sin(e)+\cos(e))} - \frac{b^2(5a+2b)(\cos(2e)-i\sin(2e))(a\cos(2(e+fx))+a+2b)\tan^{-1}\left(\right.}{a^2f(a+b)^{5/2}\sqrt{b(\cos(}}\right.}{8\left(a+b\sec^2(e+fx\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(e
 + f*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] -
 I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(a^
2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos[2*(e
 + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x])/((a + b)^2*f) + (b^2*(-((a + 2*b)*S
in[2*e]) + a*Sin[2*f*x]))/(a^2*(a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[
```

```
e]))))/(8*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.104, size = 149, normalized size = 1.2

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f a^2}-\frac{1}{f\left(a+b\right)^2\tan\left(fx+e\right)}+\frac{b^2\tan\left(fx+e\right)}{2f\left(a+b\right)^2 a\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)}+\frac{5b^2}{2f\left(a+b\right)^2 a}\arctan\left(\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/f/a^2*arctan(tan(f*x+e))-1/f/(a+b)^2/tan(f*x+e)+1/2/f*b^2/(a+b)^2/a*tan(
f*x+e)/(a+b+b*tan(f*x+e)^2)+5/2/f*b^2/(a+b)^2/a/((a+b)*b)^(1/2)*arctan(tan(
f*x+e)*b/((a+b)*b)^(1/2))+1/f*b^3/(a+b)^2/a^2/((a+b)*b)^(1/2)*arctan(tan(f*
x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.724258, size = 1384, normalized size = 11.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a^3 + a*b^2)*cos(f*x + e)^3 - (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a
*b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
```

```
e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x
 + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a
^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 4*(2*a^2*b
- a*b^2)*cos(f*x + e) + 8*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^
2*b + 2*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f
*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), -1/4*(2*(2*a^3
+ a*b^2)*cos(f*x + e)^3 + (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a*b^2)*cos(f*x +
e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
 b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(2*a^2*b - a*b^2)*cos(
f*x + e) + 4*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^2*b + 2*a*b^2
 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (
a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37533, size = 247, normalized size = 2.04

$$\frac{\frac{\left(5\,ab^2+2\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{ab+b^2}}-\frac{2\,ab\tan\left(fx+e\right)^2-b^2\tan\left(fx+e\right)^2+2\,a^2+2\,ab}{\left(b\tan\left(fx+e\right)^3+a\tan\left(fx+e\right)+b\tan\left(fx+e\right)\right)\left(a^3+2\,a^2b+ab^2\right)}-\frac{2\left(fx+e\right)}{a^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((5*a*b^2 + 2*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b + b^2)) - (2
*a*b*tan(f*x + e)^2 - b^2*tan(f*x + e)^2 + 2*a^2 + 2*a*b)/((b*tan(f*x + e)^
3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^3 + 2*a^2*b + a*b^2)) - 2*(f*x + e)
/a^2)/f
```

3.361 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=160

$$-\frac{b^{5/2}(7a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{7/2}}+\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2af(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\cot^3(e+fx)}{6af(a+b)^2}-\frac{b\cot^3(e+}{2af(a+b)\left(a+b\tan\right.}$$

```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(7/2)*f) + ((2*a^2 + 6*a*b - b^2)*Cot[e + f*x])/(2*a*(a + b)^3
*f) - ((2*a - 3*b)*Cot[e + f*x]^3)/(6*a*(a + b)^2*f) - (b*Cot[e + f*x]^3)/(
2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.353714, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 203, 205}

$$-\frac{b^{5/2}(7a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{7/2}}+\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2af(a+b)^3}+\frac{x}{a^2}-\frac{(2a-3b)\cot^3(e+fx)}{6af(a+b)^2}-\frac{b\cot^3(e+}{2af(a+b)\left(a+b\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] x/a^2 - (b^(5/2)*(7*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2
*a^2*(a + b)^(7/2)*f) + ((2*a^2 + 6*a*b - b^2)*Cot[e + f*x])/(2*a*(a + b)^3
*f) - ((2*a - 3*b)*Cot[e + f*x]^3)/(6*a*(a + b)^2*f) - (b*Cot[e + f*x]^3)/(
2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-3b-5bx^2}{x^4(1+x^2)\left(a+b+bx^2\right)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{3\left(2a^2+6ab-b^2\right)+3(2}{x^2(1+x^2)(a+b-}\right.}{6a(a}$$

$$= \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx\right.}$$

$$= \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx\right.}$$

$$= \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}f} + \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{2a(a+b)^3f} - \frac{(2a-3b)\cot^3(e-}{6a(a+b)^2f}$$

Mathematica [C] time = 6.93164, size = 1896, normalized size = 11.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((7*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*((b^3*ArcTan[S
ec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*
Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2
*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(8*a^2*Sqrt[a + b]*f*Sqrt[b*Cos[
4*e] - I*b*Sin[4*e]]) - ((I/8)*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]
*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Co
s[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]
*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^3
```

```
*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[e +
 f*x]^3*Sec[2*e]*Sec[e + f*x]^4*(-6*a^4*f*x*Cos[f*x] - 54*a^3*b*f*x*Cos[f*x
] - 126*a^2*b^2*f*x*Cos[f*x] - 114*a*b^3*f*x*Cos[f*x] - 36*b^4*f*x*Cos[f*x]
 + 3*a^4*f*x*Cos[3*f*x] - 3*a^3*b*f*x*Cos[3*f*x] - 27*a^2*b^2*f*x*Cos[3*f*x
] - 33*a*b^3*f*x*Cos[3*f*x] - 12*b^4*f*x*Cos[3*f*x] + 6*a^4*f*x*Cos[2*e - f
*x] + 54*a^3*b*f*x*Cos[2*e - f*x] + 126*a^2*b^2*f*x*Cos[2*e - f*x] + 114*a*
b^3*f*x*Cos[2*e - f*x] + 36*b^4*f*x*Cos[2*e - f*x] + 6*a^4*f*x*Cos[2*e + f*
x] + 54*a^3*b*f*x*Cos[2*e + f*x] + 126*a^2*b^2*f*x*Cos[2*e + f*x] + 114*a*b
^3*f*x*Cos[2*e + f*x] + 36*b^4*f*x*Cos[2*e + f*x] - 6*a^4*f*x*Cos[4*e + f*x
] - 54*a^3*b*f*x*Cos[4*e + f*x] - 126*a^2*b^2*f*x*Cos[4*e + f*x] - 114*a*b^
3*f*x*Cos[4*e + f*x] - 36*b^4*f*x*Cos[4*e + f*x] - 3*a^4*f*x*Cos[2*e + 3*f*
x] + 3*a^3*b*f*x*Cos[2*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[2*e + 3*f*x] + 33*a*
b^3*f*x*Cos[2*e + 3*f*x] + 12*b^4*f*x*Cos[2*e + 3*f*x] + 3*a^4*f*x*Cos[4*e
+ 3*f*x] - 3*a^3*b*f*x*Cos[4*e + 3*f*x] - 27*a^2*b^2*f*x*Cos[4*e + 3*f*x] -
 33*a*b^3*f*x*Cos[4*e + 3*f*x] - 12*b^4*f*x*Cos[4*e + 3*f*x] - 3*a^4*f*x*Co
s[6*e + 3*f*x] + 3*a^3*b*f*x*Cos[6*e + 3*f*x] + 27*a^2*b^2*f*x*Cos[6*e + 3*
f*x] + 33*a*b^3*f*x*Cos[6*e + 3*f*x] + 12*b^4*f*x*Cos[6*e + 3*f*x] - 3*a^4*
f*x*Cos[2*e + 5*f*x] - 9*a^3*b*f*x*Cos[2*e + 5*f*x] - 9*a^2*b^2*f*x*Cos[2*e
 + 5*f*x] - 3*a*b^3*f*x*Cos[2*e + 5*f*x] + 3*a^4*f*x*Cos[4*e + 5*f*x] + 9*a
^3*b*f*x*Cos[4*e + 5*f*x] + 9*a^2*b^2*f*x*Cos[4*e + 5*f*x] + 3*a*b^3*f*x*Co
s[4*e + 5*f*x] - 3*a^4*f*x*Cos[6*e + 5*f*x] - 9*a^3*b*f*x*Cos[6*e + 5*f*x]
- 9*a^2*b^2*f*x*Cos[6*e + 5*f*x] - 3*a*b^3*f*x*Cos[6*e + 5*f*x] + 3*a^4*f*x
*Cos[8*e + 5*f*x] + 9*a^3*b*f*x*Cos[8*e + 5*f*x] + 9*a^2*b^2*f*x*Cos[8*e +
5*f*x] + 3*a*b^3*f*x*Cos[8*e + 5*f*x] - 12*a^4*Sin[f*x] - 60*a^3*b*Sin[f*x]
 - 96*a^2*b^2*Sin[f*x] + 18*b^4*Sin[f*x] + 4*a^4*Sin[3*f*x] + 36*a^3*b*Sin[
3*f*x] + 80*a^2*b^2*Sin[3*f*x] - 6*a*b^3*Sin[3*f*x] + 6*b^4*Sin[3*f*x] + 4*
a^4*Sin[2*e - f*x] + 76*a^3*b*Sin[2*e - f*x] + 144*a^2*b^2*Sin[2*e - f*x] +
 18*b^4*Sin[2*e - f*x] - 4*a^4*Sin[2*e + f*x] - 76*a^3*b*Sin[2*e + f*x] - 1
44*a^2*b^2*Sin[2*e + f*x] + 6*a*b^3*Sin[2*e + f*x] + 18*b^4*Sin[2*e + f*x]
- 12*a^4*Sin[4*e + f*x] - 60*a^3*b*Sin[4*e + f*x] - 96*a^2*b^2*Sin[4*e + f*
x] - 6*a*b^3*Sin[4*e + f*x] - 18*b^4*Sin[4*e + f*x] - 12*a^4*Sin[2*e + 3*f*
x] - 24*a^3*b*Sin[2*e + 3*f*x] + 6*a*b^3*Sin[2*e + 3*f*x] - 6*b^4*Sin[2*e +
 3*f*x] + 4*a^4*Sin[4*e + 3*f*x] + 36*a^3*b*Sin[4*e + 3*f*x] + 80*a^2*b^2*S
in[4*e + 3*f*x] - 3*a*b^3*Sin[4*e + 3*f*x] - 6*b^4*Sin[4*e + 3*f*x] - 12*a^
4*Sin[6*e + 3*f*x] - 24*a^3*b*Sin[6*e + 3*f*x] + 3*a*b^3*Sin[6*e + 3*f*x] +
 6*b^4*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x] + 20*a^3*b*Sin[2*e + 5*f*x
] + 3*a*b^3*Sin[2*e + 5*f*x] - 3*a*b^3*Sin[4*e + 5*f*x] + 8*a^4*Sin[6*e + 5
*f*x] + 20*a^3*b*Sin[6*e + 5*f*x]))/(384*a^2*(a + b)^3*f*(a + b*Sec[e + f*x
]^2)^2)
```

Maple [A] time = 0.118, size = 186, normalized size = 1.2

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f a^2}-\frac{1}{3 f \left(a+b\right)^2 \left(\tan\left(fx+e\right)\right)^3}+\frac{a}{f \left(a+b\right)^3 \tan\left(fx+e\right)}+3\frac{b}{f \left(a+b\right)^3 \tan\left(fx+e\right)}-\frac{}{2 f a \left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] 1/f/a^2*arctan(tan(f*x+e))-1/3/f/(a+b)^2/tan(f*x+e)^3+1/f/(a+b)^3/tan(f*x+e
)*a+3/f/(a+b)^3/tan(f*x+e)*b-1/2/f*b^3/a/(a+b)^3*tan(f*x+e)/(a+b+b*tan(f*x+
e)^2)-7/2/f*b^3/a/(a+b)^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/
2))-1/f*b^4/a^2/(a+b)^3/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.760028, size = 2182, normalized size = 13.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(4*(8*a^4 + 20*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 8*(3*a^4 + 5*a^3*b -
 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x +
e)^4 - 7*a*b^3 - 2*b^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*
cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(
```

```
f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*
cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(
f*x + e) + 24*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^
4 + 2*a^3*b - 2*a*b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4)*f*x)*sin(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos
(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5
*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e)), 1/12*(2*(8*a^4 + 20
*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 4*(3*a^4 + 5*a^3*b - 10*a^2*b^2 + 3*a*b^
3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 - 7*a*b^3 - 2*b
^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1
/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x +
 e)))*sin(f*x + e) - 6*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(f*x + e) + 12*((a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*
b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f*x)*si
n(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6
 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3
*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3606, size = 297, normalized size = 1.86

$$-\frac{\frac{3\,b^3\tan\left(fx+e\right)}{\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{3\left(7\,ab^3+2\,b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sqrt{ab+b^2}}-\frac{6\left(fx+e\right)}{a^2}-\frac{2\left(3\,a\tan\left(fx+e\right)^2+9\,b\tan\left(fx+e\right.\right.}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\tan\left(fx\right.}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^3*tan(f*x + e)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(b*tan(f*x +
e)^2 + a + b)) + 3*(7*a*b^3 + 2*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*sqrt(a*b + b^2)) - 6*(f*x + e)/a^2 - 2*(3*a*tan(f*x + e)^2 + 9*b*tan(f
*x + e)^2 - a - b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

3.362 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

Optimal. Leaf size=207

$$\frac{b^{7/2}(9a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{9/2}}+\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6af(a+b)^3}-\frac{\left(8a^2b+2a^3+12ab^2-b^3\right)\cot(e+fx)}{2af(a+b)^4}-\frac{x}{a^2}-\frac{(2a}{}$$

```
[Out] -(x/a^2) + (b^(7/2)*(9*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(9/2)*f) - ((2*a^3 + 8*a^2*b + 12*a*b^2 - b^3)*Cot[e + f*x]
)/(2*a*(a + b)^4*f) + ((2*a^2 + 6*a*b - 3*b^2)*Cot[e + f*x]^3)/(6*a*(a + b)
^3*f) - ((2*a - 5*b)*Cot[e + f*x]^5)/(10*a*(a + b)^2*f) - (b*Cot[e + f*x]^5
)/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.437015, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 472, 583, 522, 203, 205}

$$\frac{b^{7/2}(9a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2f(a+b)^{9/2}}+\frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6af(a+b)^3}-\frac{\left(8a^2b+2a^3+12ab^2-b^3\right)\cot(e+fx)}{2af(a+b)^4}-\frac{x}{a^2}-\frac{(2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```


```
[Out] -(x/a^2) + (b^(7/2)*(9*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(2*a^2*(a + b)^(9/2)*f) - ((2*a^3 + 8*a^2*b + 12*a*b^2 - b^3)*Cot[e + f*x]
)/(2*a*(a + b)^4*f) + ((2*a^2 + 6*a*b - 3*b^2)*Cot[e + f*x]^3)/(6*a*(a + b)
^3*f) - ((2*a - 5*b)*Cot[e + f*x]^5)/(10*a*(a + b)^2*f) - (b*Cot[e + f*x]^5
)/(2*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-5b-7bx^2}{x^6(1+x^2)(a+b+bx^2)}\,dx, x, \tan(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{5(2a^2+6ab-3b^2)-}{x^4(1+x^2)(a-}\right.}{10a}$$

$$= \frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2f} - \frac{b\cot^5(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e-\right.}$$

$$= -\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{2a(a+b)^4f} + \frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\,\text{co}}{10a(a+}$$

$$= -\frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{2a(a+b)^4f} + \frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{6a(a+b)^3f} - \frac{(2a-5b)\,\text{co}}{10a(a+}$$

$$= -\frac{x}{a^2} + \frac{b^{7/2}(9a+2b)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{9/2}f} - \frac{\left(2a^3+8a^2b+12ab^2-b^3\right)\cot(e+fx)}{2a(a+b)^4f} + \frac{\left(2a^2\right.}{}$$

Mathematica [C] time = 7.32759, size = 3028, normalized size = 14.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

```
[Out] ((9*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-(b^4*ArcTan[
Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)
```

*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(8*a^2*Sqrt[a + b]*f*Sqrt[b*Cos [4*e] - I*b*Sin[4*e]]) + ((I/8)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*C os[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^ 4*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e]*Csc[e + f*x]^5*Sec[2*e]*Sec[e + f*x]^4*(75*a^5*f*x*Cos[f*x] + 900*a^4*b*f*x*Cos[f *x] + 2850*a^3*b^2*f*x*Cos[f*x] + 3900*a^2*b^3*f*x*Cos[f*x] + 2475*a*b^4*f* x*Cos[f*x] + 600*b^5*f*x*Cos[f*x] - 15*a^5*f*x*Cos[3*f*x] + 240*a^4*b*f*x*C os[3*f*x] + 1110*a^3*b^2*f*x*Cos[3*f*x] + 1740*a^2*b^3*f*x*Cos[3*f*x] + 118 5*a*b^4*f*x*Cos[3*f*x] + 300*b^5*f*x*Cos[3*f*x] - 75*a^5*f*x*Cos[2*e - f*x] - 900*a^4*b*f*x*Cos[2*e - f*x] - 2850*a^3*b^2*f*x*Cos[2*e - f*x] - 3900*a^ 2*b^3*f*x*Cos[2*e - f*x] - 2475*a*b^4*f*x*Cos[2*e - f*x] - 600*b^5*f*x*Cos[2*e - f*x] - 75*a^5*f*x*Cos[2*e + f*x] - 900*a^4*b*f*x*Cos[2*e + f*x] - 285 0*a^3*b^2*f*x*Cos[2*e + f*x] - 3900*a^2*b^3*f*x*Cos[2*e + f*x] - 2475*a*b^4 *f*x*Cos[2*e + f*x] - 600*b^5*f*x*Cos[2*e + f*x] + 75*a^5*f*x*Cos[4*e + f*x] + 900*a^4*b*f*x*Cos[4*e + f*x] + 2850*a^3*b^2*f*x*Cos[4*e + f*x] + 3900*a ^2*b^3*f*x*Cos[4*e + f*x] + 2475*a*b^4*f*x*Cos[4*e + f*x] + 600*b^5*f*x*Cos [4*e + f*x] + 15*a^5*f*x*Cos[2*e + 3*f*x] - 240*a^4*b*f*x*Cos[2*e + 3*f*x] - 1110*a^3*b^2*f*x*Cos[2*e + 3*f*x] - 1740*a^2*b^3*f*x*Cos[2*e + 3*f*x] - 1 185*a*b^4*f*x*Cos[2*e + 3*f*x] - 300*b^5*f*x*Cos[2*e + 3*f*x] - 15*a^5*f*x* Cos[4*e + 3*f*x] + 240*a^4*b*f*x*Cos[4*e + 3*f*x] + 1110*a^3*b^2*f*x*Cos[4* e + 3*f*x] + 1740*a^2*b^3*f*x*Cos[4*e + 3*f*x] + 1185*a*b^4*f*x*Cos[4*e + 3 *f*x] + 300*b^5*f*x*Cos[4*e + 3*f*x] + 15*a^5*f*x*Cos[6*e + 3*f*x] - 240*a^ 4*b*f*x*Cos[6*e + 3*f*x] - 1110*a^3*b^2*f*x*Cos[6*e + 3*f*x] - 1740*a^2*b^3 *f*x*Cos[6*e + 3*f*x] - 1185*a*b^4*f*x*Cos[6*e + 3*f*x] - 300*b^5*f*x*Cos[6 *e + 3*f*x] + 45*a^5*f*x*Cos[2*e + 5*f*x] + 120*a^4*b*f*x*Cos[2*e + 5*f*x] + 30*a^3*b^2*f*x*Cos[2*e + 5*f*x] - 180*a^2*b^3*f*x*Cos[2*e + 5*f*x] - 195* a*b^4*f*x*Cos[2*e + 5*f*x] - 60*b^5*f*x*Cos[2*e + 5*f*x] - 45*a^5*f*x*Cos[4 *e + 5*f*x] - 120*a^4*b*f*x*Cos[4*e + 5*f*x] - 30*a^3*b^2*f*x*Cos[4*e + 5*f *x] + 180*a^2*b^3*f*x*Cos[4*e + 5*f*x] + 195*a*b^4*f*x*Cos[4*e + 5*f*x] + 6 0*b^5*f*x*Cos[4*e + 5*f*x] + 45*a^5*f*x*Cos[6*e + 5*f*x] + 120*a^4*b*f*x*Co s[6*e + 5*f*x] + 30*a^3*b^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*b^3*f*x*Cos[6*e + 5*f*x] - 195*a*b^4*f*x*Cos[6*e + 5*f*x] - 60*b^5*f*x*Cos[6*e + 5*f*x] - 4 5*a^5*f*x*Cos[8*e + 5*f*x] - 120*a^4*b*f*x*Cos[8*e + 5*f*x] - 30*a^3*b^2*f* x*Cos[8*e + 5*f*x] + 180*a^2*b^3*f*x*Cos[8*e + 5*f*x] + 195*a*b^4*f*x*Cos[8 *e + 5*f*x] + 60*b^5*f*x*Cos[8*e + 5*f*x] - 15*a^5*f*x*Cos[4*e + 7*f*x] - 6 0*a^4*b*f*x*Cos[4*e + 7*f*x] - 90*a^3*b^2*f*x*Cos[4*e + 7*f*x] - 60*a^2*b^3 *f*x*Cos[4*e + 7*f*x] - 15*a*b^4*f*x*Cos[4*e + 7*f*x] + 15*a^5*f*x*Cos[6*e + 7*f*x] + 60*a^4*b*f*x*Cos[6*e + 7*f*x] + 90*a^3*b^2*f*x*Cos[6*e + 7*f*x] + 60*a^2*b^3*f*x*Cos[6*e + 7*f*x] + 15*a*b^4*f*x*Cos[6*e + 7*f*x] - 15*a^5* f*x*Cos[8*e + 7*f*x] - 60*a^4*b*f*x*Cos[8*e + 7*f*x] - 90*a^3*b^2*f*x*Cos[8 *e + 7*f*x] - 60*a^2*b^3*f*x*Cos[8*e + 7*f*x] - 15*a*b^4*f*x*Cos[8*e + 7*f* x] + 15*a^5*f*x*Cos[10*e + 7*f*x] + 60*a^4*b*f*x*Cos[10*e + 7*f*x] + 90*a^3

```
*b^2*f*x*Cos[10*e + 7*f*x] + 60*a^2*b^3*f*x*Cos[10*e + 7*f*x] + 15*a*b^4*f*
x*Cos[10*e + 7*f*x] - 10*a^5*Sin[f*x] + 860*a^4*b*Sin[f*x] + 3120*a^3*b^2*S
in[f*x] + 3600*a^2*b^3*Sin[f*x] - 300*b^5*Sin[f*x] + 46*a^5*Sin[3*f*x] - 50
8*a^4*b*Sin[3*f*x] - 2324*a^3*b^2*Sin[3*f*x] - 3120*a^2*b^3*Sin[3*f*x] + 75
*a*b^4*Sin[3*f*x] - 150*b^5*Sin[3*f*x] - 240*a^5*Sin[2*e - f*x] - 1840*a^4*
b*Sin[2*e - f*x] - 4840*a^3*b^2*Sin[2*e - f*x] - 5040*a^2*b^3*Sin[2*e - f*x
] - 300*b^5*Sin[2*e - f*x] + 240*a^5*Sin[2*e + f*x] + 1840*a^4*b*Sin[2*e +
f*x] + 4840*a^3*b^2*Sin[2*e + f*x] + 5040*a^2*b^3*Sin[2*e + f*x] - 75*a*b^4
*Sin[2*e + f*x] - 300*b^5*Sin[2*e + f*x] - 10*a^5*Sin[4*e + f*x] + 860*a^4*
b*Sin[4*e + f*x] + 3120*a^3*b^2*Sin[4*e + f*x] + 3600*a^2*b^3*Sin[4*e + f*x
] + 75*a*b^4*Sin[4*e + f*x] + 300*b^5*Sin[4*e + f*x] - 240*a^4*b*Sin[2*e +
3*f*x] - 900*a^3*b^2*Sin[2*e + 3*f*x] - 1200*a^2*b^3*Sin[2*e + 3*f*x] - 75*
a*b^4*Sin[2*e + 3*f*x] + 150*b^5*Sin[2*e + 3*f*x] + 46*a^5*Sin[4*e + 3*f*x]
 - 508*a^4*b*Sin[4*e + 3*f*x] - 2324*a^3*b^2*Sin[4*e + 3*f*x] - 3120*a^2*b^
3*Sin[4*e + 3*f*x] + 60*a*b^4*Sin[4*e + 3*f*x] + 150*b^5*Sin[4*e + 3*f*x] -
 240*a^4*b*Sin[6*e + 3*f*x] - 900*a^3*b^2*Sin[6*e + 3*f*x] - 1200*a^2*b^3*S
in[6*e + 3*f*x] - 60*a*b^4*Sin[6*e + 3*f*x] - 150*b^5*Sin[6*e + 3*f*x] - 48
*a^5*Sin[2*e + 5*f*x] - 32*a^4*b*Sin[2*e + 5*f*x] + 340*a^3*b^2*Sin[2*e + 5
*f*x] + 864*a^2*b^3*Sin[2*e + 5*f*x] - 60*a*b^4*Sin[2*e + 5*f*x] + 30*b^5*S
in[2*e + 5*f*x] - 90*a^5*Sin[4*e + 5*f*x] - 300*a^4*b*Sin[4*e + 5*f*x] - 30
0*a^3*b^2*Sin[4*e + 5*f*x] + 60*a*b^4*Sin[4*e + 5*f*x] - 30*b^5*Sin[4*e + 5
*f*x] - 48*a^5*Sin[6*e + 5*f*x] - 32*a^4*b*Sin[6*e + 5*f*x] + 340*a^3*b^2*S
in[6*e + 5*f*x] + 864*a^2*b^3*Sin[6*e + 5*f*x] - 15*a*b^4*Sin[6*e + 5*f*x]
- 30*b^5*Sin[6*e + 5*f*x] - 90*a^5*Sin[8*e + 5*f*x] - 300*a^4*b*Sin[8*e + 5
*f*x] - 300*a^3*b^2*Sin[8*e + 5*f*x] + 15*a*b^4*Sin[8*e + 5*f*x] + 30*b^5*S
in[8*e + 5*f*x] + 46*a^5*Sin[4*e + 7*f*x] + 172*a^4*b*Sin[4*e + 7*f*x] + 21
6*a^3*b^2*Sin[4*e + 7*f*x] + 15*a*b^4*Sin[4*e + 7*f*x] - 15*a*b^4*Sin[6*e +
 7*f*x] + 46*a^5*Sin[8*e + 7*f*x] + 172*a^4*b*Sin[8*e + 7*f*x] + 216*a^3*b^
2*Sin[8*e + 7*f*x]))/(7680*a^2*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

Maple [A] time = 0.125, size = 248, normalized size = 1.2

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^2}-\frac{1}{5f(a+b)^2\left(\tan\left(fx+e\right)\right)^5}+\frac{a}{3f(a+b)^3\left(\tan\left(fx+e\right)\right)^3}+\frac{b}{f(a+b)^3\left(\tan\left(fx+e\right)\right)^3}-\frac{}{f(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x)
```

```
[Out] -1/f/a^2*arctan(tan(f*x+e))-1/5/f/(a+b)^2/tan(f*x+e)^5+1/3/f/(a+b)^3/tan(f*
x+e)^3*a+1/f/(a+b)^3/tan(f*x+e)^3*b-1/f/(a+b)^4/tan(f*x+e)*a^2-4/f/(a+b)^4/
```

```
tan(f*x+e)*a*b-6/f/(a+b)^4/tan(f*x+e)*b^2+1/2/f*b^4/(a+b)^4/a*tan(f*x+e)/(a
+b+b*tan(f*x+e)^2)+9/2/f*b^4/(a+b)^4/a/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/
((a+b)*b)^(1/2))+1/f*b^5/(a+b)^4/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((
a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.847581, size = 3366, normalized size = 16.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/120*(4*(46*a^5 + 172*a^4*b + 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 4
*(70*a^5 + 234*a^4*b + 218*a^3*b^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5
 + 20*(6*a^5 + 10*a^4*b - 20*a^3*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e)^3
 - 15*((9*a^2*b^3 + 2*a*b^4)*cos(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2*b^3
 - 5*a*b^4 - 2*b^5)*cos(f*x + e)^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*cos(f*x
 + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*
a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*
b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x +
e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(2*a^4*b + 8*a^3*b^2
+ 12*a^2*b^3 - a*b^4)*cos(f*x + e) + 120*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + a*b^4)*f*x*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^
3 - 2*a*b^4 - b^5)*f*x*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*
b^3 - 7*a*b^4 - 2*b^5)*f*x*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3
+ 4*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^
3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 -
2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*
b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*
```

```
b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e)), -1/60*(2*(46*a^5 + 172*a^4*b +
 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 2*(70*a^5 + 234*a^4*b + 218*a^3*b
^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + 10*(6*a^5 + 10*a^4*b - 20*a^3
*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e)^3 + 15*((9*a^2*b^3 + 2*a*b^4)*cos
(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2*b^3 - 5*a*b^4 - 2*b^5)*cos(f*x + e)
^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x
+ e)))*sin(f*x + e) + 30*(2*a^4*b + 8*a^3*b^2 + 12*a^2*b^3 - a*b^4)*cos(f*x
 + e) + 60*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*x*cos(f*x + e
)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*x*cos(f*x
 + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*x*cos
(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*x)*sin(f*x
+ e))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
- (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x
 + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f
*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*
sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46434, size = 419, normalized size = 2.02

$$\frac{\frac{15\,b^4\tan\left(fx+e\right)}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\left(b\tan\left(fx+e\right)^2+a+b\right)}+\frac{15\left(9\,ab^4+2\,b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4\right)\sqrt{ab+b^2}}-\frac{30\left(fx+e\right)}{a^2}-\frac{2\left(15\,a^2\tan\left(fx+e\right)^4+60\right.}{}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*b^4*tan(f*x + e)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*
(b*tan(f*x + e)^2 + a + b)) + 15*(9*a*b^4 + 2*b^5)*(pi*floor((f*x + e)/pi +
 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b + b^2)) - 30*(f*x + e)/a^2 - 2*(15*
a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 - 5*a^2*
tan(f*x + e)^2 - 20*a*b*tan(f*x + e)^2 - 15*b^2*tan(f*x + e)^2 + 3*a^2 + 6*
a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^5))/
f
```

3.363 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} \, dx$

Optimal. Leaf size=78

$$\frac{(a+b)^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{a+b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

```
[Out] (a + b)^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - (a + b)/(a^3*f*(b + a*Cos[e
+ f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rubi [A] time = 0.107703, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 444, 43}

$$\frac{(a+b)^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{a+b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (a + b)^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - (a + b)/(a^3*f*(b + a*Cos[e
+ f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x\left(1-x^2\right)^2}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a+b)^2}{a^2(b+ax)^3} - \frac{2(a+b)}{a^2(b+ax)^2} + \frac{1}{a^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b)^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{a+b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

Mathematica [A] time = 2.19357, size = 136, normalized size = 1.74

$$-\frac{2\left(a^2+4ab+3b^2\right)+a^2\cos^2(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+(a+2b)^2\log(a\cos(2(e+fx))+a+2b)+2a\,c}{2a^3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] +
 a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(e +
f*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(2*a^3*f*(
a + 2*b + a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.089, size = 138, normalized size = 1.8

$$-\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}{2a^{3}f}-\frac{1}{fa^{2}\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}-\frac{b}{a^{3}f\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}+\frac{1}{4fa\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/2*ln(b+a*cos(f*x+e)^2)/a^3/f-1/f/a^2/(b+a*cos(f*x+e)^2)-b/a^3/f/(b+a*cos
(f*x+e)^2)+1/4/f/a/(b+a*cos(f*x+e)^2)^2+1/2/f/a^2/(b+a*cos(f*x+e)^2)^2*b+1/
4*b^2/a^3/f/(b+a*cos(f*x+e)^2)^2
```

Maxima [A] time = 1.02008, size = 151, normalized size = 1.94

$$\frac{\frac{4\left(a^{2}+ab\right)\sin\left(fx+e\right)^{2}-3a^{2}-6ab-3b^{2}}{a^{5}\sin\left(fx+e\right)^{4}+a^{5}+2a^{4}b+a^{3}b^{2}-2\left(a^{5}+a^{4}b\right)\sin\left(fx+e\right)^{2}}-\frac{2\log\left(a\sin\left(fx+e\right)^{2}-a-b\right)}{a^{3}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*(a^2 + a*b)*sin(f*x + e)^2 - 3*a^2 - 6*a*b - 3*b^2)/(a^5*sin(f*x +
e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*
sin(f*x + e)^2 - a - b)/a^3)/f
```

Fricas [A] time = 0.574916, size = 271, normalized size = 3.47

$$-\frac{4\left(a^{2}+ab\right)\cos\left(fx+e\right)^{2}-a^{2}+2ab+3b^{2}+2\left(a^{2}\cos\left(fx+e\right)^{4}+2ab\cos\left(fx+e\right)^{2}+b^{2}\right)\log\left(a\cos\left(fx+e\right)^{2}+b\right)}{4\left(a^{5}f\cos\left(fx+e\right)^{4}+2a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - a^2 + 2*a*b + 3*b^2 + 2*(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f
*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.91474, size = 772, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^3 - 4*log(-(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 1)/a^3 - (3*a^2 + 6*a*b + 3*b^2 + 20*a^2*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) - 12*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 50*a^2*(cos(f*x + e) - 1
)^2/(cos(f*x + e) + 1)^2 - 28*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + 18*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^2*(cos(f*x + e)
- 1)^3/(cos(f*x + e) + 1)^3 + 8*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)
^3 - 12*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^2*(cos(f*x + e)
 - 1)^4/(cos(f*x + e) + 1)^4 + 6*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1
)^4 + 3*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a + b + 2*a*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2)^2*a^3))/f
```

3.364 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal. Leaf size=81

$$-\frac{b(a+b)}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}+\frac{a+2b}{2a^3f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

```
[Out] -(b*(a + b))/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) + (a + 2*b)/(2*a^3*f*(b + a
*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rubi [A] time = 0.111637, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 77}

$$-\frac{b(a+b)}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}+\frac{a+2b}{2a^3f\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(b*(a + b))/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) + (a + 2*b)/(2*a^3*f*(b + a
*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^3\left(1-x^2\right)}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x)x}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{b(a+b)}{a^2(b+ax)^3}+\frac{a+2b}{a^2(b+ax)^2}-\frac{1}{a^2(b+ax)}\right) dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{b(a+b)}{4a^3f\left(b+a\cos^2(e+fx)\right)^2}+\frac{a+2b}{2a^3f\left(b+a\cos^2(e+fx)\right)}+\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

Mathematica [A] time = 0.935679, size = 131, normalized size = 1.62

$$\frac{2\left(a^2+3ab+3b^2\right)+a^2\cos^2(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+(a+2b)^2\log(a\cos(2(e+fx))+a+2b)+2a(a\cdots}{2a^3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] (2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] +
a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*(a + 2*b)*Co
s[2*(e + f*x)]*(1 + Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(2*a^3*f*(a + 2*b +
 a*Cos[2*(e + f*x)])^2)
```

Maple [A] time = 0.083, size = 115, normalized size = 1.4

$$\frac{\ln\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}{2a^{3}f}+\frac{1}{2fa^{2}\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}+\frac{b}{a^{3}f\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)}-\frac{b}{4fa^{2}\left(b+a\left(\cos\left(fx+e\right)\right)^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/2*ln(b+a*cos(f*x+e)^2)/a^3/f+1/2/f/a^2/(b+a*cos(f*x+e)^2)+b/a^3/f/(b+a*co
s(f*x+e)^2)-1/4/f/a^2/(b+a*cos(f*x+e)^2)^2*b-1/4*b^2/a^3/f/(b+a*cos(f*x+e)^
2)^2
```

Maxima [A] time = 0.996037, size = 153, normalized size = 1.89

$$-\frac{\frac{2\left(a^{2}+2ab\right)\sin\left(fx+e\right)^{2}-2a^{2}-5ab-3b^{2}}{a^{5}\sin\left(fx+e\right)^{4}+a^{5}+2a^{4}b+a^{3}b^{2}-2\left(a^{5}+a^{4}b\right)\sin\left(fx+e\right)^{2}}-\frac{2\log\left(a\sin\left(fx+e\right)^{2}-a-b\right)}{a^{3}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((2*(a^2 + 2*a*b)*sin(f*x + e)^2 - 2*a^2 - 5*a*b - 3*b^2)/(a^5*sin(f*x
 + e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log
(a*sin(f*x + e)^2 - a - b)/a^3)/f
```

Fricas [A] time = 0.571272, size = 262, normalized size = 3.23

$$\frac{2\left(a^{2}+2ab\right)\cos\left(fx+e\right)^{2}+ab+3b^{2}+2\left(a^{2}\cos\left(fx+e\right)^{4}+2ab\cos\left(fx+e\right)^{2}+b^{2}\right)\log\left(a\cos\left(fx+e\right)^{2}+b\right)}{4\left(a^{5}f\cos\left(fx+e\right)^{4}+2a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 3*b^2 + 2*(a^2*cos(f*x + e)^4 +
 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)
^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.99435, size = 952, normalized size = 11.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*((3*a^3 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 20*a^3*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) + 28*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*a*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*b^3*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 34*a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 22*a^2*b*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 10*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 + 18*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^3*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 28*a^2*b*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 - 4*a*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 12
*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^3*(cos(f*x + e) - 1)^4
/(cos(f*x + e) + 1)^4 + 9*a^2*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 +
 9*a*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^3*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4)/((a^4 + a^3*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)^2) - 2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^3 + 4*log(-(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 1)/a^3)/f
```

3.365 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=74

$$\frac{b^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

```
[Out] b^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - b/(a^3*f*(b + a*Cos[e + f*x]^2)) -
 Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rubi [A] time = 0.07414, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 266, 43}

$$\frac{b^2}{4a^3f\left(a\cos^2(e+fx)+b\right)^2}-\frac{b}{a^3f\left(a\cos^2(e+fx)+b\right)}-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] b^2/(4*a^3*f*(b + a*Cos[e + f*x]^2)^2) - b/(a^3*f*(b + a*Cos[e + f*x]^2)) -
 Log[b + a*Cos[e + f*x]^2]/(2*a^3*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2(b+ax)^3}-\frac{2b}{a^2(b+ax)^2}+\frac{1}{a^2(b+ax)}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{b^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2}-\frac{b}{a^3f\left(b+a\cos^2(e+fx)\right)}-\frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

Mathematica [A] time = 1.5085, size = 129, normalized size = 1.74

$$-\frac{a^2\cos^2(2(e+fx))\log(a\cos(2(e+fx))+a+2b)+(a+2b)^2\log(a\cos(2(e+fx))+a+2b)+2a\cos(2(e+fx))((a+2b}{2a^3f(a\cos(2(e+fx))+a+2b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a^2*Cos
[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(e + f*x)]*(2
*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(2*a^3*f*(a + 2*b + a*Co
s[2*(e + f*x)])^2)
```

Maple [A] time = 0.038, size = 81, normalized size = 1.1

$$-\frac{\ln\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)}{2fa^3}+\frac{1}{2fa^2\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)}+\frac{1}{4fa\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^2}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/2/f/a^3*ln(a+b*sec(f*x+e)^2)+1/2/f/a^2/(a+b*sec(f*x+e)^2)+1/4/f/a/(a+b*s
ec(f*x+e)^2)^2+1/f/a^3*ln(sec(f*x+e))
```

Maxima [A] time = 0.993384, size = 138, normalized size = 1.86

$$\frac{\frac{4\,ab\sin\left(fx+e\right)^{2}-4\,ab-3\,b^{2}}{a^{5}\sin\left(fx+e\right)^{4}+a^{5}+2\,a^{4}b+a^{3}b^{2}-2\,\left(a^{5}+a^{4}b\right)\sin\left(fx+e\right)^{2}}-\frac{2\,\log\left(a\sin\left(fx+e\right)^{2}-a-b\right)}{a^{3}}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((4*a*b*sin(f*x + e)^2 - 4*a*b - 3*b^2)/(a^5*sin(f*x + e)^4 + a^5 + 2*a
^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*sin(f*x + e)^2 -
 a - b)/a^3)/f
```

Fricas [A] time = 0.573321, size = 242, normalized size = 3.27

$$-\frac{4\,ab\cos\left(fx+e\right)^{2}+3\,b^{2}+2\left(a^{2}\cos\left(fx+e\right)^{4}+2\,ab\cos\left(fx+e\right)^{2}+b^{2}\right)\log\left(a\cos\left(fx+e\right)^{2}+b\right)}{4\left(a^{5}f\cos\left(fx+e\right)^{4}+2\,a^{4}bf\cos\left(fx+e\right)^{2}+a^{3}b^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*b*cos(f*x + e)^2 + 3*b^2 + 2*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*
cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.79737, size = 1133, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*((3*a^4 + 12*a^3*b + 18*a^2*b^2 + 12*a*b^3 + 3*b^4 + 12*a^4*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 40*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 24*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b^3*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) - 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
18*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 56*a^3*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 12*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 8*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^4*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^4*(cos(f*x + e) - 1)^3/(cos(f*x +
 e) + 1)^3 + 40*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 24*a^2*b^
2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 16*a*b^3*(cos(f*x + e) - 1)^3
/(cos(f*x + e) + 1)^3 - 12*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 +
3*a^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 12*a^3*b*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4 + 18*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) +
 1)^4 + 12*a*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^4*(cos(f*x
 + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a + b + 2*a*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2)^2) - 2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^
3 + 4*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/a^3)/f
```

3.366 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} \, dx$

Optimal. Leaf size=130

$$-\frac{b^3}{4a^3f(a+b)\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{b\left(3a^2+3ab+b^2\right)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^3}+$$

```
[Out] -b^3/(4*a^3*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a^3*
(a + b)^2*f*(b + a*Cos[e + f*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*Co
s[e + f*x]^2])/(2*a^3*(a + b)^3*f) + Log[Sin[e + f*x]]/((a + b)^3*f)
```

Rubi [A] time = 0.165065, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4138, 446, 88}

$$-\frac{b^3}{4a^3f(a+b)\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^2(3a+2b)}{2a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{b\left(3a^2+3ab+b^2\right)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -b^3/(4*a^3*(a + b)*f*(b + a*Cos[e + f*x]^2)^2) + (b^2*(3*a + 2*b))/(2*a^3*
(a + b)^2*f*(b + a*Cos[e + f*x]^2)) + (b*(3*a^2 + 3*a*b + b^2)*Log[b + a*Co
s[e + f*x]^2])/(2*a^3*(a + b)^3*f) + Log[Sin[e + f*x]]/((a + b)^3*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^7}{(1-x^2)\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(1-x)(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{b^3}{a^2(a+b)(b+ax)^3} + \frac{b^2(3a+2b)}{a^2(a+b)^2(b+ax)^2} - \frac{b\left(3a^2+3ab+b^2\right)}{a^2(a+b)^3(b+ax)}\right) dx, x, \cos^2(e+fx\right.}{2f}$$

$$= -\frac{b^3}{4a^3(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2f\left(b+a\cos^2(e+fx)\right)} + \frac{b\left(3a^2+3ab+b\right.}{2}$$

Mathematica [A] time = 1.09815, size = 158, normalized size = 1.22

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(-\frac{b^3(a+b)^2}{a^3\left(-a\sin^2(e+fx)+a+b\right)^2} + \frac{2b^2(a+b)(3a+2b)}{a^3\left(-a\sin^2(e+fx)+a+b\right)} + \frac{2b\left(3a^2+3ab+b^2\right)\log\left(-a\sin^2(e+fx)+a+b\right)}{a^3} + \right.}{32f(a+b)^3\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(4*Log[Sin[e + f*x]] + (2*
b*(3*a^2 + 3*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3 - (b^3*(a + b)^2
)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^3*(a
+ b - a*Sin[e + f*x]^2))))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.102, size = 304, normalized size = 2.3

$$\frac{3 b \ln \left(b+a \left(\cos \left(f x+e\right)\right)^{2}\right)}{2 f \left(a+b\right)^{3} a}+\frac{3 b^{2} \ln \left(b+a \left(\cos \left(f x+e\right)\right)^{2}\right)}{2 f \left(a+b\right)^{3} a^{2}}+\frac{b^{3} \ln \left(b+a \left(\cos \left(f x+e\right)\right)^{2}\right)}{2 f \left(a+b\right)^{3} a^{3}}+\frac{3 b^{2}}{2 f \left(a+b\right)^{3} a \left(b+a \left(\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 3/2/f*b/(a+b)^3/a*ln(b+a*cos(f*x+e)^2)+3/2/f*b^2/(a+b)^3/a^2*ln(b+a*cos(f*x
+e)^2)+1/2/f*b^3/(a+b)^3/a^3*ln(b+a*cos(f*x+e)^2)+3/2/f*b^2/(a+b)^3/a/(b+a*
cos(f*x+e)^2)+5/2/f*b^3/(a+b)^3/a^2/(b+a*cos(f*x+e)^2)+1/f*b^4/(a+b)^3/a^3/
(b+a*cos(f*x+e)^2)-1/4/f*b^3/(a+b)^3/a/(b+a*cos(f*x+e)^2)^2-1/2/f*b^4/(a+b)
^3/a^2/(b+a*cos(f*x+e)^2)^2-1/4*b^5/a^3/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+1/2/
f/(a+b)^3*ln(1+cos(f*x+e))+1/2/f/(a+b)^3*ln(-1+cos(f*x+e))
```

Maxima [A] time = 1.01423, size = 328, normalized size = 2.52

$$\frac{\frac{2 \left(3 a^{2} b+3 a b^{2}+b^{3}\right) \log\left(a \sin\left(f x+e\right)^{2}-a-b\right)}{a^{6}+3 a^{5} b+3 a^{4} b^{2}+a^{3} b^{3}}+\frac{6 a^{2} b^{2}+9 a b^{3}+3 b^{4}-2 \left(3 a^{2} b^{2}+2 a b^{3}\right) \sin\left(f x+e\right)^{2}}{a^{7}+4 a^{6} b+6 a^{5} b^{2}+4 a^{4} b^{3}+a^{3} b^{4}+\left(a^{7}+2 a^{6} b+a^{5} b^{2}\right) \sin\left(f x+e\right)^{4}-2 \left(a^{7}+3 a^{6} b+3 a^{5} b^{2}+a^{4} b^{3}\right) \sin\left(f x+e\right)^{2}}+\frac{2 \log}{a^{3}+3}}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 3*a^5
*b + 3*a^4*b^2 + a^3*b^3) + (6*a^2*b^2 + 9*a*b^3 + 3*b^4 - 2*(3*a^2*b^2 + 2
*a*b^3)*sin(f*x + e)^2)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 +
(a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2 + a
^4*b^3)*sin(f*x + e)^2) + 2*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3))/f
```

Fricas [B] time = 1.92186, size = 670, normalized size = 5.15

$$\frac{5 a^{2} b^{3}+8 a b^{4}+3 b^{5}+2 \left(3 a^{3} b^{2}+5 a^{2} b^{3}+2 a b^{4}\right) \cos\left(f x+e\right)^{2}+2 \left(3 a^{2} b^{3}+3 a b^{4}+b^{5}+\left(3 a^{4} b+3 a^{3} b^{2}+a^{2} b^{3}\right) \cos\left(\right.\right.}{4 \left(\left(a^{8}+3 a^{7} b+3 a^{6} b^{2}+a^{5} b^{3}\right) f \cos\left(f x+e\right)^{4}+2 \left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^2*b^3 + 8*a*b^4 + 3*b^5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*cos(
f*x + e)^2 + 2*(3*a^2*b^3 + 3*a*b^4 + b^5 + (3*a^4*b + 3*a^3*b^2 + a^2*b^3)
*cos(f*x + e)^4 + 2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*log(a*c
os(f*x + e)^2 + b) + 4*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b
^2)*log(1/2*sin(f*x + e)))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x
 + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a
^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53223, size = 1100, normalized size = 8.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(
a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) + 2*log(-(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (9*a^3*b + 18*a^2*b^2 + 12*a*b^
3 + 3*b^4 + 36*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 24*a^2*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b^3*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) - 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 54*a^3*b*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*
```

```
x + e) + 1)^2 + 8*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^4*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 36*a^3*b*(cos(f*x + e) - 1)^3/(
cos(f*x + e) + 1)^3 + 24*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
- 16*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 12*b^4*(cos(f*x + e)
 - 1)^3/(cos(f*x + e) + 1)^3 + 9*a^3*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) +
 1)^4 + 18*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 12*a*b^3*(co
s(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^4*(cos(f*x + e) - 1)^4/(cos(f*
x + e) + 1)^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
+ e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)^2) - 4*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/a^3)/f
```

3.367 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{b^4}{4a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{b^3(2a+b)}{a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)}-\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^4}-$$

```
[Out] b^4/(4*a^3*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) - (b^3*(2*a + b))/(a^3*(a
+ b)^3*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a + b)^3*f) - (b^2*(6
*a^2 + 4*a*b + b^2)*Log[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^4*f) - ((a +
4*b)*Log[Sin[e + f*x]])/((a + b)^4*f)
```

Rubi [A] time = 0.208823, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$\frac{b^4}{4a^3f(a+b)^2\left(a\cos^2(e+fx)+b\right)^2}-\frac{b^3(2a+b)}{a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)}-\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cos^2(e+fx)+b\right)}{2a^3f(a+b)^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] b^4/(4*a^3*(a + b)^2*f*(b + a*Cos[e + f*x]^2)^2) - (b^3*(2*a + b))/(a^3*(a
+ b)^3*f*(b + a*Cos[e + f*x]^2)) - Csc[e + f*x]^2/(2*(a + b)^3*f) - (b^2*(6
*a^2 + 4*a*b + b^2)*Log[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^4*f) - ((a +
4*b)*Log[Sin[e + f*x]])/((a + b)^4*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^9}{\left(1-x^2\right)^2\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x)^2(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3(-1+x)^2} + \frac{a+4b}{(a+b)^4(-1+x)} + \frac{b^4}{a^2(a+b)^2(b+ax)^3} - \frac{2b^3(2a+b)}{a^2(a+b)^3(b+ax)^2} + \frac{b^2\left(6a^2+4ab+b^2\right)}{a^2(a+b)^4(b+ax)}\right) d\ldots}{2f}$$

$$= \frac{b^4}{4a^3(a+b)^2 f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b^3(2a+b)}{a^3(a+b)^3 f\left(b+a\cos^2(e+fx)\right)} - \frac{\csc^2(e+fx)}{2(a+b)^3 f} - \ldots$$

Mathematica [A] time = 1.93355, size = 176, normalized size = 1.14

$$-\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(-\frac{b^4(a+b)^2}{a^3\left(-a\sin^2(e+fx)+a+b\right)^2} + \frac{4b^3(a+b)(2a+b)}{a^3\left(-a\sin^2(e+fx)+a+b\right)} + \frac{2b^2\left(6a^2+4ab+b^2\right)\log\left(-a\sin^2(e+fx)+a+\ldots\right)}{a^3}\ldots\right.}{32f(a+b)^4\left(a+b\sec^2(e+fx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*Csc[e + f*x]^2
 + 4*(a + 4*b)*Log[Sin[e + f*x]] + (2*b^2*(6*a^2 + 4*a*b + b^2)*Log[a + b -
 a*Sin[e + f*x]^2])/a^3 - (b^4*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2
) + (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(32*(a + b
```

```
)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.115, size = 389, normalized size = 2.5

$$-3\frac{b^2\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{f\left(a+b\right)^4a}-2\frac{b^3\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{f\left(a+b\right)^4a^2}-\frac{b^4\ln\left(b+a\left(\cos\left(fx+e\right)\right)^2\right)}{2f\left(a+b\right)^4a^3}-2\frac{b^3}{f\left(a+b\right)^4a\left(b+a\left(c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -3/f*b^2/(a+b)^4/a*ln(b+a*cos(f*x+e)^2)-2/f*b^3/(a+b)^4/a^2*ln(b+a*cos(f*x+
e)^2)-1/2/f*b^4/(a+b)^4/a^3*ln(b+a*cos(f*x+e)^2)-2/f*b^3/(a+b)^4/a/(b+a*cos
(f*x+e)^2)-3/f*b^4/(a+b)^4/a^2/(b+a*cos(f*x+e)^2)-1/f*b^5/(a+b)^4/a^3/(b+a*
cos(f*x+e)^2)+1/4/f*b^4/(a+b)^4/a/(b+a*cos(f*x+e)^2)^2+1/2/f*b^5/(a+b)^4/a^
2/(b+a*cos(f*x+e)^2)^2+1/4/f*b^6/(a+b)^4/a^3/(b+a*cos(f*x+e)^2)^2-1/4/f/(a+
b)^3/(1+cos(f*x+e))-1/2/f/(a+b)^4*ln(1+cos(f*x+e))*a-2/f/(a+b)^4*ln(1+cos(f
*x+e))*b+1/4/f/(a+b)^3/(-1+cos(f*x+e))-1/2/f/(a+b)^4*ln(-1+cos(f*x+e))*a-2/
f/(a+b)^4*ln(-1+cos(f*x+e))*b
```

Maxima [B] time = 1.04002, size = 464, normalized size = 3.01

$$-\frac{\frac{2\left(6a^2b^2+4ab^3+b^4\right)\log\left(a\sin\left(fx+e\right)^2-a-b\right)}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4}+\frac{2\left(a+4b\right)\log\left(\sin\left(fx+e\right)^2\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{2a^5+4a^4b+2a^3b^2+2\left(a^5-4a^2b^3-2ab^4\right)\sin\left(fx+e\right)^4-\left(4a^5\right.}{\left(a^8+3a^7b+3a^6b^2+a^5b^3\right)\sin\left(fx+e\right)^6-2\left(a^8+4a^7b+6a^6b^2+4a^5b^3+a^4b^4\right)\sin\left(f\right.}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^7 + 4*
a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 2*(a + 4*b)*log(sin(f*x + e)^2)/
(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*a^5 + 4*a^4*b + 2*a^3*b^2
+ 2*(a^5 - 4*a^2*b^3 - 2*a*b^4)*sin(f*x + e)^4 - (4*a^5 + 4*a^4*b - 8*a^2*b
^3 - 11*a*b^4 - 3*b^5)*sin(f*x + e)^2)/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^
3)*sin(f*x + e)^6 - 2*(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*sin
(f*x + e)^4 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^
```

```
5)*sin(f*x + e)^2))/f
```

Fricas [B] time = 3.33967, size = 1234, normalized size = 8.01

$$\frac{2\,a^4b^2+2\,a^3b^3+7\,a^2b^4+10\,ab^5+3\,b^6+2\left(a^6+a^5b-4\,a^3b^3-6\,a^2b^4-2\,ab^5\right)\cos\left(fx+e\right)^4+\left(4\,a^5b+4\,a^4b^2+8\,a^3b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^4*b^2 + 2*a^3*b^3 + 7*a^2*b^4 + 10*a*b^5 + 3*b^6 + 2*(a^6 + a^5*b
- 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 + (4*a^5*b + 4*a^4*b^2 +
8*a^3*b^3 + 5*a^2*b^4 - 6*a*b^5 - 3*b^6)*cos(f*x + e)^2 - 2*((6*a^4*b^2 + 4
*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (6*a^4*b^2
 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (12*a^3*b^3 + 2*a^2*b^
4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 4*((a^6 + 4*
a^5*b)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - (a^6 + 2*a^5*b - 8*a^4*b^2)*c
os(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 - 4*a^3*b^3)*cos(f*x + e)^2)*log(1/2*s
in(f*x + e)))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x
+ e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*
cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 -
 a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
 a^3*b^6)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60081, size = 1530, normalized size = 9.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 4*(a + 4*b)*log(-(cos
(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4) - (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*b*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*(cos(f*x + e) - 1)) - (cos(f*x + e) - 1)/((a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*(cos(f*x + e) + 1)) - 2*(18*a^4*b^2 + 48*a^3*b^3 + 45*a^2*b
^4 + 18*a*b^5 + 3*b^6 + 72*a^4*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
80*a^3*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^2*b^4*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 40*a*b^5*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
 12*b^6*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 108*a^4*b^2*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 64*a^3*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 46*a^2*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 44*a*b^5*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^6*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 + 72*a^4*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 80
*a^3*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 20*a^2*b^4*(cos(f*x +
e) - 1)^3/(cos(f*x + e) + 1)^3 - 40*a*b^5*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3 - 12*b^6*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 18*a^4*b^2*(c
os(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 48*a^3*b^3*(cos(f*x + e) - 1)^4/(
cos(f*x + e) + 1)^4 + 45*a^2*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4
+ 18*a*b^5*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^6*(cos(f*x + e)
- 1)^4/(cos(f*x + e) + 1)^4)/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*
b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 8*log(-(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 1)/a^3)/f
```

3.368 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=192

$$-\frac{b^5}{4a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^4(5a+2b)}{2a^3f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2+5ab+10b^2\right)\log(\sin(e+fx))}{f(a+b)^5}+\frac{b^3\left(10\right.}{}$$

```
[Out] -b^5/(4*a^3*(a + b)^3*f*(b + a*Cos[e + f*x]^2)^2) + (b^4*(5*a + 2*b))/(2*a^
3*(a + b)^4*f*(b + a*Cos[e + f*x]^2)) + ((2*a + 5*b)*Csc[e + f*x]^2)/(2*(a
+ b)^4*f) - Csc[e + f*x]^4/(4*(a + b)^3*f) + (b^3*(10*a^2 + 5*a*b + b^2)*Lo
g[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^5*f) + ((a^2 + 5*a*b + 10*b^2)*Log[
Sin[e + f*x]])/((a + b)^5*f)
```

Rubi [A] time = 0.269266, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4138, 446, 88}

$$-\frac{b^5}{4a^3f(a+b)^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{b^4(5a+2b)}{2a^3f(a+b)^4\left(a\cos^2(e+fx)+b\right)}+\frac{\left(a^2+5ab+10b^2\right)\log(\sin(e+fx))}{f(a+b)^5}+\frac{b^3\left(10\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] -b^5/(4*a^3*(a + b)^3*f*(b + a*Cos[e + f*x]^2)^2) + (b^4*(5*a + 2*b))/(2*a^
3*(a + b)^4*f*(b + a*Cos[e + f*x]^2)) + ((2*a + 5*b)*Csc[e + f*x]^2)/(2*(a
+ b)^4*f) - Csc[e + f*x]^4/(4*(a + b)^3*f) + (b^3*(10*a^2 + 5*a*b + b^2)*Lo
g[b + a*Cos[e + f*x]^2])/(2*a^3*(a + b)^5*f) + ((a^2 + 5*a*b + 10*b^2)*Log[
Sin[e + f*x]])/((a + b)^5*f)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^{11}}{\left(1-x^2\right)^3\left(b+ax^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^5}{(1-x)^3(b+ax)^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)^3}+\frac{-2a-5b}{(a+b)^4(-1+x)^2}+\frac{-a^2-5ab-10b^2}{(a+b)^5(-1+x)}-\frac{b^5}{a^2(a+b)^3(b+ax)^3}+\frac{b^4(5a+2b)}{a^2(a+b)^4(b+ax)^2}-\frac{b^3}{a}\right.\right.}{2f}$$

$$= -\frac{b^5}{4a^3(a+b)^3f\left(b+a\cos^2(e+fx)\right)^2}+\frac{b^4(5a+2b)}{2a^3(a+b)^4f\left(b+a\cos^2(e+fx)\right)}+\frac{(2a+5b)\csc^2(}{2(a+b)^4}$$

Mathematica [A] time = 5.42182, size = 208, normalized size = 1.08

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(-\frac{b^5(a+b)^2}{a^3\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{2b^4(a+b)(5a+2b)}{a^3\left(-a\sin^2(e+fx)+a+b\right)}+\frac{2b^3\left(10a^2+5ab+b^2\right)\log\left(-a\sin^2(e+fx)+a+b\right)}{a^3}\right.}{32f(a+b)^5\left(a+b\sec^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*(2*a + 5*b)*Csc
[e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 5*a*b + 10*b^2)*Log[Sin[e
```

```
 + f*x]] + (2*b^3*(10*a^2 + 5*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3
 - (b^5*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^4*(a + b)*(5*a
 + 2*b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(32*(a + b)^5*f*(a + b*Sec[e +
f*x]^2)^3)
```

Maple [B] time = 0.126, size = 522, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 5/f*b^3/(a+b)^5/a*ln(b+a*cos(f*x+e)^2)+5/2/f*b^4/(a+b)^5/a^2*ln(b+a*cos(f*x
+e)^2)+1/2/f*b^5/(a+b)^5/a^3*ln(b+a*cos(f*x+e)^2)+5/2/f*b^4/(a+b)^5/a/(b+a*
cos(f*x+e)^2)+7/2/f*b^5/(a+b)^5/a^2/(b+a*cos(f*x+e)^2)+1/f*b^6/(a+b)^5/a^3/
(b+a*cos(f*x+e)^2)-1/4/f*b^5/(a+b)^5/a/(b+a*cos(f*x+e)^2)^2-1/2/f*b^6/(a+b)
^5/a^2/(b+a*cos(f*x+e)^2)^2-1/4/f*b^7/(a+b)^5/a^3/(b+a*cos(f*x+e)^2)^2-1/16
/f/(a+b)^3/(1+cos(f*x+e))^2+7/16/f/(a+b)^4/(1+cos(f*x+e))*a+19/16/f/(a+b)^4
/(1+cos(f*x+e))*b+1/2/f/(a+b)^5*ln(1+cos(f*x+e))*a^2+5/2/f/(a+b)^5*ln(1+cos
(f*x+e))*a*b+5/f/(a+b)^5*ln(1+cos(f*x+e))*b^2-1/16/f/(a+b)^3/(-1+cos(f*x+e)
)^2-7/16/f/(a+b)^4/(-1+cos(f*x+e))*a-19/16/f/(a+b)^4/(-1+cos(f*x+e))*b+1/2/
f/(a+b)^5*ln(-1+cos(f*x+e))*a^2+5/2/f/(a+b)^5*ln(-1+cos(f*x+e))*a*b+5/f/(a+
b)^5*ln(-1+cos(f*x+e))*b^2
```

Maxima [B] time = 1.05227, size = 613, normalized size = 3.19

$$\frac{\frac{2\,(10\,a^2b^3+5\,ab^4+b^5)\log\left(a\sin(fx+e)^2-a-b\right)}{a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5}+\frac{2\,(a^2+5\,ab+10\,b^2)\log\left(\sin(fx+e)^2\right)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}+\frac{2\,(2\,a^6+5\,a^5b-5\,a^2b^4-2\,ab^5)\sin(fx+e)^6-a^6-3\,a^5b-3\,a^4b^2-a^3b^3-(9}{(a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4)\sin(fx+e)^8-2\,(a^9+5\,a^8b+10\,a^7b^2-}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a*sin(f*x + e)^2 - a - b)/(a^8 + 5*
a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 2*(a^2 + 5*a*b + 1
0*b^2)*log(sin(f*x + e)^2)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5) + (2*(2*a^6 + 5*a^5*b - 5*a^2*b^4 - 2*a*b^5)*sin(f*x + e)^6 - a^6
```

```
- 3*a^5*b - 3*a^4*b^2 - a^3*b^3 - (9*a^6 + 29*a^5*b + 20*a^4*b^2 - 10*a^2*
b^4 - 13*a*b^5 - 3*b^6)*sin(f*x + e)^4 + 2*(3*a^6 + 11*a^5*b + 13*a^4*b^2 +
 5*a^3*b^3)*sin(f*x + e)^2)/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b
^4)*sin(f*x + e)^8 - 2*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4
 + a^4*b^5)*sin(f*x + e)^6 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*
a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*sin(f*x + e)^4))/f
```

Fricas [B] time = 5.70836, size = 1867, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^5*b^2 + 12*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 + 12*a*b^6 + 3*b^7 - 2*
(2*a^7 + 7*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 7*a^2*b^5 - 2*a*b^6)*cos(f*x + e
)^6 + (3*a^7 + 4*a^6*b - 19*a^5*b^2 - 20*a^4*b^3 - 20*a^3*b^4 - 19*a^2*b^5
+ 4*a*b^6 + 3*b^7)*cos(f*x + e)^4 + 2*(3*a^6*b + 10*a^5*b^2 + 2*a^4*b^3 - 2
*a^2*b^5 - 10*a*b^6 - 3*b^7)*cos(f*x + e)^2 + 2*((10*a^4*b^3 + 5*a^3*b^4 +
a^2*b^5)*cos(f*x + e)^8 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(10*a^4*b^3 - 5*a^
3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (10*a^4*b^3 - 35*a^3*b^4 - 9*a^
2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(10*a^3*b^4 - 5*a^2*b^5 - 4*a*b^6 -
 b^7)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*((a^7 + 5*a^6*b + 10*a^
5*b^2)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 - 2*(a^7 + 4*a^6*b
 + 5*a^5*b^2 - 10*a^4*b^3)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 35*a
^4*b^3 + 10*a^3*b^4)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 10
*a^3*b^4)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^10 + 5*a^9*b + 10*a^8*
b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*
b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 +
 a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^
7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*
b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^
5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.64494, size = 2751, normalized size = 14.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(32*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a + b + 2*a*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2)/(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 32*(a
^2 + 5*a*b + 10*b^2)*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/(a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - (12*a^3*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 60*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 84*
a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 36*b^3*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) + a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*a^2*b*(
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*a*b^2*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 + b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 6
*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) - (a^7 + 4*a
^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + 16*a^7*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 80*a^6*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 144*a^5*b^2*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 112*a^4*b^3*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 32*a^3*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 70*a^7*(
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 312*a^6*b*(cos(f*x + e) - 1)^2/(
cos(f*x + e) + 1)^2 + 436*a^5*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + 536*a^4*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 822*a^3*b^4*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 672*a^2*b^5*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 + 224*a*b^6*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
32*b^7*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 140*a^7*(cos(f*x + e) -
1)^3/(cos(f*x + e) + 1)^3 + 568*a^6*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
1)^3 + 672*a^5*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 1096*a^4*b^3
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 852*a^3*b^4*(cos(f*x + e) - 1)
^3/(cos(f*x + e) + 1)^3 - 384*a^2*b^5*(cos(f*x + e) - 1)^3/(cos(f*x + e) +
1)^3 - 512*a*b^6*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 128*b^7*(cos(f
*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 145*a^7*(cos(f*x + e) - 1)^4/(cos(f*x
 + e) + 1)^4 + 612*a^6*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 838*a^
5*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 1572*a^4*b^3*(cos(f*x + e
```

```
) - 1)^4/(cos(f*x + e) + 1)^4 + 1777*a^3*b^4*(cos(f*x + e) - 1)^4/(cos(f*x
+ e) + 1)^4 + 704*a^2*b^5*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 576*a
*b^6*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 192*b^7*(cos(f*x + e) - 1)
^4/(cos(f*x + e) + 1)^4 + 76*a^7*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5
+ 392*a^6*b*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 720*a^5*b^2*(cos(f*
x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 1080*a^4*b^3*(cos(f*x + e) - 1)^5/(cos
(f*x + e) + 1)^5 + 676*a^3*b^4*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 -
384*a^2*b^5*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 - 512*a*b^6*(cos(f*x
+ e) - 1)^5/(cos(f*x + e) + 1)^5 - 128*b^7*(cos(f*x + e) - 1)^5/(cos(f*x +
e) + 1)^5 + 16*a^7*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 112*a^6*b*(c
os(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 336*a^5*b^2*(cos(f*x + e) - 1)^6/
(cos(f*x + e) + 1)^6 + 720*a^4*b^3*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^
6 + 960*a^3*b^4*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 672*a^2*b^5*(co
s(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 224*a*b^6*(cos(f*x + e) - 1)^6/(co
s(f*x + e) + 1)^6 + 32*b^7*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((a^8
 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*(a*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*(
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(f*x +
e) - 1)^3/(cos(f*x + e) + 1)^3)^2) - 64*log(-(cos(f*x + e) - 1)/(cos(f*x +
e) + 1) + 1)/a^3)/f
```

3.369 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=147

$$-\frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f}-\frac{x}{a^3}-\frac{(a+b)\tan^3(e+fx)}{4abf\,(a+b\tan^2(e+fx)+b}$$

```
[Out] -(x/a^3) + (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(8*a^3*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(4*a*b*f*(a +
 b + b*Tan[e + f*x]^2)^2) - ((3*a - 4*b)*(a + b)*Tan[e + f*x])/(8*a^2*b^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.292408, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 578, 522, 203, 205}

$$-\frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b\tan^2(e+fx)+b\right)}+\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f}-\frac{x}{a^3}-\frac{(a+b)\tan^3(e+fx)}{4abf\,(a+b\tan^2(e+fx)+b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(8*a^3*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(4*a*b*f*(a +
 b + b*Tan[e + f*x]^2)^2) - ((3*a - 4*b)*(a + b)*Tan[e + f*x])/(8*a^2*b^2*f
*(a + b + b*Tan[e + f*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3(a+b)+(3a-b)x^2\right)}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{-(3a-4b)(a}{(1}\right.}{}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{a^3f}$$

$$= -\frac{x}{a^3} + \frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} -$$

Mathematica [C] time = 6.52417, size = 760, normalized size = 5.17

$$\frac{\sec(2e)\sec^6(e+fx)(a\cos(2e+2fx)+a+2b)\left(-20a^2b^2\sin(4e+2fx)+6a^2b^2\sin(2e+4fx)-24a^2b^2fx\cos(2e)-16a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((-3*a^3 + a^2*b - 4*a*b^2 - 8*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
+ f*x]^6*((ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*S
in[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))
*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*b^2*S
qrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*ArcTan[Sec[f*x]*(Co
```

```
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*b^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*
b*Sin[4*e]])))/(a + b*Sec[e + f*x]^2)^3 + ((a + 2*b + a*Cos[2*e + 2*f*x])*S
ec[2*e]*Sec[e + f*x]^6*(-24*a^2*b^2*f*x*Cos[2*e] - 64*a*b^3*f*x*Cos[2*e] -
64*b^4*f*x*Cos[2*e] - 16*a^2*b^2*f*x*Cos[2*f*x] - 32*a*b^3*f*x*Cos[2*f*x] -
 16*a^2*b^2*f*x*Cos[4*e + 2*f*x] - 32*a*b^3*f*x*Cos[4*e + 2*f*x] - 4*a^2*b^
2*f*x*Cos[2*e + 4*f*x] - 4*a^2*b^2*f*x*Cos[6*e + 4*f*x] + 9*a^4*Sin[2*e] +
15*a^3*b*Sin[2*e] - 18*a^2*b^2*Sin[2*e] - 72*a*b^3*Sin[2*e] - 48*b^4*Sin[2*
e] - 9*a^4*Sin[2*f*x] - 13*a^3*b*Sin[2*f*x] + 28*a^2*b^2*Sin[2*f*x] + 32*a*
b^3*Sin[2*f*x] + 3*a^4*Sin[4*e + 2*f*x] - a^3*b*Sin[4*e + 2*f*x] - 20*a^2*b
^2*Sin[4*e + 2*f*x] - 16*a*b^3*Sin[4*e + 2*f*x] - 3*a^4*Sin[2*e + 4*f*x] +
3*a^3*b*Sin[2*e + 4*f*x] + 6*a^2*b^2*Sin[2*e + 4*f*x]))/(128*a^3*b^2*f*(a +
 b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.106, size = 356, normalized size = 2.4

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^3}-\frac{\left(\tan\left(fx+e\right)\right)^3}{8fa\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)^2}-\frac{5\left(\tan\left(fx+e\right)\right)^3}{8f\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)^2b}-\frac{3a\tan\left(fx\cdots\right.}{8f\left(a+b+b\left(\tan\left(f\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/f/a^3*arctan(tan(f*x+e))-1/8/f/a/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-5/8
/f/(a+b+b*tan(f*x+e)^2)^2/b*tan(f*x+e)^3-3/8/f*a/(a+b+b*tan(f*x+e)^2)^2/b^2
*tan(f*x+e)-1/4/f/(a+b+b*tan(f*x+e)^2)^2/b*tan(f*x+e)+5/8*tan(f*x+e)/a/f/(a
+b+b*tan(f*x+e)^2)^2+3/8/f/b^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b
)^(1/2))-1/8/f/a/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2
/f/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+1/2/f/a^2*b/(a+
b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2
)^2+1/f/a^3*b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.677676, size = 1519, normalized size = 10.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*a^2*b^2*f*x*cos(f*x + e)^4 + 64*a*b^3*f*x*cos(f*x + e)^2 + 32*b^
4*f*x - ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 - 4*a*b^3
 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-(a + b)/
b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)
*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4
*(3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3
)*cos(f*x + e))*sin(f*x + e))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f
*x + e)^2 + a^3*b^4*f), -1/16*(16*a^2*b^2*f*x*cos(f*x + e)^4 + 32*a*b^3*f*x
*cos(f*x + e)^2 + 16*b^4*f*x + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^
4 + 3*a^2*b^2 - 4*a*b^3 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x
 + e)^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a
+ b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(a^4 - a^3*b - 2*a^2*b^
2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3)*cos(f*x + e))*sin(f*x + e
))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 + a^3*b^4*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.87869, size = 285, normalized size = 1.94

$$-\frac{\frac{8\left(fx+e\right)}{a^3}-\frac{\left(3\,a^3-a^2b+4\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^3b^2}+\frac{5\,a^2b\tan\left(fx+e\right)^3+ab^2\tan\left(fx+e\right)^3-4\,b^3\tan\left(fx+e\right)^3+3\,a^3\tan\left(fx+e\right)+2\,a^2}{\left(b\tan\left(fx+e\right)^2+a+b\right)^2a^2b^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(f*x + e)/a^3 - (3*a^3 - a^2*b + 4*a*b^2 + 8*b^3)*(pi*floor((f*x +
e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b
^2)*a^3*b^2) + (5*a^2*b*tan(f*x + e)^3 + a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f
*x + e)^3 + 3*a^3*tan(f*x + e) + 2*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x + e
) - 4*b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b^2))/f
```

3.370 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=137

$$\frac{\left(a^2-4ab-8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}f\sqrt{a+b}}+\frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{(a+b)\tan(e+fx)}{4abf\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(8*a^3*b^(3/2)*Sqrt[a + b]*f) - ((a + b)*Tan[e + f*x])/(4*a*b*f*(a + b + b*
Tan[e + f*x]^2)^2) + ((a - 4*b)*Tan[e + f*x])/(8*a^2*b*f*(a + b + b*Tan[e +
 f*x]^2))
```

Rubi [A] time = 0.257351, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 470, 527, 522, 203, 205}

$$\frac{\left(a^2-4ab-8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}f\sqrt{a+b}}+\frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{(a+b)\tan(e+fx)}{4abf\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] x/a^3 + ((a^2 - 4*a*b - 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/
(8*a^3*b^(3/2)*Sqrt[a + b]*f) - ((a + b)*Tan[e + f*x])/(4*a*b*f*(a + b + b*
Tan[e + f*x]^2)^2) + ((a - 4*b)*Tan[e + f*x])/(8*a^2*b*f*(a + b + b*Tan[e +
 f*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b+(a-3b)x^2}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{(a+b)(a+4b}{(1+x^2)}\right.}{}$$

$$= -\frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{a^3f}$$

$$= \frac{x}{a^3} + \frac{\left(a^2-4ab-8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}f} - \frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-}{8a^2bf\left(a\right.}$$

Mathematica [C] time = 15.0373, size = 1473, normalized size = 10.75

$$\frac{(\cos(2e+2fx)a+a+2b)^3\left(\frac{\left(3a^2+8ba+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}\left(3a^2+16ba+3(a+2b)\cos(2(e+fx))a+16b^2\right)\sin(2(e+fx))}{(a+b)^2(\cos(2(e+fx))a+a+2b)^2}\right)\sec^6(e+fx)}{1024b^{5/2}f\left(b\sec^2(e+fx)+a\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a
^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((
a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec[e +
 f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a +
2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]*(
```

```
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2
048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^
4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]
) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]
*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b
)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3
*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*
b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4
*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*C
os[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*
e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 28
80*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^
6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x
] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*f*
x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2*S
in[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e +
 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b^2
*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e +
2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4
096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e +
 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*
b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x
)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 +
 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*
(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2048*b^2*(a + b)^2*f*(a + b*Sec[e + f*
x]^2)^3)
```

Maple [B] time = 0.099, size = 264, normalized size = 1.9

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^3}+\frac{\left(\tan\left(fx+e\right)\right)^3}{8fa\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)^2}-\frac{b\left(\tan\left(fx+e\right)\right)^3}{2fa^2\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)^2}-\frac{\tan\left(fx+e\right.}{8f\left(a+b+b\left(\tan\left(f\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f/a^3*arctan(tan(f*x+e))+1/8/f/a/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-1/2/
f/a^2*b/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-1/8/f/(a+b+b*tan(f*x+e)^2)^2/b*
tan(f*x+e)-5/8*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2-1/2*b*tan(f*x+e)/a^2/f
/(a+b+b*tan(f*x+e)^2)^2+1/8/f/a/b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b
)*b)^(1/2))-1/2/f/a^2/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-
1/f/a^3*b/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.705592, size = 1686, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^2*b^3 + a*b^4)*f*x
*cos(f*x + e)^2 + 32*(a*b^4 + b^5)*f*x + ((a^4 - 4*a^3*b - 8*a^2*b^2)*cos(f
*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8*a*b^3)*cos
(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x +
 e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f
*x + e)^2 + b^2)) - 4*((a^4*b + 7*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^
3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^2 + a^5*b^
3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^4*b^4 + a
^3*b^5)*f), 1/16*(16*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 32*(a^2*b^3 +
 a*b^4)*f*x*cos(f*x + e)^2 + 16*(a*b^4 + b^5)*f*x - ((a^4 - 4*a^3*b - 8*a^2
*b^2)*cos(f*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8
*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((a^4*b + 7*a^3*b^2
 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e)
```

```
)*sin(f*x + e))/((a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^
4)*f*cos(f*x + e)^2 + (a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.27803, size = 227, normalized size = 1.66

$$\frac{\frac{8\left(fx+e\right)}{a^3}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(a^2-4\,ab-8\,b^2\right)}{\sqrt{ab+b^2}a^3b}+\frac{ab\tan\left(fx+e\right)^3-4\,b^2\tan\left(fx+e\right)^3-a^2\tan\left(fx+e\right)-5\,ab\tan\left(fx+e\right)-4\,b^2\tan\left(fx+e\right)}{\left(b\tan\left(fx+e\right)^2+a+b\right)^2a^2b}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(f*x + e)/a^3 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(
f*x + e)/sqrt(a*b + b^2)))*(a^2 - 4*a*b - 8*b^2)/(sqrt(a*b + b^2)*a^3*b) +
(a*b*tan(f*x + e)^3 - 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) - 5*a*b*tan(f
*x + e) - 4*b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b))/f
```

3.371 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3} dx$

Optimal. Leaf size=138

$$\frac{\left(3a^2+12ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}f(a+b)^{3/2}}+\frac{(3a+4b)\tan(e+fx)}{8a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{x}{a^3}+\frac{\tan(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^2}$$

```
[Out] -(x/a^3) + ((3*a^2 + 12*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*f) + Tan[e + f*x]/(4*a*f*(a + b + b*Tan[
e + f*x]^2)^2) + ((3*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)*f*(a + b + b*Tan
[e + f*x]^2))
```

Rubi [A] time = 0.227172, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4141, 1975, 471, 527, 522, 203, 205}

$$\frac{\left(3a^2+12ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}f(a+b)^{3/2}}+\frac{(3a+4b)\tan(e+fx)}{8a^2f(a+b)\left(a+b\tan^2(e+fx)+b\right)}-\frac{x}{a^3}+\frac{\tan(e+fx)}{4af\left(a+b\tan^2(e+fx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x/a^3) + ((3*a^2 + 12*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/(8*a^3*Sqrt[b]*(a + b)^(3/2)*f) + Tan[e + f*x]/(4*a*f*(a + b + b*Tan[
e + f*x]^2)^2) + ((3*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)*f*(a + b + b*Tan
[e + f*x]^2))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b+bx^2\right)^3}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2}-\frac{\text{Subst}\left(\int \frac{1-3x^2}{(1+x^2)\left(a+b+bx^2\right)^2}\,dx,x,\tan(e+fx)\right)}{4af}$$

$$=\frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}-\frac{\text{Subst}\left(\int \frac{5a+4b+}{(1+x^2)}\right.}{}$$

$$=\frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.}{a}$$

$$=-\frac{x}{a^3}+\frac{\left(3a^2+12ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}(a+b)^{3/2}f}+\frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2}+\frac{}{8a^2(a}$$

<u>Mathematica</u> [C] time = 13.2285, size = 1473, normalized size = 10.67

$$\frac{(\cos(2e+2fx)a+a+2b)^3\left(\frac{\left(3a^2+8ba+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{a\sqrt{b}\left(3a^2+16ba+3(a+2b)\cos(2(e+fx))a+16b^2\right)\sin(2(e+fx))}{(a+b)^2(\cos(2(e+fx))a+a+2b)^2}\right)\sec^6(e+fx)}{1024b^{5/2}f\left(b\sec^2(e+fx)+a\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)*
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3*a
^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((
a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec[e +
 f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*(a +
2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqrt[b]*(
```

```
3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2
048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3
*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3 + 640*a*b^
4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]
) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Sec[2*e]
*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 512*a*b^2*(a + b
)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*f*x)] + 256*a^3
*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*f*x)] + 512*a^4*
b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*x] + 2560*a^2*b^4
*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] + 128*a^4*b^2*f*x*C
os[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128*a^2*b^4*f*x*Cos[6*
e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 684*a^4*b^2*Sin[2*e] + 28
80*a^3*b^3*Sin[2*e] + 5280*a^2*b^4*Sin[2*e] + 4608*a*b^5*Sin[2*e] + 1536*b^
6*Sin[2*e] + 9*a^6*Sin[2*f*x] - 14*a^5*b*Sin[2*f*x] - 608*a^4*b^2*Sin[2*f*x
] - 2112*a^3*b^3*Sin[2*f*x] - 2560*a^2*b^4*Sin[2*f*x] - 1024*a*b^5*Sin[2*f*
x] + 3*a^6*Sin[2*(e + 2*f*x)] - 12*a^5*b*Sin[2*(e + 2*f*x)] - 204*a^4*b^2*S
in[2*(e + 2*f*x)] - 384*a^3*b^3*Sin[2*(e + 2*f*x)] - 192*a^2*b^4*Sin[2*(e +
 2*f*x)] - 3*a^6*Sin[4*e + 2*f*x] + 10*a^5*b*Sin[4*e + 2*f*x] + 304*a^4*b^2
*Sin[4*e + 2*f*x] + 1056*a^3*b^3*Sin[4*e + 2*f*x] + 1280*a^2*b^4*Sin[4*e +
2*f*x] + 512*a*b^5*Sin[4*e + 2*f*x]))/(a + 2*b + a*Cos[2*(e + f*x)])^2))/(4
096*a^3*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e +
 2*f*x])^3*Sec[e + f*x]^6*((-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (a*Sec[2*e]*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*
b^4)*Sin[2*f*x] + a*(-3*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x
)] + (3*a^4 - 64*a^2*b^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 +
 18*a^4*b - 64*a^3*b^2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*
(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(2048*b^2*(a + b)^2*f*(a + b*Sec[e + f*
x]^2)^3)
```

Maple [B] time = 0.096, size = 263, normalized size = 1.9

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^{3}}+\frac{3\,b\left(\tan\left(fx+e\right)\right)^{3}}{8\,fa\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}(a+b)}+\frac{b^{2}\left(\tan\left(fx+e\right)\right)^{3}}{2\,fa^{2}\left(a+b+b\left(\tan\left(fx+e\right)\right)^{2}\right)^{2}(a+b)}+\frac{\quad}{8\,fa\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/f/a^3*arctan(tan(f*x+e))+3/8/f/a/(a+b+b*tan(f*x+e)^2)^2*b/(a+b)*tan(f*x+
e)^3+1/2/f/a^2/(a+b+b*tan(f*x+e)^2)^2*b^2/(a+b)*tan(f*x+e)^3+5/8*tan(f*x+e)
/a/f/(a+b+b*tan(f*x+e)^2)^2+1/2*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+3
/8/f/a/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))+3/2/f/a^2
/(a+b)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*b+1/f/a^3/(a+b)
/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.713179, size = 1910, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(32*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^3*b^2 +
 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 32*(a^2*b^3 + 2*a*b^4 + b^5)*f*x +
 ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 12*a*b^3 + 8*
b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x +
 e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((5*a^4*b
 + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^
4)*cos(f*x + e))*sin(f*x + e))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e
)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4
*b^4 + a^3*b^5)*f), -1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e
)^4 + 32*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^2*b^3 + 2
*a*b^4 + b^5)*f*x + ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*
b^2 + 12*a*b^3 + 8*b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)
*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)
```

```
*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*
x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((
a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^
4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64476, size = 244, normalized size = 1.77

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)\left(3a^2+12ab+8b^2\right)}{\left(a^4+a^3b\right)\sqrt{ab+b^2}}+\frac{3ab\tan\left(fx+e\right)^3+4b^2\tan\left(fx+e\right)^3+5a^2\tan\left(fx+e\right)+9ab\tan\left(fx+e\right)+4b^2\tan\left(fx+e\right)}{\left(a^3+a^2b\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}-\frac{8\left(fx\right.}{a^3}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
+ b^2)))*(3*a^2 + 12*a*b + 8*b^2)/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (3*a*b*
tan(f*x + e)^3 + 4*b^2*tan(f*x + e)^3 + 5*a^2*tan(f*x + e) + 9*a*b*tan(f*x
+ e) + 4*b^2*tan(f*x + e))/((a^3 + a^2*b)*(b*tan(f*x + e)^2 + a + b)^2) - 8
*(f*x + e)/a^3)/f
```

3.372 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e\right.}$$

```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.177944, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4128, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{5/2}}-\frac{b(7a+4b)\tan(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}+\frac{x}{a^3}-\frac{b\tan(e+fx)}{4af(a+b)\left(a+b\tan^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```


```
[Out] x/a^3 - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(5/2)*f) - (b*Tan[e + f*x])/(4*a*(a + b)*f*(a +
b + b*Tan[e + f*x]^2)^2) - (b*(7*a + 4*b)*Tan[e + f*x])/(8*a^2*(a + b)^2*f*
(a + b + b*Tan[e + f*x]^2))
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a+b-3bx^2}{(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= -\frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= \frac{x}{a^3} - \frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}$$

Mathematica [C] time = 5.82632, size = 332, normalized size = 2.31

$$\frac{\sec^6(e+fx)(a\cos(2(e+fx))+a+2b)\left(\frac{b\left(\left(9a^2+28ab+16b^2\right)\sin(2e)-3a(3a+2b)\sin(2fx)\right)(a\cos(2(e+fx))+a+2b)}{f(a+b)^2(\cos(e)-\sin(e))(\sin(e)+\cos(e))} + \frac{b\left(15a^2+20ab+8b^2\right)(\cos(2e)}{}\right.}{64a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(C
os[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2
*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2*b)*S
in[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) +
(b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e] - 3*a
*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))
))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.089, size = 321, normalized size = 2.2

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{f a^{3}}-\frac{7 b^{2} \left(\tan\left(fx+e\right)\right)^{3}}{8 f a \left(a+b+b \left(\tan\left(fx+e\right)\right)^{2}\right)^{2} \left(a^{2}+2 ab+b^{2}\right)}-\frac{b^{3} \left(\tan\left(fx+e\right)\right)^{3}}{2 f a^{2} \left(a+b+b \left(\tan\left(fx+e\right)\right)^{2}\right)^{2} \left(a^{2}+2 a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f/a^3*arctan(tan(f*x+e))-7/8/f/a*b^2/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^
2)*tan(f*x+e)^3-1/2/f/a^2*b^3/(a+b+b*tan(f*x+e)^2)^2/(a^2+2*a*b+b^2)*tan(f*
x+e)^3-9/8*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/2/f/a^2*b^2/(a+b
+b*tan(f*x+e)^2)^2/(a+b)*tan(f*x+e)-15/8/f/a*b/(a^2+2*a*b+b^2)/((a+b)*b)^(1
/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-5/2/f/a^2*b^2/(a^2+2*a*b+b^2)/((a+
b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))-1/f/a^3*b^3/(a^2+2*a*b+b^2
)/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.723028, size = 1854, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2*
b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15*a
```

```
^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^4 +
 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log((
(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4
*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/
(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a*b^3)*co
s(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(
a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*
b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b
+ 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3 + b^4)*f*x
+ ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 +
 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b)
)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 +
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x +
 e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4
*b^3 + a^3*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41983, size = 277, normalized size = 1.92

$$-\frac{\frac{\left(15\,a^{2}b+20\,ab^{2}+8\,b^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{5}+2\,a^{4}b+a^{3}b^{2}\right)\sqrt{ab+b^{2}}}+\frac{7\,ab^{2}\tan\left(fx+e\right)^{3}+4\,b^{3}\tan\left(fx+e\right)^{3}+9\,a^{2}b\tan\left(fx+e\right)+13\,ab^{2}\tan\left(fx+e\right)+4\,b^{3}\tan(fx+}{\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)\left(b\tan\left(fx+e\right)^{2}+a+b\right)^{2}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b
 + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x
 + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a^2*b
^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

3.373 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=181

$$\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{7/2}}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2f(a+b)^3}-\frac{b(9a+4b)\cot(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}$$

```
[Out] -(x/a^3) + (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*f) - ((8*a^2 - 11*a*b - 4*b^2)*Cot[e +
f*x])/(8*a^2*(a + b)^3*f) - (b*Cot[e + f*x])/(4*a*(a + b)*f*(a + b + b*Tan[
e + f*x]^2)^2) - (b*(9*a + 4*b)*Cot[e + f*x])/(8*a^2*(a + b)^2*f*(a + b + b
*Tan[e + f*x]^2))
```

Rubi [A] time = 0.380387, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 203, 205}

$$\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{7/2}}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2f(a+b)^3}-\frac{b(9a+4b)\cot(e+fx)}{8a^2f(a+b)^2\left(a+b\tan^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] -(x/a^3) + (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(7/2)*f) - ((8*a^2 - 11*a*b - 4*b^2)*Cot[e +
f*x])/(8*a^2*(a + b)^3*f) - (b*Cot[e + f*x])/(4*a*(a + b)*f*(a + b + b*Tan[
e + f*x]^2)^2) - (b*(9*a + 4*b)*Cot[e + f*x])/(8*a^2*(a + b)^2*f*(a + b + b
*Tan[e + f*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b+bx^2\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-b-5bx^2}{x^2(1+x^2)\left(a+b+bx^2\right)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= -\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2(a+b)^3f} - \frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(9a+}{8a^2(a+b)^2f\,(}$$

$$= -\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2(a+b)^3f} - \frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(9a+}{8a^2(a+b)^2f\,(}$$

$$= -\frac{x}{a^3} + \frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}f} - \frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{8a^2(a+b)^3f} -$$

Mathematica [C] time = 6.99894, size = 2089, normalized size = 11.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*
(-(b^2*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4
*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(
a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*Sqrt[a +
b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^2*ArcTan[Sec[f*x]*(Cos[2*
e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt
[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a
*Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*
e]])))/((a + b)^3*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x
])*Csc[e]*Csc[e + f*x]*Sec[2*e]*Sec[e + f*x]^6*(8*a^5*f*x*Cos[f*x] + 56*a^4
*b*f*x*Cos[f*x] + 184*a^3*b^2*f*x*Cos[f*x] + 296*a^2*b^3*f*x*Cos[f*x] + 224
*a*b^4*f*x*Cos[f*x] + 64*b^5*f*x*Cos[f*x] - 12*a^5*f*x*Cos[3*f*x] - 68*a^4*
b*f*x*Cos[3*f*x] - 132*a^3*b^2*f*x*Cos[3*f*x] - 108*a^2*b^3*f*x*Cos[3*f*x]
- 32*a*b^4*f*x*Cos[3*f*x] - 8*a^5*f*x*Cos[2*e - f*x] - 56*a^4*b*f*x*Cos[2*e
 - f*x] - 184*a^3*b^2*f*x*Cos[2*e - f*x] - 296*a^2*b^3*f*x*Cos[2*e - f*x] -
 224*a*b^4*f*x*Cos[2*e - f*x] - 64*b^5*f*x*Cos[2*e - f*x] - 8*a^5*f*x*Cos[2
*e + f*x] - 56*a^4*b*f*x*Cos[2*e + f*x] - 184*a^3*b^2*f*x*Cos[2*e + f*x] -
296*a^2*b^3*f*x*Cos[2*e + f*x] - 224*a*b^4*f*x*Cos[2*e + f*x] - 64*b^5*f*x*
Cos[2*e + f*x] + 8*a^5*f*x*Cos[4*e + f*x] + 56*a^4*b*f*x*Cos[4*e + f*x] + 1
84*a^3*b^2*f*x*Cos[4*e + f*x] + 296*a^2*b^3*f*x*Cos[4*e + f*x] + 224*a*b^4*
f*x*Cos[4*e + f*x] + 64*b^5*f*x*Cos[4*e + f*x] + 12*a^5*f*x*Cos[2*e + 3*f*x
] + 68*a^4*b*f*x*Cos[2*e + 3*f*x] + 132*a^3*b^2*f*x*Cos[2*e + 3*f*x] + 108*
a^2*b^3*f*x*Cos[2*e + 3*f*x] + 32*a*b^4*f*x*Cos[2*e + 3*f*x] - 12*a^5*f*x*C
os[4*e + 3*f*x] - 68*a^4*b*f*x*Cos[4*e + 3*f*x] - 132*a^3*b^2*f*x*Cos[4*e +
 3*f*x] - 108*a^2*b^3*f*x*Cos[4*e + 3*f*x] - 32*a*b^4*f*x*Cos[4*e + 3*f*x]
+ 12*a^5*f*x*Cos[6*e + 3*f*x] + 68*a^4*b*f*x*Cos[6*e + 3*f*x] + 132*a^3*b^2
*f*x*Cos[6*e + 3*f*x] + 108*a^2*b^3*f*x*Cos[6*e + 3*f*x] + 32*a*b^4*f*x*Cos
[6*e + 3*f*x] - 4*a^5*f*x*Cos[2*e + 5*f*x] - 12*a^4*b*f*x*Cos[2*e + 5*f*x]
- 12*a^3*b^2*f*x*Cos[2*e + 5*f*x] - 4*a^2*b^3*f*x*Cos[2*e + 5*f*x] + 4*a^5*
f*x*Cos[4*e + 5*f*x] + 12*a^4*b*f*x*Cos[4*e + 5*f*x] + 12*a^3*b^2*f*x*Cos[4
*e + 5*f*x] + 4*a^2*b^3*f*x*Cos[4*e + 5*f*x] - 4*a^5*f*x*Cos[6*e + 5*f*x] -
 12*a^4*b*f*x*Cos[6*e + 5*f*x] - 12*a^3*b^2*f*x*Cos[6*e + 5*f*x] - 4*a^2*b^
3*f*x*Cos[6*e + 5*f*x] + 4*a^5*f*x*Cos[8*e + 5*f*x] + 12*a^4*b*f*x*Cos[8*e
+ 5*f*x] + 12*a^3*b^2*f*x*Cos[8*e + 5*f*x] + 4*a^2*b^3*f*x*Cos[8*e + 5*f*x]
 - 32*a^5*Sin[f*x] - 64*a^4*b*Sin[f*x] - 30*a^2*b^3*Sin[f*x] - 120*a*b^4*Si
n[f*x] - 48*b^5*Sin[f*x] + 32*a^5*Sin[3*f*x] + 64*a^4*b*Sin[3*f*x] + 26*a^3
*b^2*Sin[3*f*x] + 86*a^2*b^3*Sin[3*f*x] + 32*a*b^4*Sin[3*f*x] - 48*a^5*Sin[
2*e - f*x] - 128*a^4*b*Sin[2*e - f*x] - 128*a^3*b^2*Sin[2*e - f*x] - 30*a^2
*b^3*Sin[2*e - f*x] - 120*a*b^4*Sin[2*e - f*x] - 48*b^5*Sin[2*e - f*x] + 48
*a^5*Sin[2*e + f*x] + 128*a^4*b*Sin[2*e + f*x] + 102*a^3*b^2*Sin[2*e + f*x]
```

```
 - 86*a^2*b^3*Sin[2*e + f*x] - 136*a*b^4*Sin[2*e + f*x] - 48*b^5*Sin[2*e +
f*x] - 32*a^5*Sin[4*e + f*x] - 64*a^4*b*Sin[4*e + f*x] + 26*a^3*b^2*Sin[4*e
 + f*x] + 86*a^2*b^3*Sin[4*e + f*x] + 136*a*b^4*Sin[4*e + f*x] + 48*b^5*Sin
[4*e + f*x] - 8*a^5*Sin[2*e + 3*f*x] - 26*a^3*b^2*Sin[2*e + 3*f*x] - 86*a^2
*b^3*Sin[2*e + 3*f*x] - 32*a*b^4*Sin[2*e + 3*f*x] + 32*a^5*Sin[4*e + 3*f*x]
 + 64*a^4*b*Sin[4*e + 3*f*x] - 13*a^3*b^2*Sin[4*e + 3*f*x] - 36*a^2*b^3*Sin
[4*e + 3*f*x] - 16*a*b^4*Sin[4*e + 3*f*x] - 8*a^5*Sin[6*e + 3*f*x] + 13*a^3
*b^2*Sin[6*e + 3*f*x] + 36*a^2*b^3*Sin[6*e + 3*f*x] + 16*a*b^4*Sin[6*e + 3*
f*x] + 8*a^5*Sin[2*e + 5*f*x] + 13*a^3*b^2*Sin[2*e + 5*f*x] + 6*a^2*b^3*Sin
[2*e + 5*f*x] - 13*a^3*b^2*Sin[4*e + 5*f*x] - 6*a^2*b^3*Sin[4*e + 5*f*x] +
8*a^5*Sin[6*e + 5*f*x]))/(512*a^3*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

Maple [B] time = 0.113, size = 337, normalized size = 1.9

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^3}-\frac{1}{f\left(a+b\right)^3\tan\left(fx+e\right)}+\frac{11\,b^3\left(\tan\left(fx+e\right)\right)^3}{8\,f\left(a+b\right)^3a\left(a+b+b\left(\tan\left(fx+e\right)\right)^2\right)^2}+\frac{b^4\left(\tan\left(\right.\right.}{2\,f\left(a+b\right)^3a^2\left(a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/f/a^3*arctan(tan(f*x+e))-1/f/(a+b)^3/tan(f*x+e)+11/8/f*b^3/(a+b)^3/a/(a+
b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3+1/2/f*b^4/(a+b)^3/a^2/(a+b+b*tan(f*x+e)^2)
^2*tan(f*x+e)^3+13/8/f*b^2/(a+b)^3/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)+17/8/f
*b^3/(a+b)^3/a/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)+1/2/f*b^4/(a+b)^3/a^2/(a+b
+b*tan(f*x+e)^2)^2*tan(f*x+e)+35/8/f*b^2/(a+b)^3/a/((a+b)*b)^(1/2)*arctan(t
an(f*x+e)*b/((a+b)*b)^(1/2))+7/2/f*b^3/(a+b)^3/a^2/((a+b)*b)^(1/2)*arctan(t
an(f*x+e)*b/((a+b)*b)^(1/2))+1/f*b^4/(a+b)^3/a^3/((a+b)*b)^(1/2)*arctan(tan
(f*x+e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [B] time = 0.811265, size = 2372, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^5 + 13*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 4*(16*a^4*b - 1
3*a^3*b^2 + 5*a^2*b^3 + 4*a*b^4)*cos(f*x + e)^3 - (35*a^2*b^3 + 28*a*b^4 +
8*b^5 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2
+ 28*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b
+ 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*
b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(
f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x
+ e) + 4*(8*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 32*((a^5 + 3*a^4
*b + 3*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*
x)*sin(f*x + e))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 +
 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 +
3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), -1/16*(2*(8*a^5 + 13*a^3
*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 2*(16*a^4*b - 13*a^3*b^2 + 5*a^2*b^3 + 4
*a*b^4)*cos(f*x + e)^3 + (35*a^2*b^3 + 28*a*b^4 + 8*b^5 + (35*a^4*b + 28*a^
3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 28*a^2*b^3 + 8*a*b^4)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(8*a^3*b^2 -
11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 16*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b
^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*f*x*cos(
f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2
+ 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
+ a^3*b^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44737, size = 347, normalized size = 1.92

$$\frac{\frac{\left(35\,a^2b^2+28\,ab^3+8\,b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(fx+e\right)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sqrt{ab+b^2}}+\frac{11\,ab^3\tan\left(fx+e\right)^3+4\,b^4\tan\left(fx+e\right)^3+13\,a^2b^2\tan\left(fx+e\right)+17\,ab^3\tan\left(fx+e\right)+4\,b^4\tan\left(f\right.}{\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\left(b\tan\left(fx+e\right)^2+a+b\right)^2}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*sqrt(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 4*b^4*tan(f*x + e)^3 + 1
3*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e) + 4*b^4*tan(f*x + e))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e
)/a^3 - 8/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

3.374 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=230

$$-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{9/2}}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2f(a+b)^3}+\frac{\left(32a^2b+8a^3-15ab^2-4b^3\right)\cot(e}{8a^2f(a+b)^4}$$

```
[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(9/2)*f) + ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b^3
)*Cot[e + f*x])/(8*a^2*(a + b)^4*f) - ((8*a^2 - 39*a*b - 12*b^2)*Cot[e + f*
x]^3)/(24*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^3)/(4*a*(a + b)*f*(a + b + b*T
an[e + f*x]^2)^2) - (b*(11*a + 4*b)*Cot[e + f*x]^3)/(8*a^2*(a + b)^2*f*(a +
 b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.46112, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 203, 205}

$$-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{9/2}}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2f(a+b)^3}+\frac{\left(32a^2b+8a^3-15ab^2-4b^3\right)\cot(e}{8a^2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] x/a^3 - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(8*a^3*(a + b)^(9/2)*f) + ((8*a^3 + 32*a^2*b - 15*a*b^2 - 4*b^3
)*Cot[e + f*x])/(8*a^2*(a + b)^4*f) - ((8*a^2 - 39*a*b - 12*b^2)*Cot[e + f*
x]^3)/(24*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^3)/(4*a*(a + b)*f*(a + b + b*T
an[e + f*x]^2)^2) - (b*(11*a + 4*b)*Cot[e + f*x]^3)/(8*a^2*(a + b)^2*f*(a +
 b + b*Tan[e + f*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-7bx^2}{x^4(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\text{Subst}}{}$$

$$= -\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a}{8a^2(a+b)^2}$$

$$= \frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{}{4a(a+b)}$$

$$= \frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{8a^2(a+b)^4f} - \frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{24a^2(a+b)^3f} - \frac{}{4a(a+b)}$$

$$= \frac{x}{a^3} - \frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}f} + \frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+}{8a^2(a+b)^4f}$$

Mathematica [C] time = 7.61893, size = 3340, normalized size = 14.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] ((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*
((b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*
e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a
```

```
*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*Sqrt[a + b
]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*b^3*ArcTan[Sec[f*x]*(Cos[2*e
]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[
a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*
Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e
]]))))/((a + b)^4*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x]
)*Csc[e]*Csc[e + f*x]^3*Sec[2*e]*Sec[e + f*x]^6*(-36*a^6*f*x*Cos[f*x] - 336
*a^5*b*f*x*Cos[f*x] - 1560*a^4*b^2*f*x*Cos[f*x] - 3600*a^3*b^3*f*x*Cos[f*x]
 - 4260*a^2*b^4*f*x*Cos[f*x] - 2496*a*b^5*f*x*Cos[f*x] - 576*b^6*f*x*Cos[f*
x] + 36*a^6*f*x*Cos[3*f*x] + 240*a^5*b*f*x*Cos[3*f*x] + 408*a^4*b^2*f*x*Cos
[3*f*x] - 48*a^3*b^3*f*x*Cos[3*f*x] - 732*a^2*b^4*f*x*Cos[3*f*x] - 672*a*b^
5*f*x*Cos[3*f*x] - 192*b^6*f*x*Cos[3*f*x] + 36*a^6*f*x*Cos[2*e - f*x] + 336
*a^5*b*f*x*Cos[2*e - f*x] + 1560*a^4*b^2*f*x*Cos[2*e - f*x] + 3600*a^3*b^3*
f*x*Cos[2*e - f*x] + 4260*a^2*b^4*f*x*Cos[2*e - f*x] + 2496*a*b^5*f*x*Cos[2
*e - f*x] + 576*b^6*f*x*Cos[2*e - f*x] + 36*a^6*f*x*Cos[2*e + f*x] + 336*a^
5*b*f*x*Cos[2*e + f*x] + 1560*a^4*b^2*f*x*Cos[2*e + f*x] + 3600*a^3*b^3*f*x
*Cos[2*e + f*x] + 4260*a^2*b^4*f*x*Cos[2*e + f*x] + 2496*a*b^5*f*x*Cos[2*e
+ f*x] + 576*b^6*f*x*Cos[2*e + f*x] - 36*a^6*f*x*Cos[4*e + f*x] - 336*a^5*b
*f*x*Cos[4*e + f*x] - 1560*a^4*b^2*f*x*Cos[4*e + f*x] - 3600*a^3*b^3*f*x*Co
s[4*e + f*x] - 4260*a^2*b^4*f*x*Cos[4*e + f*x] - 2496*a*b^5*f*x*Cos[4*e + f
*x] - 576*b^6*f*x*Cos[4*e + f*x] - 36*a^6*f*x*Cos[2*e + 3*f*x] - 240*a^5*b*
f*x*Cos[2*e + 3*f*x] - 408*a^4*b^2*f*x*Cos[2*e + 3*f*x] + 48*a^3*b^3*f*x*Co
s[2*e + 3*f*x] + 732*a^2*b^4*f*x*Cos[2*e + 3*f*x] + 672*a*b^5*f*x*Cos[2*e +
 3*f*x] + 192*b^6*f*x*Cos[2*e + 3*f*x] + 36*a^6*f*x*Cos[4*e + 3*f*x] + 240*
a^5*b*f*x*Cos[4*e + 3*f*x] + 408*a^4*b^2*f*x*Cos[4*e + 3*f*x] - 48*a^3*b^3*
f*x*Cos[4*e + 3*f*x] - 732*a^2*b^4*f*x*Cos[4*e + 3*f*x] - 672*a*b^5*f*x*Cos
[4*e + 3*f*x] - 192*b^6*f*x*Cos[4*e + 3*f*x] - 36*a^6*f*x*Cos[6*e + 3*f*x]
- 240*a^5*b*f*x*Cos[6*e + 3*f*x] - 408*a^4*b^2*f*x*Cos[6*e + 3*f*x] + 48*a^
3*b^3*f*x*Cos[6*e + 3*f*x] + 732*a^2*b^4*f*x*Cos[6*e + 3*f*x] + 672*a*b^5*f
*x*Cos[6*e + 3*f*x] + 192*b^6*f*x*Cos[6*e + 3*f*x] - 12*a^6*f*x*Cos[2*e + 5
*f*x] - 144*a^5*b*f*x*Cos[2*e + 5*f*x] - 456*a^4*b^2*f*x*Cos[2*e + 5*f*x] -
 624*a^3*b^3*f*x*Cos[2*e + 5*f*x] - 396*a^2*b^4*f*x*Cos[2*e + 5*f*x] - 96*a
*b^5*f*x*Cos[2*e + 5*f*x] + 12*a^6*f*x*Cos[4*e + 5*f*x] + 144*a^5*b*f*x*Cos
[4*e + 5*f*x] + 456*a^4*b^2*f*x*Cos[4*e + 5*f*x] + 624*a^3*b^3*f*x*Cos[4*e
+ 5*f*x] + 396*a^2*b^4*f*x*Cos[4*e + 5*f*x] + 96*a*b^5*f*x*Cos[4*e + 5*f*x]
 - 12*a^6*f*x*Cos[6*e + 5*f*x] - 144*a^5*b*f*x*Cos[6*e + 5*f*x] - 456*a^4*b
^2*f*x*Cos[6*e + 5*f*x] - 624*a^3*b^3*f*x*Cos[6*e + 5*f*x] - 396*a^2*b^4*f*
x*Cos[6*e + 5*f*x] - 96*a*b^5*f*x*Cos[6*e + 5*f*x] + 12*a^6*f*x*Cos[8*e + 5
*f*x] + 144*a^5*b*f*x*Cos[8*e + 5*f*x] + 456*a^4*b^2*f*x*Cos[8*e + 5*f*x] +
 624*a^3*b^3*f*x*Cos[8*e + 5*f*x] + 396*a^2*b^4*f*x*Cos[8*e + 5*f*x] + 96*a
*b^5*f*x*Cos[8*e + 5*f*x] - 12*a^6*f*x*Cos[4*e + 7*f*x] - 48*a^5*b*f*x*Cos[
4*e + 7*f*x] - 72*a^4*b^2*f*x*Cos[4*e + 7*f*x] - 48*a^3*b^3*f*x*Cos[4*e + 7
*f*x] - 12*a^2*b^4*f*x*Cos[4*e + 7*f*x] + 12*a^6*f*x*Cos[6*e + 7*f*x] + 48*
a^5*b*f*x*Cos[6*e + 7*f*x] + 72*a^4*b^2*f*x*Cos[6*e + 7*f*x] + 48*a^3*b^3*f
*x*Cos[6*e + 7*f*x] + 12*a^2*b^4*f*x*Cos[6*e + 7*f*x] - 12*a^6*f*x*Cos[8*e
```

```
+ 7*f*x] - 48*a^5*b*f*x*Cos[8*e + 7*f*x] - 72*a^4*b^2*f*x*Cos[8*e + 7*f*x]
- 48*a^3*b^3*f*x*Cos[8*e + 7*f*x] - 12*a^2*b^4*f*x*Cos[8*e + 7*f*x] + 12*a^
6*f*x*Cos[10*e + 7*f*x] + 48*a^5*b*f*x*Cos[10*e + 7*f*x] + 72*a^4*b^2*f*x*C
os[10*e + 7*f*x] + 48*a^3*b^3*f*x*Cos[10*e + 7*f*x] + 12*a^2*b^4*f*x*Cos[10
*e + 7*f*x] - 128*a^6*Sin[f*x] - 440*a^5*b*Sin[f*x] - 1152*a^4*b^2*Sin[f*x]
 - 1920*a^3*b^3*Sin[f*x] + 228*a^2*b^4*Sin[f*x] + 1320*a*b^5*Sin[f*x] + 432
*b^6*Sin[f*x] + 48*a^6*Sin[3*f*x] + 104*a^5*b*Sin[3*f*x] + 640*a^4*b^2*Sin[
3*f*x] + 1511*a^3*b^3*Sin[3*f*x] - 528*a^2*b^4*Sin[3*f*x] + 264*a*b^5*Sin[3
*f*x] + 144*b^6*Sin[3*f*x] - 32*a^6*Sin[2*e - f*x] + 384*a^5*b*Sin[2*e - f*
x] + 2048*a^4*b^2*Sin[2*e - f*x] + 3072*a^3*b^3*Sin[2*e - f*x] + 228*a^2*b^
4*Sin[2*e - f*x] + 1320*a*b^5*Sin[2*e - f*x] + 432*b^6*Sin[2*e - f*x] + 32*
a^6*Sin[2*e + f*x] - 384*a^5*b*Sin[2*e + f*x] - 2048*a^4*b^2*Sin[2*e + f*x]
 - 2919*a^3*b^3*Sin[2*e + f*x] + 642*a^2*b^4*Sin[2*e + f*x] + 1416*a*b^5*Si
n[2*e + f*x] + 432*b^6*Sin[2*e + f*x] - 128*a^6*Sin[4*e + f*x] - 440*a^5*b*
Sin[4*e + f*x] - 1152*a^4*b^2*Sin[4*e + f*x] - 2073*a^3*b^3*Sin[4*e + f*x]
- 642*a^2*b^4*Sin[4*e + f*x] - 1416*a*b^5*Sin[4*e + f*x] - 432*b^6*Sin[4*e
+ f*x] - 144*a^6*Sin[2*e + 3*f*x] - 672*a^5*b*Sin[2*e + 3*f*x] - 960*a^4*b^
2*Sin[2*e + 3*f*x] + 153*a^3*b^3*Sin[2*e + 3*f*x] + 528*a^2*b^4*Sin[2*e + 3
*f*x] - 264*a*b^5*Sin[2*e + 3*f*x] - 144*b^6*Sin[2*e + 3*f*x] + 48*a^6*Sin[
4*e + 3*f*x] + 104*a^5*b*Sin[4*e + 3*f*x] + 640*a^4*b^2*Sin[4*e + 3*f*x] +
1664*a^3*b^3*Sin[4*e + 3*f*x] - 66*a^2*b^4*Sin[4*e + 3*f*x] - 408*a*b^5*Sin
[4*e + 3*f*x] - 144*b^6*Sin[4*e + 3*f*x] - 144*a^6*Sin[6*e + 3*f*x] - 672*a
^5*b*Sin[6*e + 3*f*x] - 960*a^4*b^2*Sin[6*e + 3*f*x] + 66*a^2*b^4*Sin[6*e +
 3*f*x] + 408*a*b^5*Sin[6*e + 3*f*x] + 144*b^6*Sin[6*e + 3*f*x] + 80*a^6*Si
n[2*e + 5*f*x] + 480*a^5*b*Sin[2*e + 5*f*x] + 832*a^4*b^2*Sin[2*e + 5*f*x]
+ 294*a^2*b^4*Sin[2*e + 5*f*x] + 96*a*b^5*Sin[2*e + 5*f*x] - 48*a^6*Sin[4*e
 + 5*f*x] - 120*a^5*b*Sin[4*e + 5*f*x] - 294*a^2*b^4*Sin[4*e + 5*f*x] - 96*
a*b^5*Sin[4*e + 5*f*x] + 80*a^6*Sin[6*e + 5*f*x] + 480*a^5*b*Sin[6*e + 5*f*
x] + 832*a^4*b^2*Sin[6*e + 5*f*x] - 51*a^3*b^3*Sin[6*e + 5*f*x] - 132*a^2*b
^4*Sin[6*e + 5*f*x] - 48*a*b^5*Sin[6*e + 5*f*x] - 48*a^6*Sin[8*e + 5*f*x] -
 120*a^5*b*Sin[8*e + 5*f*x] + 51*a^3*b^3*Sin[8*e + 5*f*x] + 132*a^2*b^4*Sin
[8*e + 5*f*x] + 48*a*b^5*Sin[8*e + 5*f*x] + 32*a^6*Sin[4*e + 7*f*x] + 104*a
^5*b*Sin[4*e + 7*f*x] + 51*a^3*b^3*Sin[4*e + 7*f*x] + 18*a^2*b^4*Sin[4*e +
7*f*x] - 51*a^3*b^3*Sin[6*e + 7*f*x] - 18*a^2*b^4*Sin[6*e + 7*f*x] + 32*a^6
*Sin[8*e + 7*f*x] + 104*a^5*b*Sin[8*e + 7*f*x]))/(6144*a^3*(a + b)^4*f*(a +
 b*Sec[e + f*x]^2)^3)
```

Maple [A] time = 0.122, size = 374, normalized size = 1.6

$$\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^3}-\frac{1}{3\,f\,(a+b)^3\left(\tan\left(fx+e\right)\right)^3}+\frac{a}{f\,(a+b)^4\tan\left(fx+e\right)}+4\,\frac{b}{f\,(a+b)^4\tan\left(fx+e\right)}-\frac{}{8\,fa\,(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] 1/f/a^3*arctan(tan(f*x+e))-1/3/f/(a+b)^3/tan(f*x+e)^3+1/f/(a+b)^4/tan(f*x+e
)*a+4/f/(a+b)^4/tan(f*x+e)*b-15/8/f*b^4/a/(a+b)^4/(a+b+b*tan(f*x+e)^2)^2*ta
n(f*x+e)^3-1/2/f*b^5/a^2/(a+b)^4/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)^3-17/8/f
*b^3/(a+b)^4/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)-21/8/f*b^4/a/(a+b)^4/(a+b+b*
tan(f*x+e)^2)^2*tan(f*x+e)-1/2/f*b^5/a^2/(a+b)^4/(a+b+b*tan(f*x+e)^2)^2*tan
(f*x+e)-63/8/f*b^3/a/(a+b)^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^
(1/2))-9/2/f*b^4/a^2/(a+b)^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^
(1/2))-1/f*b^5/a^3/(a+b)^4/((a+b)*b)^(1/2)*arctan(tan(f*x+e)*b/((a+b)*b)^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.949497, size = 3640, normalized size = 15.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(4*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 4*
(24*a^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*cos(
f*x + e)^5 - 4*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a*b^
5)*cos(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6
 - 63*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b^4 -
16*a*b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^6)*cos
(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
```

```
*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 -
 (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*
x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(8*a^4*b^2 + 32*a
^3*b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 96*((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 -
8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2
+ 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 - (a^4*b^2 + 4*
a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a^9 + 4*a^8*b +
6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*
b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^
7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^
7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e)), 1/48
*(2*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 2*(24*a
^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*cos(f*x +
 e)^5 - 2*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a*b^5)*co
s(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 - 63
*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b^4 - 16*a*
b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^6)*cos(f*x
+ e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a
 + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(8*a^4*b^2 + 32*a^3*
b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 48*((a^6 + 4*a^5*b + 6*a^4*b^2 +
 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a
^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8
*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3
*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a^9 + 4*a^8*b + 6*a
^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2
 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b
^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b
^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4433, size = 424, normalized size = 1.84

$$-\frac{\frac{3\left(63\,a^{2}b^{3}+36\,ab^{4}+8\,b^{5}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{7}+4\,a^{6}b+6\,a^{5}b^{2}+4\,a^{4}b^{3}+a^{3}b^{4}\right)\sqrt{ab+b^{2}}}+\frac{3\left(15\,ab^{4}\tan(fx+e)^{3}+4\,b^{5}\tan(fx+e)^{3}+17\,a^{2}b^{3}\tan(fx+e)+21\,ab^{4}\tan(fx+e)+4\,b^{5}\,\mathrm{ta}\right.}{\left(a^{6}+4\,a^{5}b+6\,a^{4}b^{2}+4\,a^{3}b^{3}+a^{2}b^{4}\right)\left(b\tan(fx+e)^{2}+a+b\right)^{2}}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(
b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 4*a^6*b + 6*a^5*b^2 +
4*a^4*b^3 + a^3*b^4)*sqrt(a*b + b^2)) + 3*(15*a*b^4*tan(f*x + e)^3 + 4*b^5*
tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) + 21*a*b^4*tan(f*x + e) + 4*b^5*ta
n(f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*(b*tan(f*x +
 e)^2 + a + b)^2) - 24*(f*x + e)/a^3 - 8*(3*a*tan(f*x + e)^2 + 12*b*tan(f*x
 + e)^2 - a - b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x + e)^
3))/f
```

3.375 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

Optimal. Leaf size=285

$$\frac{b^{7/2}\left(99a^2+44ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{11/2}}-\frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{40a^2f(a+b)^3}+\frac{\left(32a^2b+8a^3-51ab^2-12b^3\right)\cot^3(}{24a^2f(a+b)^4}$$

```
[Out] -(x/a^3) + (b^(7/2)*(99*a^2 + 44*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(11/2)*f) - ((8*a^4 + 40*a^3*b + 80*a^2*b^2 -
 19*a*b^3 - 4*b^4)*Cot[e + f*x])/(8*a^2*(a + b)^5*f) + ((8*a^3 + 32*a^2*b -
 51*a*b^2 - 12*b^3)*Cot[e + f*x]^3)/(24*a^2*(a + b)^4*f) - ((8*a^2 - 75*a*b
 - 20*b^2)*Cot[e + f*x]^5)/(40*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^5)/(4*a*(
a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(13*a + 4*b)*Cot[e + f*x]^5)/(8
*a^2*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```

Rubi [A] time = 0.60672, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4141, 1975, 472, 579, 583, 522, 203, 205}

$$\frac{b^{7/2}\left(99a^2+44ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3f(a+b)^{11/2}}-\frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{40a^2f(a+b)^3}+\frac{\left(32a^2b+8a^3-51ab^2-12b^3\right)\cot^3(}{24a^2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```


```
[Out] -(x/a^3) + (b^(7/2)*(99*a^2 + 44*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])
/Sqrt[a + b]])/(8*a^3*(a + b)^(11/2)*f) - ((8*a^4 + 40*a^3*b + 80*a^2*b^2 -
 19*a*b^3 - 4*b^4)*Cot[e + f*x])/(8*a^2*(a + b)^5*f) + ((8*a^3 + 32*a^2*b -
 51*a*b^2 - 12*b^3)*Cot[e + f*x]^3)/(24*a^2*(a + b)^4*f) - ((8*a^2 - 75*a*b
 - 20*b^2)*Cot[e + f*x]^5)/(40*a^2*(a + b)^3*f) - (b*Cot[e + f*x]^5)/(4*a*(
a + b)*f*(a + b + b*Tan[e + f*x]^2)^2) - (b*(13*a + 4*b)*Cot[e + f*x]^5)/(8
*a^2*(a + b)^2*f*(a + b + b*Tan[e + f*x]^2))
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x^6(1+x^2)(a+b+bx^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{4a-5b-9bx^2}{x^6(1+x^2)(a+b+bx^2)^2}\,dx, x, \tan(e+fx)\right)}{4a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(13a}{8a^2(a+b)^2f}$$

$$= \frac{\left(8a^3+32a^2b-51ab^2-12b^3\right)\cot^3(e+fx)}{24a^2(a+b)^4f} - \frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{40a^2(a+b)^3f} - \frac{}{4a(a+b}$$

$$= -\frac{\left(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4\right)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{\left(8a^3+32a^2b-51ab^2-12b^3\right)\cot^3(}{24a^2(a+b)^4f}$$

$$= -\frac{\left(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4\right)\cot(e+fx)}{8a^2(a+b)^5f} + \frac{\left(8a^3+32a^2b-51ab^2-12b^3\right)\cot^3(}{24a^2(a+b)^4f}$$

$$= -\frac{x}{a^3} + \frac{b^{7/2}\left(99a^2+44ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{11/2}f} - \frac{\left(8a^4+40a^3b+80a^2b^2-19ab^3-4b\right.}{8a^2(a+b)^5f}$$

Mathematica [C] time = 8.20968, size = 976, normalized size = 3.42

$$\frac{\left(99a^2+44ba+8b^2\right)\left(\cos(2e+2fx)a+a+2b\right)^3\left(\frac{ib^4\tan^{-1}\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{i\sin(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}\right)(-a\sin(fx)-2b\sin(fx)\right.}{64a^3\sqrt{a+b}f\sqrt{b\cos(4e)-ib\sin(4e)}}\right.}{(a+b)^5\left(b\sec\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

```
[Out] -(x*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(8*a^3*(a + b*Sec[e +
f*x]^2)^3) + ((11*a*Cos[e] + 26*b*Cos[e])*(a + 2*b + a*Cos[2*e + 2*f*x])^3*
Csc[e]*Csc[e + f*x]^2*Sec[e + f*x]^6)/(120*(a + b)^4*f*(a + b*Sec[e + f*x]^
2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Cot[e]*Csc[e + f*x]^4*Sec[e + f*x
]^6)/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + ((99*a^2 + 44*a*b + 8*b^2)
*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-(b^4*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(
Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x]
 + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b
*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*C
os[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] -
I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e]
)/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((a + b)^5*(a + b*S
ec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]^5
*Sec[e + f*x]^6*Sin[f*x])/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + ((a +
 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]^3*Sec[e + f*x]^6*(-11*a*Si
n[f*x] - 26*b*Sin[f*x]))/(120*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3) + ((a +
 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]*Sec[e + f*x]^6*(23*a^2*Sin
[f*x] + 106*a*b*Sin[f*x] + 173*b^2*Sin[f*x]))/(120*(a + b)^5*f*(a + b*Sec[e
 + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(a*
b^5*Sin[2*e] + 2*b^6*Sin[2*e] - a*b^5*Sin[2*f*x]))/(16*a^3*(a + b)^4*f*(a +
 b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[2*e]*Sec[e +
f*x]^6*(-21*a^2*b^4*Sin[2*e] - 52*a*b^5*Sin[2*e] - 16*b^6*Sin[2*e] + 21*a^2
*b^4*Sin[2*f*x] + 6*a*b^5*Sin[2*f*x]))/(64*a^3*(a + b)^5*f*(a + b*Sec[e + f
*x]^2)^3)
```

Maple [A] time = 0.131, size = 437, normalized size = 1.5

$$-\frac{\arctan\left(\tan\left(fx+e\right)\right)}{fa^3}-\frac{1}{5\,f\,(a+b)^3\left(\tan\left(fx+e\right)\right)^5}+\frac{a}{3\,f\,(a+b)^4\left(\tan\left(fx+e\right)\right)^3}+\frac{4\,b}{3\,f\,(a+b)^4\left(\tan\left(fx+e\right)\right)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x)
```

```
[Out] -1/f/a^3*arctan(tan(f*x+e))-1/5/f/(a+b)^3/tan(f*x+e)^5+1/3/f/(a+b)^4/tan(f*
x+e)^3*a+4/3/f/(a+b)^4/tan(f*x+e)^3*b-1/f/(a+b)^5/tan(f*x+e)*a^2-5/f/(a+b)^
5/tan(f*x+e)*a*b-10/f/(a+b)^5/tan(f*x+e)*b^2+19/8/f*b^5/a/(a+b)^5/(a+b+b*ta
```

```
n(f*x+e)^2)^2*tan(f*x+e)^3+1/2/f*b^6/a^2/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan
(f*x+e)^3+21/8/f*b^4/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)+25/8/f*b^5/a
/(a+b)^5/(a+b+b*tan(f*x+e)^2)^2*tan(f*x+e)+1/2/f*b^6/a^2/(a+b)^5/(a+b+b*tan
(f*x+e)^2)^2*tan(f*x+e)+99/8/f*b^4/a/(a+b)^5/((a+b)*b)^(1/2)*arctan(tan(f*x
+e)*b/((a+b)*b)^(1/2))+11/2/f*b^5/a^2/(a+b)^5/((a+b)*b)^(1/2)*arctan(tan(f*
x+e)*b/((a+b)*b)^(1/2))+1/f*b^6/a^3/(a+b)^5/((a+b)*b)^(1/2)*arctan(tan(f*x+
e)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.10918, size = 5045, normalized size = 17.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/480*(4*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)*
cos(f*x + e)^9 - 4*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 945
*a^3*b^4 - 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 4*(120*a^7 + 40*a^6*b -
1416*a^5*b^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*cos(f
*x + e)^5 + 20*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 153*a^
2*b^5 + 36*a*b^6)*cos(f*x + e)^3 - 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a^2*b^5
)*cos(f*x + e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 55*a^3*b
^4 - 36*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b^4 - 69*
a^2*b^5 + 12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^2*b^5 - 3
6*a*b^6 - 8*b^7)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2
)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b
^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e
) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) +
60*(8*a^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^6)*cos(f*x + e
```

```
) + 480*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*
x*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a
*b^6)*f*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b
^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5
*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4
*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)*sin(f*x + e))/(((a^10
+ 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8
 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f
*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5
 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 -
 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e)), -1/240*(2
*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)*cos(f*x +
e)^9 - 2*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 945*a^3*b^4 -
 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 2*(120*a^7 + 40*a^6*b - 1416*a^5*b
^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*cos(f*x + e)^5
+ 10*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 153*a^2*b^5 + 36
*a*b^6)*cos(f*x + e)^3 + 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a^2*b^5)*cos(f*x
+ e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 55*a^3*b^4 - 36*a^
2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b^4 - 69*a^2*b^5 +
12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^2*b^5 - 36*a*b^6 -
8*b^7)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(8*a
^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^6)*cos(f*x + e) + 240
*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*x*cos(f
*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f
*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*
a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^
3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 +
10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)*sin(f*x + e))/(((a^10 + 5*a^9
*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a
^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)
^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*
b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*
b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6
*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64043, size = 545, normalized size = 1.91

$$\frac{\frac{15\left(99\,a^{2}b^{4}+44\,ab^{5}+8\,b^{6}\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^{2}}}\right)\right)}{\left(a^{8}+5\,a^{7}b+10\,a^{6}b^{2}+10\,a^{5}b^{3}+5\,a^{4}b^{4}+a^{3}b^{5}\right)\sqrt{ab+b^{2}}}+\frac{15\left(19\,ab^{5}\tan(fx+e)^{3}+4\,b^{6}\tan(fx+e)^{3}+21\,a^{2}b^{4}\tan(fx+e)+25\,ab^{5}\tan(fx+e)+4\,b^{6}\,\mathrm{ta}\right.}{\left(a^{7}+5\,a^{6}b+10\,a^{5}b^{2}+10\,a^{4}b^{3}+5\,a^{3}b^{4}+a^{2}b^{5}\right)\left(b\tan(fx+e)^{2}+a+b\right)^{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 5*a^7*b + 10*a^6*b^2
+ 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt(a*b + b^2)) + 15*(19*a*b^5*tan(f*x
 + e)^3 + 4*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) + 25*a*b^5*tan(f*x
 + e) + 4*b^6*tan(f*x + e))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a
^3*b^4 + a^2*b^5)*(b*tan(f*x + e)^2 + a + b)^2) - 120*(f*x + e)/a^3 - 8*(15
*a^2*tan(f*x + e)^4 + 75*a*b*tan(f*x + e)^4 + 150*b^2*tan(f*x + e)^4 - 5*a^
2*tan(f*x + e)^2 - 25*a*b*tan(f*x + e)^2 - 20*b^2*tan(f*x + e)^2 + 3*a^2 +
6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*t
an(f*x + e)^5))/f
```

3.376 $\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$

Optimal. Leaf size=111

$$\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f - ((a + 2*b)*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2*f) + (a +
b*Sec[e + f*x]^2)^(5/2)/(5*b^2*f)
```

Rubi [A] time = 0.141112, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4139, 446, 88, 50, 63, 208}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```


```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f - ((a + 2*b)*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2*f) + (a +
b*Sec[e + f*x]^2)^(5/2)/(5*b^2*f)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\sqrt{a+bx^2}}{x}\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^2\sqrt{a+bx}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)\sqrt{a+bx}}{b}+\frac{\sqrt{a+bx}}{x}+\frac{(a+bx)^{3/2}}{b}\right)dx,x,\sec^2(e+fx)\right)}{2f}\\
&= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\right.}{}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\left(a+b\sec^2(e+fx)\right.}{5b^2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\left(a+b\sec^2(e+fx)\right.}{5b^2f}\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+2b)\left(a+b\sec^2(e\right.}{3b^2f}
\end{aligned}
$$

Mathematica [F] time = 2.41833, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5, x]
```

Maple [B] time = 0.439, size = 924, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/30/f/(a+b)^(1/2)/b^2*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-1+
cos(f*x+e))*(15*cos(f*x+e)^5*(a+b)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2))*a^(1/2)*b^2+15*cos(f*x+e)^5*ln(-2/(a+b)^(1/2)*
(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+b)/sin(f*x+e)^2)*a*b^2-15*cos(f*x+e)^5*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))
*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e
)^2)*a*b^2+2*cos(f*x+e)^5*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a^2+10*cos(f*x+e)^5*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a*b-15*cos(f*x+e)^5*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^2+2*cos(f*x+e)^4*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^2+10*cos(f*x+e)^4*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a*b-15*cos(f*x+e)^4*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^2-cos(f*x+e)^3*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a*b+10*cos(f*x+e)^3*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^2-cos(f*x+e)^2*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a*b+10*cos(f*x+e)^2*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^2-3*cos(f*x+e)*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^2-3*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)/s
in(f*x+e)^2/cos(f*x+e)^4/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

Fricas [B] time = 6.67986, size = 1123, normalized size = 10.12

$$\left[\frac{15\sqrt{a}b^2\cos\left(fx+e\right)^4\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/120*(15*sqrt(a)*b^2*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - 8*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(f
*x + e)^2 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(
f*x + e)^4), 1/60*(15*sqrt(-a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b
*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 -
4*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(f*x + e)^2
 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^
4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec^2\left(e+fx\right)}\tan^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

3.377 $\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$

Optimal. Leaf size=80

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - Sqrt[a + b*Sec[e
+ f*x]^2]/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b*f)
```

Rubi [A] time = 0.104652, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4139, 446, 80, 50, 63, 208}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}-\frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```


```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - Sqrt[a + b*Sec[e
+ f*x]^2]/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b*f)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)\sqrt{a+bx^2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{se}\right.}{2f}$$

$$= -\frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}{bf}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3bf}$$

Mathematica [F] time = 1.07293, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3, x]
```

Maple [B] time = 0.36, size = 648, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] -1/6/f/(a+b)^(1/2)/b*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*(-1+co
s(f*x+e))*(3*a^(1/2)*cos(f*x+e)^3*(a+b)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+cos(f*x+e)^3*(a+b)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-3*cos(f*x+e)^3*(a+b)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b-3*cos(f*x+e)^3*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+
e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*c
os(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*
x+e)^2)*a*b+3*cos(f*x+e)^3*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b+cos(f*
x+e)^2*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-3*cos(f*x+
e)^2*(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+cos(f*x+e)*(
a+b)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b*(a+b)^(1/2))/sin(f*x+e)^2/cos(f*x+e)^2/((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

Fricas [B] time = 1.888, size = 969, normalized size = 12.11

$$\left[\frac{3\,\sqrt{a}b\cos\left(fx+e\right)^{2}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\left(fx+e\right)^{4}+32\,ab^{3}\cos\left(fx+e\right)^{2}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*co
s(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 +
 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)
^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)) + 8*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(b*f*cos(f*x + e)^2), -1/12*(3*sqrt(-a)*b*arctan(1/4*(8*a^2*cos(f*x
 + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*co
s(f*x + e)^2 - 4*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(b*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\tan^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\tan\left(fx + e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

3.378 $\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f
```

Rubi [A] time = 0.0647038, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 266, 50, 63, 208}

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sec
[e + f*x]^2]/f
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [B] time = 0.486268, size = 119, normalized size = 2.2

$$\frac{\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{b}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{b}}-2\sqrt{a}\cos(e+fx)\sinh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)\right)}{\sqrt{2}\sqrt{b}f\sqrt{\frac{a\cos(2(e+fx))+a+2b}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] ((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]]*Cos[e + f*x] + Sqrt[2]
*Sqrt[b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])*Sqrt[a + b*Sec[e + f*x]^2]
)/(Sqrt[2]*Sqrt[b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])
```

Maple [A] time = 0.079, size = 61, normalized size = 1.1

$$-\frac{1}{f}\sqrt{a}\ln\left(\frac{1}{\sec\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}\right)\right)+\frac{1}{f}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

```
[Out] -1/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))+(a+b*s
ec(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e), x)
```

Fricas [B] time = 0.834996, size = 779, normalized size = 14.43

$$\left[\frac{\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+b^4-8\left(16\,a^3\cos\right.\right.}{8\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec^2\left(e+fx\right)}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x), x)
```

Giac [B] time = 1.50058, size = 537, normalized size = 9.94

$$2\left(\frac{a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{}{\left(\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\right.}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 2*(a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + 2*((sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b))*b + sqrt(a + b)*b)/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a + b) + a -
 3*b))*sgn(cos(f*x + e))/f
```

3.379 $\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (Sqrt[a + b]*ArcT
anh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

Rubi [A] time = 0.110015, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 446, 83, 63, 208}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (Sqrt[a + b]*ArcT
anh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \text{se}\right.}{2f}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{bf}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Mathematica [F] time = 1.79822, size = 0, normalized size = 0.

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] time = 0.408, size = 570, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f/(a+b)^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)*4^(1/2)*cos(f*x+
e)*(2*a^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
)*(a+b)^(1/2)+a*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)+ln(-2/(a+b)^(1/2)*(-1
+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
b)/sin(f*x+e)^2)*b-ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a-ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b)*(-1+cos(f*x+e))/sin(f*x+e
)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.0962, size = 2398, normalized size = 34.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(a + b)*l
og(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)
^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2
 + 1)))/f, 1/8*(4*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)) + sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(
f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8
*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
)/f, -1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2
 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x
 + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, -1/4*(
sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqr
t(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3
*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(
f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec^2\left(e + fx\right)} \cot\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e), x)
```

3.380 $\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=109

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}$$

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a + b)*Arc
Tanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]*f) - (Cot[e +
f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.148657, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4139, 446, 99, 156, 63, 208}

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a + b)*Arc
Tanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*Sqrt[a + b]*f) - (Cot[e +
f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^2x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{-a-\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \cdots$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+f\cdots}\right.}{bf}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}f} - \frac{\cot^2(e+fx\cdots}{}$$

Mathematica [C] time = 6.13886, size = 527, normalized size = 4.83

$$\frac{e^{i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{1+e^{2i(e+fx)}}{\left(-1+e^{2i(e+fx)}\right)^2} - \frac{(2a+b)\log\left(1-e^{2i(e+fx)}\right)+\sqrt{a}\sqrt{a+b}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}\cdots\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*((1 + E^((2*I)*(e + f*x)))/(-1 + E^((2*I)*(e + f*x)))^2
- ((-2*I)*Sqrt[a]*Sqrt[a + b]*f*x + (2*a + b)*Log[1 - E^((2*I)*(e + f*x))]
+ Sqrt[a]*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a]*Sqrt[a +
b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*a*Log[a + b +
a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - b*Log[a + b + a*E^((2*I)*
```

```
(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f
*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2
]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

Maple [B] time = 0.454, size = 2528, normalized size = 23.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f/(a+b)^(5/2)*(-1+cos(f*x+e))*(4*4^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)*a^(3/2)-2*cos(f*x+e)^2*4^(1/2)
*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3-cos(f*x+e)^2*4^(1/2)*ln(-2/(a+b)
^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/sin(f*x+e)^2)*b^3+2*cos(f*x+e)^2*4^(1/2)*ln(-4*(cos(f*x+e)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3+cos(
f*x+e)^2*4^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^3-4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*4^(1/2)*(a+b)^(3/2)*b+5*4^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos
(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e
)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*
a^2*b-4*cos(f*x+e)^2*4^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2))*(a+b)^(3/2)*a^(1/2)*b+4*4^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(
f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/si
n(f*x+e)^2)*a*b^2-5*4^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b-4*4^(1/2)*ln(-4*(cos(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b^2
-5*cos(f*x+e)^2*4^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f
```

```
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b-4*cos(f
*x+e)^2*4^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b^2+5*cos(f*x+e)^2*
4^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
+b)/(-1+cos(f*x+e)))*a^2*b+4*cos(f*x+e)^2*4^(1/2)*ln(-4*(cos(f*x+e)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b^2+4*4^(1
/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a
*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3
/2)*a^(1/2)*b+8*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)-2*c
os(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*(a+b)^(3/2)
*a+2*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*(a+b)
^(3/2)*b+2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/2)*(
a+b)^(3/2)*a+2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*4^(1/
2)*(a+b)^(3/2)*b-4*cos(f*x+e)^2*4^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)*a^(3/2)+8*cos(f*x+e)^2*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)+16*cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(3/2)*(a+b)^(3/2)+2*4^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(
f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/si
n(f*x+e)^2)*a^3+4^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^3-2*4^(1/2)
*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1
+cos(f*x+e)))*a^3-4^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^3)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/
cos(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/sin(f*x+e)^
4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^2 + a} \cot \left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

Fricas [B] time = 1.68552, size = 3353, normalized size = 30.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
+ ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256
*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos
(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a + b)*log(2*((8*a^
2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 +
4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a
 + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a + b)*cos(f*x + e)^2 - 2*a - b
)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
b^2)) + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8
 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
 + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + 2*((a + b)*cos(
f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f
*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^
3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + ((2*a + b)*cos(f*x +
e)^2 - 2*a - b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2
*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos
(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(
f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
 + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a
 + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x
+ e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2
```

```
+ a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2\left(e+fx\right)}\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

3.381 $\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=161

$$-\frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)}$$

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 12*a*b
+ 3*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)*
f) + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f)
- (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.229269, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4139, 446, 99, 151, 156, 63, 208}

$$-\frac{(8a^2+12ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+\frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 12*a*b
+ 3*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(3/2)*
f) + ((4*a + 3*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)*f)
- (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{(-1+x)^3 x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{-2a-\frac{3bx}{2}}{(-1+x)^2 x\sqrt{a+bx}}\,dx, x, \sec^2(e+f$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx}}{4f}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx}}{4f}$$

$$= \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx}}{4f}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\left(8a^2+12ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} + \frac{(}{}$$

Mathematica [F] time = 5.31879, size = 0, normalized size = 0.

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] time = 0.447, size = 7346, normalized size = 45.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^{2}+a} \cot \left(fx+e\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

Fricas [B] time = 4.82482, size = 4782, normalized size = 29.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*
b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
+ e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) + ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b
^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^
2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f
*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)
, 1/16*(((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b^
```

```
2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a +
 b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sq
rt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*c
os(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 +
24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((6*a^2 + 11*a*b + 5*
b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(
a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/32*(8*((a^
2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^
2 + 2*a*b + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x
+ e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^2 + 12*a*b + 3*b^2
)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a
*b + 3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*
a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
 + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^
4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)
*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(4*((a^2 + 2*a*b + b^2)*c
os(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*s
qrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt
(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*
a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 -
 2*(8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-
a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) +
2*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos(f*
x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)
*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b +
 b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^{2}+a}\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

3.382 $\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx$

Optimal. Leaf size=219

$$\frac{(5a^2b+a^3+15ab^2-5b^3)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{\tan^5(e+$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - b)*(a + 5*b)*Tan[e + f*
x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 5*b)*Tan[e + f*x]^3*S
qrt[a + b + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(6*f)
```

Rubi [A] time = 0.435785, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4141, 1975, 478, 582, 523, 217, 206, 377, 203}

$$\frac{(5a^2b+a^3+15ab^2-5b^3)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{5/2}f}-\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16b^2f}+\frac{\tan^5(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```


```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b + b*Tan[e + f*x]^2]])/(16*b^(5/2)*f) - ((a - b)*(a + 5*b)*Tan[e + f*
x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(16*b^2*f) + ((a - 5*b)*Tan[e + f*x]^3*S
qrt[a + b + b*Tan[e + f*x]^2])/(24*b*f) + (Tan[e + f*x]^5*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(6*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b(1+x^2)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} - \frac{\text{Subst}\left(\int \frac{x^4(5(a+b)+(-a+5b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan\right.}{6f}$$

$$= \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} + \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2}}{6f}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(e+fx)\sqrt{}}{24}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(e+fx)\sqrt{}}{24}$$

$$= -\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} + \frac{(a-5b)\tan^3(e+fx)\sqrt{}}{24}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+5a^2b+15ab^2-5b^3)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e}{\sqrt{a+b+b\tan}}\right.}{16b^{5/2}f}$$

Mathematica [A] time = 3.77023, size = 263, normalized size = 1.2

$$-\frac{\tan(e+fx)\sec^4(e+fx)\left(4\left(3a^2+12ab-7b^2\right)\cos(2(e+fx))+\left(3a^2+14ab-33b^2\right)\cos(4(e+fx))+9a^2+34ab-59b\right.}{384b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] -((16*Sqrt[a]*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]
^2]] - ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/S
qrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*
x]^2])/(8*Sqrt[2]*b^2*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) - ((9*a^2 + 34*
a*b - 59*b^2 + 4*(3*a^2 + 12*a*b - 7*b^2)*Cos[2*(e + f*x)] + (3*a^2 + 14*a*
b - 33*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Tan
[e + f*x])/(384*b^2*f)
```

Maple [C] time = 0.612, size = 2756, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x)
```

```
[Out] -1/48/f/b^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*((b+a*cos(f*
x+e)^2)/cos(f*x+e)^2)^(1/2)*(15*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(
-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2))*a^3+30*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)
/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3+3*sin(f*x+e)*cos
(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3-15*sin(f*x+
e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF(
```

(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3-33*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+33*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+14*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-33*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-14*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+33*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+40*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-40*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+26*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-26*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+3*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-8*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-10*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+10*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2+96*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2-30*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-90*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)/cos(f*x+e)^5

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx+e\right)^{2}+a}\tan\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

Fricas [A] time = 14.2267, size = 4315, normalized size = 19.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/192*(24*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(b)
*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
- 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 - 13*b
^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(b^3*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4
*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*c
os(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 3*(a^3 + 5*a^2*b +
15*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2*((3*a^2*b + 14*a*b^2 - 33*b
^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos
```

```
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/19
2*(48*sqrt(a)*b^3*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b
)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^5 - 3*(a^3 + 5*a^2*b + 15*a*b
^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^
3 - 2*(a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/96*(24*sqrt(a)*b^3*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e)^5 + 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e)^5 - 2*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(
a*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(b^3*f*cos(f*x + e)^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\tan^6\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\tan\left(fx + e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

3.383 $\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$

Optimal. Leaf size=165

$$-\frac{(a^2+6ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}+\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2}}{8bf}$$

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.314065, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4141, 1975, 478, 582, 523, 217, 206, 377, 203}

$$-\frac{(a^2+6ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{3/2}f}+\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}+\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/(8*b*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b(1+x^2)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(-a+3b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan\right.}{4f}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(}}{4f}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(}}{4f}$$

$$= \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(}}{4f}$$

$$= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+6ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} + \cdots$$

Mathematica [A] time = 2.81687, size = 208, normalized size = 1.26

$$\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(8\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)-\frac{(a^2+6ab-3b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{b}}\right)}{4\sqrt{2}bf\sqrt{a\cos(2e+2fx)+a+2b}}+\frac{\tan(e+fx)\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] ((8*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]
 - ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin
[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(4*Sqrt[2]
*b*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + ((a - b + (a - 5*b)*Cos[2*(e + f
*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(16*b*f)
```

Maple [C] time = 0.447, size = 2005, normalized size = 12.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/8/f/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)
^2)/cos(f*x+e)^2)^(1/2)*(16*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(
1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+
sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
^2-2*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
```

```
2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a*b-3*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^2-2*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+
b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-12*sin(f*x+e)*
cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(
1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(
2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2))*b^2+cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-5*cos(f*x+
e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^4*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+5*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*a*b+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-5
*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-3*cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+5*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*b^2+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/(b+a
*cos(f*x+e)^2)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Fricas [B] time = 4.30914, size = 3947, normalized size = 23.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*b^2*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
- a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((a*b - 5*b^2
)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*b^2*cos(f*x + e)^3*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b -
 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b
^2)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x
+ e)^3), -1/32*(8*sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a
*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 + (a^2 + 6*a*b
- 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8
*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/
cos(f*x + e)^4) - 4*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3), -1/16*(4*
sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
```

```
 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(
f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 + (a^2 + 6*a*b - 3*b^2)*sqrt(-b)*
arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))
)*cos(f*x + e)^3 - 2*((a*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\tan^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\tan\left(fx + e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

3.384 $\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}f}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.223441, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4141, 1975, 478, 523, 217, 206, 377, 203}

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```



```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/(2*Sqrt[b]*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b\left(1+x^2\right)}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a+b+(-a+b)x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan(e \cdots\right.}{2f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan(e \cdots\right.}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+f \cdots}}\right.}{f}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e + \cdots}{}$$

Mathematica [C] time = 4.51664, size = 526, normalized size = 4.46

$$\frac{e^{i(e+fx)}\cos(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{i\sqrt{a}\sqrt{b}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)-i\sqrt{a}\sqrt{b}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i \cdots}\right.}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e + f*
x)))^2 + (-2*Sqrt[a]*Sqrt[b]*f*x + I*Sqrt[a]*Sqrt[b]*Log[a + 2*b + a*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
```

```
 f*x)))^2]] - I*Sqrt[a]*Sqrt[b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I
)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] - a*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((
2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 + E^((2*I)
*(e + f*x))))] + b*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*
E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 + E^((
2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*f*Sqrt[a + 2*b + a*C
os[2*e + 2*f*x]])
```

Maple [C] time = 0.345, size = 1331, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] 1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*((b+a*cos(f*x+e)^2
)/cos(f*x+e)^2)^(1/2)*(2*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-2*cos(
f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b-4*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+
b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2))*a+cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)
^2)^(1/2))*a+cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)
```

```
*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+
e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b
)^2)^(1/2))*b+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*
x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(-1+c
os(f*x+e))/(b+a*cos(f*x+e)^2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Fricas [B] time = 1.75934, size = 3605, normalized size = 30.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)
*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3
+ 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*(a - b)*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
```

```
e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f
*x + e) + sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x +
e) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 +
 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2
)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/(b*f*cos(f*x + e)), 1/4*(sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + (a
- b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(b*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec^2\left(e + fx\right)} \tan^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx + e\right)^2 + a} \tan\left(fx + e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

3.385 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rubi [A] time = 0.0508672, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\sec^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\
&= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}
\end{aligned}$$

Mathematica [F] time = 0.0948179, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.429, size = 588, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*2^(1/2)*(EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a+EllipticF((-1+cos(
f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b
^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b-2*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-
(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2))*b)*cos(f*x+e)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)^2*((
b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.17246, size = 2984, normalized size = 37.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*co
s(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
)/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e)
)*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a
^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
 + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - sqrt(b)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/f, -1/4*(sqrt(a)*arctan
(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*
a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e))) - 2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

3.386 $\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rubi [A] time = 0.178692, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4141, 1975, 475, 12, 377, 203}

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^2(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{a}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+}}\right.}{f}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

Mathematica [A] time = 0.633579, size = 130, normalized size = 1.88

$$-\frac{\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{a}\sin(e+fx)\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a+b}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}\right)}{f\sqrt{a+b}\sqrt{\frac{a\cos(2(e+fx))+a+2b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[a + b]*Sqrt[(a + 2*b + a*C
os[2*(e + f*x)])/(a + b)] + Sqrt[2]*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/S
qrt[a + b]]*Sin[e + f*x]))/(Sqrt[a + b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)
])/(a + b)]))
```

Maple [C] time = 0.495, size = 1004, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)
^2)^(1/2)*cos(f*x+e)*(a*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*s
in(f*x+e)*cos(f*x+e)-2*a*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/
2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)+2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*
a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*
a*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(b+a*cos(f*x+e)^2)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Fricas [B] time = 0.946086, size = 1211, normalized size = 17.55

$$\left[\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+a^4-28\,a^3b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))*sin(f*x + e) - 8*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e))/(f*sin(f*x + e)), 1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a
^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^
2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec^2\left(e+fx\right)}\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

3.387 $\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}$$

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f) -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rubi [A] time = 0.250015, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 475, 583, 12, 377, 203}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a + 2*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f) -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^4(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{b-3(a+b)-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan\right.}{3f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2}}{3f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2}}{3f}$$

$$= \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2}}{3f}$$

$$= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

Mathematica [A] time = 0.79228, size = 176, normalized size = 1.54

$$-\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{\csc^3(e+fx)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{a+b} - 3\csc(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sqrt{a}\sin(e+fx)\ldots}{\sqrt{a+b}\sqrt{1\ldots}}\right.\right.}{3f\sqrt{a\cos(2e+2fx)+a+2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(Sqrt[2]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((Csc[e + f*x]^3*(a + b -
 a*Sin[e + f*x]^2)^(3/2))/(a + b) - 3*Csc[e + f*x]*Sqrt[a + b - a*Sin[e + f
*x]^2]*(1 + (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x
])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))))/(3*f*Sqrt[a + 2*b
```

```
+ a*Cos[2*e + 2*f*x]])
```

Maple [C] time = 0.416, size = 3855, normalized size = 33.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)*(-6*cos(f*x+e)^3*sin(f
*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f
*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b
^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2))*a^2-6*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a
^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a
*b+3*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/
2))*a^2+3*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*a*b-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a
+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-6*cos(f*x+e)^
2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^
```

```
(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^
(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*
(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/
2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+6*cos(f*x+e)*sin(f*x+e)*2^(1/
2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)
/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)
*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2))*a^2+6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2
)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b-3*cos(f*x+
e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-3*cos(f
*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+4*co
s(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f*x+e)^4*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)
-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+
6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*
x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(
1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
```

```
)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a^2*sin(f*x+e)-3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1
/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a*b*sin(f*x+e)-3*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a^2+2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*cos(f*x+e)
^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)
*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(b+a*cos(f*x+e)^2)/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)^2+a}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Fricas [B] time = 2.20932, size = 1526, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*log(128*a^4*cos(f*x + e)
^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*
a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(
a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
```

```
 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((4*a + 3*b)*cos(
f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), -1/12*(3*((
a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*
a + 3*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec^2\left(e + fx\right)}\cot^4\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx + e\right)^2 + a}\cot\left(fx + e\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

3.388 $\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e}{5f}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 25*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)^2*f) - ((b - 5*(a + b))*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(15*(a + b)*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(
5*f)
```

Rubi [A] time = 0.333206, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 475, 583, 12, 377, 203}

$$-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^2}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 25*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)^2*f) - ((b - 5*(a + b))*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(15*(a + b)*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(
5*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+b(1+x^2)}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b+bx^2}}{x^6(1+x^2)}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{b-5(a+b)-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e\right.}{5f}$$

$$= -\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} - \frac{\cot^5(e+fx)\sqrt{a+b+b}}{5f}$$

$$= -\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot}{}$$

$$= -\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot}{}$$

$$= -\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{(b-5(a+b))\cot}{}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b+b\,\text{ta}}}{15(a+b)^2f}$$

Mathematica [A] time = 1.76971, size = 178, normalized size = 1.07

$$-\frac{\cot(e+fx)\left(-\left(11a^2+21ab+10b^2\right)\csc^2(e+fx)+23a^2+3(a+b)^2\csc^4(e+fx)+40ab+15b^2\right)\sqrt{a+b\sec^2(e+fx)}}{15f(a+b)^2} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x
]^2]]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e
+ 2*f*x]])) - (Cot[e + f*x]*(23*a^2 + 40*a*b + 15*b^2 - (11*a^2 + 21*a*b +
10*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sqrt[a + b*Sec[e + f*x
]^2])/(15*(a + b)^2*f)
```

Maple [C] time = 0.556, size = 8605, normalized size = 51.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^2 + a} \cot \left(fx+e\right)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

Fricas [B] time = 7.95205, size = 2071, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(
f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
```

```
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8*((23*a^2 + 40*a*b + 15*b^2)
*cos(f*x + e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^2 + 25*
a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((
a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^
2 + (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/60*(15*((a^2 + 2*a*b + b^2)*cos
(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2
 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e
)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^2 + 40*a*b + 15*b^2)*cos(f*x +
e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^2 + 25*a*b + 8*b^2
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b
 + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 +
2*a*b + b^2)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec \left(fx+e\right)^2 + a} \cot \left(fx+e\right)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

3.389 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$

Optimal. Leaf size=135

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+a$$

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) - ((a + 2*b)*(a + b
*Sec[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Sec[e + f*x]^2)^(7/2)/(7*b^2*f)
```

Rubi [A] time = 0.164087, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4139, 446, 88, 50, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) - ((a + 2*b)*(a + b
*Sec[e + f*x]^2)^(5/2))/(5*b^2*f) + (a + b*Sec[e + f*x]^2)^(7/2)/(7*b^2*f)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}{x}\,dx,x,\sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2(a+bx)^{3/2}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)(a+bx)^{3/2}}{b}+\frac{(a+bx)^{3/2}}{x}+\frac{(a+bx)^{5/2}}{b}\right)dx,x,\sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2f}+\frac{\text{Subst}\left(\int \frac{(a+\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}+\frac{\left(a+b\sec^2(e+\ldots\right.}{7b^2f}$$

$$= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\sec^2(e+\ldots\right.}{5b^2f}$$

$$= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\left(a+b\sec^2(e+\ldots\right.}{5b^2f}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+f\ldots\right.}{3f}$$

Mathematica [F] time = 3.20422, size = 0, normalized size = 0.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]

[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5, x]
```

Maple [B] time = 0.5, size = 2606, normalized size = 19.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/420/f/(a+b)^(9/2)/b^2*4^(1/2)*(-1+cos(f*x+e))^3*(-210*cos(f*x+e)^7*ln(-4/
(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^6*b^2-945*cos(f*x+e)^7*ln(-4/(a+b)^(1/2)*
(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+b)/sin(f*x+e)^2)*a^5*b^3-1575*cos(f*x+e)^7*ln(-4/(a+b)^(1/2)*(-1+cos(f*x
+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f
*x+e)^2)*a^4*b^4-1155*cos(f*x+e)^7*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f
*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^
3*b^5-315*cos(f*x+e)^7*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^6+210*co
s(f*x+e)^7*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^6*b^2+945*cos(f*x+e)^7*l
n(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^5*b^3-70*cos(f*x+e)^5*(a+b)^(7/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4-70*cos(f*x+e)^4*(a+b)^(7/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4+84*cos(f*x+e)^3*(a+b)^(7/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4+84*cos(f*x+e)^2*(a+b)^(7/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4-30*cos(f*x+e)*(a+b)^(7/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4-30*(a+b)^(7/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3+1575*cos(f*x+e)^7*ln(-2/(a+b)^(1/2)*(
-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b
)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+b)/sin(f*x+e)^2)*a^4*b^4+210*cos(f*x+e)^7*(a+b)^(7/2)*a^(3/2)*ln(4*cos(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^3+96*cos(f*x+e)^7*(a+
b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b-196*cos(f*x+e)^7
*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2-280*cos(f*
x+e)^7*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3+96*cos
(f*x+e)^6*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b-196
*cos(f*x+e)^6*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b
^2-280*cos(f*x+e)^6*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a*b^3-6*cos(f*x+e)^5*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^3*b+162*cos(f*x+e)^5*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
```

```
^(1/2)*a^2*b^2+98*cos(f*x+e)^5*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a*b^3-6*cos(f*x+e)^4*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a^3*b+162*cos(f*x+e)^4*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a^2*b^2+98*cos(f*x+e)^4*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a*b^3-48*cos(f*x+e)^3*(a+b)^(7/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2+1155*cos(f*x+e)^7*ln(-2/(a+b)^(1/2)*(-1+c
os(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)
/sin(f*x+e)^2)*a^3*b^5+315*cos(f*x+e)^7*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(
cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^
2)*a^2*b^6+12*cos(f*x+e)^7*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a^4+12*cos(f*x+e)^6*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^4-30*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^4
+36*cos(f*x+e)^3*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*
b^3-48*cos(f*x+e)^2*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^2*b^2+36*cos(f*x+e)^2*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a*b^3-30*cos(f*x+e)*(a+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a*b^3+210*cos(f*x+e)^7*(a+b)^(7/2)*a^(5/2)*ln(4*cos(f*x+e)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2
)^(3/2)/cos(f*x+e)^4/sin(f*x+e)^6/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

Fricas [B] time = 24.0816, size = 1289, normalized size = 9.55

$$\left[\frac{105\, a^{\frac{3}{2}} b^2 \cos\left(fx+e\right)^6 \log\left(128\, a^4 \cos\left(fx+e\right)^8 + 256\, a^3 b \cos\left(fx+e\right)^6 + 160\, a^2 b^2 \cos\left(fx+e\right)^4 + 32\, a b^3 \cos\left(fx+e\right)^2\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/840*(105*a^(3/2)*b^2*cos(f*x + e)^6*log(128*a^4*cos(f*x + e)^8 + 256*a^3
*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 +
b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
 + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)) - 8*(2*(3*a^3 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84
*a*b^2 + 35*b^3)*cos(f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)
^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6), 1/
420*(105*sqrt(-a)*a*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos
(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^6 - 4*(2*(3*a^3
 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84*a*b^2 + 35*b^3)*cos(
f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

3.390 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx$

Optimal. Leaf size=104

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (a*Sqrt[a + b*Sec
[e + f*x]^2])/f - (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) + (a + b*Sec[e + f*x]^
2)^(5/2)/(5*b*f)
```

Rubi [A] time = 0.126954, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4139, 446, 80, 50, 63, 208}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf}-\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}-\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (a*Sqrt[a + b*Sec
[e + f*x]^2])/f - (a + b*Sec[e + f*x]^2)^(3/2)/(3*f) + (a + b*Sec[e + f*x]^
2)^(5/2)/(5*b*f)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+bx^2)^{3/2}}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x,\right.}{2f}$$
$$= -\frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf} -$$
$$= -\frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5bf} -$$
$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}$$

Mathematica [F] time = 2.05455, size = 0, normalized size = 0.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3, x]
```

Maple [B] time = 0.407, size = 2150, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x)
```

```
[Out] -1/60/f/b/(a+b)^(9/2)*4^(1/2)*(-1+cos(f*x+e))^3*(6*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^3-34*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^2*b-40*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^2-34*cos(f*x+e)^4*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^2*b-40*cos(f*x+e)^4*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^2+12*cos(f*x+e)^3*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^2*b+2*cos(f*x+e)^3*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^2+12*cos(f*x+e)^2*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^2*b+2*cos(f*x+e)^2*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^2+6*cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^2+30*cos(f*x+e)^5*l
n(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(
f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^(5/2)*(a+b)
^(7/2)*b+30*cos(f*x+e)^5*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2))*a^(3/2)*(a+b)^(7/2)*b^2+6*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^3+6*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a^3-30*cos(f*x+e)^5*ln(-4/(a+b)^(1/2)*(-1+
cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b
)/sin(f*x+e)^2)*a^6*b-135*cos(f*x+e)^5*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(c
os(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x
+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2
)*a^5*b^2-225*cos(f*x+e)^5*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4*b^3-16
5*cos(f*x+e)^5*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b^4-45*cos(f*x+e)^
5*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^5+30*cos(f*x+e)^5*ln(-2/(a+b)
^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/sin(f*x+e)^2)*a^6*b+135*cos(f*x+e)^5*ln(-2/(a+b)^(1/2)*(-1+cos
(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/s
in(f*x+e)^2)*a^5*b^2+225*cos(f*x+e)^5*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(co
s(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+
e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)
*a^4*b^3+165*cos(f*x+e)^5*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos
```

```
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b^4+45*
cos(f*x+e)^5*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^5-10*cos(f*x+e)^3*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^3-10*cos(f*x+e)^2
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^3+6*cos(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*b^3+6*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)*a*b^2)*((b+a*cos(f*x+e)^2)/cos(f*
x+e)^2)^(3/2)/sin(f*x+e)^6/cos(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

Fricas [B] time = 6.62635, size = 1095, normalized size = 10.53

$$\left[\frac{15\,a^{\frac{3}{2}}b\cos\left(fx+e\right)^4\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/120*(15*a^(3/2)*b*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*
cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
```

```
 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) + 8*((3*a^2 - 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2
+ 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4),
 -1/60*(15*sqrt(-a)*a*b*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos
(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 - 4*((3*a^2 -
 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2 + 3*b^2)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

3.391 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f)
```

Rubi [A] time = 0.0824348, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 266, 50, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```


```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sec[e + f*x]^2])/f + (a + b*Sec[e + f*x]^2)^(3/2)/(3*f)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x}\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\right.}{2f}\\
&= \frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\right.}{bf}\\
&= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sec^2(e+fx)}}{f}+\frac{\left(a+b\sec^2(e+fx\right.}{3f}
\end{aligned}
$$

Mathematica [C] time = 0.278951, size = 84, normalized size = 1.08

$$\frac{2b\left(a+b\sec^2(e+fx)\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{a\cos^2(e+fx)}{b}\right)}{3f\sqrt{\frac{a\cos^2(e+fx)}{b}+1}(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] (2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cos[e + f*x]^2)/b)]*(a + b*Se
c[e + f*x]^2)^(3/2))/(3*f*Sqrt[1 + (a*Cos[e + f*x]^2)/b]*(a + 2*b + a*Cos[2
*(e + f*x)]))
```

Maple [A] time = 0.056, size = 81, normalized size = 1.

$$\frac{1}{3f}\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^{\frac{3}{2}}-\frac{1}{f}a^{\frac{3}{2}}\ln\left(\frac{1}{\sec\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}\right)\right)+\frac{a}{f}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x)
```

```
[Out] 1/3*(a+b*sec(f*x+e)^2)^(3/2)/f-1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e
)^2)^(1/2))/sec(f*x+e))+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

Fricas [B] time = 1.85961, size = 944, normalized size = 12.1

$$\left[\frac{3\,a^{\frac{3}{2}}\cos\left(fx+e\right)^{2}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\left(fx+e\right)^{4}+32\,ab^{3}\cos\left(fx+e\right)^{2}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/24*(3*a^(3/2)*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(
f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8
*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
 + 8*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8
*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^
2 + 4*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
/(f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

3.392 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((a + b)^(3/2)*Ar
cTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f + (b*Sqrt[a + b*Sec[e + f*
x]^2])/f
```

Rubi [A] time = 0.136138, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4139, 446, 84, 156, 63, 208}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{b\sqrt{a+b\sec^2(e+fx)}}{f}-\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((a + b)^(3/2)*Ar
cTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f + (b*Sqrt[a + b*Sec[e + f*
x]^2])/f
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(-1+x^2\right)}dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)x}dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\text{Subst}\left(\int \frac{a^2+b(2a+b)x}{(-1+x)x\sqrt{a+bx}}dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}dx, x, \sec^2(e+fx)\right)}{2f} + \frac{(a+b)^2\,\text{S}}{}$$

$$= \frac{b\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} + \frac{(a}{}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f} + \frac{b\sqrt{a+b}}{}$$

Mathematica [C] time = 5.68721, size = 506, normalized size = 5.56

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{a^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+a^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i}\right.}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*((2*b)/(1 + E^((2*I)*(e + f*x))) + ((-2*I)*a^(
3/2)*f*x + 2*(a + b)^(3/2)*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Log[a + 2
*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] + a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((
2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
 + f*x)))^2]] - 2*a*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2
*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]] - 2*b*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^
((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
```

```
*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3
/2))
```

Maple [B] time = 0.339, size = 2424, normalized size = 26.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f/(a+b)^(9/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*cos(f*x+e)^2*(-1
+cos(f*x+e))^3*(2*cos(f*x+e)*(a+b)^(7/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2))*a^(5/2)+2*cos(f*x+e)*(a+b)^(7/2)*ln(4*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^(3/2)*b+2*cos(f*x+e)*(a
+b)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+2*cos(f*x+e)*(a+b
)^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+2*a*b*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(7/2)+2*b^2*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(7/2)+3*cos(f*x+e)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e)
)*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+
e)^2)*a^5*b+9*cos(f*x+e)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4*b^2+9*co
s(f*x+e)*ln(-4/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b^3+3*cos(f*x+e)*ln(-4/(
a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^4+cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos
(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/s
in(f*x+e)^2)*a^6+3*cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^5*b+6
*cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4*b^2+11*cos(f*x+e)*ln(
-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
```

```
))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b^3+12*cos(f*x+e)*ln(-2/(a+b)^(1/2)*
(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+b)/sin(f*x+e)^2)*a^2*b^4+6*cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*
(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)
^2)*a*b^5+cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^6-cos(f*x+e)*l
n(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+c
os(f*x+e)))*a^6-6*cos(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^5*b-15*cos(f*x+e)*ln(-4*(cos(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*
a^4*b^2-20*cos(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b^3-15*cos(f*x+e)*ln(-4*(cos(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b
^4-6*cos(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b^5-cos(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^6)*4^(1/2)/sin
(f*x+e)^6/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

Fricas [B] time = 1.83206, size = 2700, normalized size = 29.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*(a + b)^(3/2)
*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x +
e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)) + 8*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/8*(4*(a +
b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) + a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 16
0*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*
x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*b*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, -1/4*(sqrt(-a)*a*arctan(1/4*(8*a^2*
cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b
^2)) - (a + b)^(3/2)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b
 + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x +
e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e
)^4 - 2*cos(f*x + e)^2 + 1)) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/f, -1/4*(sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*co
s(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*(a + b)*sqrt(-a - b)*ar
ctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - 4*b*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

3.393 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=114

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}+\frac{(2a-b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}$$

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a - b)*Sqr
t[a + b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*f) - ((a + b)*
Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.170203, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4139, 446, 98, 156, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}+\frac{(2a-b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f) + ((2*a - b)*Sqr
t[a + b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*f) - ((a + b)*
Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*f)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(-1+x^2\right)^2}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)^2 x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{a^2+\frac{1}{2}(a-b)bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec\right.}{2f}$$

$$= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e\right.}{2f}$$

$$= -\frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{bf}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a-b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f} - \frac{(a}{}$$

Mathematica [C] time = 5.84379, size = 622, normalized size = 5.46

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{(a+b)\left(1+e^{2i(e+fx)}\right)}{\left(-1+e^{2i(e+fx)}\right)^2} - \frac{\left(2a^2+ab-b^2\right)\log\left(1-e^{2i(e+fx)}\right)-2a^2\log\left(\sqrt{a+b}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((a + b)*(1 + E^((2*I)*(e + f*x))))/(-1 + E^(
(2*I)*(e + f*x)))^2 - ((-2*I)*a^(3/2)*Sqrt[a + b]*f*x + (2*a^2 + a*b - b^2)
*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I
)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + a^(3/2)*Sqrt[a + b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2]] - 2*a^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x
)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))
```

```
^2]] - a*b*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt
[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + b^
2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*S
qrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]
*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*(a + b*Sec
[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] time = 0.458, size = 1609, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/f/(a+b)^(3/2)*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*(-1+cos(f
*x+e))^2*cos(f*x+e)^3*(4*cos(f*x+e)*(a+b)^(3/2)*a^(3/2)*ln(4*cos(f*x+e)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))-4*a^(3/2)*ln(4*cos(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)-2*cos(f*x+e)*(a+b)^(3
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-2*cos(f*x+e)*(a+b)^(3/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b-2*cos(f*x+e)*ln(-4*(cos(f*x+e
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3
-3*cos(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b+cos(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^3+2*cos(f*x+e)*l
n(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3+3*cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1
+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
b)/sin(f*x+e)^2)*a^2*b-cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^3
+2*a^3*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
b)/(-1+cos(f*x+e)))+3*a^2*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
```

```
^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b-b^3*ln(-4*(cos(f*x+e)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))-2*a^3*ln(-2/(a+b)
^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/sin(f*x+e)^2)-3*a^2*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b+b^
3*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2))/sin(f*x+e)^6/((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot \left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

Fricas [B] time = 1.94054, size = 3213, normalized size = 28.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
+ (a*cos(f*x + e)^2 - a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos
(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 -
8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^
4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b
+ b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a +
b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 +
```

```
b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x +
e)^2 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)^2 - 2*((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a - b)*arctan(1/2*
((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a*cos(f*x + e)^2
- a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^
2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x +
e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)
^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/(f*cos(f*x + e)^2
 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e)^2 + 2*(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a -
b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
 e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x + e)^2 - f), 1
/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 +
(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((2*a - b)*cos(f*x
 + e)^2 - 2*a + b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*s
qrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*
x + e)^2 + a*b + b^2)))/(f*cos(f*x + e)^2 - f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

3.394 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$-\frac{\left(8a^2+4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f\sqrt{a+b}}+\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+$$

```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 4*a*b -
 b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*Sqrt[a + b]*f) +
((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - ((a + b)*Cot[
e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```

Rubi [A] time = 0.24287, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4139, 446, 98, 151, 156, 63, 208}

$$-\frac{\left(8a^2+4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f\sqrt{a+b}}+\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}-\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] (a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - ((8*a^2 + 4*a*b -
 b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*Sqrt[a + b]*f) +
((4*a - b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*f) - ((a + b)*Cot[
e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*f)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x\left(-1+x^2\right)^3}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{(-1+x)^3 x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f} - \frac{\operatorname{Subst}\left(\int \frac{2a^2+\frac{1}{2}(3a-b)bx}{(-1+x)^2 x\sqrt{a+bx}}\,dx, x, \mathrm{s}\right.}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec}}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec}}{4f}$$

$$= \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec}}{4f}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\left(8a^2+4ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}f} +$$

Mathematica [C] time = 6.0669, size = 684, normalized size = 4.3

$$\frac{e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{\left(8a^2+4ab-b^2\right)\log\left(1-e^{2i(e+fx)}\right)-8a^2\log\left(\sqrt{a+b}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(-(((1 + E^((2*I)*(e + f*x)))*(b*(1 + 6*E^((2*I)*(e +
```

```
f*x)) + E^((4*I)*(e + f*x))) + a*(6 - 4*E^((2*I)*(e + f*x)) + 6*E^((4*I)*(e
 + f*x)))))/(-1 + E^((2*I)*(e + f*x)))^4) + ((-8*I)*a^(3/2)*Sqrt[a + b]*f*x
 + (8*a^2 + 4*a*b - b^2)*Log[1 - E^((2*I)*(e + f*x))] + 4*a^(3/2)*Sqrt[a +
b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + 4*a^(3/2)*Sqrt[a + b]*Log[a + a*E^((
2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 8*a^2*Log[a + b + a*E^((2*I)*(e +
 f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) +
 a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*a*b*Log[a + b + a*E^((2*I)*(e + f*x))
+ b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
 E^((2*I)*(e + f*x)))^2]] + b^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2
*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*f*(a + 2*b + a
*Cos[2*e + 2*f*x])^(3/2))
```

Maple [B] time = 0.456, size = 4955, normalized size = 31.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/32/f/(a+b)^(5/2)*4^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)*(-1+cos(
f*x+e))^2*cos(f*x+e)^3*(-8*cos(f*x+e)^3*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-b^4*ln(-2/(a+b)^(1
/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/sin(f*x+e)^2)-8*a^4*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))+b^4*ln(-4*(cos(f*x+e)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))+8*a^4*ln(-2/(a+
b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)+b)/sin(f*x+e)^2)-2*cos(f*x+e)^3*ln(-4*(cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b^3-20*cos(f*x+e
)^2*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
```

```
e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b-15*cos(f*x+e)^2*ln(-2/(a+b)
^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^2-2*cos(f*x+e)^2*ln(-2/(a+b)^(1/2)*(-1+cos
(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/s
in(f*x+e)^2)*a*b^3+20*cos(f*x+e)^2*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b+15*cos(f*x+e)^2*ln(-4
*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f
*x+e)))*a^2*b^2+2*cos(f*x+e)^2*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a*b^3-20*cos(f*x+e)*ln(-2/(a+b)
^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b-15*cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*
x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a
*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(
f*x+e)^2)*a^2*b^2-2*cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b^3+
20*cos(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b+15*cos(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2+2*cos(f
*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b
)/(-1+cos(f*x+e)))*a*b^3-12*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(5/2)*a-2*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(5/2)*b+8*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(5/2)*a-2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(5/2)*b+16*cos(f*x+e)^3*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2))*(a+b)^(5/2)*a^(3/2)-16*cos(f*x+e)^2*ln(4*cos(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(5/2)*a^(3/2)-16*cos(f*x+e)
*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*co
s(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(5/2)
*a^(3/2)+20*cos(f*x+e)^3*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^3*b+15*cos
(f*x+e)^3*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+co
```

```
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^2*b^2+2*cos(f*x+e)^3*ln(-
2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a*b^3-20*cos(f*x+e)^3*ln(-4*(cos(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^3*b-
15*cos(f*x+e)^3*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+b)/(-1+cos(f*x+e)))*a^2*b^2+cos(f*x+e)^3*ln(-4*(cos(f*x+e)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^4-8*cos(f*
x+e)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4-cos(f*x+e)^3*ln(-2/(a+b)^(1/
2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+b)/sin(f*x+e)^2)*b^4-8*cos(f*x+e)^2*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))
*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e
)^2)*a^4-cos(f*x+e)^2*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^4+cos(f*x+e)*ln(-2/(a+b)^(1/2)*(-1+cos
(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/s
in(f*x+e)^2)*b^4+16*a^(3/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2))*(a+b)^(5/2)+8*cos(f*x+e)^2*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4+8*cos(f*x+e)^3*
ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^4+8*cos(f*x+e)*ln(-4*(cos(f*x+e)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*a^4-cos
(f*x+e)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)
+b)/(-1+cos(f*x+e)))*b^4+20*a^3*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b+15*
a^2*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^2+2*b^3*ln(-2/(a+b)^(1/2)*(-1+c
os(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)
/sin(f*x+e)^2)*a-20*a^3*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
```

```
)^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b-15*a^2*ln(-4*(cos(f*x+e)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^2-2*b^3*ln(-4*
(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*
x+e)))*a+cos(f*x+e)^2*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^4)/sin(f*x+e)
^8/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cot \left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

Fricas [B] time = 5.70292, size = 4382, normalized size = 27.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 +
 a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a
^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
 e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - ((8*a^2 + 4*a*b
 - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4
*a*b - b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*
a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
 + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 -
```

```
(4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f
), 1/16*(((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*co
s(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(
f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + a*b)*cos(f*x + e)^4 - 2
*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8
 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
 + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)) - 2*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 - (4*a^2 + 3*a*
b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f), -1/32*(8*((
a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*
b*cos(f*x + e)^2 + a*b^2)) + ((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a
^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(a + b)*log(2*(
(8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
 + 4*((6*a^2 + 7*a*b + b^2)*cos(f*x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e
)^4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f), -1/16*(4*((a^2 + a*b)*cos(f*
x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(-a)*arctan(1/4*(8
*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2
+ a*b^2)) - ((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)
*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*c
os(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((6*a^2 + 7*a*b + b^2)*cos(f
*x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2
+ (a + b)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

3.395 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx$

Optimal. Leaf size=290

$$\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{192bf}-\frac{\left(17a^2b+3a^3-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f}$$

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3 + 17
*a^2*b - 55*a*b^2 - 5*b^3)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 50*a*b - 5*b^2)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(192*b*f) + ((9*a + b)*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^
2])/(48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b + b*Tan[e + f*x]^2])/(8*f)
```

Rubi [A] time = 0.570578, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4141, 1975, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{192bf}-\frac{\left(17a^2b+3a^3-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{128b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```


```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(128*b^(5/2)*f) - ((3*a^3 + 17
*a^2*b - 55*a*b^2 - 5*b^3)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(12
8*b^2*f) + ((3*a^2 - 50*a*b - 5*b^2)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e +
f*x]^2])/(192*b*f) + ((9*a + b)*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^
2])/(48*f) + (b*Tan[e + f*x]^7*Sqrt[a + b + b*Tan[e + f*x]^2])/(8*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\text{Subst}\left(\int \frac{x^6\left((a+b)(8a+b)+b(9a+b)x^2\right)}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\right.}{8f}$$

$$= \frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f} + \frac{b\tan^7(e+fx)\sqrt{a+b+b}}{8f}$$

$$= \frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf} + \frac{(9a+b)\tan^5(}{}$$

$$= -\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2\right.}{}$$

$$= -\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2\right.}{}$$

$$= -\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2\right.}{}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4\right)\text{ta}}{128b^{5/2}f}$$

Mathematica [A] time = 6.58032, size = 353, normalized size = 1.22

$$-\frac{\tan(e+fx)\sec^6(e+fx)\left(\left(759a^2b+135a^3-2303ab^2+513b^3\right)\cos(2(e+fx))+2\left(159a^2b+27a^3-523ab^2-191b^3\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

```
[Out] -((128*a^(3/2)*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x
]^2]] - ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(Sqrt[b
]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]^3*(a
 + b*Sec[e + f*x]^2)^(3/2))/(32*Sqrt[2]*b^2*f*(a + 2*b + a*Cos[2*e + 2*f*x]
)^(3/2)) - ((90*a^3 + 498*a^2*b - 1594*a*b^2 - 626*b^3 + (135*a^3 + 759*a^2
*b - 2303*a*b^2 + 513*b^3)*Cos[2*(e + f*x)] + 2*(27*a^3 + 159*a^2*b - 523*a
*b^2 - 191*b^3)*Cos[4*(e + f*x)] + 9*a^3*Cos[6*(e + f*x)] + 57*a^2*b*Cos[6*
(e + f*x)] - 337*a*b^2*Cos[6*(e + f*x)] + 15*b^3*Cos[6*(e + f*x)])*Sec[e +
f*x]^6*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(12288*b^2*f)
```

Maple [C] time = 0.89, size = 3583, normalized size = 12.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x)
```

```
[Out] 1/384/f/b^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(120*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^3*b+118*cos(f*x+e)^5*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*b^4-118*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*b^4+136*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-4
8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-768*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*
x+e)*cos(f*x+e)^8*a^2*b^2+120*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f
*x+e)^3*a*b^3+48*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*b^4-57*
cos(f*x+e)^9*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b+337*cos(f*x+e)^9
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-15*cos(f*x+e)^9*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+57*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*a^3*b-337*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^2*b^2+15*cos(f*x+e)^8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+
```

540*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^2*b^2-360*2^(1/2)*(1/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*sin(f*x+e)*cos(f*x+e)^8*a*b^3-60*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^3*b+114*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^2*b^2+180*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a*b^3-9*cos(f*x+e)^9*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^4+9*cos(f*x+e)^8*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^4-15*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*b^4+15*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-136
*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^4-3*cos(f*x+e)^7*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3*b-301*cos(f*x+e)^7*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2+455*cos(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*a*b^3+3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^3*b+301*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-45
5*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3+78*cos(f*x+e)^
5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b^2-380*cos(f*x+e)^5*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^3-78*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a^2*b^2+380*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b^3-120*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b
^3-9*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos
(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1
/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos(f*x+e

```
)^8*a^4+15*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)*cos
(f*x+e)^8*b^4+18*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticP
i((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)*cos(f*x+e)^8*a^4-30*2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*sin(f*x+e)*cos(f*x+e)^8*b^4)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/
2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos(f*x+e)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 46.7945, size = 4788, normalized size = 16.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] [1/1536*(192*sqrt(-a)*a*b^3*cos(f*x + e)^7*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
```

```
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
- 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*
b^3 - 5*b^4)*sqrt(b)*cos(f*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*
x + e)^6 - 2*(3*a^2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(
9*a*b^3 - 17*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7), 1/768*(96*sqrt(-a)*a*b^3*cos(f*x +
 e)^7*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 3*
(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(-b)*arctan(-1/2*((a
 - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^
7 - 2*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^
2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
b^3*f*cos(f*x + e)^7), 1/1536*(384*a^(3/2)*b^3*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqr
t(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a
^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^
7 - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(b)*cos(f*x +
e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((9*a^3
*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 - 122*a
*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x + e)^
2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x
 + e)^7), 1/768*(192*a^(3/2)*b^3*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2
- a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 -
 (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^7 + 3*(3*a^4 +
 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^7 - 2*((
9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 -
122*a*b^3 + 59*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*co
s(f*x + e)^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

3.396 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=214

$$\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16bf}-\frac{(a-b)\left(a^2+10ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{a^{3/2}\tan^{-1}}{}$$

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 8*a*b - b^2)*Tan[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a + b)*Tan[e + f*x]^3*Sqrt[a + b
 + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x
]^2])/(6*f)
```

Rubi [A] time = 0.478218, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4141, 1975, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{16bf}-\frac{(a-b)\left(a^2+10ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{16b^{3/2}f}+\frac{a^{3/2}\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```


```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f -
 ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(16*b^(3/2)*f) + ((a^2 - 8*a*b - b^2)*Tan[e + f*x]*Sqrt
[a + b + b*Tan[e + f*x]^2])/(16*b*f) + ((7*a + b)*Tan[e + f*x]^3*Sqrt[a + b
 + b*Tan[e + f*x]^2])/(24*f) + (b*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x
]^2])/(6*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f} + \frac{\text{Subst}\left(\int \frac{x^4\left((a+b)(6a+b)+b(7a+b)x^2\right)}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx\right.}{6f}$$

$$= \frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} + \frac{b\tan^5(e+fx)\sqrt{a+b+b\,\mathrm{t}}}{6f}$$

$$= \frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3(e+fx}{}$$

$$= \frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3(e+fx}{}$$

$$= \frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} + \frac{(7a+b)\tan^3(e+fx}{}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a-b)\left(a^2+10ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+}{\sqrt{a+b+b\tan^2(}}\right.}{16b^{3/2}f}$$

Mathematica [A] time = 4.32997, size = 258, normalized size = 1.21

$$\frac{\tan(e+fx)\sec^4(e+fx)\left(4\left(3a^2-24ab-11b^2\right)\cos(2(e+fx)) + \left(3a^2-38ab+3b^2\right)\cos(4(e+fx)) + 9a^2-58ab+17b^2\right.}{384bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] ((16*a^(3/2)*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
] - ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b
 - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)
)/(4*Sqrt[2]*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((9*a^2 - 58*a*b +
 17*b^2 + 4*(3*a^2 - 24*a*b - 11*b^2)*Cos[2*(e + f*x)] + (3*a^2 - 38*a*b +
3*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e +
f*x])/(384*b*f)
```

Maple [C] time = 0.578, size = 2757, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/48/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b*sin(f*x+e)*(-21*sin(f*x+e)
*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b-6*sin
(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^6*2^(1/2
)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(
a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2))*b^3+3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)
/(a+b)^2)^(1/2))*a^3-3*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
```

```
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*b^3+3*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*b^3-3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-38*cos
(f*x+e)^7*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+3*cos(f*x+e)^7*((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+38*cos(f*x+e)^6*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*a^2*b-3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a*b^2-52*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^
2+52*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+96*sin(f*x+
e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^
(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)
*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I
*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-14*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*b^3+14*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*b^3-8*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+3*cos(f*x+e)^7*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-3*cos(f*x+e)^6*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^3+8*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*b^3+17*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-17*cos
(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+22*cos(f*x+e)^3*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-22*cos(f*x+e)^2*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b^2-27*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-54*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b)
,(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2))*a^2*b+54*sin(f*x+e)*cos(f*x+e)^6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-
a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2)*((b+a*cos
(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2/cos(f*x
+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Fricas [A] time = 14.4802, size = 4290, normalized size = 20.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(24*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(b)*
cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(
f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) +
 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
b^2*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3
2*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos
(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*
a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e
))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^
2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 + 2*((3*a^2*b - 38*a*b^2 + 3*b^3)*co
s(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e
```

```
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^5), -1/192*(48*a
^(3/2)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3
+ (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f
*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^5 + 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3
)*sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) - 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b
^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b^2*f*cos(f*x + e)^5), -1/96*(24*a^(3/2)*b^2*arctan(1/4*(8*a^2*co
s(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f
*x + e)^5 + 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 - 2
*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*b^2 - b^3)*co
s(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^
2*f*cos(f*x + e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \tan \left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

3.397 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=166

$$\frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}f}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}+$$

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^2 - 6*a*b - b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f
)
```

Rubi [A] time = 0.364926, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4141, 1975, 477, 582, 523, 217, 206, 377, 203}

$$\frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8\sqrt{b}f}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4f}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + ((3*a^2 - 6*a*b - b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(8*Sqrt[b]*f) + ((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[
e + f*x]^2])/(8*f) + (b*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*f
)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f} + \frac{\text{Subst}\left(\int \frac{x^2\left((a+b)(4a+b)+b(5a+b)x^2\right)}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx\right.}{4f}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\,\text{ta}}}{4f}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\,\text{ta}}}{4f}$$

$$= \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\,\text{ta}}}{4f}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\left(3a^2-6ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f}$$

Mathematica [C] time = 6.58653, size = 703, normalized size = 4.23

$$\frac{e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-3a^2\log\left(\frac{4f\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}\right)}{\left(3a^2-6ab-b^2\right)\left(1+e^{2i(e+fx)}\right)}\right)+6ab\log\left(\frac{4f\left(\sqrt{b}\left(-1+e^{2i(e+fx}\cdots\right.\right.}{\left(3a^2-\cdots\right.}\right.\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f
*x))]*Cos[e + f*x]^3*(((-I)*(-1 + E^((2*I)*(e + f*x)))*(5*a*(1 + E^((2*I)*(
e + f*x)))^2 - b*(1 - 6*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(1 + E
^((2*I)*(e + f*x)))^4 + (-8*a^(3/2)*Sqrt[b]*f*x + (4*I)*a^(3/2)*Sqrt[b]*Log
[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a
*(1 + E^((2*I)*(e + f*x)))^2]] - (4*I)*a^(3/2)*Sqrt[b]*Log[a + a*E^((2*I)*(
e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x))
+ a*(1 + E^((2*I)*(e + f*x)))^2]] - 3*a^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e
 + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]
)*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x))))] + 6*a*b*Log[(4*(Sqr
t[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E
^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x)))
)] + b^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 +
E^((2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((
2*I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*f*(a + 2*b +
 a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.402, size = 2002, normalized size = 12.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x)
```

```
[Out] 1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(6*sin(f*x+e)*cos(
f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*a^2-12*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*a*b-2*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+5*sin(f*x+e)*cos(f*x+e
)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2+6*sin(f*x+e)*cos(
f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+sin(f*x+e)*c
os(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/
2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-16*sin(f*
x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
Pi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2
*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*
I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+5*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^2-cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a*b-5*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e
)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+7*cos(f*x+e)^3*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*b^2-7*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+co
s(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*cos(f*x+e)*((2*I*a
```

```
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*b^2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/(-1+cos(f*x+e))/(b+a*cos(f
*x+e)^2)^2/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan \left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Fricas [B] time = 4.39122, size = 3929, normalized size = 23.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
- a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
+ a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^
2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) - (3*a^2 - 6*a*b - b^2)*sqrt(b)*cos(f*x + e)^3*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b - b^2
)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(b*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*
a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*
b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^
4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f
*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2
)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
```

```
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + (3*a^2 - 6*a*b -
 b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2
)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)
^3), 1/32*(8*a^(3/2)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos
(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*
a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^3 - (3*a^2 - 6*a*b - b^2
)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) + 4*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3), 1/16*(4*a^(3/2)*b*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*
a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*
sin(f*x + e)))*cos(f*x + e)^3 + (3*a^2 - 6*a*b - b^2)*sqrt(-b)*arctan(-1/2*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x +
e)^3 + 2*((5*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\tan^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx + e\right)^2 + a\right)^{\frac{3}{2}}\tan\left(fx + e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

3.398 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0959582, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4128, 416, 523, 217, 206, 377, 203}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2f}+\frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 (Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]])/(2*f) + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*f)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{(a+b)(2a+b)+b(3a+b)x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2f}$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} +$$

$$= \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(b(}{}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{}}{}$$

Mathematica [C] time = 1.98768, size = 527, normalized size = 4.47

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-ia^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)+ia^{3/2}\log\left(\sqrt{a}\sqrt{a\left(1+e^{2}\right.}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*((((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((
2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x))
+ Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] -
3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)*(1 + E^
```

```
((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f
 + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(
b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*
Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.313, size = 1557, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(6*sin(f*x+e)*cos(f*x+e)^2*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-
b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2))*a*b+2*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(
1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-2*cos(f
*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-3*
sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
*b-sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*b^2+4*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
```

```
/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+cos(f*x+e)^3*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)*a*b+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)*((b+a*cos(f*x+e)^2)/
cos(f*x+e)^2)^(3/2)*sin(f*x+e)/(-1+cos(f*x+e))/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 1.7838, size = 3602, normalized size = 30.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b
)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqrt(-b)*arcta
```

```
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos
(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
 a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*co
s(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(f*cos(f*x + e)), -1/8*(2*a^(3/2)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e
) - (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
+ 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^
2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(f*cos(f*x + e)), -1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 -
8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b +
a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) - (3*a
+ b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)
*sin(f*x + e)))*cos(f*x + e) - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec^2\left(e + fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx + e\right)^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.399 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/f - ((a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rubi [A] time = 0.227552, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4141, 1975, 474, 523, 217, 206, 377, 203}

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) + (b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/f - ((a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b\left(1+x^2\right)\right)^{3/2}}{x^2\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int\frac{\left(a+b+bx^2\right)^{3/2}}{x^2\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}+\frac{\text{Subst}\left(\int\frac{-a^2+b^2+b^2x^2}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx\right.}{f}$$

$$=-\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{a^2\,\text{Subst}\left(\int\frac{1}{\left(1+x^2\right)\sqrt{a+b+bx^2}}\right.}{f}$$

$$=-\frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}-\frac{a^2\,\text{Subst}\left(\int\frac{1}{1+ax^2}\,dx,x,\right.}{f}$$

$$=-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}-\frac{(a+b)}{}$$

Mathematica [C] time = 6.64334, size = 410, normalized size = 3.69

$$\frac{\sqrt{2}e^{i(e+fx)}\cos^3(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-2\left(a^{3/2}fx+b^{3/2}\log\left(\frac{f\left(\sqrt{b}\left(-1+e^{2i(e+fx)}\right)-i\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}\right)}{b^2\left(1+e^{2i(e+fx)}\right)}\right)\right)+ia^{3/2}\log\left(\sqrt{}\right.}{}\right.}{f(a\cos(2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I
)*(e + f*x))]*Cos[e + f*x]^3*(((-2*I)*(a + b))/(-1 + E^((2*I)*(e + f*x))) +
 (I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)
*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - I*a^(3/2)*Log[a + a*E^((2*I
```

```
)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*(a^(3/2)*f*x + b^(3/2)*Log[((Sqrt[
b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2
))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

Maple [C] time = 0.534, size = 1952, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((b+a*cos(f*x+e)^2)/cos(f*x+e)
^2)^(3/2)*cos(f*x+e)^3*(cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*a^2-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*E
llipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e)
,(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*
b^2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+2*
2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+
a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2))*b^2*sin(f*x+e)*cos(f*x+e)+2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^
```

```
2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)-2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^2*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+2*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-
(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2))*b^2*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^
2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*a*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)/si
n(f*x+e)/(b+a*cos(f*x+e)^2)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [B] time = 2.02692, size = 3586, normalized size = 32.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e
)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5
*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)
*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))*sin(f*x + e) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8
*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/
cos(f*x + e)^4)*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*cos(f*x + e))/(f*sin(f*x + e)), 1/8*(4*sqrt(-b)*b*arctan(-1/2*((a
 - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e)
+ sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
+ 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos
(f*x + e))/(f*sin(f*x + e)), 1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b
+ a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) + b^(
3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2
 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e
) - 4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*
sin(f*x + e)), 1/4*(a^(3/2)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt(-b)*b*arct
an(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*si
n(f*x + e) - 4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e))/(f*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

3.400 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=112

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a - b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f) - ((a + b)*C
ot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rubi [A] time = 0.275149, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 474, 583, 12, 377, 203}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f +
 ((3*a - b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f) - ((a + b)*C
ot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^{3/2}}{x^4\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^4\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} + \frac{\text{Subst}\left(\int \frac{-(3a-b)(a+b)-(2a-b)}{x^2\left(1+x^2\right)\sqrt{a+b+bx^2}}\right.}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+fx)\sqrt{a+b}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+fx)\sqrt{a+b}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+fx)\sqrt{a+b}}{3f}$$

$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

Mathematica [C] time = 0.356267, size = 100, normalized size = 0.89

$$-\frac{2(a+b)\cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{a\sin^2(e+fx)}{a+b}\right)}{3f\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + b)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (a*Sin[e + f
*x]^2)/(a + b)]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*(e +
f*x)])*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

Maple [C] time = 0.351, size = 2015, normalized size = 18.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(6*cos(f*x+e)^3*sin(f*x+e)*2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a
-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2))*a^2-3*cos(f*x+e)^3*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a^2+6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(
1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)
-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-
3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))
*a^2-6*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/
2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f
*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))
^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+3*cos(f*x+e)*sin(f*x+e)
*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))
*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-
4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-6*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
```

```
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I
*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))
*a^2*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*
b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elliptic
F((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin
(f*x+e)-4*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+3*cos(f*
x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-5*cos(f*x+e)^2*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)^3*((b+a*cos(f
*x+e)^2)/cos(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)^2/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [B] time = 2.66525, size = 1428, normalized size = 12.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a*cos(f*x + e)^2 - a)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*(4*a*cos(f*x + e)^3 - (3*a
- b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x
```

```
 + e)^2 - f)*sin(f*x + e)), -1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(a)*arctan(
1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b
^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
 + e)))*sin(f*x + e) - 4*(4*a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^2 - f)*sin(f*x +
e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

3.401 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=165

$$-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^5(e+fx)\sqrt{a+b}}{5f}$$

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 10*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*f) - ((a + b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5*f)
```

Rubi [A] time = 0.355179, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 474, 583, 12, 377, 203}

$$-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)}-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}-\frac{(a+b)\cot^5(e+fx)\sqrt{a+b}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
) - ((15*a^2 + 10*a*b - 2*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*(a + b)*f) + ((5*a - b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])
/(15*f) - ((a + b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^{3/2}}{x^6\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^{3/2}}{x^6\left(1+x^2\right)}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f} + \frac{\text{Subst}\left(\int \frac{-(5a-b)(a+b)-(4a-b)}{x^4\left(1+x^2\right)\sqrt{a+b+bx^2}}\right.}{5f}$$

$$= \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f} - \frac{(a+b)\cot^5(e+fx)\sqrt{a+b}}{5f}$$

$$= -\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} + \frac{(5a-b)\cot^3(e}{}$$

$$= -\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} + \frac{(5a-b)\cot^3(e}{}$$

$$= -\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} + \frac{(5a-b)\cot^3(e}{}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b+b}}{15(a+b)f}$$

Mathematica [C] time = 1.50507, size = 139, normalized size = 0.84

$$\frac{2\cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(\frac{5(a+b)^2\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{a\sin^2(e+fx)}{a+b}\right)}{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}} - \frac{3}{4}\csc^2(e+fx)(a\cos(2(e+fx))+a+\right.}{15f(a+b)(a\cos(2(e+fx))+a+2b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-3*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Csc[e + f*x]^2)/4 + (5*(a + b)^2*Hypergeometric2F1[-3/2, -3/2, -1
```

```
/2, (a*Sin[e + f*x]^2)/(a + b)])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))
/(15*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

Maple [C] time = 0.494, size = 5850, normalized size = 35.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

Fricas [B] time = 10.3436, size = 1882, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/120*(15*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2
 + a*b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e
)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5
```

```
*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)
*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))*sin(f*x + e) - 8*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2 + 24*a*
b - 5*b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4 - 2*(a + b
)*f*cos(f*x + e)^2 + (a + b)*f)*sin(f*x + e)), 1/60*(15*((a^2 + a*b)*cos(f*
x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*arctan(1/4*(8*
a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(
f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
*sin(f*x + e) - 4*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2 + 24*a*b - 5*
b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*co
s(f*x + e)^2 + (a + b)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

3.402 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) - ((a + 2*b)*Sqr
t[a + b*Sec[e + f*x]^2])/(b^2*f) + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b^2*f)
```

Rubi [A] time = 0.133072, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4139, 446, 88, 63, 208}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}-\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) - ((a + 2*b)*Sqr
t[a + b*Sec[e + f*x]^2])/(b^2*f) + (a + b*Sec[e + f*x]^2)^(3/2)/(3*b^2*f)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{-a-2b}{b\sqrt{a+bx}}+\frac{1}{x\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \sec^2(e+fx)\right)}{2f}\\
&= -\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+f\right.}{2f}\\
&= -\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec}\right.}{bf}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}-\frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f}
\end{aligned}
$$

Mathematica [F] time = 2.1216, size = 0, normalized size = 0.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] time = 0.44, size = 358, normalized size = 4.

$$\frac{\left(\sin\left(fx+e\right)\right)^2}{3\,fb^2\left(\cos\left(fx+e\right)\right)^4\left(\left(\cos\left(fx+e\right)\right)^2-1\right)}\left(2\,\left(\cos\left(fx+e\right)\right)^4a^{5/2}+3\,\sqrt{\frac{b+a\left(\cos\left(fx+e\right)\right)^2}{\left(1+\cos\left(fx+e\right)\right)^2}}\ln\left(4\,\cos\left(fx+e\right)\sqrt{\frac{b+}{(1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3/f/b^2/a^(1/2)*sin(f*x+e)^2*(2*cos(f*x+e)^4*a^(5/2)+3*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2))*cos(f*x+e)^4*b^2+6*cos(f*x+e)^4*a^(3/2)*b+3*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2))*cos(f*x+e)^3*b^2+cos(f*x+e)^2*a^(3/2)*b+6*cos(f*x+e)^2*a^(1/
2)*b^2-a^(1/2)*b^2)/cos(f*x+e)^4/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(c
os(f*x+e)^2-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.07901, size = 1006, normalized size = 11.3

$$\left[\frac{3\,\sqrt{a}b^2\cos\left(fx+e\right)^2\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(a)*b^2*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*
cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) - 8*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*b^2*arctan(1/4
*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)
^2 + a*b^2))*cos(f*x + e)^2 - 4*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.403 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) + Sqrt[a + b*Sec[e
+ f*x]^2]/(b*f)
```

Rubi [A] time = 0.0954163, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4139, 446, 80, 63, 208}

$$\frac{\sqrt{a+b\sec^2(e+fx)}}{bf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) + Sqrt[a + b*Sec[e
+ f*x]^2]/(b*f)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{-1+x}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sec^2(e+fx)}}{bf} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}
\end{aligned}$$

Mathematica [F] time = 1.65595, size = 0, normalized size = 0.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] time = 0.389, size = 303, normalized size = 5.4

$$-\frac{\left(\sin\left(fx+e\right)\right)^2}{fb\left(\cos\left(fx+e\right)\right)^2\left(\left(\cos\left(fx+e\right)\right)^2-1\right)}\left(\left(\cos\left(fx+e\right)\right)^2\sqrt{\frac{b+a\left(\cos\left(fx+e\right)\right)^2}{\left(1+\cos\left(fx+e\right)\right)^2}}\ln\left(4\cos\left(fx+e\right)\sqrt{\frac{b+a\left(\cos\left(fx+\right.\right.}{\left(1+\cos\left(fx+\right.\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/a^(1/2)/b*sin(f*x+e)^2*(cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1
/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b
+cos(f*x+e)^2*a^(3/2)+cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*c
os(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+a^(1/2)*
b)/cos(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(cos(f*x+e)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 0.902168, size = 807, normalized size = 14.41

$$\left[\frac{\sqrt{a}b\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx+e\right)^2+b^4+8\left(16\,a^3\,\mathrm{c}\right.\right.}{8\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*a*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f), -1/4*(sqrt(-a)*b*arctan(1/4*(8
*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2
+ a*b^2)) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [B] time = 2.51108, size = 541, normalized size = 9.66

$$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{a+b-\frac{2a}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}+\frac{2b}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}+\frac{a}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}+\frac{b}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}}-\sqrt{a+b}-\frac{\sqrt{a+b}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\cdots}{\left(\sqrt{a+b-\frac{2a}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}+\frac{2b}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}+\cdots}\right.}\right)}{f\mathrm{sgn}\left(\tan\left(\frac{1}{2}f\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(arctan(1/2*(sqrt(a + b - 2*a/tan(1/2*f*x + 1/2*e)^2 + 2*b/tan(1/2*f*x +
 1/2*e)^2 + a/tan(1/2*f*x + 1/2*e)^4 + b/tan(1/2*f*x + 1/2*e)^4) - sqrt(a +
 b) - sqrt(a + b)/tan(1/2*f*x + 1/2*e)^2)/sqrt(-a))/sqrt(-a) + 2*(sqrt(a +
b - 2*a/tan(1/2*f*x + 1/2*e)^2 + 2*b/tan(1/2*f*x + 1/2*e)^2 + a/tan(1/2*f*x
 + 1/2*e)^4 + b/tan(1/2*f*x + 1/2*e)^4) - sqrt(a + b) - sqrt(a + b)/tan(1/2
*f*x + 1/2*e)^2)/((sqrt(a + b - 2*a/tan(1/2*f*x + 1/2*e)^2 + 2*b/tan(1/2*f*
x + 1/2*e)^2 + a/tan(1/2*f*x + 1/2*e)^4 + b/tan(1/2*f*x + 1/2*e)^4) - sqrt(
a + b)/tan(1/2*f*x + 1/2*e)^2)^2 + 2*sqrt(a + b)*(sqrt(a + b - 2*a/tan(1/2*
f*x + 1/2*e)^2 + 2*b/tan(1/2*f*x + 1/2*e)^2 + a/tan(1/2*f*x + 1/2*e)^4 + b/
tan(1/2*f*x + 1/2*e)^4) - sqrt(a + b)/tan(1/2*f*x + 1/2*e)^2) + a - 3*b))/(
f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))
```

3.404 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rubi [A] time = 0.0546615, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4139, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}\end{aligned}$$

Mathematica [F] time = 0.123238, size = 0, normalized size = 0.

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]

[Out] Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [A] time = 0.072, size = 42, normalized size = 1.3

$$-\frac{1}{f}\ln\left(\frac{1}{\sec\left(fx+e\right)}\left(2\,a+2\,\sqrt{a}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^{2}}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 0.68355, size = 652, normalized size = 19.76

$$\left[\frac{\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\cos\left(fx+e\right)^{4}+32\,ab^{3}\cos\left(fx+e\right)^{2}+b^{4}-8\left(16\,a^{3}\cos\left(f\right.\right.\right.}{8\,\sqrt{a}f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*co
s(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 2
4*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt
(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(sqrt(a)*f), 1/4*sqrt(-a)*
arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*co
s(f*x + e)^2 + a*b^2))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [B] time = 1.85015, size = 167, normalized size = 5.06

$$-\frac{2\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}f\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*f*sgn(tan(1/2
*f*x + 1/2*e)^2 - 1))
```

3.405 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Rubi [A] time = 0.106442, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}f}
\end{aligned}$$

Mathematica [F] time = 2.35647, size = 0, normalized size = 0.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] time = 0.407, size = 376, normalized size = 5.4

$$\frac{\left(\sin\left(fx+e\right)\right)^{2}}{2f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{b+a\left(\cos\left(fx+e\right)\right)^{2}}{\left(1+\cos\left(fx+e\right)\right)^{2}}}\left(\ln\left(-4\frac{1}{-1+\cos\left(fx+e\right)}\left(\cos\left(fx+e\right)\sqrt{\frac{b+a\left(\cos\left(fx+\right.\right.}{\left(1+\cos\left(fx+\right.\right.}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/(a+b)^(1/2)/a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(ln(-
4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos
(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(
f*x+e)))*a^(1/2)-ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*a^(1/2)-2*ln(4*cos(f
*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*
a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(1/2))*sin(f*x+e
)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 1.1719, size = 2527, normalized size = 36.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(
a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*co
s(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a
 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1)))/((a^2 + a*b)*f), 1/8*(4*a*sqrt(-a - b)*arctan(1/2*((2*a +
 b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a + b)*sqrt(a)*log(128*a
^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 +
 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*
x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^2 + a*b)*f), -1/4*(sqrt(-a)*(a + b)
*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*c
os(f*x + e)^2 + a*b^2)) - sqrt(a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*
x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^2 + a*b)*f), -1/4*(sqr
t(-a)*(a + b)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^
4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 2*a*sqrt(-a - b)*arctan(1/2*((2*a +
b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^2 + a*b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.406 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ((2*a + 3*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)*f) - (Cot[
e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*f)
```

Rubi [A] time = 0.164005, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4139, 446, 103, 156, 63, 208}

$$-\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f(a+b)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f)) + ((2*a + 3*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/2)*f) - (Cot[
e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*f)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{bx}{2}}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f} - \frac{(2a+3b)\,\text{Subs}}{}$$

$$= -\frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{bf} - \frac{(2a+3}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e}{2(a+b)f}$$

Mathematica [F] time = 5.02927, size = 0, normalized size = 0.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] time = 0.48, size = 4245, normalized size = 36.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x)
```

[Out] -1/4/f/(a+b)^(5/2)/a^(1/2)*(2*cos(f*x+e)^3*(a+b)^(3/2)*a^(3/2)+2*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)-2*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))+4*cos(f*x+e)*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a+4*cos(f*x+e)*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+5*cos(f*x+e)*a^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b-5*cos(f*x+e)*a^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b+3*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^2-3*cos(f*x+e)*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^2-4*cos(f*x+e)^2*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a-4*cos(f*x+e)^2*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b-5*cos(f*x+e)^2*a^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b+5*cos(f*x+e)^2*a^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b-3*cos(f*x+e)^2*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^2+3*cos(f*x+e)^2*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^2-4*cos(f*x+e)^3*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x

+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
)*a-4*cos(f*x+e)^3*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos
(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b-5*cos(f*x+
e)^3*a^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*
(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)+b)/sin(f*x+e)^2)*b-2*cos(f*x+e)^2*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)+2*cos(f*x+e)^2*a^(5/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))+2*cos(f*x+e)*(a+b)
^(3/2)*a^(1/2)*b+4*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos
(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a+4*(a+b)^(3
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+5*a^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b-5*a^(3/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b+3*a^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)*b^2-3*a
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^2-2*cos(f
*x+e)*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))-2*co
s(f*x+e)^3*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^
(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)+b)/sin(f*x+e)^2)+2*cos(f*x+e)^3*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)+b)/(-1+cos(f*x+e)))+2*cos(f*x+e)*a^(5/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+

```
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/sin(f*x+e)^2)+5*cos(f*x+e)^3*a
^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+b)/(-1+cos(f*x+e)))*b-3*cos(f*x
+e)^3*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-2/(a+b)^(1/2)
*(-1+cos(f*x+e))*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)-a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)+b)/sin(f*x+e)^2)*b^2+3*cos(f*x+e)^3*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(-4*(cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*(a+b)^(1/2)+a*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)+b)/(-1+cos(f*x+e)))*b^2)*sin(f*x+e)^2/(-1+cos(f*x+e))^2/cos(f*x+e
)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/(1+cos(f*x+e))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{3}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 1.95909, size = 3742, normalized size = 32.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)^2 + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e
)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*c
os(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^2 + 3*a*b)*cos(f*x + e)^2 -
 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2
```

```
*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos
(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(
f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e
)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a^2 + 3*a*b)*cos(f*x + e)^2 -
2*a^2 - 3*a*b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(
-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x +
e)^2 + a*b + b^2)) + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^
2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*
b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)
^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2
)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^3 + 2*a^2*b + a
*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + 2*((a^2 + 2*a*b +
 b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*
x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) +
 ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2
 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4
*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^
3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/4*(2*
(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + ((
a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)*arctan(1/4*
(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^
2 + a*b^2)) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(-a - b)
*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^3 +
 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{3}}{\sqrt{b\sec\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.407 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(8a^2+20ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}+\frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)^2}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ((8*a^2 + 20*a*b
+ 15*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(5/2)
*f) + ((4*a + 7*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)^2*
f) - (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*(a + b)*f)
```

Rubi [A] time = 0.244155, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4139, 446, 103, 151, 156, 63, 208}

$$-\frac{\left(8a^2+20ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}-\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f(a+b)}+\frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```


```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) - ((8*a^2 + 20*a*b
+ 15*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(5/2)
*f) + ((4*a + 7*b)*Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2])/(8*(a + b)^2*
f) - (Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2])/(4*(a + b)*f)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^3\sqrt{a+bx^2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{3bx}{2}}{(-1+x)^2x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{\left(8a^2+20ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} + \frac{(4a+7b)\cot^2(e}{8}$$

Mathematica [F] time = 8.48484, size = 0, normalized size = 0.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [B] time = 0.564, size = 10441, normalized size = 62.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 4.97435, size = 5370, normalized size = 32.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b +
3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)*l
og(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x
+ e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*
b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((8*a^3 + 20*a^2*b + 15*a*b^2)
*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*
b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^
4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 +
b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)
```

```
*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos
(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4
 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), 1/16*(((8*a^3 + 20*a^2*b + 15*a*b^2)*co
s(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2
)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
 + e)^2 + a*b + b^2)) + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 +
 a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x
+ e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(3*(2*a^3 +
 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x +
 e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*co
s(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/32*(8*((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*co
s(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2
)) - ((8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*
a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*(
(8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
 + 4*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*
a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/1
6*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*
b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-a)*arct
an(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*
x + e)^2 + a*b^2)) - ((8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3
+ 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqr
t(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2))
 + 2*(3*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*
a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 +
 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.408 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=173

$$\frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b\,\mathrm{t}}}{4bf}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b +
b*Tan[e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^
2])/(4*b*f)
```

Rubi [A] time = 0.318719, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4141, 1975, 479, 582, 523, 217, 206, 377, 203}

$$\frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{8b^{5/2}f}-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{8b^2f}+\frac{\tan^3(e+fx)\sqrt{a+b\,\mathrm{t}}}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]])/(8*b^(5/2)*f) - ((3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b +
b*Tan[e + f*x]^2])/(8*b^2*f) + (Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^
2])/(4*b*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} - \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a+7b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{4bf}$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} +$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} -$$

$$= -\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - (3a+7b)\,\text{ta}$$

Mathematica [A] time = 5.21273, size = 230, normalized size = 1.33

$$-\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\left(\frac{8b^2\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{a}}-\frac{(3a^2+10ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{b}}\right)}{8\sqrt{2}b^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\tan(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(((8*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sq
rt[a] - ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a +
b - a*Sin[e + f*x]^2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e +
 f*x])/(8*Sqrt[2]*b^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - ((a + 2*b + a*Cos[2*(
e + f*x)])*(3*a + 5*b + 3*(a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*x]^4*Tan[e
+ f*x])/(32*b^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.504, size = 1993, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2*sin(f*x+e)*(6*sin(f*x+e)*
cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(
1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2+20*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(
f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*
cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-
(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2))*a*b+30*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(
```

```
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*sin(f*x+e)*cos(
f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-10*sin(f*x+e
)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((
-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^
(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-7*sin(
f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2
)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipt
icF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4
*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2-1
6*sin(f*x+e)*cos(f*x+e)^4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2-3*cos(f*x+e)^5*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-9*cos(f*x+e)^5*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*a*b+3*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2
+9*cos(f*x+e)^4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-cos(f*x+e)^3*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-9*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*b^2+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*a*b+9*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+2*cos(f*
x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2-2*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f
*x+e)^2)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [B] time = 4.89076, size = 4045, normalized size = 23.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")

```
[Out] [-1/32*(4*sqrt(-a)*b^3*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4
 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2
*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*cos(f*x
 + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e
)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(2*a
*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), -1/16*(2*sqrt(-a)*b^3*c
os(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6
 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a
^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^
3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*co
s(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x +
 e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(2*a*b^2
- 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), 1/32*(8*sqrt(a)*b^3*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*cos(f*x + e)^3 + (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*cos(f*x + e)^
3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 +
4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(2*a*b^2 -
 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), 1/16*(4*sqrt(a)*b^3*arctan(1/
```

```
4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2
)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3
*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x +
 e)))*cos(f*x + e)^3 + (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2*(
(a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e
)^3 + 2*(2*a*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.409 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=120

$$-\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) -
 ((a + 3*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rubi [A] time = 0.230063, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4141, 1975, 479, 523, 217, 206, 377, 203}

$$-\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}f}+\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) -
 ((a + 3*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/(2*b^(3/2)*f) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{a+b+(a+3b)x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{2bf}$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{(a+$$

$$= \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a+3b)}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\,\mathrm{t}}}{2bf}$$

Mathematica [A] time = 3.29694, size = 196, normalized size = 1.63

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)}{4bf\sqrt{a+b\sec^2(e+fx)}} + \frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\left(\frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{a}} - \right.}{2\sqrt{2}bf\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (((2*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[
a] - ((a + 3*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^
2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(2*Sqrt[2]*b
*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*
x]^2*Tan[e + f*x])/(4*b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.41, size = 1320, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/f/b/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*(4*sin(f*x+e)*co
s(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+
cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1
/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2))*b+cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a+cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f
*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)
-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b-2*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2
*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
))*a-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b)
)^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b+cos(f*x+e)^3*((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*a-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b-((2*I*a^(1/
2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f*x+
e)^2)/cos(f*x+e)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.06177, size = 3686, normalized size = 30.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^
3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f
*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*
x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*(sqrt(-a)*b
^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
+ e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*
a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*
cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e)) + 2*(a^2 + 3*a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*
cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*co
s(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*a*b*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*(2*sqrt
(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
```

```
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x
+ e)^2)*sin(f*x + e)))*cos(f*x + e) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*lo
g(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/
4*(sqrt(a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e) + (a^2 + 3*a*b)*sqrt(-b)*arctan(
-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f
*x + e) - 2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(
a*b^2*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.410 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rubi [A] time = 0.19681, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 483, 217, 206, 377, 203}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]
*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

Mathematica [F] time = 2.97924, size = 0, normalized size = 0.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2], x]
```

Maple [C] time = 0.435, size = 404, normalized size = 5.1

$$-2\,\frac{\sqrt{2}\left(\sin\left(fx+e\right)\right)^2}{f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{i\cos\left(fx+e\right)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+a\cos\left(fx+e\right)+b}{(a+b)\left(1+\cos\left(fx+e\right)\right)}}\sqrt{-2\,\frac{i\cos\left(fx+e\right)\sqrt{a}\sqrt{b}-}{(a+b)\left(1\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*(EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^
(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))-EllipticPi
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*
a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x
+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 1.31023, size = 3036, normalized size = 37.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b +
70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3
)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
```

```
x + e)) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^
2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x +
e)^4))/(a*b*f), 1/8*(4*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*c
os(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8
 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos
(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^
3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 -
 (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*b*f), 1/4*(sqrt(a)*b*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
))) + a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))
/(a*b*f), 1/4*(sqrt(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*c
os(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 2*a*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/(a*b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.411 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rubi [A] time = 0.0290335, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4128, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [B] time = 0.122145, size = 87, normalized size = 2.23

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2])
```

Maple [C] time = 0.418, size = 380, normalized size = 9.7

$$-\frac{\sqrt{2}\left(\sin\left(fx+e\right)\right)^2}{f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\sqrt{\frac{1}{(a+b)\left(1+\cos\left(fx+e\right)\right)}\left(i\cos\left(fx+e\right)\sqrt{a}\sqrt{b}-i\sqrt{a}\sqrt{b}+a\cos\left(fx+e\right)+b\right)}\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-
2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+
cos(f*x+e)))^(1/2)*(EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b
)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)
*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*sin(f*
x+e)^2/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.767014, size = 976, normalized size = 25.03

$$\left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4-14\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+a^4-28\,a^3b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*
```

```
a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
 e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*c
os(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x +
 e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)
*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.412 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f)
```

Rubi [A] time = 0.191736, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4141, 1975, 480, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```



```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} + \frac{\text{Subst}\left(\int \frac{-a-b}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{(a+b)f}$$
$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

Mathematica [A] time = 0.230536, size = 127, normalized size = 1.72

$$-\frac{\sec(e+fx)\sqrt{a\cos(2(e+fx))+a+2b}\left((a+b)\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)+\sqrt{a}\csc(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}\right)}{\sqrt{2}\sqrt{a}f(a+b)\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] + Sqrt[a]*Csc[e + f*x]*Sqrt[
a + b - a*Sin[e + f*x]^2]))/(Sqrt[2]*Sqrt[a]*(a + b)*f*Sqrt[a + b*Sec[e + f
*x]^2]))
```

Maple [C] time = 0.518, size = 1865, normalized size = 25.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x)

[Out] -1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b)*(2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)*cos(f*x+e)+2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)*cos(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)*cos(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)*cos(f*x+e)+2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e

```
))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/
2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)/cos(f*x+e)/sin(f*x
+e)/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot \left(fx+e\right)^{2}}{\sqrt{b\sec \left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

Fricas [B] time = 0.978228, size = 1272, normalized size = 17.19

$$\left[-\frac{\sqrt{-a}(a+b)\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\left(fx+e\right)^{6}+32\left(5\,a^{4}-14\,a^{3}b+5\,a^{2}b^{2}\right)\cos\left(fx+e\right)^{4}+a^{4}-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^
3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*
cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
```

```
sin(f*x + e))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), 1/4*((a + b)*sqrt(a)*arctan(1
/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^
3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e)))*sin(f*x + e) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e))/((a^2 + a*b)*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.413 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}+\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) +
 ((3*a + 5*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)^2*f)
- (Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rubi [A] time = 0.249387, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 480, 583, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)}+\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f) +
 ((3*a + 5*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)^2*f)
- (Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \frac{\text{Subst}\left(\int \frac{-3a-5b-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{3(a+b)f}$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \cdots$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \cdots$$

$$= \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} + \cdots$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2f} - \frac{\cot^3(e+fx)\cdots}{\cdots}$$

<u>Mathematica</u> [A] time = 1.95576, size = 168, normalized size = 1.41

$$\frac{\sec(e+fx)\sqrt{a\cos(2e+2fx)+a+2b}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}} - \frac{\csc^3(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+2b)(\cdots}{6f(a+b)^2\sqrt{a+b\sec^2(e\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e +
f*x]^2]) - ((a + 2*b + a*Cos[2*(e + f*x)])*(-a - 2*b + (2*a + 3*b)*Cos[2*(e
 + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*
x]^2])
```

Maple [C] time = 0.489, size = 5619, normalized size = 47.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.07552, size = 1705, normalized size = 14.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a)
*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4
- 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a
*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*
a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b
+ 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*s
qrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x +
 e) - 8*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b^2)*f*cos
```

```
(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e)), -1/12*(3*((a^2 + 2*
a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*co
s(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f
*x + e) - 4*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b^2)*f
*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.414 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

Optimal. Leaf size=172

$$-\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+}}{5f(a+b)}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - ((15*a^2 + 40*a*b + 33*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(15*(a + b)^3*f) + ((5*a + 9*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5
*(a + b)*f)
```

Rubi [A] time = 0.344836, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 480, 583, 12, 377, 203}

$$-\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15f(a+b)^3}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+}}{5f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
) - ((15*a^2 + 40*a*b + 33*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(15*(a + b)^3*f) + ((5*a + 9*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*(a + b)^2*f) - (Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(5
*(a + b)*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+b(1+x^2)}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} + \frac{\text{Subst}\left(\int \frac{-5a-9b-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{5(a+b)f}$$

$$= \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f} - \text{S}$$

$$= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+}}{15(a+b)^2}$$

$$= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+}}{15(a+b)^2}$$

$$= -\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+}}{15(a+b)^2}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f} + ($$

Mathematica [A] time = 4.5927, size = 199, normalized size = 1.16

$$-\frac{\csc(e+fx)\sec(e+fx)(a\cos(2(e+fx))+a+2b)\left(-\left(11a^2+26ab+15b^2\right)\csc^2(e+fx)+23a^2+3(a+b)^2\csc^4(e+fx}{30f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

```
[Out] -((ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2
*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])) - ((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*(23*a^2 + 60*a*b
 + 45*b^2 - (11*a^2 + 26*a*b + 15*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e +
 f*x]^4)*Sec[e + f*x])/(30*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Maple [C] time = 0.641, size = 11267, normalized size = 65.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 6.61052, size = 2334, normalized size = 13.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b
 + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(-
a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
```

```
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x
 + e) + 8*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a^3 + 94*a^2
*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b + 3*a^2*b^2
 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), 1/60*(15*(
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*
cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*c
os(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e
)))*sin(f*x + e) - 4*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a
^3 + 94*a^2*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.415 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + (a + b)^2/(a*b
^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Sqrt[a + b*Sec[e + f*x]^2]/(b^2*f)
```

Rubi [A] time = 0.150769, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4139, 446, 87, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + (a + b)^2/(a*b
^2*f*Sqrt[a + b*Sec[e + f*x]^2]) + Sqrt[a + b*Sec[e + f*x]^2]/(b^2*f)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}+\frac{1}{ax\sqrt{a+bx}}\right)dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$
$$= \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

Mathematica [F] time = 5.26377, size = 0, normalized size = 0.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] time = 0.529, size = 6593, normalized size = 74.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.83191, size = 1095, normalized size = 12.44

$$\left[\frac{\left(ab^2\cos\left(fx+e\right)^2+b^3\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+e\right)^4+32\,ab^3\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256
*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos
(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + 8*(a^2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
, 1/4*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(a^
2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)
```

```
/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^5}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.416 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - (a + b)/(a*b*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.116453, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4139, 446, 78, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - (a + b)/(a*b*f*Sq
rt[a + b*Sec[e + f*x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx=\frac{\text{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{3/2}}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{3/2}}\,dx,x,\sec^2(e+fx)\right)}{2f}$$

$$=-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\sec^2(e+fx)\right)}{2af}$$

$$=-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sec^2(e+fx)}\right)}{abf}$$

$$=\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] time = 4.26875, size = 187, normalized size = 2.97

$$\frac{3(a+b)\tan^4(e+fx)F_1\left(2;\frac{1}{2},\frac{3}{2};3;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)}{2f\left(a+b\sec^2(e+fx)\right)^{3/2}\left(\sin^2(e+fx)\left(3aF_1\left(3;\frac{1}{2},\frac{5}{2};4;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)F_1\left(3;\frac{3}{2},\frac{3}{2};4;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\right)+\cdots\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a + b)*AppellF1[2, 1/2, 3/2, 3, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Tan[e + f*x]^4)/(2*f*(a + b*Sec[e + f*x]^2)^(3/2)*(6*(a + b)*AppellF1[
2, 1/2, 3/2, 3, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1
[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*Appe
llF1[3, 3/2, 3/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2))
```

Maple [B] time = 0.372, size = 2371, normalized size = 37.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)

[Out] 1/f/a^(3/2)/((-a*b)^(1/2)-a)/((-a*b)^(1/2)+a)/b*(-1+cos(f*x+e))^2*(1+cos(f*x+e))^2*(3*cos(f*x+e)^5*a^(11/2)*b-4*cos(f*x+e)^3*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^3-2*cos(f*x+e)^3*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^4-2*cos(f*x+e)^2*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b^2-4*cos(f*x+e)^2*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^3-2*cos(f*x+e)^2*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^4-cos(f*x+e)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^3-2*cos(f*x+e)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^4-cos(f*x+e)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^5-cos(f*x+e)^5*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^5*b-2*cos(f*x+e)^5*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b^2-cos(f*x+e)^5*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^3-cos(f*x+e)^4*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^5*b-2*cos(f*x+e)^4*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b^2-cos(f*x+e)^4*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^3-2*cos(f*x+e)^3*ln(4*cos(f*x+e)*((b+a*cos(f*x

```
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^4*b^2+6*cos(f*x+e)^3*a^(9/2)*b^2+6*cos(f*x+e)^3*a^(7/2)*b^3+2*cos(f*x+
e)^3*a^(5/2)*b^4+cos(f*x+e)*a^(9/2)*b^2+3*cos(f*x+e)*a^(7/2)*b^3+3*cos(f*x+
e)*a^(5/2)*b^4+cos(f*x+e)*a^(3/2)*b^5+3*cos(f*x+e)^5*a^(9/2)*b^2+cos(f*x+e)
^5*a^(7/2)*b^3+2*cos(f*x+e)^3*a^(11/2)*b-ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^
3*b^3-2*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)
+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^4-ln(4*cos(f*x+e)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*a*cos(f*x+e)+4*a^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a*b^5+cos(f*x+e)^5*a^(13/2))*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/cos(f*x+
e)^2)^(3/2)/(b+a*cos(f*x+e)^2)^4/(a+b)/sin(f*x+e)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.03414, size = 1022, normalized size = 16.22

$$\left[-\frac{8\left(a^2+ab\right)\sqrt{\frac{a\cos\left(fx+e\right)^2+b}{\cos\left(fx+e\right)^2}}\cos\left(fx+e\right)^2-\left(ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)^2 - (a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*
a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(
f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*f), -1/4*(4*(a^2 + a*b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*b*cos(f*x + e)^2
 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 +
b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x +
e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*
f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.417 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{af\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.0721779, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 266, 51, 63, 208}

$$\frac{1}{af\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sec[e + f*x]^2])
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}\\
&= \frac{1}{af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af}\\
&= \frac{1}{af\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sec^2(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 7.15968, size = 382, normalized size = 6.7

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)^{3/2}\left(-\frac{2}{b\sqrt{a\cos(2(e+fx))+a+2b}}+\frac{\sqrt{2}e^{i(e+fx)}\sec(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{\sqrt{a}(a+4b)\left(1+e^{2\ldots}\right.}{b\left(a\left(1+e^{2i(e+fx)}\right)^2+\ldots\right.}\right.}{\ldots}\right.}{16f\left(a+b\sec^2(e+fx\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sec[e + f*x]^2*(-2/(b*Sqrt[a + 2*b +
a*Cos[2*(e + f*x)]]) + (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)
*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*((Sqrt[a]*(a + 4*b)*(1 + E^((2*I)*(e +
 f*x))))/(b*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)) + ((
4*I)*f*x - 2*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*Log[a + a*E^((2*I)*(e +
f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*
(1 + E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2])*Sec[e + f*x])/a^(3/2)))/(16*f*(a + b*Sec[e + f*x]^2)^(3/
2))
```

Maple [A] time = 0.056, size = 64, normalized size = 1.1

$$\frac{1}{af}\frac{1}{\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}}-\frac{1}{f}\ln\left(\frac{1}{\sec\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^2}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/a/f/(a+b*sec(f*x+e)^2)^(1/2)-1/f/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e
)^2)^(1/2))/sec(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 0.952492, size = 971, normalized size = 17.04

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)^{2}+\left(a\cos\left(fx+e\right)^{2}+b\right)\sqrt{a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}+256\,a^{3}b\cos\left(fx+e\right)^{6}+160\,a^{2}b^{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*c
os(f*x + e)^2 + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x +
 e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*
a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^
3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/(a^
3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e)^2 + (a*cos(f*x + e)^2 + b)*sqrt(-a)*arctan(1/4*(8*a^2
*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*
b^2)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [A] time = 11.0727, size = 53, normalized size = 0.93

$$\frac{1}{af\sqrt{a+b\sec^2\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\sec^2\left(e+fx\right)}}{\sqrt{-a}}\right)}{af\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] 1/(a*f*sqrt(a + b*sec(e + f*x)**2)) + atan(sqrt(a + b*sec(e + f*x)**2)/sqrt
(-a))/(a*f*sqrt(-a))
```

Giac [B] time = 2.07094, size = 336, normalized size = 5.89

$$\frac{\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}-\frac{1}{a\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}}-\frac{2\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e \cdots}{2\sqrt{-a \cdots}}\right.}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2} \cdots\right.\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((tan(1/2*f*x + 1/2*e)^2/(a*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)) - 1/(a*sgn(tan
(1/2*f*x + 1/2*e)^2 - 1)))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
 - 2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1
/2*f*x + 1/2*e)^2 - 1)))/f
```

3.418 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - b/(a*(a + b)*f*Sqrt[a +
b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.147423, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4139, 446, 85, 156, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b}{af(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ArcTanh[Sqrt[a +
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - b/(a*(a + b)*f*Sqrt[a +
b*Sec[e + f*x]^2])
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(-1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+fx)\right)}{2af} + \frac{\text{Subst}\left(\int \frac{}{(-1+x}\right.}{}$$

$$= -\frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sec^2(e+fx)}\right)}{abf} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [F] time = 6.81818, size = 0, normalized size = 0.

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] time = 0.512, size = 9693, normalized size = 96.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.16591, size = 3750, normalized size = 37.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)^2 - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)
*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^
2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8
 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(a^3*cos(f*x + e)^
2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a
*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
+ e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2
 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 4*(a^3*cos(f*x + e)^2 + a^2*
b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*s
```

```
qrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*
cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 +
 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^5 + 2*a^4*b + a^3*b
^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/4*(4*(a^2*b + a
*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a^2*b +
 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/
4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e
)^2 + a*b^2)) - (a^3*cos(f*x + e)^2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*
a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2
*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/4
*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)^2 + (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt
(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2
*b*cos(f*x + e)^2 + a*b^2)) - 2*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-a - b)*a
rctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)))/((a^5 + 2
*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.419 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b(a-2b)}{2af(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cot^2(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ((2*a + 5*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(5/2)*f) - ((a -
 2*b)*b)/(2*a*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - Cot[e + f*x]^2/(2*(
a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.242316, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4139, 446, 103, 152, 156, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{b(a-2b)}{2af(a+b)^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cot^2(e+fx)}{2f(a+b)\sqrt{a+b\sec^2(e+fx)}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ((2*a + 5*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(5/2)*f) - ((a -
 2*b)*b)/(2*a*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - Cot[e + f*x]^2/(2*(
a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{3bx}{2}}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b)^2-\frac{1}{4}}{(-1+x)x\sqrt{}}\right.}{a(}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.}{2af}$$

$$= -\frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(}}$$

Mathematica [F] time = 10.7717, size = 0, normalized size = 0.

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] time = 0.967, size = 19968, normalized size = 130.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 5.61965, size = 5342, normalized size = 34.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b
^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(a
)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f
*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a
^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^4 + 5*a^3*b)*cos(f*x
+ e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2
)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^
2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*s
qrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2
*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*cos(f*x + e)
```

```
^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
- (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b
^2 + 3*a^3*b^3 + a^2*b^4)*f), -1/8*(2*((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2
*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(-a
- b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - ((
a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a
*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(a)*log(12
8*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^
4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos
(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b
^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f
*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 -
(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), 1/8*(2*((a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 +
2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) +
((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b
- 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x +
 e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*
b^2 + 2*a*b^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 +
 a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*
x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), 1/4*(((a^4 + 3*a^3
*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4
- (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^
2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a
*b^2)) - ((2*a^4 + 5*a^3*b)*cos(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 +
 3*a^3*b - 5*a^2*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*co
s(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/
((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^4 + a^3*b + 2*a^2*b^2 + 2
*a*b^3)*cos(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^
2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.420 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=213

$$\frac{b\left(4a^2+11ab-8b^2\right)}{8af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(8a^2+28ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot}{4f(a+b)\sqrt{}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ((8*a^2 + 28*a*b
+ 35*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(7/2)
*f) + (b*(4*a^2 + 11*a*b - 8*b^2))/(8*a*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]
^2]) + ((4*a + 9*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^
2]) - Cot[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.333689, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4139, 446, 103, 151, 152, 156, 63, 208}

$$\frac{b\left(4a^2+11ab-8b^2\right)}{8af(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{\left(8a^2+28ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}-\frac{\cot}{4f(a+b)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f) - ((8*a^2 + 28*a*b
+ 35*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(7/2)
*f) + (b*(4*a^2 + 11*a*b - 8*b^2))/(8*a*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]
^2]) + ((4*a + 9*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^
2]) - Cot[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^3\left(a+bx^2\right)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{5bx}{2}}{(-1+x)^2x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{2(a+b)^2+\frac{3}{4}b(4}{(-1+x)x(a+l}\right.}{4(a}$$

$$= \frac{b\left(4a^2+11ab-8b^2\right)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+f}{4(a+b)f\sqrt{a+b\,\text{se}}}$$

$$= \frac{b\left(4a^2+11ab-8b^2\right)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+f}{4(a+b)f\sqrt{a+b\,\text{se}}}$$

$$= \frac{b\left(4a^2+11ab-8b^2\right)}{8a(a+b)^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+f}{4(a+b)f\sqrt{a+b\,\text{se}}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\left(8a^2+28ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} + \frac{b\left(4a^2\right.}{8a(a+b)^3f\sqrt{}}$$

Mathematica [F] time = 14.2711, size = 0, normalized size = 0.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

Maple [B] time = 1.522, size = 32565, normalized size = 152.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 17.2826, size = 7887, normalized size = 37.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 +
a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^
2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2
- 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x
+ e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*c
os(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 +
10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)) + ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 +
 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a
^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f
*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a
```

*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), 1/16*(((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), -1/32*(8*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((6*a^5 + 19*a^4*b + 13*a^3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6*a^5*b

```
^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 +
 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5
*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5
*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f), -1/16*(4*((a^5 + 4*a^4*b + 6*a^
3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*
cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*
cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*co
s(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - ((8*a^5 + 28*a^4*b + 35*a
^3*b^2)*cos(f*x + e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a
^4*b + 42*a^3*b^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3
*b^2 - 70*a^2*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f
*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a
^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((6*a^5 + 19*a^4*b + 13*a^3*b^2
+ 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2 + 3*a^
2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^3 - 8*a*
b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4
*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6
*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 +
 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2
+ (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.421 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2ab^2f}-\frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}-\frac{}{ab}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - ((3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]
^2]])/(2*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(a*b*f*Sqrt[a + b + b*Tan[e
+ f*x]^2]) + ((3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a
*b^2*f)
```

Rubi [A] time = 0.348505, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4141, 1975, 470, 582, 523, 217, 206, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2ab^2f}-\frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{5/2}f}-\frac{}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```



```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - ((3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]
^2]])/(2*b^(5/2)*f) - ((a + b)*Tan[e + f*x]^3)/(a*b*f*Sqrt[a + b + b*Tan[e
+ f*x]^2]) + ((3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a
*b^2*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+(3a+2b)x^2)}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} - \text{Subs}\ldots$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} - \text{Subs}\ldots$$

$$= -\frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f} - \text{Subs}\ldots$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{(3a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{(a+b)\tan^3(e \ldots}{abf\sqrt{a+b+b\tan \ldots}}$$

Mathematica [A] time = 10.0612, size = 247, normalized size = 1.44

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)\left((3a^2+4ab+2b^2)\cos(2(e+fx))+3a^2+6ab+2b^2\right)}{8ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\sec^3(e+f \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((2*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sq
rt[a] + (a*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
+ f*x]^2]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(
4*Sqrt[2]*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e +
f*x)])*(3*a^2 + 6*a*b + 2*b^2 + (3*a^2 + 4*a*b + 2*b^2)*Cos[2*(e + f*x)])*S
ec[e + f*x]^4*Tan[e + f*x])/(8*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.433, size = 3860, normalized size = 22.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/f/a/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^(5/2)*(4*2^(1/2)*b^(5/2)*
(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*EllipticF(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a
^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a-8*2^(1/
2)*b^(5/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)
*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x
+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(
1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-20*2^(1/2)*b^(5/2)*(1/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos
(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*
b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*a+6*2^(1/2)*a^3*b^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos
(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-12*2^(1/2)*a^3*b^(1/2)*(1/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*EllipticPi((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(
```

1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2))+10*2^(1/2)*b^(5/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2
)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a+10*2^(1/2)*b^(3/2)*(1/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*EllipticF((-1+cos(f*x+e))*((2*I
*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^
(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-20*2^(1/2)*b^(3/2)*(1/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*co
s(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*EllipticPi((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)
b^(1/2)+a-b)(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*a^2+6*2^(1/2)*b^(3/2)*(1/(a+b)*(I*cos(f*x+e)*a^(
1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a
+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(
f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticF((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2-12*2^(1/2)*b^(3/2)*(1/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticPi((-1+cos(f*x
+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*a^2-8*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
*cos(f*x+e)^4*a^2-6*a^3*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos
(f*x+e)^4+8*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a^
2+8*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a^2+2*b^(5
/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)*a+4*b^(5/2)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5*a+8*b^(5/2)*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^3*a-8*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*cos(f*x+e)^2*a^2-4*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)*cos(f*x+e)^4*a-8*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*
x+e)^2*a+6*a^3*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^5
-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(5/2)*a-4*b^(7/2)*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^2+4*b^(7/2)*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*cos(f*x+e)^3+7*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*arctanh(1/8
*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2
)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^2-7*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2

```
)*sin(f*x+e)*cos(f*x+e)^2*arctanh(1/4*b^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4
^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2))*a*b^2+4*2^(1/2)*b^(7/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1
/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))+7*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*arctanh(1/8
*b^(1/2)*4^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2
)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b-7*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*cos(f*x+e)^4*sin(f*x+e)*arctanh(1/4*b^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4
^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2))*a^2*b-8*2^(1/2)*b^(7/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1
/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)))*s
in(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3
/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [B] time = 6.32243, size = 4514, normalized size = 26.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(f
*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2
*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a
^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - ((3*a^4 + 5*a^3*b)*cos(f*
x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b +
b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(a^2*b^2 + (3*a^3*b + 4*a^2*b^2
+ 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)), -1/8*(2*((3*
a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(-b
)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e
))) + (a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(f*
x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*
b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^
4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
- 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
+ e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 4*(a^2*b^2 + (3*a^3*b + 4*
a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)), 1/8*
(2*(a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + ((3*a
^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*
log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*(a^2*b^2 + (
3*a^3*b + 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x
 + e)), 1/4*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(a)*arctan(1/4*(
8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*c
os(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*co
s(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)
)) - ((3*a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e)
)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*si
n(f*x + e))) + 2*(a^2*b^2 + (3*a^3*b + 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f
*x + e)^3 + a^2*b^4*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.422 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) +
 ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
 - ((a + b)*Tan[e + f*x])/(a*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.249025, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4141, 1975, 470, 523, 217, 206, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}f}-\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) +
 ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(3/2)*f)
 - ((a + b)*Tan[e + f*x])/(a*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+b+ax^2}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af} + \frac{\text{Subst}\left(\int \frac{}{1-}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [A] time = 4.69041, size = 201, normalized size = 1.73

$$\frac{\sec^3(e+fx)(a\cos(2e+2fx)+a+2b)^{3/2}\left(\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{a}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{\sqrt{b}}\right)}{2\sqrt{2}abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(a+b)\tan(e+fx)\sec^2}{2abf\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (((b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[a]
 + (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[
b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a*b*f*(
a + b*Sec[e + f*x]^2)^(3/2)) - ((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[
e + f*x]^2*Tan[e + f*x])/(2*a*b*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.529, size = 2662, normalized size = 23.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/b^(3/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a*(2*2^(1/2)*sin(f*x
+e)*cos(f*x+e)^2*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2)*a-4*(1/(
a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos
(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a
*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*b^(3/2)*Ellipt
icPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/
(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1/2)*a+2*2^(1/2)*sin(f*x+e)*cos(f
*x+e)^2*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4
*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(1/2)*a^2-4*2^(1/2)*sin
(f*x+e)*cos(f*x+e)^2*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(
1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(1/2)*a^2+2*2^(1/2)*sin(f*x+e)*(1/(a+b)*(I
*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)
))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*
x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-
a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(5/2)-4*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(
```

```
1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))
^(1/2)*sin(f*x+e)*b^(5/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1
/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*2^(1
/2)+2*2^(1/2)*sin(f*x+e)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^(3/2)*
a-4*2^(1/2)*sin(f*x+e)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(
1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi(
(-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a
^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^(3/2)*a-sin(f*x+e)*cos(f*x+e)^2*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(f*x
+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2
+sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctan
h(1/4*b^(1/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/s
in(f*x+e)^2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*a^2+2*cos(f*x+e)^3*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a+2*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1
/2)*a^2-2*cos(f*x+e)^2*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+2*
cos(f*x+e)*b^(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-sin(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/8*b^(1/2)*4^(1/2)*(-1+cos(
f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^2/((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a*b+sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*arctanh(1/4*b^(1
/2)*(-1+cos(f*x+e))*(cos(f*x+e)*4^(1/2)-2*cos(f*x+e)-4^(1/2)-2)/sin(f*x+e)^
2/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a*b-2*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(1
/2)*a^2+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^(3/2)*a-2*b^
(5/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)-2*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)*b^(3/2)*a)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos(f
*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.31401, size = 3985, normalized size = 34.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 2*(a^3*cos(f*x + e)^2 +
 a^2*b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))
/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f), -1/8*(8*(a^2*b + a*b^2)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - 4*(a^3*cos(f*
x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e))) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
```

```
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a^3*b^2*f*cos(
f*x + e)^2 + a^2*b^3*f), -1/4*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*
sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x
+ e)^2)*sin(f*x + e))) - (a^3*cos(f*x + e)^2 + a^2*b)*sqrt(b)*log(((a^2 - 6
*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/(a^3*b^2*f*cos(f*x + e)^2 +
a^2*b^3*f), -1/4*(4*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*arcta
n(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b +
 b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2
*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f
*x + e))) - 2*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))))/(a^3*b^2*f*cos(f*x +
 e)^2 + a^2*b^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.423 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{\tan(e+fx)}{af\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) + Tan[e + f*x]/(a*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.209005, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4141, 1975, 471, 377, 203}

$$\frac{\tan(e+fx)}{af\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) + Tan[e + f*x]/(a*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
```

```
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}$$

$$= \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}}$$

Mathematica [B] time = 2.64007, size = 169, normalized size = 2.38

$$-\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\sqrt{2}\sqrt{a}\sqrt{a+b}\sin(e+fx)\sqrt{\cdots}\right.}{4a^{3/2}f\sqrt{a+b}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(ArcSin[(Sqrt[a]*Sin[e + f*
x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a]*Sqrt[a +
b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*S
qrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(
a + b)])
```

Maple [C] time = 0.434, size = 569, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a*(b+a*cos(f*x+e)^2)*(2^(1/2)*(
1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2
)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2
)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^
(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*si
n(f*x+e)+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*cos(f*x+e)-((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2))*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^3/((b+a*cos
(f*x+e)^2)/cos(f*x+e)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.06647, size = 1322, normalized size = 18.62

$$\left[\frac{8\,a\sqrt{\frac{a\cos\left(fx+e\right)^{2}+b}{\cos\left(fx+e\right)^{2}}}\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(a\cos\left(fx+e\right)^{2}+b\right)\sqrt{-a}\log\left(128\,a^{4}\cos\left(fx+e\right)^{8}-256\left(a^{4}-a^{3}b\right)\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
+ e) - (a*cos(f*x + e)^2 + b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a*co
s(f*x + e)^2 + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{2}\left(e+fx\right)}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.424 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0464997, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4128, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Tan[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}\\
&= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{af}\\
&= -\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}+\frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{af}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 1.45234, size = 168, normalized size = 2.18

$$\frac{\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\sqrt{a+b}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a\cos(2(e+fx))+a+2b)-\sqrt{2}\sqrt{a}b\sin(e+fx)\sqrt{\frac{a\cos\cdots}{\cdots}}\right.}{4a^{3/2}f(a+b)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{a+b}}\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a]
*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a
]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/2)*
(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a +
 b)])
```

Maple [C] time = 0.433, size = 1007, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a*(b+a*cos(f*x+e)^2)*(2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*sin(f*x+e)+2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(
f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1
/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-
b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/
2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*sin
(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+c
os(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/
2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+
a-b)/(a+b))^(1/2)*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b)*sin(f*x+e)/(
```

```
-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/cos(f*x+e)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.05175, size = 1438, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f*cos(f*x
 + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e
)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a
^2*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

3.425 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{af(a+b)^2}-\frac{b\cot(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((a - b
)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(a + b)^2*f)
```

Rubi [A] time = 0.272158, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 472, 583, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{af(a+b)^2}-\frac{b\cot(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```


```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((a - b
)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(a + b)^2*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{x^2(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f} - \text{Sub}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f} - \text{Sub}$$

$$= -\frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2f} - \text{Sub}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a}}{a(a+}$$

Mathematica [A] time = 4.44028, size = 182, normalized size = 1.53

$$-\frac{\csc(e+fx)\sec^3(e+fx)(a\cos(2(e+fx))+a+2b)\left(\left(a^2+b^2\right)\cos(2(e+fx))+a^2+2ab-b^2\right)}{4af(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\sec^3(e+fx)(a\cos(2e}{2\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b +
a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a^(3/2)*f*(a + b*Sec[e
```

```
 + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(a^2 + 2*a*b - b^2 + (a
^2 + b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x]^3)/(4*a*(a + b)^2*f*(
a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.556, size = 2836, normalized size = 23.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f/(a+b)^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a/(b+a*cos(f*x+e)^2)^2
*(cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)
*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*El
lipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),
(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a
^2+2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)
*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)*a*b+2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*co
s(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I
*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*
x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(
1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)*cos(
f*x+e)-2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-4*cos(f*x+e)*sin(f*x+
e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^
(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1
/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2))*a*b-2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f
*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b
```

```
)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2
)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2+2^
(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)
+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*sin(f*x+e)+2*2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1
/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+2^(1/2)*(1/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x
+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a
^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^2*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/
(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b
^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*sin(f
*x+e)-4*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*
cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)
-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)
*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*
a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+c
os(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1
/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^2*sin(f*x+
e)+cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2+cos(f*x+e)^2*((
2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a*b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2)*cos(f*x+e)^3*((b+a*
cos(f*x+e)^2)/cos(f*x+e)^2)^(3/2)/sin(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.46159, size = 1728, normalized size = 14.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqr
t(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(
f*x + e) + 8*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*cos(f*x + e))*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b + a^3*b^2)*f*
cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), 1/4*((a^2*
b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(
1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b
^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
 + e)))*sin(f*x + e) - 4*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b +
 a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.426 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3af(a+b)^2}-\frac{b\cot^3(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-}{}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Cot[e + f*x]^3)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*a -
b)*(a + 3*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^3*f)
 - ((a - 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^2
*f)
```

Rubi [A] time = 0.362018, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 472, 583, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}f}-\frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3af(a+b)^2}-\frac{b\cot^3(e+fx)}{af(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```



```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f) -
 (b*Cot[e + f*x]^3)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*a -
b)*(a + 3*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^3*f)
 - ((a - 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a*(a + b)^2
*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-3b-4bx^2}{x^4(1+x^2)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f} - \ldots$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+\ldots}}{3a(a+b)^3f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+\ldots}}{3a(a+b)^3f}$$

$$= -\frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+\ldots}}{3a(a+b)^3f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a-b)(a+3b)\cot(e\ldots}{3a\ldots}$$

Mathematica [A] time = 5.65957, size = 224, normalized size = 1.29

$$\frac{\sec^3(e+fx)(a\cos(2e+2fx)+a+2b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{2\sqrt{2}a^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\sec^3(e+fx)(a\cos(2e+2fx)+a+2b)^2\left(-\frac{\ldots}{2af(a+\ldots}\right.}{\left(a+b\sec^2\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b + a
*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a^(3/2)*f*(a + b*Sec[e
+ f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^3*(((4*a
+ 9*b)*Csc[e + f*x])/(12*(a + b)^3*f) - Csc[e + f*x]^3/(12*(a + b)^2*f) - (
b^3*Sin[e + f*x])/(2*a*(a + b)^3*f*(a + 2*b + a*Cos[2*e + 2*f*x]))))/(a + b
*Sec[e + f*x]^2)^(3/2)
```

Maple [C] time = 0.489, size = 7541, normalized size = 43.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 7.52485, size = 2388, normalized size = 13.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a
^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sq
rt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(
5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
- 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 +
8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*
a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin
(f*x + e) - 8*((4*a^4 + 9*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - (3*a^4 + 4*a^3*
b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (3*a^3*b + 8*a^2*b^2 - 3*a*b^3)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*
b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*
x + e)), -1/12*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3
*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x +
e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)
^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*co
s(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*a^4 + 9*a^3*b + 3*a*b^3)*
cos(f*x + e)^5 - (3*a^4 + 4*a^3*b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (
3*a^3*b + 8*a^2*b^2 - 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (
a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2
+ 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot \left(fx+e\right)^{4}}{\left(b \sec \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.427 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15af(a+b)^3}-\frac{\left(55a^2b+15a^3+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e}{15af(a+b)^4}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x]^5)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((15*
a^3 + 55*a^2*b + 73*a*b^2 - 15*b^3)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*a*(a + b)^4*f) + ((5*a^2 + 14*a*b - 15*b^2)*Cot[e + f*x]^3*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(15*a*(a + b)^3*f) - ((a - 5*b)*Cot[e + f*x]^5*Sq
rt[a + b + b*Tan[e + f*x]^2])/(5*a*(a + b)^2*f)
```

Rubi [A] time = 0.471311, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 472, 583, 12, 377, 203}

$$\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{15af(a+b)^3}-\frac{\left(55a^2b+15a^3+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e}{15af(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(3/2)*f)
) - (b*Cot[e + f*x]^5)/(a*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((15*
a^3 + 55*a^2*b + 73*a*b^2 - 15*b^3)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(15*a*(a + b)^4*f) + ((5*a^2 + 14*a*b - 15*b^2)*Cot[e + f*x]^3*Sqrt[a
 + b + b*Tan[e + f*x]^2])/(15*a*(a + b)^3*f) - ((a - 5*b)*Cot[e + f*x]^5*Sq
rt[a + b + b*Tan[e + f*x]^2])/(5*a*(a + b)^2*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6\left(1+x^2\right)\left(a+b\left(1+x^2\right)\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6\left(1+x^2\right)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a-5b-6bx^2}{x^6\left(1+x^2\right)\sqrt{a+b+bx^2}}\,dx, x, \tan(e+fx)\right)}{a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f} - $$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b+b\tan}}{15a(a+b)^3f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+}}{15a(a+b)^4f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+}}{15a(a+b)^4f}$$

$$= -\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+}}{15a(a+b)^4f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\left(15a^3+55a^2b+73a\right.}{}$$

Mathematica [A] time = 11.1437, size = 237, normalized size = 0.98

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)^2\left(-\left(23a^2+80ab+90b^2\right)\csc^2(e+fx)+\frac{30b^4}{a(a\cos(2(e+fx))+a+2b)}-3(a+\right.}{60f(a+b)^4\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b +
a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a^(3/2)*f*(a + b*Sec[e
 + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^2*((30*b^4)/(a*(a + 2*b
 + a*Cos[2*(e + f*x)])) - (23*a^2 + 80*a*b + 90*b^2)*Csc[e + f*x]^2 + (a +
b)*(11*a + 20*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x]^
2*Tan[e + f*x])/(60*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.697, size = 14137, normalized size = 58.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 24.112, size = 3425, normalized size = 14.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6
 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3
*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b
^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(
a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*
x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^5
+ 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - (35*a^5 + 106*a^4*b +
80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + (15*a^5 + 20*a^4*b - 5
6*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^3 + (15*a^4*b + 55*a^3*b^2
 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e
)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos
(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^
5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)
*f)*sin(f*x + e)), 1/60*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4
)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 +
 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2
*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*a
rctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a
*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*s
in(f*x + e)))*sin(f*x + e) - 4*((23*a^5 + 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)
*cos(f*x + e)^7 - (35*a^5 + 106*a^4*b + 80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)
*cos(f*x + e)^5 + (15*a^5 + 20*a^4*b - 56*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)
*cos(f*x + e)^3 + (15*a^4*b + 55*a^3*b^2 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x +
 e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*
b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2
+ 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^
5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^
5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.428 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + (a + b)^2/(3*a
*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (a^(-2) - b^(-2))/(f*Sqrt[a + b*Sec[
e + f*x]^2])
```

Rubi [A] time = 0.166095, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4139, 446, 87, 63, 208}

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + (a + b)^2/(3*a
*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (a^(-2) - b^(-2))/(f*Sqrt[a + b*Sec[
e + f*x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{ab(a+bx)^{5/2}} + \frac{a^2-b^2}{a^2b(a+bx)^{3/2}} + \frac{1}{a^2x\sqrt{a+bx}}\right) dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e\right.}{2a^2f}$$

$$= \frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\mathrm{s}}\right.}{a^2bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] time = 8.61337, size = 187, normalized size = 1.93

$$\frac{4(a+b)\tan^6(e+fx)F_1\left(3;\frac{1}{2},\frac{5}{2};4;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)}{3f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(\sin^2(e+fx)\left(5aF_1\left(4;\frac{1}{2},\frac{7}{2};5;\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)+(a+b)F_1\left(4;\frac{3}{2},\frac{5}{2};5;\sin^2(e+fx),\frac{a\,\mathrm{s}}{}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (4*(a + b)*AppellF1[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Tan[e + f*x]^6)/(3*f*(a + b*Sec[e + f*x]^2)^(5/2)*(8*(a + b)*AppellF1[
3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1
[4, 1/2, 7/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*Appe
llF1[4, 3/2, 5/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2))
```

Maple [B] time = 2.303, size = 10947, normalized size = 112.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.29382, size = 1332, normalized size = 13.73

$$\left[\frac{3\left(a^2b^2\cos\left(fx+e\right)^4+2\,ab^3\cos\left(fx+e\right)^2+b^4\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b^4)*sqrt(a)*lo
g(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x +
 e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b
```

```
*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(2*(a^4 - a^3*b - 2*a^2*b^2)*
cos(f*x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2
 + a^3*b^4*f), 1/12*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b
^4)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4
 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 4*(2*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*
x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 + a^3
*b^4*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.429 $\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}} dx$

Optimal. Leaf size=89

$$-\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - (a + b)/(3*a*b*f*
(a + b*Sec[e + f*x]^2)^(3/2)) - 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.128431, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4139, 446, 78, 51, 63, 208}

$$-\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - (a + b)/(3*a*b*f*
(a + b*Sec[e + f*x]^2)^(3/2)) - 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{-1+x}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$

$$= -\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e\right.}{2a^2f}$$

$$= -\frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{a^2bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] time = 10.4458, size = 613, normalized size = 6.89

$$-\frac{e^{i(e+fx)}\sec^5(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-12\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)-12\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right.}\right.}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}}\right.}{96\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e
 + f*x]^4)/(48*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*
x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e +
 2*f*x])^(5/2)*Sec[e + f*x]^4)/(96*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/
2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((
2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/
2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*x)) + a^3
```

```
*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))^2 - 6*a^2
*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E^((2*I)*(e
+ f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12*Log[a + 2*b
 + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^
((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec[
e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [B] time = 2.165, size = 10839, normalized size = 121.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.8122, size = 1264, normalized size = 14.2

$$\left[\frac{3\left(a^{2} b \cos\left(f x + e\right)^{4} + 2\, a b^{2} \cos\left(f x + e\right)^{2} + b^{3}\right)\sqrt{a} \log\left(128\, a^{4} \cos\left(f x + e\right)^{8} + 256\, a^{3} b \cos\left(f x + e\right)^{6} + 160\, a^{2} b^{2} \cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e
)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*c
os(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(3*a*b^2*cos(f*x + e)^2 + (a^3
+ 4*a^2*b)*cos(f*x + e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^
5*b*f*cos(f*x + e)^4 + 2*a^4*b^2*f*cos(f*x + e)^2 + a^3*b^3*f), -1/12*(3*(a
^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*
a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 +
 a*b^2)) + 4*(3*a*b^2*cos(f*x + e)^2 + (a^3 + 4*a^2*b)*cos(f*x + e)^4)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*b*f*cos(f*x + e)^4 + 2*a^4*b^
2*f*cos(f*x + e)^2 + a^3*b^3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.430 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sec[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.0883175, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 266, 51, 63, 208}

$$\frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sec[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2af}$$

$$= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sec^2(e+\ldots\right.}{2a^2f}$$

$$= \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\text{se}\ldots}\right.}{a^2bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

Mathematica [C] time = 7.6078, size = 613, normalized size = 7.39

$$\frac{e^{i(e+fx)}\sec^5(e+fx)\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\left(\frac{-12\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}+ae^{2i(e+fx)}+a+2b\right)-12\log\left(\sqrt{a}\sqrt{a\left(1+e^{2i(e+fx)}\right)^2}\ldots\right.}{\sqrt{a\left(1+e^{2i(e+fx)}\right)^2+4be^{2i(e+fx)}}}\right.}{96\sqrt{2}\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e
 + f*x]^4)/(48*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e + f*
x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e +
 2*f*x])^(5/2)*Sec[e + f*x]^4)/(32*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/
2)*(a + b*Sec[e + f*x]^2)^(5/2)) + (E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((
2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/
2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*x)) + a^3
```

```
*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))^2 - 6*a^2
*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E^((2*I)*(e
+ f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12*Log[a + 2*b
 + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^
((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
 + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec[
e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [A] time = 0.056, size = 86, normalized size = 1.

$$\frac{1}{3af}\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{-\frac{3}{2}}+\frac{1}{a^{2}f}\frac{1}{\sqrt{a+b\left(\sec\left(fx+e\right)\right)^{2}}}-\frac{1}{f}\ln\left(\frac{1}{\sec\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sec\left(fx+e\right)\right)^{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/a/f/(a+b*sec(f*x+e)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/f/a^(5/
2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 2.36484, size = 1208, normalized size = 14.55

$$\left[\frac{3\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{a}\log\left(128\,a^4\cos\left(fx+e\right)^8+256\,a^3b\cos\left(fx+e\right)^6+160\,a^2b^2\cos\left(fx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*
a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f
*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*(4*a^2*cos(f*x + e)^4 + 3*a*b*cos(f
*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*f*cos(f*x + e)
^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), 1/12*(3*(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*
b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(4*a^2*cos(f
*x + e)^4 + 3*a*b*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [A] time = 66.105, size = 78, normalized size = 0.94

$$\frac{1}{3af\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}+\frac{1}{a^2f\sqrt{a+b\sec^2\left(e+fx\right)}}+\frac{\operatorname{atan}\left(\frac{\sqrt{a+b\sec^2\left(e+fx\right)}}{\sqrt{-a}}\right)}{a^2f\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] 1/(3*a*f*(a + b*sec(e + f*x)**2)**(3/2)) + 1/(a**2*f*sqrt(a + b*sec(e + f*x
)**2)) + atan(sqrt(a + b*sec(e + f*x)**2)/sqrt(-a))/(a**2*f*sqrt(-a))
```

Giac [B] time = 2.04053, size = 613, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((((4*a^9*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 3*a^8*b^3*sgn(tan(1/2*
f*x + 1/2*e)^2 - 1))*tan(1/2*f*x + 1/2*e)^2/(a^10*b^2) - 3*(4*a^9*b^2*sgn(t
an(1/2*f*x + 1/2*e)^2 - 1) - a^8*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^10
*b^2))*tan(1/2*f*x + 1/2*e)^2 + 3*(4*a^9*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
) - a^8*b^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1))/(a^10*b^2))*tan(1/2*f*x + 1/2*
e)^2 - (4*a^9*b^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 3*a^8*b^3*sgn(tan(1/2*f
*x + 1/2*e)^2 - 1))/(a^10*b^2))/(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
)^(3/2) - 6*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a^2*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1)))/f
```

3.431 $\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{b(2a+b)}{a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ArcTanh[Sqrt[a + 
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - b/(3*a*(a + b)*f*(a + b*
Sec[e + f*x]^2)^(3/2)) - (b*(2*a + b))/(a^2*(a + b)^2*f*Sqrt[a + b*Sec[e + 
f*x]^2])
```

Rubi [A] time = 0.206542, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4139, 446, 85, 152, 156, 63, 208}

$$-\frac{b(2a+b)}{a^2f(a+b)^2\sqrt{a+b\sec^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b}{3af(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ArcTanh[Sqrt[a + 
b*Sec[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - b/(3*a*(a + b)*f*(a + b*
Sec[e + f*x]^2)^(3/2)) - (b*(2*a + b))/(a^2*(a + b)^2*f*Sqrt[a + b*Sec[e + 
f*x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x], 
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b-bx}{(-1+x)x(a+bx)^{3/2}}\,dx, x, \sec^2(e+fx)\right)}{2a(a+b)f}$$

$$= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(a+b}{(-1+}\right.}{}$$

$$= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{2}$$

$$= -\frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} -$$

Mathematica [F] time = 9.14549, size = 0, normalized size = 0.

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [B] time = 2.566, size = 68989, normalized size = 503.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 5.58353, size = 5253, normalized size = 38.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
+ a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f
*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 +
160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(
f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*
x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 6*(a^5*cos
(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((8*a^2 +
 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*(
(2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 8*((7
*a^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3
```

```
+ a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8
 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 +
 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 +
 a^3*b^5)*f), 1/24*(12*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b
^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)) + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b
^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*co
s(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*c
os(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos
(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*((7*a
^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 +
a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 +
 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3
*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a
^3*b^5)*f), -1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 +
a*b^4)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3*(a^5*cos(f*x + e
)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b +
 b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b
)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((7*a^4*b +
 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 + 3*a^7
*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b
^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5
)*f), -1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^
3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x
+ e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 6*(a^5*cos(f*x + e)^4 +
2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*
x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^
2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 4*((7*a^4*b + 11*a^3*b^2 + 4*a^2*b^
3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b
^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*
x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.432 $\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{b\left(a^2-6ab-2b^2\right)}{2a^2f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b(3a-2b)}{6af(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cot^2(e+}{2f(a+b)\left(a+b\sec\right.}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ((2*a + 7*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(7/2)*f) - ((3*a
 - 2*b)*b)/(6*a*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/
(2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (b*(a^2 - 6*a*b - 2*b^2))/(2*a
^2*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rubi [A] time = 0.319628, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4139, 446, 103, 152, 156, 63, 208}

$$-\frac{b\left(a^2-6ab-2b^2\right)}{2a^2f(a+b)^3\sqrt{a+b\sec^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}-\frac{b(3a-2b)}{6af(a+b)^2\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\cot^2(e+}{2f(a+b)\left(a+b\sec\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + ((2*a + 7*b)*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(7/2)*f) - ((3*a
 - 2*b)*b)/(6*a*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) - Cot[e + f*x]^2/
(2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) - (b*(a^2 - 6*a*b - 2*b^2))/(2*a
^2*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
```

```
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^2x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b+\frac{5bx}{2}}{(-1+x)x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}(a \cdots}{(- \cdots}\right.}{\cdots}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b\left(a^2 - \cdots\right.}{2a^2(a+b)^3f\sqrt{\cdots}}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b\left(a^2 - \cdots\right.}{2a^2(a+b)^3f\sqrt{\cdots}}$$

$$= -\frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b\left(a^2 - \cdots\right.}{2a^2(a+b)^3f\sqrt{\cdots}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(2a+7b)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(3a-2b)b}{6a(a+b)^2f\left(a+b\sec^2(\cdots\right.}$$

Mathematica [F] time = 19.8624, size = 0, normalized size = 0.

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [B] time = 3.917, size = 105237, normalized size = 526.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 18.0088, size = 7761, normalized size = 38.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
- a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*
```

b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 3*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), -1/24*(6*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) - 3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), 1/24*(6*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 3*((2*a^6 + 7*a^5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2

```
)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(
a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(
f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1)) + 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)
*cos(f*x + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)
*cos(f*x + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e
)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b
^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8
*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 +
 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 +
 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f), 1/12*(3*((a^6 + 4*a^5*b +
 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*
a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^
4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4
- 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3*((2*a^
6 + 7*a^5*b)*cos(f*x + e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14
*a^4*b^2)*cos(f*x + e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^
2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b +
 b^2)) + 2*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*x
 + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f*x
 + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a
^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3
 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b
^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b
^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{3}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.433 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$\frac{b\left(15a^2b+4a^3-32ab^2-8b^3\right)}{8a^2f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}+\frac{b\left(12a^2+39ab-8b^2\right)}{24af(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+36ab+63b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ((8*a^2 + 36*a*b
+ 63*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(9/2)
*f) + (b*(12*a^2 + 39*a*b - 8*b^2))/(24*a*(a + b)^3*f*(a + b*Sec[e + f*x]^2
)^(3/2)) + ((4*a + 11*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sec[e + f*x]
^2)^(3/2)) - Cot[e + f*x]^4/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (b
*(4*a^3 + 15*a^2*b - 32*a*b^2 - 8*b^3))/(8*a^2*(a + b)^4*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

Rubi [A] time = 0.448714, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4139, 446, 103, 151, 152, 156, 63, 208}

$$\frac{b\left(15a^2b+4a^3-32ab^2-8b^3\right)}{8a^2f(a+b)^4\sqrt{a+b\sec^2(e+fx)}}+\frac{b\left(12a^2+39ab-8b^2\right)}{24af(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+36ab+63b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f) - ((8*a^2 + 36*a*b
+ 63*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/(8*(a + b)^(9/2)
*f) + (b*(12*a^2 + 39*a*b - 8*b^2))/(24*a*(a + b)^3*f*(a + b*Sec[e + f*x]^2
)^(3/2)) + ((4*a + 11*b)*Cot[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sec[e + f*x]
^2)^(3/2)) - Cot[e + f*x]^4/(4*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)) + (b
*(4*a^3 + 15*a^2*b - 32*a*b^2 - 8*b^3))/(8*a^2*(a + b)^4*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
```

```
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)^3\left(a+bx^2\right)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(-1+x)^3x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{2(a+b)+\frac{7bx}{2}}{(-1+x)^2x(a+bx)^{5/2}}\,dx, x, \sec^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{2(a+b)^2+\cdots}{(-1+x\cdots}\right.}{\cdots}$$

$$= \frac{b\left(12a^2+39ab-8b^2\right)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot\cdots}{4(a+b)f\left(a+\cdots\right.}$$

$$= \frac{b\left(12a^2+39ab-8b^2\right)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot\cdots}{4(a+b)f\left(a+\cdots\right.}$$

$$= \frac{b\left(12a^2+39ab-8b^2\right)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot\cdots}{4(a+b)f\left(a+\cdots\right.}$$

$$= \frac{b\left(12a^2+39ab-8b^2\right)}{24a(a+b)^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot\cdots}{4(a+b)f\left(a+\cdots\right.}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\left(8a^2+36ab+63b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} + \frac{b\left(12a^2\cdots\right.}{24a(a+b)^3f\left(\cdots\right.}$$

Mathematica [F] time = 30.5189, size = 0, normalized size = 0.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]

[Out] Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

Maple [B] time = 15.336, size = 145925, normalized size = 544.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [B] time = 64.4681, size = 10710, normalized size = 39.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [1/96*(12*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*
cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 +
b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x
 + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 +
```

a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f), 1/48*(3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 6*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b

^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4
- 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^
7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a
^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x +
 e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^
7)*f), -1/96*(24*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^
2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a
b^6 + b^7 - 2(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*
cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2
*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 3
*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8*a^5*b^2 + 36*a^4*b^3 +
 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63*a^4*b^3)*cos(f*x + e)^6
 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 63*a^3*b^4)*cos(f*x + e)^4
 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*b^4)*cos(f*x + e)^2)*sqrt(
a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(
f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*
x + e)^2 + 1)) + 4*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4*b^3 + 136*a^3
*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a^5*b^2 + 106*a
^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6 - (24*a^6*b +
96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)*cos(f*x + e)^
4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a*b^6)*cos(f*x
+ e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 + 5*a^9*b + 10*
a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*
a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^
10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^
3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*
a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 1
0*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f), -1/48*(12*((a^7 + 5*a^6*b + 10*a^5*b^2
 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 +
 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5
*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 2
5*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b
 + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*
b*cos(f*x + e)^2 + a*b^2)) - 3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e
)^8 + 8*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^
2 - 63*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^
3 + 63*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*
a^3*b^4)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2
+ b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*

```
cos(f*x + e)^2 + a*b + b^2)) + 2*((18*a^7 + 69*a^6*b + 51*a^5*b^2 + 104*a^4
*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7 + 21*a^6*b - 93*a
^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^6)*cos(f*x + e)^6
- (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 208*a^2*b^5 + 48*a*b^6)
*cos(f*x + e)^4 - 3*(4*a^5*b^2 + 19*a^4*b^3 - 17*a^3*b^4 - 40*a^2*b^5 - 8*a
*b^6)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^10 +
 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8
- 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*
x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5
+ a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 -
5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.434 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)
*f) - ((a + b)*Tan[e + f*x]^3)/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) +
 ((a^(-2) - b^(-2))*Tan[e + f*x])/(f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.344652, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4141, 1975, 470, 578, 523, 217, 206, 377, 203}

$$\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{5/2}f}-\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(b^(5/2)
*f) - ((a + b)*Tan[e + f*x]^3)/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) +
 ((a^(-2) - b^(-2))*Tan[e + f*x])/(f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{x^2(3(a+b)+3ax^2)}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3abf}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{-3(a^2-b^2)\cdots}{(1+x^2)\sqrt{a+\cdots}}\right.}{3\cdots}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+\cdots}}\right.}{\cdots}$$

$$= -\frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{(a^2-b^2)\tan(e+fx)}{a^2b^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,\cdots\right.}{a\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3\cdots}}$$

Mathematica [B] time = 11.6583, size = 316, normalized size = 2.01

$$\frac{\sec^5(e+fx)(a\cos(2e+2fx)+a+2b)^3\left(\frac{a^2(-\sin(e+fx))-2ab\sin(e+fx)-b^2\sin(e+fx)}{6a^2bf(a\cos(2e+2fx)+a+2b)^2} + \frac{-3a^2\sin(e+fx)+ab\sin(e+fx)+4b^2\sin(e+fx)}{12a^2b^2f(a\cos(2e+2fx)+a+2b)}\right)}{\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{\sec\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(((b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt
[a] - (a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/
Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^
2*b^2*f*(a + b*Sec[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*S
ec[e + f*x]^5*((-(a^2*Sin[e + f*x]) - 2*a*b*Sin[e + f*x] - b^2*Sin[e + f*x]
)/(6*a^2*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + (-3*a^2*Sin[e + f*x] + a*b
*Sin[e + f*x] + 4*b^2*Sin[e + f*x])/(12*a^2*b^2*f*(a + 2*b + a*Cos[2*e + 2*
f*x]))))/(a + b*Sec[e + f*x]^2)^(5/2)
```

Maple [C] time = 0.469, size = 2256, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b^2/a^2*sin(f*x+e)*(b+a*cos(f
*x+e)^2)*(6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*
b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(
I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e
)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/
(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^3-3*cos(f*x+e)^2*si
n(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2
)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos
(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*
b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^3+3*sin(f*x+e)
*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b
^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-
1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(
3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-6*cos
(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/
2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^
(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellip
ticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1
/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+6*sin(f*x+e)*2^(1/2)*(1/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x
+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos
```

```
(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b
^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*a^2*b-3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+co
s(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)
*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e
)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b^3*sin(f*x+e)-6*2^(1
/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b
)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b
)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2))*b^3*sin(f*x+e)-3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^3+cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*
cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*cos(f*x+e)^2*(
(2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^3-cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/
2)+a-b)/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))
^(1/2)*a*b^2-4*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b-cos
(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*cos(f*x+e)*((2*I*a^
(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3+4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)*a^2*b+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-3*((2*I*a^(1/2)*b^(1
/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2
)^(5/2)/cos(f*x+e)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 7.53061, size = 4811, normalized size = 30.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*
log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*
b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a
^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 6*(a^5*c
os(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x
+ e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*((3*a^4*b - a^3*b^2 - 4*a^2*b
^3)*cos(f*x + e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4
 + 2*a^4*b^4*f*cos(f*x + e)^2 + a^3*b^5*f), 1/24*(12*(a^5*cos(f*x + e)^4 +
2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(a^2*b^3*cos(f*x + e)^4 +
2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^
2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)
*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - 8*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x +
 e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4*
f*cos(f*x + e)^2 + a^3*b^5*f), 1/12*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*co
s(f*x + e)^2 + b^5)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 3*(a^5*cos(f*x + e)^4 + 2*a^4*
b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x +
 e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b
^2)/cos(f*x + e)^4) - 4*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x + e)^3 + (
4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
```

```
s(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4*f*cos(f*x
 + e)^2 + a^3*b^5*f), 1/12*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e
)^2 + b^5)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*
b)*cos(f*x + e)^2)*sin(f*x + e))) + 6*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x
 + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f
*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
 + e)^2 + b^2)*sin(f*x + e))) - 4*((3*a^4*b - a^3*b^2 - 4*a^2*b^3)*cos(f*x
+ e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*cos(f*x + e)^4 + 2*a^4*b^4
*f*cos(f*x + e)^2 + a^3*b^5*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.435 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}} dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\tan(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 ((a + b)*Tan[e + f*x])/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((a -
3*b)*Tan[e + f*x])/(3*a^2*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.266793, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 470, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b\tan^2(e+fx)+b}}-\frac{(a+b)\tan(e+fx)}{3abf\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```



```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 ((a + b)*Tan[e + f*x])/(3*a*b*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((a -
3*b)*Tan[e + f*x])/(3*a^2*b*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+b+(a-2b)x^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3abf}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3b(}{(1+x^2)\sqrt{}}\right.}{3}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(1+x^2)\sqrt{}}\right.}{}$$

$$= -\frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx}{3a^2bf\sqrt{a+b+b\tan^2(e}}$$

Mathematica [B] time = 6.54829, size = 409, normalized size = 3.41

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^{5/2}\left(\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{16\left(-a\sin^2(e+fx)+a+b\right)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(\frac{a^2(a+b)\sin^4(e+fx)}{\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\sqrt{a}\sqrt{a+b}\sin(e+fx)\,\mathrm{s}\cdots}{\sqrt{\frac{-a\sin^2(\cdots}{a\cdots}}}\cdots\right)}{a^3}\cdots\right)}{\left(-a\sin^2(e+fx\cdots}\cdots\right)}{384f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*((Sqrt[2]*Csc[e + f*x]
*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin
[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e
 + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/
(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Si
n[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt
[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(a
+ b - a*Sin[e + f*x]^2)^(3/2) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])*Tan[e +
 f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (12*(b + (3*a + 2
*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)
])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] time = 0.401, size = 1142, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2*sin(f*x+e)*(b+a*cos(f*x+e
)^2)*(6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1
```

```
/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*co
s(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^
(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/si
n(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+
b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a-3*cos(f*x+e)^2*sin(f*x
+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a
^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+
e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/
2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a+6*2^(1/2)*(1/(a+b)*
(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+
e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(
f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*
I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)
)*b*sin(f*x+e)-3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b
^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2
)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF
((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*
a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*b*sin(f*
x+e)-4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+4*cos(f*x+e)^
2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+cos(f*x+e)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a-3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
b-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a+3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*b)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2
)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.11215, size = 1584, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(4*a^2*cos(f
*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a
^3*b^2*f), -1/12*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(
a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e))) + 4*(4*a^2*cos(f*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e
)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.436 $\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(2a+3b)\tan(e+fx)}{3a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + Tan[e + f*x]/(3*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((2*a + 3*b)*Ta
n[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.257275, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4141, 1975, 471, 527, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{(2a+3b)\tan(e+fx)}{3a^2f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3af\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) + Tan[e + f*x]/(3*a*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((2*a + 3*b)*Ta
n[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+b+bx^2)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1-2x^2}{(1+x^2)(a+b+bx^2)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3af}$$

$$= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{3}{(1+x^2)\ldots}\right.}{\ldots}$$

$$= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\ldots}{(1+x^2)\ldots}\right.}{\ldots}$$

$$= \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\ldots\right.}{\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+\ldots}{3a^2(a+b)f\sqrt{a+b+b\,\text{ta}\ldots}}$$

Mathematica [B] time = 4.83869, size = 410, normalized size = 3.45

$$\frac{\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^{5/2}\left(-\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{16\left(-a\sin^2(e+fx)+a+b\right)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(\frac{a^2(a+b)\sin^4(e+fx)}{\left(-a\sin^2(e+fx)+a+b\right)^2}+\frac{3\sqrt{a}\sqrt{a+b}\sin(e+\cdots}{\sqrt{-a\,s\cdots}}\cdots\right)}{a^3}\cdots\right)}{\left(-a\sin^2(e\cdots\right)}\cdots\right)}{384\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*(-((Sqrt[2]*Csc[e + f*
x]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*S
in[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin
[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2
)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*
Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sq
rt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(
a + b - a*Sin[e + f*x]^2)^(3/2)) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])*Tan[
e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (4*(b + (3*a +
 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*
x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] time = 0.418, size = 2112, normalized size = 17.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/f/(a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a^2*sin(f*x+e)*(b+a*cos
(f*x+e)^2)*(3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2
```

```
)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)
*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x
+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/
2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+
b)^2)^(1/2))*a^2+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2
)/(a+b)^2)^(1/2))*a*b-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x
+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/
(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*
b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2-6*co
s(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1
/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Elli
pticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-
1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/
((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(
1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b
-b^2)/(a+b)^2)^(1/2))*a*b*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/
2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b
)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*
x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1
/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a
+b)^2)^(1/2))*b^2*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/
2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos
(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(
1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin
(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b
))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-6*2^(1/2)*
(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1
+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/
2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a
+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2))*b^2*sin(f*x+e)+3*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^
(1/2)*a^2+4*cos(f*x+e)^3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*cos(
f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2-4*cos(f*x+e)^2*((2*I*a
^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b+2*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b
```

```
)/(a+b))^(1/2)*a*b+3*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^2
-2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b-3*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*b^2)/(-1+cos(f*x+e))/cos(f*x+e)^5/((b+a*cos(f*x+e)^2)/cos(f*x
+e)^2)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.68327, size = 1804, normalized size = 15.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(a^2*b + a*b^2)*c
os(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^
3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*
cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e)) - 8*((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^
6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b
^2 + a^3*b^3)*f), 1/12*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(
a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8
*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a
*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*((3*a^3 + 4*a^2*b
)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^
```

```
5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b^2 + a^3*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.437 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rubi [A] time = 0.0989862, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4128, 414, 527, 12, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{b(5a+3b)\tan(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{b\tan(e+fx)}{3af(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Tan[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(5*
a + 3*b)*Tan[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a+b-2bx^2}{(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right.}{3a(a+b)f}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\,\text{t}}{3a^2(a+b)^2f\sqrt{a+}}$$

<u>Mathematica</u> [C] time = 6.47174, size = 1927, normalized size = 15.42

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(
a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5
/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2,
-1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*
((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Sin[e + f*
```

```
x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*S
in[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x
]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/
2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5
)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[2]*(a + b - a*Sin[
e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*Cos[e
 + f*x]^4*Sin[e + f*x]*((5*a*f*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)) - (4*f*A
ppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[
e + f*x]*Sin[e + f*x])/3))/(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3
*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^4*Sin[e + f*x]*
(2*f*(5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f
*x]^2)/(a + b)])*Cos[e + f*x]*Sin[e + f*x] + 3*(a + b)*((5*a*f*AppellF1[3/2
, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Si
n[e + f*x])/(3*(a + b)) - (4*f*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/3) + Sin[e + f*x]^2*
(5*a*((21*a*f*AppellF1[5/2, -2, 9/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(5*(a + b)) - (12*f*AppellF1[5/2, -1,
 7/2, 7/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e +
 f*x])/5) - 4*(a + b)*((3*a*f*AppellF1[5/2, -1, 7/2, 7/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (6*(a + b)^
3*f*Cot[e + f*x]*Csc[e + f*x]^4*(-1 + (a*Sin[e + f*x]^2)/(a + b))^2*((Sqrt[
a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/(Sqrt[a + b]*Sq
rt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (a^2*Sin[e + f*x]^4)/(3*(a + b)^2*(-1
 + (a*Sin[e + f*x]^2)/(a + b))^2) + (a*Sin[e + f*x]^2)/((a + b)*(-1 + (a*Si
n[e + f*x]^2)/(a + b)))))/(a^3*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2))))))/
(4*Sqrt[2]*f*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2,
5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2,
-2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*Appel
lF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e +
 f*x]^2)^2)))
```

Maple [C] time = 0.421, size = 3024, normalized size = 24.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3/f/(a^2+2*a*b+b^2)/a^2/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*sin(f*x+
e)*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2
)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)
/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a
*b-b^2)/(a+b)^2)^(1/2))*a^3+6*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+b)*(I*c
os(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+
e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^
2+6*a*b-b^2)/(a+b)^2)^(1/2))*a^2*b+3*sin(f*x+e)*cos(f*x+e)^2*2^(1/2)*(1/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*c
os(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*
b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(
3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b^2-6*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)
*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1
/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*
a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(
a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2))*a^3-12*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a
^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/
(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+co
s(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b
))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/
2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b-6*cos(f
*x+e)^2*sin(f*x+e)*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)
*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(
2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2
```

```
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a*b^2+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e
)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(
-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1
+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-
b^2)/(a+b)^2)^(1/2))*a^2*b*sin(f*x+e)+6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1
/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+
b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f
*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)/sin(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))*a*b^2*sin(f*x+e)+3*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^
(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*
cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e))
)^(1/2)*EllipticF((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/s
in(f*x+e),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))*b^3*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*
a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+
e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*
EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+
e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1
/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*a^2*b*sin(f*x+e)-12*2^(1/2)*(1
/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+c
os(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)
-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(
1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b
),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2))*a*b^2*sin(f*x+e)-6*2^(1/2)*(1/(a+b)*(I*cos(f*x+e)*a^(1/2)*b^(1/2)-
I*a^(1/2)*b^(1/2)+a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(-2/(a+b)*(I*cos(f*
x+e)*a^(1/2)*b^(1/2)-I*a^(1/2)*b^(1/2)-a*cos(f*x+e)-b)/(1+cos(f*x+e)))^(1/2
)*EllipticPi((-1+cos(f*x+e))*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/sin(f*
x+e),-1/(2*I*a^(1/2)*b^(1/2)+a-b)*(a+b),(-(2*I*a^(1/2)*b^(1/2)-a+b)/(a+b))^
(1/2)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2))*b^3*sin(f*x+e)+6*cos(f*x+e)^
3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^2*b+4*cos(f*x+e)^3*((2*I*a^(1/2
)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2-6*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a^2*b-4*cos(f*x+e)^2*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*
a*b^2+5*cos(f*x+e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b^2+3*cos(f*x+
e)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3-5*((2*I*a^(1/2)*b^(1/2)+a-b)
/(a+b))^(1/2)*a*b^2-3*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*b^3)/(-1+cos(
f*x+e))/((b+a*cos(f*x+e)^2)/cos(f*x+e)^2)^(5/2)/cos(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29074, size = 2021, normalized size = 16.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos
(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*
(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)
^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b + 2*a^2*b
^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*
x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*
a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
 a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^
2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b
^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b
^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\sec^2\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sec \left(fx + e\right)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

3.438 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}-\frac{b(7a+3b)\cot(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)}}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*
(7*a + 3*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) - ((a - 3*b)*(3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a^
2*(a + b)^3*f)
```

Rubi [A] time = 0.377997, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{5/2}f}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}-\frac{b(7a+3b)\cot(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*
(7*a + 3*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]
) - ((a - 3*b)*(3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*a^
2*(a + b)^3*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
```

```
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-b-4bx^2}{x^2(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \text{Subst}$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - (a-3$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - (a-3$$

$$= -\frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - (a-3$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)}{3a^2(a+b)^2f\sqrt{a+}}$$

Mathematica [A] time = 7.82932, size = 247, normalized size = 1.42

$$-\frac{\csc(e+fx)\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)\left(4\left(6a^3b+3a^4+8ab^3+3b^4\right)\cos(2(e+fx))+a\left(3a^3+9ab^2+4b^3\right)}{48a^2f(a+b)^3\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b +
a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e
 + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(3*a^4 + 8*a^3*b + 5
*a^2*b^2 - 12*a*b^3 - 4*b^4) + 4*(3*a^4 + 6*a^3*b + 8*a*b^3 + 3*b^4)*Cos[2*
(e + f*x)] + a*(3*a^3 + 9*a*b^2 + 4*b^3)*Cos[4*(e + f*x)])*Csc[e + f*x]*Sec
[e + f*x]^5)/(48*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] time = 0.595, size = 7586, normalized size = 43.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 7.65613, size = 2473, normalized size = 14.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(
f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x
 + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b
 + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos
(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -
b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))*sin(f*x + e) + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^5
 + (6*a^4*b - 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (3*a^3*b^2
- 8*a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b
+ 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3
+ 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), 1/12*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a
*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*
b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^
2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f
*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*
x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*s
in(f*x + e) - 4*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^5 + (6*a^4*b
- 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (3*a^3*b^2 - 8*a^2*b^3
- 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^
8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2
+ 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4
+ a^3*b^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.439 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}+\frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{3a^2f(a+b)^4}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Cot[e + f*x]^3)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(
3*a + b)*Cot[e + f*x]^3)/(a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
 ((a - b)*(3*a^2 + 14*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]
^2])/(3*a^2*(a + b)^4*f) - ((a^2 - 10*a*b - 3*b^2)*Cot[e + f*x]^3*Sqrt[a +
b + b*Tan[e + f*x]^2])/(3*a^2*(a + b)^3*f)
```

Rubi [A] time = 0.481733, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 203}

$$-\frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3a^2f(a+b)^3}+\frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{3a^2f(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f) -
 (b*Cot[e + f*x]^3)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (b*(
3*a + b)*Cot[e + f*x]^3)/(a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*x]^2]) +
 ((a - b)*(3*a^2 + 14*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]
^2])/(3*a^2*(a + b)^4*f) - ((a^2 - 10*a*b - 3*b^2)*Cot[e + f*x]^3*Sqrt[a +
b + b*Tan[e + f*x]^2])/(3*a^2*(a + b)^3*f)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
```

```
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^4(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3(a-b)-6bx^2}{x^4(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx\right.}{3a(a+b)f}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a^2-}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-b}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-b}{}$$

$$= -\frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-b}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\,\text{co}}{a^2(a+b)^2f\sqrt{a+b}}$$

Mathematica [A] time = 14.5803, size = 234, normalized size = 0.99

$$\frac{\sec^5(e+fx)(a\cos(2e+2fx)+a+2b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{4\sqrt{2}a^{5/2}f\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{\sec^5(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\frac{4b^3\sin(e}{}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b + a
*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e
+ f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^5*(-4*(a
+ 3*b)*Csc[e + f*x] + (a + b)*Csc[e + f*x]^3 + (4*b^3*(6*a^2 + 13*a*b + 3*b
^2 + 2*a*(3*a + b)*Cos[2*(e + f*x)])*Sin[e + f*x])/(a^2*(a + 2*b + a*Cos[2*
(e + f*x)])^2)))/(24*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] time = 0.708, size = 15128, normalized size = 64.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 24.4938, size = 3449, normalized size = 14.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^4
*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b
^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*log(12
8*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos
(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b
^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8
*(4*(a^6 + 3*a^5*b + 3*a^3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^6 + a^5*b -
 8*a^4*b^2 + 8*a^3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a^5*b + 3*a^4
*b^2 - 4*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*b^2 + 11*a^3*
b^3 - 11*a^2*b^4 - 3*a*b^5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/(((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x +
 e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*c
os(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 -
a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 +
a^3*b^6)*f)*sin(f*x + e)), -1/12*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3
 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^
6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x +
 e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*
b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*(4*(a^6 + 3*a^5*b + 3*a^
3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^6 + a^5*b - 8*a^4*b^2 + 8*a^3*b^3 -
a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a^5*b + 3*a^4*b^2 - 4*a^3*b^3 + 3*a^2*
b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*b^2 + 11*a^3*b^3 - 11*a^2*b^4 - 3*a*b^
5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^9 + 4*a^
8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b -
2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b
+ 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2
 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.440 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=315

$$-\frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5a^2f(a+b)^3}+\frac{\left(19a^2b+5a^3-65ab^2-15b^3\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x]^5)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (
b*(11*a + 3*b)*Cot[e + f*x]^5)/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - ((15*a^4 + 70*a^3*b + 128*a^2*b^2 - 70*a*b^3 - 15*b^4)*Cot[e + f*x
]*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^2*(a + b)^5*f) + ((5*a^3 + 19*a^2*b
 - 65*a*b^2 - 15*b^3)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^
2*(a + b)^4*f) - ((a^2 - 20*a*b - 5*b^2)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[
e + f*x]^2])/(5*a^2*(a + b)^3*f)
```

Rubi [A] time = 0.604344, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4141, 1975, 472, 579, 583, 12, 377, 203}

$$-\frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5a^2f(a+b)^3}+\frac{\left(19a^2b+5a^3-65ab^2-15b^3\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```


```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(a^(5/2)*f)
) - (b*Cot[e + f*x]^5)/(3*a*(a + b)*f*(a + b + b*Tan[e + f*x]^2)^(3/2)) - (
b*(11*a + 3*b)*Cot[e + f*x]^5)/(3*a^2*(a + b)^2*f*Sqrt[a + b + b*Tan[e + f*
x]^2]) - ((15*a^4 + 70*a^3*b + 128*a^2*b^2 - 70*a*b^3 - 15*b^4)*Cot[e + f*x
]*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^2*(a + b)^5*f) + ((5*a^3 + 19*a^2*b
 - 65*a*b^2 - 15*b^3)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(15*a^
2*(a + b)^4*f) - ((a^2 - 20*a*b - 5*b^2)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[
e + f*x]^2])/(5*a^2*(a + b)^3*f)
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b(1+x^2)\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^6(1+x^2)\left(a+b+bx^2\right)^{5/2}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-5b-8bx^2}{x^6(1+x^2)\left(a+b+bx^2\right)^{3/2}}\,dx, x, \tan(e+fx)\right)}{3a(a+b)f}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\text{Subst}}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a^2-}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a^3+}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^4}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^4}{}$$

$$= -\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(15a^4}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(11a+3b)\,c}{3a^2(a+b)^2f\sqrt{a+}}$$

Mathematica [A] time = 26.6946, size = 272, normalized size = 0.86

$$\frac{\tan(e+fx)\sec^4(e+fx)(a\cos(2(e+fx))+a+2b)^3\left(\frac{10b^4(15a+4b)}{a^2(a\cos(2(e+fx))+a+2b)}-\frac{20b^5(a+b)}{a^2(a\cos(2(e+fx))+a+2b)^2}-\left(23a^2+100ab+15\right.\right.}{120f(a+b)^5\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b +
a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e
 + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^3*((-20*b^5*(a + b))/(a
^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (10*b^4*(15*a + 4*b))/(a^2*(a + 2*b
+ a*Cos[2*(e + f*x)])) - (23*a^2 + 100*a*b + 150*b^2)*Csc[e + f*x]^2 + (a +
 b)*(11*a + 25*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x]
^4*Tan[e + f*x])/(120*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

Maple [C] time = 0.924, size = 22712, normalized size = 72.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 74.0467, size = 4632, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/120*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5
)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f
*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5
+ a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^
5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 25
6*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^7 + 100*a^6*b + 1
50*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*cos(f*x + e)^9 - (35*a^7 + 118*a^6*b
+ 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b^4 - 10*a^2*b^5 - 15*a*b^6)*cos(f*x +
 e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^4*b^3 + 125*a^3*b^4 - 50*a^2*b^5 - 15
*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 105*a^5*b^2 + 92*a^4*b^3 - 350*a^3*b^4
 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e)^3 + (15*a^5*b^2 + 70*a^4*b^3 + 128*
a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 +
a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a
^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3
 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b
+ 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2
 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
*sin(f*x + e)), 1/60*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*
b^4 + a^2*b^5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b
^5 + 5*a*b^6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 -
 a*b^6)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b
^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
+ e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
 e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(
f*x + e) - 4*((23*a^7 + 100*a^6*b + 150*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*
cos(f*x + e)^9 - (35*a^7 + 118*a^6*b + 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b
```

```
^4 - 10*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^
4*b^3 + 125*a^3*b^4 - 50*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 1
05*a^5*b^2 + 92*a^4*b^3 - 350*a^3*b^4 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e
)^3 + (15*a^5*b^2 + 70*a^4*b^3 + 128*a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 1
0*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 +
4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x + e)^6 + (
a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 + a^4*b^6 +
a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5*a^5*b^5 -
4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 + 10*a^6*b^4 +
 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.441 $\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$

Optimal. Leaf size=105

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{df(m+1)}$$

```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(d*Tan[e + f*x])^(1 + m)*(a + b + b*Tan[e + f*x]^2)^p)/(d*f*(1
+ m)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.202328, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4141, 1975, 511, 510}

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{m+1}{2};1,-p;\frac{m+3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```


```
[Out] (AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))]*(d*Tan[e + f*x])^(1 + m)*(a + b + b*Tan[e + f*x]^2)^p)/(d*f*(1
+ m)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^m\left(a+b\left(1+x^2\right)\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(dx)^m\left(a+b+bx^2\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{(dx)^m\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx\right.}{f}$$

$$= \frac{F_1\left(\frac{1+m}{2};1,-p;\frac{3+m}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)(d\tan(e+fx))^{1+m}\left(a\right.}{df(1+m)}$$

Mathematica [B] time = 3.80146, size = 259, normalized size = 2.47

$$\frac{\sin(e+fx)\cos(e+fx)(d\tan(e+fx))^m\left(a+b\sec^2(e+fx)\right)^p F_1\left(\frac{m+1}{2};-p,1;\frac{m+3}{2};-\frac{b\tan^2(e+fx)}{a+b},-\right.}{f(m+1)\left(\frac{2\tan^2(e+fx)\left(bpF_1\left(\frac{m+3}{2};1-p,1;\frac{m+5}{2};-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)-(a+b)F_1\left(\frac{m+3}{2};-p,2;\frac{m+5}{2};-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)\right)}{(m+3)(a+b)}+F_1\left(\frac{m+1}{2};-p,1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*Tan[e + f
*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] + (2*(b*p*AppellF1[(3 + m)/2, 1 - p, 1, (5 +
 m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[(
3 + m)/2, -p, 2, (5 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
)*Tan[e + f*x]^2)/((a + b)*(3 + m))))
```

Maple [F] time = 0.804, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec \left(fx+e\right)\right)^{2}\right)^{p} \left(d\tan \left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p} \left(d\tan \left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(fx+e\right)^{2}+a\right)^{p} \left(d\tan \left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

3.442 $\int \left(a+b\sec^2(e+fx)\right)^p \tan^5(e+fx)\,dx$

Optimal. Leaf size=122

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{(a+ \ldots}{}$$

```
[Out] -((a + 2*b)*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*b^2*f*(1 + p)) - (Hypergeome
tric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(
1 + p))/(2*a*f*(1 + p)) + (a + b*Sec[e + f*x]^2)^(2 + p)/(2*b^2*f*(2 + p))
```

Rubi [A] time = 0.14756, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4139, 446, 88, 65}

$$-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{p+1}}{2b^2f(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+2}}{2b^2f(p+2)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```


```
[Out] -((a + 2*b)*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*b^2*f*(1 + p)) - (Hypergeome
tric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(
1 + p))/(2*a*f*(1 + p)) + (a + b*Sec[e + f*x]^2)^(2 + p)/(2*b^2*f*(2 + p))
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2\left(a+bx^2\right)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1+x)^2(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(-a-2b)(a+bx)^p}{b}+\frac{(a+bx)^p}{x}+\frac{(a+bx)^{1+p}}{b}\right)dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}+\frac{\left(a+b\sec^2(e+fx)\right)^{2+p}}{2b^2f(2+p)}+\frac{\text{Subst}\left(\int \frac{(a+}{}\right.}{}$$
$$= -\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2b^2f(1+p)}-\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+\right.}{2af(1+p)}$$

Mathematica [A] time = 0.599999, size = 94, normalized size = 0.77

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\left(b^2(p+2)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)+a\left(a-b(p+1)\sec^2(e+fx)\right.\right.}{2ab^2f(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

```
[Out] -((a + b*Sec[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 + p, 2
 + p, 1 + (b*Sec[e + f*x]^2)/a] + a*(a + 2*b*(2 + p) - b*(1 + p)*Sec[e + f*
x]^2)))/(2*a*b^2*f*(1 + p)*(2 + p))
```

Maple [F] time = 0.487, size = 0, normalized size = 0.

$$\int \left(a + b\left(\sec\left(fx + e\right)\right)^{2}\right)^{p} \left(\tan\left(fx + e\right)\right)^{5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx + e\right)^{2} + a\right)^{p} \tan\left(fx + e\right)^{5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx + e\right)^{2} + a\right)^{p} \tan\left(fx + e\right)^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

3.443 $\int \left(a+b\sec^2(e+fx)\right)^p \tan^3(e+fx)\,dx$

Optimal. Leaf size=86

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

```
[Out] (a + b*Sec[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rubi [A] time = 0.0906664, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4139, 446, 80, 65}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{2bf(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

```
[Out] (a + b*Sec[e + f*x]^2)^(1 + p)/(2*b*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f*
(1 + p))
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\tan^3(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{(-1+x^2)(a+bx^2)^p}{x}\,dx,x,\sec(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int\frac{(-1+x)(a+bx)^p}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$

$$=\frac{\left(a+b\sec^2(e+fx)\right)^{1+p}}{2bf(1+p)}-\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}$$

$$=\frac{\left(a+b\sec^2(e+fx)\right)^{1+p}}{2bf(1+p)}+\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e-\right.}{2af(1+p)}$$

Mathematica [A] time = 0.166738, size = 61, normalized size = 0.71

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\left(b\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)+a\right)}{2abf(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

```
[Out] ((a + b*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a])*(a +
b*Sec[e + f*x]^2)^(1 + p))/(2*a*b*f*(1 + p))
```

Maple [F] time = 0.396, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\left(\tan\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

3.444 $\int \left(a+b\sec^2(e+fx)\right)^p \tan(e+fx)\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e
 + f*x]^2)^(1 + p))/(2*a*f*(1 + p))
```

Rubi [A] time = 0.0556833, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4139, 266, 65}

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e
 + f*x]^2)^(1 + p))/(2*a*f*(1 + p))
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$

$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}$$

Mathematica [A] time = 0.0534677, size = 54, normalized size = 1.

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e
 + f*x]^2)^(1 + p))/(2*a*f*(1 + p))
```

Maple [F] time = 0.477, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p \tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)^2 + a\right)^p \tan \left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec \left(fx+e\right)^2 + a\right)^p \tan \left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

3.445 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=114

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometr}}{2f(p+1)(}$$

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(a + b
*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f
*(1 + p))
```

Rubi [A] time = 0.123828, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4139, 446, 86, 68, 65}

$$\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)+a}{a+b}\right)}{2f(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(a + b
*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)*f*(1 + p)) + (Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*a*f
*(1 + p))
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$
\begin{aligned}
\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x\left(-1+x^2\right)}\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{(-1+x)x}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{-1+x}\,dx,x,\sec^2(e+fx)\right)}{2f}-\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sec^2(e+fx)\right)}{2f}\\
&= -\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f(1+p)}+\frac{{}_2F_1\left(1,1+p;2\right.}{}
\end{aligned}
$$

Mathematica [A] time = 2.15204, size = 115, normalized size = 1.01

$$\frac{\sec^2(e+fx)(a\cos(2(e+fx))+a+2b)\left(a+b\sec^2(e+fx)\right)^p\left((a+b)\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+fx)+}{a}\right.\right.}{4af(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Hypergeometric2F1[1, 1 + p, 2 + p,
 (a + b + b*Tan[e + f*x]^2)/a] - a*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*Tan[e + f*x]^2)/(a + b)])*Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(4*a*(
a + b)*f*(1 + p))
```

Maple [F] time = 0.442, size = 0, normalized size = 0.

$$\int \cot\left(fx+e\right)\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

3.446 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=157

$$\frac{(a-bp+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sec^2(e+fx)}{a+b}\right)}{2f(p+1)(a+b)^2}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hype}}{}$$

```
[Out] -(Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)*f) + ((a + b -
b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(a
+ b*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)^2*f*(1 + p)) - (Hypergeometric2F1[1
, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(
2*a*f*(1 + p))
```

Rubi [A] time = 0.174266, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4139, 446, 103, 156, 68, 65}

$$\frac{(a-bp+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2(e+fx)+a}{a+b}\right)}{2f(p+1)(a+b)^2}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sec^2}{}\right)}{2af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] -(Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)*f) + ((a + b -
b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*(a
+ b*Sec[e + f*x]^2)^(1 + p))/(2*(a + b)^2*f*(1 + p)) - (Hypergeometric2F1[1
, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(
2*a*f*(1 + p))
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{x(-1+x^2)^2}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{(-1+x)^2x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} - \frac{\text{Subst}\left(\int \frac{(a+bx)^p(a+b-bpx)}{(-1+x)x}\,dx, x, \sec^2(e\right.}{2(a+b)f}$$
$$= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} + \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sec^2(e+fx)\right)}{2f}$$
$$= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} + \frac{(a+b-bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\,\mathrm{se}}{}\right.}{2(a+b)^2f(}$$

Mathematica [A] time = 3.53619, size = 139, normalized size = 0.89

$$-\frac{\tan^2(e+fx)\left((a+b)\cot^2(e+fx)+b\right)\left(a+b\sec^2(e+fx)\right)^p\left((a+b)^2\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\tan^2(e+f}{a}\right.\right.}{2af(p+1)(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((b + (a + b)*Cot[e + f*x]^2)*(a*(a + b)*(1 + p)*Cot[e + f*x]^2 + (a + b)^
2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b + b*Tan[e + f*x]^2)/a] - a*(a +
 b - b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/(a + b)
])*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2)/(2*a*(a + b)^2*f*(1 + p))
```

Maple [F] time = 0.354, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^3\left(a+b\left(\sec\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}\cot \left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

3.447 $\int \left(a+b\sec^2(e+fx)\right)^p \tan^4(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

```
[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rubi [A] time = 0.148573, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4141, 1975, 511, 510}

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```


```
[Out] (AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^4(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a+b\left(1+x^2\right)\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\left(a+b+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p \left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^4\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2};1,-p;\frac{7}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\left(a+b+b\tan^2(e+\right.}{5f}$$

Mathematica [B] time = 18.5323, size = 2777, normalized size = 31.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)
^p*Tan[e + f*x]^5*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/
2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)
*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])
*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)
/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f*(((a + 2*b + a*C
os[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1 + p)*((9*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p,
 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b
*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))
^p))/3 - (2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*
Sin[2*(e + f*x)]*Tan[e + f*x]*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan
[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^
2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 + (2*p*
(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]^2*((9*(a +
 b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hyp
ergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometri
c2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*T
an[e + f*x]^2)/(a + b))^p))/3 + ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
f*x]^2)^p*Tan[e + f*x]*((-18*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] +
2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*(a + b)*Cos[e + f*x]^2*((2*b*p*Appe
llF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
```

```
 + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (2*b*p*Sec[e + f*x]^2*Tan
[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p)*(-3*Hypergeometric2F1[1
/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/
2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2))/(a + b) - (9*(a + b)*App
ellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[
e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^
2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f
*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*
b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*App
ellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*S
ec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((
b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5
))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)^2 + (2*Hypergeometric2F
1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]
- 3*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e
+ f*x]^2)/(a + b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p) + 3*Sec[e + f*x]^
2*Tan[e + f*x]*(-Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a +
b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/(a + b)
)^p))/3))
```

Maple [F] time = 0.425, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\left(\tan\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^2+a\right)^p\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

3.448 $\int \left(a+b\sec^2(e+fx)\right)^p \tan^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```

Rubi [A] time = 0.146644, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4141, 1975, 511, 510}

$$\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```


```
[Out] (AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)/(
a + b))^p)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p \tan^2(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a+b\left(1+x^2\right)\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a+b+bx^2\right)^p}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx, x, \tan(\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};1,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(a+b+b\tan^2(e+\right.}{3f}$$

Mathematica [B] time = 16.9192, size = 2465, normalized size = 28.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2)
^p*Tan[e + f*x]^3*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a
+ b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1,
 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a
 + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x
]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*((a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^(1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e
+ f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e +
f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
 + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 2*a*p*(a + 2*b +
a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*Tan[e + f*
x]*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*
Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*A
ppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
 - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2])*Tan[e + f*x]^2)) + 2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
 f*x]^2)^p*Tan[e + f*x]^2*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x
]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^
2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + (a + 2*b + a*Cos[2*(e
+ f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((-2*b*p*Hypergeometric2F1[1/2,
-p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b
*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) + (6*(a + b)*AppellF1[1/2, -p,
1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*Sin[e
+ f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
 + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Cos[e +
 f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*AppellF1[3/
2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]
^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (Csc[
```

```
e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2
)/(a + b))] + (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/
(a + b))^p + (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p, 1, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p,
 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*x]^2*Ta
n[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) -
 (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*AppellF1[5/
2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f
*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a +
 b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) - (12*Ap
pellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec
[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]
^2)^2)))
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\left(\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="maxima")
```

[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="giac")

[Out] integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)

3.449 $\int \left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```

Rubi [A] time = 0.0533354, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4128, 430, 429}

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*
Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a +
b))^p)
```


Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\cdots}{\cdots}\right.}{f}$$

Mathematica [B] time = 6.26908, size = 2137, normalized size = 25.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*
(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*Appel
lF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2
```

```
*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2
*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1[1/2, -p, 1, 3/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x
)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^(-
1 + p)*(Sec[e + f*x]^2)^p*Sin[e + f*x]*Sin[2*(e + f*x)])/(3*(a + b)*AppellF
1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p
*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2])*Tan[e + f*x]^2) + (3*(a + b)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*((2*b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e
 + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a
 + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*Appell
F1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*
p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(
e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]*(4*(b*p*AppellF1[3/2, 1 - p, 1
, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3
/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e + f*
x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a
+ b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*Appe
llF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2, -(
(b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/
(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)) -
 (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]
^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 -
 p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e
 + f*x]^2)^2))
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec^{2}\left(e+fx\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Integral((a + b*sec(e + f*x)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)^{2}+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.450 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=84

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
 + b))^p))
```

Rubi [A] time = 0.14318, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4141, 1975, 511, 510}

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
 + b))^p))
```

Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\left(1+x^2\right)\right)^p}{x^2\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b+bx^2\right)^p}{x^2\left(1+x^2\right)}\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^2\left(1+x^2\right)}\,dx, x, \tan(e\right.}{f}\\
&= -\frac{F_1\left(-\frac{1}{2};1,-p;\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot(e+fx)\left(a+b+b\tan^2(e\right.}{f}
\end{aligned}$$

Mathematica [B] time = 17.2943, size = 2469, normalized size = 29.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*(a + b*
Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/
(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(
3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)
), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*(2*p*(a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*T
an[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Si
n[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b
*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - (a + 2*b +
a*Cos[2*(e + f*x)])^p*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F
1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -
((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e +
 f*x]^2)) - 2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*Cot[e + f*x]*(Sec
[e + f*x]^2)^p*Sin[2*(e + f*x)]*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Ta
n[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*Ap
pellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin
[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) + (a + 2*b + a
*Cos[2*(e + f*x)])^p*Cot[e + f*x]*(Sec[e + f*x]^2)^p*((2*b*p*Hypergeometric
2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*
x]*(1 + (b*Tan[e + f*x]^2)/(a + b))^(-1 - p))/(a + b) - (6*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f
*x]*Sin[e + f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(a +
b)*Sin[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) - (2*A
ppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Se
c[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2,
 -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^
```

```
2) - (Csc[e + f*x]*Sec[e + f*x]*(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[
e + f*x]^2)/(a + b))] - (1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e
+ f*x]^2)/(a + b))^p + (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*
x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*(4*(b*p*AppellF1[3/2, 1 - p
, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF
1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Sec[e +
 f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*
Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*
(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]^2*(b*p*((-6*A
ppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2, 2 - p, 1, 7/2,
 -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x
])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(5*(a + b)
) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2,
1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Ap
pellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Ta
n[e + f*x]^2)^2)))
```

Maple [F] time = 0.366, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^{2}\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

3.451 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

```
[Out] -(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2
)/(a + b))^p)
```

Rubi [A] time = 0.146109, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4141, 1975, 511, 510}

$$-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```


```
[Out] -(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)]*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2
)/(a + b))^p)
```


Rule 4141

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/f, Subst[Int[((d*ff*x)^m*(a + b*(1 + ff^2*x^2)^(n/2))^p)/(1 + ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Integ
erQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
```

```
BinomialMatchQ[{u, v}, x]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b\left(1+x^2\right)\right)^p}{x^4\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int\frac{\left(a+b+bx^2\right)^p}{x^4\left(1+x^2\right)}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a+b}\right)^p}{x^4\left(1+x^2\right)}\,dx,x,\tan(\right.}{f}$$

$$=-\frac{F_1\left(-\frac{3}{2};1,-p;-\frac{1}{2};-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot^3(e+fx)\left(a+b+b\tan\right.}{3f}$$

Mathematica [B] time = 18.8349, size = 3033, normalized size = 34.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

```
[Out] ((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^7*(Sec[e + f*x]^2)^p*(a + b*
Sec[e + f*x]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*Appe
llF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(
b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*T
an[e + f*x]^2)/(a + b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e +
f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f
*((2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^2*(Sec[e + f*x]^2)^p*(
(9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1
 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*App
ellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan
[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a +
 b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*T
an[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 - (a + 2*b + a*Cos[2*
(e + f*x)])^p*Cot[e + f*x]^2*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*((9*(a + b)*
AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*S
in[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2,
-p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2
) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]
^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (2*a*p*(a + 2*b + a*Cos[2*(e + f*
x)])^(-1 + p)*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((9*(a + b
)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2
, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]
^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] - 3
*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*
x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 + ((a + 2*b + a*Cos[2*(e + f*x
)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*((18*(a + b)*AppellF1[1/2, -p, 1, 3
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e +
f*x])/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -
Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/
(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (18*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Tan[e + f
*x]^3)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
-Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)
```

```
/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e
+ f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (9*(a + b)*Sin[e +
f*x]^2*Tan[e + f*x]^2*((2*b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)) -
(2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
]*Sec[e + f*x]^2*Tan[e + f*x])/3))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((
b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p,
1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[
3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f
*x]^2) + (2*b*p*Sec[e + f*x]^2*Tan[e + f*x]*(1 + (b*Tan[e + f*x]^2)/(a + b)
)^(-1 - p)*(Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))
] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e
 + f*x]^2))/(a + b) - (9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2*(4*(b*p*Appel
lF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (
a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2])*Sec[e + f*x]^2*Tan[e + f*x] + 3*(a + b)*((2*b*p*AppellF1[3/2, 1 - p,
 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan
[e + f*x])/(3*(a + b)) - (2*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/
(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/3) + 2*Tan[e + f*x]
^2*(b*p*((-6*AppellF1[5/2, 1 - p, 2, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -T
an[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5 - (6*b*(1 - p)*AppellF1[5/2,
2 - p, 1, 7/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]
^2*Tan[e + f*x])/(5*(a + b))) - (a + b)*((6*b*p*AppellF1[5/2, 1 - p, 2, 7/2
, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*
x])/(5*(a + b)) - (12*AppellF1[5/2, -p, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/5))))/(3*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*
AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2
] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Tan[e + f*x]^2)^2 - (-6*Hypergeometric2F1[-1/2, -p, 1/2, -((b*T
an[e + f*x]^2)/(a + b))]*Sec[e + f*x]^2*Tan[e + f*x] - 3*Sec[e + f*x]^2*Tan
[e + f*x]*(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]
- (1 + (b*Tan[e + f*x]^2)/(a + b))^p) - 3*Csc[e + f*x]*Sec[e + f*x]*(-Hyper
geometric2F1[-3/2, -p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] + (1 + (b*Tan[e
 + f*x]^2)/(a + b))^p))/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3))
```

Maple [F] time = 0.4, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^{4}\left(a+b\left(\sec\left(fx+e\right)\right)^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

```
[Out] int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)^{2}+a\right)^{p}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

3.452 $\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx$

Optimal. Leaf size=92

$$\frac{a\sec^4(e+fx)}{4f}-\frac{a\sec^2(e+fx)}{f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^7(e+fx)}{7f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^3(e+fx)}{3f}$$

```
[Out] -((a*Log[Cos[e + f*x]])/f) - (a*Sec[e + f*x]^2)/f + (b*Sec[e + f*x]^3)/(3*f
) + (a*Sec[e + f*x]^4)/(4*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]
^7)/(7*f)
```

Rubi [A] time = 0.0689336, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 1802}

$$\frac{a\sec^4(e+fx)}{4f}-\frac{a\sec^2(e+fx)}{f}-\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^7(e+fx)}{7f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) - (a*Sec[e + f*x]^2)/f + (b*Sec[e + f*x]^3)/(3*f
) + (a*Sec[e + f*x]^4)/(4*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]
^7)/(7*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(b+ax^3\right)}{x^8}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^8}-\frac{2b}{x^6}+\frac{a}{x^5}+\frac{b}{x^4}-\frac{2a}{x^3}+\frac{a}{x}\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{a\log(\cos(e+fx))}{f}-\frac{a\sec^2(e+fx)}{f}+\frac{b\sec^3(e+fx)}{3f}+\frac{a\sec^4(e+fx)}{4f}-\frac{2b}{}$$

Mathematica [A] time = 0.270366, size = 87, normalized size = 0.95

$$-\frac{a\left(-\tan^4(e+fx)+2\tan^2(e+fx)+4\log(\cos(e+fx))\right)}{4f}+\frac{b\sec^7(e+fx)}{7f}-\frac{2b\sec^5(e+fx)}{5f}+\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

```
[Out] (b*Sec[e + f*x]^3)/(3*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]^7)/
(7*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan[e + f*x]^4))/(4*f)
```

Maple [B] time = 0.054, size = 183, normalized size = 2.

$$\frac{\left(\tan\left(fx+e\right)\right)^4 a}{4f}-\frac{\left(\tan\left(fx+e\right)\right)^2 a}{2f}-\frac{a\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{b\left(\sin\left(fx+e\right)\right)^6}{7f\left(\cos\left(fx+e\right)\right)^7}+\frac{b\left(\sin\left(fx+e\right)\right)^6}{35f\left(\cos\left(fx+e\right)\right)^5}-\frac{b\left(\sin\left(f\right.\right.}{105f\left(\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x)
```

```
[Out] 1/4/f*tan(f*x+e)^4*a-1/2/f*a*tan(f*x+e)^2-a*ln(cos(f*x+e))/f+1/7/f*b*sin(f*
x+e)^6/cos(f*x+e)^7+1/35/f*b*sin(f*x+e)^6/cos(f*x+e)^5-1/105/f*b*sin(f*x+e)
^6/cos(f*x+e)^3+1/35/f*b*sin(f*x+e)^6/cos(f*x+e)+8/105/f*b*cos(f*x+e)+1/35/
f*b*cos(f*x+e)*sin(f*x+e)^4+4/105/f*b*cos(f*x+e)*sin(f*x+e)^2
```

Maxima [A] time = 0.994856, size = 99, normalized size = 1.08

$$-\frac{420\,a\log\left(\cos\left(fx+e\right)\right)+\frac{420\,a\cos\left(fx+e\right)^5-140\,b\cos\left(fx+e\right)^4-105\,a\cos\left(fx+e\right)^3+168\,b\cos\left(fx+e\right)^2-60\,b}{\cos\left(fx+e\right)^7}}{420\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/420*(420*a*log(cos(f*x + e)) + (420*a*cos(f*x + e)^5 - 140*b*cos(f*x + e
)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/cos(f*x + e)^7)/f
```

Fricas [A] time = 0.543684, size = 227, normalized size = 2.47

$$-\frac{420\,a\cos\left(fx+e\right)^7\log\left(-\cos\left(fx+e\right)\right)+420\,a\cos\left(fx+e\right)^5-140\,b\cos\left(fx+e\right)^4-105\,a\cos\left(fx+e\right)^3+168\,b\cos\left(f\right.}{420\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/420*(420*a*cos(f*x + e)^7*log(-cos(f*x + e)) + 420*a*cos(f*x + e)^5 - 14
0*b*cos(f*x + e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/(f
*cos(f*x + e)^7)
```

Sympy [A] time = 16.7713, size = 119, normalized size = 1.29

$$\begin{cases}\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^4\left(e+fx\right)}{4f}-\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^4\left(e+fx\right)\sec^3\left(e+fx\right)}{7f}-\frac{4b\tan^2\left(e+fx\right)\sec^3\left(e+fx\right)}{35f}+\frac{8b\sec^3\left(e+fx\right)}{105f} & \text{for } f\neq 0\\ x\left(a+b\sec^3\left(e\right)\right)\tan^5\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*t
an(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**3/(7*f) - 4*b*tan(e
```

```
+ f*x)**2*sec(e + f*x)**3/(35*f) + 8*b*sec(e + f*x)**3/(105*f), Ne(f, 0)),
(x*(a + b*sec(e)**3)*tan(e)**5, True))
```

Giac [B] time = 3.083, size = 495, normalized size = 5.38

$$420\,a\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)-420\,a\log\left(\left|-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-1\right|\right)+\frac{1089\,a+64\,b+\frac{8463\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{448\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{28749\,a\left(\cos\left(fx+e\right)-1\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/420*(420*a*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 420*a*log(ab
s(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (1089*a + 64*b + 8463*a*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + 448*b*(cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 28749*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 1344*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 51555*a*(cos(f*x + e) - 1)^3/(cos(f*x
+ e) + 1)^3 - 2240*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 51555*a*(c
os(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 4480*b*(cos(f*x + e) - 1)^4/(cos(
f*x + e) + 1)^4 + 28749*a*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)^5 + 8463*
a*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 1089*a*(cos(f*x + e) - 1)^7/(
cos(f*x + e) + 1)^7)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^7)/f
```

3.453 $\int \left(a+b \sec^3(e+f x)\right) \tan^3(e+f x) \, dx$

Optimal. Leaf size=61

$$\frac{a \sec^2(e+f x)}{2 f}+\frac{a \log(\cos(e+f x))}{f}+\frac{b \sec^5(e+f x)}{5 f}-\frac{b \sec^3(e+f x)}{3 f}$$

```
[Out] (a*Log[Cos[e + f*x]])/f + (a*Sec[e + f*x]^2)/(2*f) - (b*Sec[e + f*x]^3)/(3*
f) + (b*Sec[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0544285, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 1802}

$$\frac{a \sec^2(e+f x)}{2 f}+\frac{a \log(\cos(e+f x))}{f}+\frac{b \sec^5(e+f x)}{5 f}-\frac{b \sec^3(e+f x)}{3 f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

```
[Out] (a*Log[Cos[e + f*x]])/f + (a*Sec[e + f*x]^2)/(2*f) - (b*Sec[e + f*x]^3)/(3*
f) + (b*Sec[e + f*x]^5)/(5*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+b\sec^3(e+fx)\right)\tan^3(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)(b+ax^3)}{x^6}\,dx,x,\cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^6}-\frac{b}{x^4}+\frac{a}{x^3}-\frac{a}{x}\right)dx,x,\cos(e+fx)\right)}{f}$$
$$= \frac{a\log(\cos(e+fx))}{f}+\frac{a\sec^2(e+fx)}{2f}-\frac{b\sec^3(e+fx)}{3f}+\frac{b\sec^5(e+fx)}{5f}$$

Mathematica [A] time = 0.135557, size = 59, normalized size = 0.97

$$\frac{a\left(\tan^2(e+fx)+2\log(\cos(e+fx))\right)}{2f}+\frac{b\sec^5(e+fx)}{5f}-\frac{b\sec^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

```
[Out] -(b*Sec[e + f*x]^3)/(3*f) + (b*Sec[e + f*x]^5)/(5*f) + (a*(2*Log[Cos[e + f*
x]] + Tan[e + f*x]^2))/(2*f)
```

Maple [B] time = 0.05, size = 126, normalized size = 2.1

$$\frac{\left(\tan\left(fx+e\right)\right)^2 a}{2f}+\frac{a\ln\left(\cos\left(fx+e\right)\right)}{f}+\frac{b\left(\sin\left(fx+e\right)\right)^4}{5f\left(\cos\left(fx+e\right)\right)^5}+\frac{b\left(\sin\left(fx+e\right)\right)^4}{15f\left(\cos\left(fx+e\right)\right)^3}-\frac{b\left(\sin\left(fx+e\right)\right)^4}{15f\cos\left(fx+e\right)}-\frac{b\cos\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x)
```

```
[Out] 1/2/f*a*tan(f*x+e)^2+a*ln(cos(f*x+e))/f+1/5/f*b*sin(f*x+e)^4/cos(f*x+e)^5+1
/15/f*b*sin(f*x+e)^4/cos(f*x+e)^3-1/15/f*b*sin(f*x+e)^4/cos(f*x+e)-1/15/f*b
*cos(f*x+e)*sin(f*x+e)^2-2/15/f*b*cos(f*x+e)
```

Maxima [A] time = 1.39281, size = 69, normalized size = 1.13

$$\frac{30\,a\log\left(\cos\left(fx+e\right)\right)+\frac{15\,a\cos\left(fx+e\right)^{3}-10\,b\cos\left(fx+e\right)^{2}+6\,b}{\cos\left(fx+e\right)^{5}}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/30*(30*a*log(cos(f*x + e)) + (15*a*cos(f*x + e)^3 - 10*b*cos(f*x + e)^2 +
 6*b)/cos(f*x + e)^5)/f
```

Fricas [A] time = 0.526663, size = 157, normalized size = 2.57

$$\frac{30\,a\cos\left(fx+e\right)^{5}\log\left(-\cos\left(fx+e\right)\right)+15\,a\cos\left(fx+e\right)^{3}-10\,b\cos\left(fx+e\right)^{2}+6\,b}{30\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(30*a*cos(f*x + e)^5*log(-cos(f*x + e)) + 15*a*cos(f*x + e)^3 - 10*b*c
os(f*x + e)^2 + 6*b)/(f*cos(f*x + e)^5)
```

Sympy [A] time = 4.4262, size = 82, normalized size = 1.34

$$\begin{cases} -\frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^{2}\left(e+fx\right)}{2f}+\frac{b\tan^{2}\left(e+fx\right)\sec^{3}\left(e+fx\right)}{5f}-\frac{2b\sec^{3}\left(e+fx\right)}{15f} & \text{for}\ f\neq 0\\ x\left(a+b\sec^{3}\left(e\right)\right)\tan^{3}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b*
tan(e + f*x)**2*sec(e + f*x)**3/(5*f) - 2*b*sec(e + f*x)**3/(15*f), Ne(f, 0
```

```
)), (x*(a + b*sec(e)**3)*tan(e)**3, True))
```

Giac [B] time = 1.8401, size = 394, normalized size = 6.46

$$-\frac{60\,a\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)-60\,a\log\left(\left|-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-1\right|\right)+\frac{137\,a+16\,b+\frac{805\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{80\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{1730\,a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{80\,b(\cdots}{(\cdots}}{}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 60*a*log(abs(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (137*a + 16*b + 805*a*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 80*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 1730*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 1730*a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)
^3 + 240*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 805*a*(cos(f*x + e)
- 1)^4/(cos(f*x + e) + 1)^4 + 137*a*(cos(f*x + e) - 1)^5/(cos(f*x + e) + 1)
^5)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^5)/f
```

3.454 $\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx$

Optimal. Leaf size=30

$$\frac{b\sec^3(e+fx)}{3f}-\frac{a\log(\cos(e+fx))}{f}$$

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.023017, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4138, 14}

$$\frac{b\sec^3(e+fx)}{3f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx = -\frac{\text{Subst}\left(\int \frac{b+ax^3}{x^4}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b}{x^4}+\frac{a}{x}\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{a\log(\cos(e+fx))}{f}+\frac{b\sec^3(e+fx)}{3f}$$

Mathematica [A] time = 0.0134052, size = 30, normalized size = 1.

$$\frac{b\sec^3(e+fx)}{3f}-\frac{a\log(\cos(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

```
[Out] -((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

Maple [A] time = 0.02, size = 28, normalized size = 0.9

$$\frac{b\left(\sec\left(fx+e\right)\right)^3}{3f}+\frac{a\ln\left(\sec\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e)^3)*tan(f*x+e),x)
```

```
[Out] 1/3*b*sec(f*x+e)^3/f+1/f*a*ln(sec(f*x+e))
```

Maxima [A] time = 1.03585, size = 38, normalized size = 1.27

$$-\frac{a\log\left(\cos\left(fx+e\right)^3\right)-\frac{b}{\cos\left(fx+e\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/3*(a*log(cos(f*x + e)^3) - b/cos(f*x + e)^3)/f
```

Fricas [A] time = 0.508701, size = 93, normalized size = 3.1

$$-\frac{3\,a\cos\left(fx+e\right)^{3}\log\left(-\cos\left(fx+e\right)\right)-b}{3\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*cos(f*x + e)^3*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^3)
```

Sympy [A] time = 2.32186, size = 42, normalized size = 1.4

$$\begin{cases} \frac{a\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f}+\frac{b\sec^{3}\left(e+fx\right)}{3f} & \text{for}\ f\neq 0\\ x\left(a+b\sec^{3}\left(e\right)\right)\tan\left(e\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)**3)*tan(f*x+e),x)
```

```
[Out] Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**3/(3*f), Ne(f
, 0)), (x*(a + b*sec(e)**3)*tan(e), True))
```

Giac [B] time = 1.30283, size = 262, normalized size = 8.73

$$\frac{6\,a\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)-6\,a\log\left(\left|-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}-1\right|\right)+\frac{11\,a+4\,b+\frac{33\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{33\,a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{12\,b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{11\,a\left(\cos\left(fx+e\right)-1\right)}{\left(\cos\left(fx+e\right)+1\right)^{3}}}{\left(\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 6*a*log(abs(-(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (11*a + 4*b + 33*a*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 33*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 12*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 11*a*(cos(f*x + e) - 1)^3
/(cos(f*x + e) + 1)^3)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

3.455 $\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(\cos(e+fx)+1)}{2f}+\frac{b\sec(e+fx)}{f}$$

```
[Out] ((a + b)*Log[1 - Cos[e + f*x]])/(2*f) + ((a - b)*Log[1 + Cos[e + f*x]])/(2*
f) + (b*Sec[e + f*x])/f
```

Rubi [A] time = 0.069843, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4138, 1802}

$$\frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(\cos(e+fx)+1)}{2f}+\frac{b\sec(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ((a + b)*Log[1 - Cos[e + f*x]])/(2*f) + ((a - b)*Log[1 + Cos[e + f*x]])/(2*
f) + (b*Sec[e + f*x])/f
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)dx = -\frac{\text{Subst}\left(\int \frac{b+ax^3}{x^2(1-x^2)}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{-a-b}{2(-1+x)}+\frac{b}{x^2}+\frac{-a+b}{2(1+x)}\right)dx, x, \cos(e+fx)\right)}{f}$$
$$= \frac{(a+b)\log(1-\cos(e+fx))}{2f}+\frac{(a-b)\log(1+\cos(e+fx))}{2f}+\frac{b\sec(e+fx)}{f}$$

Mathematica [A] time = 0.0627475, size = 65, normalized size = 1.2

$$\frac{a(\log(\tan(e+fx))+\log(\cos(e+fx)))}{f}+\frac{b\sec(e+fx)}{f}+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((b*Log[Cos[(e + f*x)/2]])/f) + (b*Log[Sin[(e + f*x)/2]])/f + (a*(Log[Cos[
e + f*x]] + Log[Tan[e + f*x]]))/f + (b*Sec[e + f*x])/f
```

Maple [A] time = 0.05, size = 48, normalized size = 0.9

$$\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}+\frac{b}{f\cos\left(fx+e\right)}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sec(f*x+e)^3),x)
```

```
[Out] a*ln(sin(f*x+e))/f+1/f*b/cos(f*x+e)+1/f*b*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 0.998987, size = 61, normalized size = 1.13

$$\frac{(a-b)\log\left(\cos\left(fx+e\right)+1\right)+(a+b)\log\left(\cos\left(fx+e\right)-1\right)+\frac{2b}{\cos\left(fx+e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] 1/2*((a - b)*log(cos(f*x + e) + 1) + (a + b)*log(cos(f*x + e) - 1) + 2*b/co
s(f*x + e))/f
```

Fricas [A] time = 0.513617, size = 177, normalized size = 3.28

$$\frac{(a-b)\cos\left(fx+e\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+(a+b)\cos\left(fx+e\right)\log\left(-\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)+2\,b}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) + (a + b)*cos(f*x + e
)*log(-1/2*cos(f*x + e) + 1/2) + 2*b)/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec^{3}\left(e+fx\right)\right)\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral((a + b*sec(e + f*x)**3)*cot(e + f*x), x)
```

Giac [A] time = 1.22243, size = 122, normalized size = 2.26

$$-\frac{2\,a\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)-(a+b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-\frac{4\,b}{\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - (a + b)*log(-(c
os(f*x + e) - 1)/(cos(f*x + e) + 1)) - 4*b/((cos(f*x + e) - 1)/(cos(f*x + e
) + 1) + 1))/f
```

3.456 $\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)dx$

Optimal. Leaf size=72

$$-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(\cos(e+fx)+1)}{4f}-\frac{\csc^2(e+fx)(a+b\cos(e+fx))}{2f}$$

```
[Out] -((a + b*Cos[e + f*x])*Csc[e + f*x]^2)/(2*f) - ((2*a - b)*Log[1 - Cos[e + f
*x]])/(4*f) - ((2*a + b)*Log[1 + Cos[e + f*x]])/(4*f)
```

Rubi [A] time = 0.0626343, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4138, 1814, 633, 31}

$$-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)\log(\cos(e+fx)+1)}{4f}-\frac{\csc^2(e+fx)(a+b\cos(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((a + b*Cos[e + f*x])*Csc[e + f*x]^2)/(2*f) - ((2*a - b)*Log[1 - Cos[e + f
*x]])/(4*f) - ((2*a + b)*Log[1 + Cos[e + f*x]])/(4*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)dx=-\frac{\text{Subst}\left(\int \frac{b+ax^3}{\left(1-x^2\right)^2}\,dx,x,\cos(e+fx)\right)}{f}$$

$$=-\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}+\frac{\text{Subst}\left(\int \frac{-b+2ax}{1-x^2}\,dx,x,\cos(e+fx)\right)}{2f}$$

$$=-\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}+\frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\cos(e+fx)\right)}{4f}$$

$$=-\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f}-\frac{(2a-b)\log(1-\cos(e+fx))}{4f}-\frac{(2a+b)}{}$$

Mathematica [A] time = 1.09186, size = 114, normalized size = 1.58

$$-\frac{a\left(\cot^2(e+fx)+2\log(\tan(e+fx))+2\log(\cos(e+fx))\right)}{2f}-\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\log\left(\sin\left(\frac{1}{2}(\right.\right.}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -(b*Csc[(e + f*x)/2]^2)/(8*f) - (b*Log[Cos[(e + f*x)/2]])/(2*f) + (b*Log[Si
n[(e + f*x)/2]])/(2*f) - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Log[T
an[e + f*x]]))/(2*f) + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

Maple [A] time = 0.054, size = 69, normalized size = 1.

$$-\frac{\left(\cot\left(fx+e\right)\right)^{2}a}{2f}-\frac{a\ln\left(\sin\left(fx+e\right)\right)}{f}-\frac{b\csc\left(fx+e\right)\cot\left(fx+e\right)}{2f}+\frac{b\ln\left(\csc\left(fx+e\right)-\cot\left(fx+e\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/2/f*a*cot(f*x+e)^2-a*ln(sin(f*x+e))/f-1/2/f*b*csc(f*x+e)*cot(f*x+e)+1/2/
f*b*ln(csc(f*x+e)-cot(f*x+e))
```

Maxima [A] time = 1.00388, size = 84, normalized size = 1.17

$$-\frac{(2a+b)\log\left(\cos\left(fx+e\right)+1\right)+(2a-b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(b\cos\left(fx+e\right)+a\right)}{\cos\left(fx+e\right)^{2}-1}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/4*((2*a + b)*log(cos(f*x + e) + 1) + (2*a - b)*log(cos(f*x + e) - 1) - 2
*(b*cos(f*x + e) + a)/(cos(f*x + e)^2 - 1))/f
```

Fricas [A] time = 0.518424, size = 254, normalized size = 3.53

$$\frac{2b\cos\left(fx+e\right)-\left((2a+b)\cos\left(fx+e\right)^{2}-2a-b\right)\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-\left((2a-b)\cos\left(fx+e\right)^{2}-2a+b\right)\log\left(-\frac{1}{2}\right.}{4\left(f\cos\left(fx+e\right)^{2}-f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*cos(f*x + e) - ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*log(1/2*cos(f*
x + e) + 1/2) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*log(-1/2*cos(f*x + e)
```

```
+ 1/2) + 2*a)/(f*cos(f*x + e)^2 - f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**3),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41699, size = 244, normalized size = 3.39

$$\frac{8\,a\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)-2\,(2\,a-b)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)+\frac{\left(a+b+\frac{4\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{2\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}\right)\left(\cos\left(fx+e\right)+1\right)}{\cos\left(fx+e\right)-1}+\frac{a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] 1/8*(8*a*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) - 2*(2*a - b)*log(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + (a + b + 4*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e
) + 1)/(cos(f*x + e) - 1) + a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1))/f
```

3.457 $\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=219

$$\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}b^{4/3}f}-\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{4/3}f}-\frac{\left(a^{2/3}+2b^{2/3}\right.}{}$$

```
[Out] -(((a^(2/3) + 2*b^(2/3))*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]
*b^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)*f)) - ((a^(2/3) - 2*b^(2/3))*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*b^(4/3)*f) + ((a^(2/3) - 2*b^(2/3))*
Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2])/(6*a^
(1/3)*b^(4/3)*f) - Log[b + a*Cos[e + f*x]^3]/(3*a*f) + Sec[e + f*x]/(b*f)
```

Rubi [A] time = 0.320877, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4138, 1834, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}b^{4/3}f}-\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{4/3}f}-\frac{\left(a^{2/3}+2b^{2/3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```


```
[Out] -(((a^(2/3) + 2*b^(2/3))*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]
*b^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)*f)) - ((a^(2/3) - 2*b^(2/3))*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*b^(4/3)*f) + ((a^(2/3) - 2*b^(2/3))*
Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2])/(6*a^
(1/3)*b^(4/3)*f) - Log[b + a*Cos[e + f*x]^3]/(3*a*f) + Sec[e + f*x]/(b*f)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2\left(b+ax^3\right)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{bx^2}+\frac{-2b-ax+bx^2}{b\left(b+ax^3\right)}\right)dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{-2b-ax+bx^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{bf}$$

$$= \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{-2b-ax}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{bf}$$

$$= -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\left(-a\sqrt[3]{b}-4\sqrt[3]{a}b\right)+\sqrt[3]{a}\left(-a\sqrt[3]{b}+2\sqrt[3]{a}b\right)x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \cos\right.}{3\sqrt[3]{a}b^{5/3}f}$$

$$= -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f} - \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\sec(e+fx)}{bf} + \frac{\left(a^{2/3}-\right.}{}$$

$$= -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f} + \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/}\right.}{6\sqrt[3]{a}b^{4/3}f}$$

$$= -\frac{\left(a^{2/3}+2b^{2/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}f} - \frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f} + \frac{\left(a^{2/3}-2b\right.}{}$$

Mathematica [C] time = 0.365992, size = 251, normalized size = 1.15

$$-\text{RootSum}\left[\#1^3a-6\#1^2a-\#1^3b+12\#1a-8a\&,\frac{\#1^2ab\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)-\#1^2b^2\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)-4a^2\log\left(-\#1+\text{ta}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (3*b*Log[Sec[(e + f*x)/2]^2] - RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3 -
 b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 + T
an[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 8*a*b*Log[
1 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2
 - b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1
^2) & ] + 3*a*Sec[e + f*x])/(3*a*b*f)
```

Maple [A] time = 0.063, size = 274, normalized size = 1.3

$$\frac{2}{3fa}\ln\left(\cos\left(fx+e\right)+\sqrt[3]{\frac{b}{a}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}-\frac{1}{3fa}\ln\left(\left(\cos\left(fx+e\right)\right)^2-\sqrt[3]{\frac{b}{a}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}}{3fa}\arctan\left(\frac{\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x)
```

```
[Out] 2/3/f/a/(b/a)^(2/3)*ln(cos(f*x+e)+(b/a)^(1/3))-1/3/f/a/(b/a)^(2/3)*ln(cos(f
*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+(b/a)^(2/3))+2/3/f/a/(b/a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(b/a)^(1/3)*cos(f*x+e)-1))-1/3/f/b/(b/a)^(1/3)*ln(cos(f*
x+e)+(b/a)^(1/3))+1/6/f/b/(b/a)^(1/3)*ln(cos(f*x+e)^2-(b/a)^(1/3)*cos(f*x+e
)+(b/a)^(2/3))+1/3/f/b*3^(1/2)/(b/a)^(1/3)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3
)*cos(f*x+e)-1))-1/3*ln(b+a*cos(f*x+e)^3)/a/f+1/f/b/cos(f*x+e)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.62116, size = 10531, normalized size = 48.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/
27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2
*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) +
1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(
a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))
*a*b*f*cos(f*x + e)*log(1/36*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)
/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2
*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/
3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^
3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 - ((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^
2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f
^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4
*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^
3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f + 4*a^2*b + 5*b^3 + (a^3 + 8*a*b^2)*co
s(f*x + e)) - (((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2)
)/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a
^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt
(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 +
b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/
(a*f))*a*b*f*cos(f*x + e) + 3*sqrt(1/3)*a*b*f*sqrt(-(((-I*sqrt(3) + 1)*(1/(
a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^
2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 +
 b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2
 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^
2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3
) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*
(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 -
2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3)
```

```
+ 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*
(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 +
 36*b^2)/(a^2*b^2*f^2))*cos(f*x + e) - 18*b*cos(f*x + e))*log(1/36*((-I*sqr
t(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/
54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4
 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^
3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/
54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 -
((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f
^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1
/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27
/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f
^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f
+ 4*a^2*b + 5*b^3 - 1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2
 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^
3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*
f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3
*b^4*f^3))^(1/3) + 6/(a*f))*a^2*b^3*f^2 + 18*a*b^3*f)*sqrt(-(((-I*sqrt(3) +
 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^
2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a
^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1
/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^
4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-
I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3)
 + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54
*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a
^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3)
 - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 2
88*a^2 + 36*b^2)/(a^2*b^2*f^2)) - 2*(a^3 + 8*a*b^2)*cos(f*x + e)) - (((-I*s
qrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) +
1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a
^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*
f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) -
1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b*f*cos(f*x
+ e) - 3*sqrt(1/3)*a*b*f*sqrt(-(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b
^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18
*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^
(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3)
 + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4
*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3) + 1)*(1/(a^2*f^2) -
 (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*
f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3
*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/
(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^
```

```
4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 + 36*b^2)/(a^2*b^2*f^2
))*cos(f*x + e) - 18*b*cos(f*x + e))*log(-1/36*((-I*sqrt(3) + 1)*(1/(a^2*f^
2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*
b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/
(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b
^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2
+ b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 + ((-I*sqrt(3) + 1)*(1
/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*
b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2
 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a
^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*
a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f - 4*a^2*b - 5*b^3 -
1/12*sqrt(1/3)*(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2
))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(
a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqr
t(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 +
 b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6
/(a*f))*a^2*b^3*f^2 + 18*a*b^3*f)*sqrt(-(((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (
2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^
3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b
^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a
*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)
/(a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^2*f^2 - 12*((-I*sqrt(3) + 1)*(1/(a
^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2
)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 +
b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2
+ 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2
*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^2*f + 288*a^2 + 36*b^2)/(a^
2*b^2*f^2)) + 2*(a^3 + 8*a*b^2)*cos(f*x + e)) - 36*a)/(a*b*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**3),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{5}}{b\sec\left(fx+e\right)^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/(b*sec(f*x + e)^3 + a), x)
```

3.458 $\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=166

$$\frac{\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}f}-\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}f}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}f}+\frac{\log\left(a\cos\right.}{3}$$

```
[Out] ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/
3)*b^(2/3)*f) - Log[b^(1/3) + a^(1/3)*Cos[e + f*x]]/(3*a^(1/3)*b^(2/3)*f) +
 Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2]/(6*a^
(1/3)*b^(2/3)*f) + Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```

Rubi [A] time = 0.146733, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4138, 1871, 200, 31, 634, 617, 204, 628, 260}

$$\frac{\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}f}-\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}f}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}f}+\frac{\log\left(a\cos\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```


```
[Out] ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/
3)*b^(2/3)*f) - Log[b^(1/3) + a^(1/3)*Cos[e + f*x]]/(3*a^(1/3)*b^(2/3)*f) +
 Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2]/(6*a^
(1/3)*b^(2/3)*f) + Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```


Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx, x, \cos(e+fx)\right)}{3b^{2/3}f} - \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\right.}{3b^{2/3}f}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \text{co}\right.}{6\sqrt[3]{a}b^{2/3}f}$$

$$= -\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b+\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}f} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\text{co}\right.}{6\sqrt[3]{a}b^{2/3}f}$$

Mathematica [C] time = 0.244182, size = 242, normalized size = 1.46

$$\frac{\text{RootSum}\left[\#1^3a-3\#1^2a-\#1^3b-3\#1^2b+3\#1a-3\#1b-a-b\&, \frac{\#1^2a\log\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\#1\right)-\#1^2b\log\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-\#1\right)-4\#1a\,\text{lo}}{}\right.}{3af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (-3*Log[Sec[(e + f*x)/2]^2] + RootSum[-a - b + 3*a*#1 - 3*b*#1 - 3*a*#1^2 -
 3*b*#1^2 + a*#1^3 - b*#1^3 & , (-(a*Log[-#1 + Tan[(e + f*x)/2]^2]) - b*Log
[-#1 + Tan[(e + f*x)/2]^2] - 4*a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1 - 2*b*Log
[-#1 + Tan[(e + f*x)/2]^2]*#1 + a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2 - b*Lo
g[-#1 + Tan[(e + f*x)/2]^2]*#1^2)/(a - b - 2*a*#1 - 2*b*#1 + a*#1^2 - b*#1^
```

```
2) & ])/(3*a*f)
```

Maple [A] time = 0.066, size = 141, normalized size = 0.9

$$-\frac{1}{3fa}\ln\left(\cos\left(fx+e\right)+\sqrt[3]{\frac{b}{a}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}+\frac{1}{6fa}\ln\left(\left(\cos\left(fx+e\right)\right)^{2}-\sqrt[3]{\frac{b}{a}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}-\frac{\sqrt{3}}{3fa}\arctan\left(\frac{\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/3/f/a/(b/a)^(2/3)*ln(cos(f*x+e)+(b/a)^(1/3))+1/6/f/a/(b/a)^(2/3)*ln(cos(
f*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+(b/a)^(2/3))-1/3/f/a/(b/a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*cos(f*x+e)-1))+1/3*ln(b+a*cos(f*x+e)^3)/a/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 72.2434, size = 5415, normalized size = 32.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(6*sqrt(1/3)*a*f*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*
b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(
3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a
```

```
^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2))*arctan(-1/8*(2*sqrt(1/3)*
sqrt((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b
^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)^2 -
4*a*b*cos(f*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3) + 1
)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/
3) - 2/(a*f)) + 4*b^2)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f
^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*b*f^2 + 2*b*f)*sqr
t(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2
)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(
a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*
f))*a*f + 4)/(a^2*f^2)) + sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) +
1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b
^2*f^3 - 8*a*b*f*cos(f*x + e) + 4*b^2*f - 4*(a^2*b*f^2*cos(f*x + e) - a*b^2
*f^2)*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 -
b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f)))*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*
f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^
2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54
*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)))/a) - 6*sq
rt(1/3)*a*f*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) -
1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3
) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3)
)^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2))*arctan(-1/8*(2*sqrt(1/3)*sqrt((3*(I*
sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^
2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)^2 - 4*a*b*cos(f
*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3) + 1)*(-1/54/(a
^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f
)) + 4*b^2)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*
(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*b*f^2 + 2*b*f)*sqrt(((3*(I*sq
rt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*
f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) +
1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4
)/(a^2*f^2)) - sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2
*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^3 - 8*
a*b*f*cos(f*x + e) + 4*b^2*f - 4*(a^2*b*f^2*cos(f*x + e) - a*b^2*f^2)*(3*(I
*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b
^2*f^3))^(1/3) - 2/(a*f)))*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54
/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 +
 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2
)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)))/a) + (3*(I*sqrt(3) +
 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(
1/3) - 2/(a*f))*a*f*log(1/4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b
^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 +
a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + (a^2*b*f*cos(f*x + e) + a*b^2*f)*
(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(
a^3*b^2*f^3))^(1/3) - 2/(a*f)) + b^2) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3
```

```
) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f
 + 6)*log((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^
2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*b^2*f^2 + 4*a^2*cos(f*x + e)
^2 - 4*a*b*cos(f*x + e) - 2*(a^2*b*f*cos(f*x + e) - 2*a*b^2*f)*(3*(I*sqrt(3
) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3)
)^(1/3) - 2/(a*f)) + 4*b^2))/(a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e+fx\right)}{a+b\sec^3\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

Giac [B] time = 1.88372, size = 1077, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*(-1/(a*b^2))^(1/3)*arctan((((a^2*b)^(2/3)*(sqrt(3)*a^2 - sqr
t(3)*a*b) - (sqrt(3)*a^3 - sqrt(3)*a^2*b)*(a^2*b)^(1/3))*cos(f*x + e) + (a^
2*b)^(2/3)*(sqrt(3)*a^2 - sqrt(3)*a*b) - (sqrt(3)*a^2*b - sqrt(3)*a*b^2)*(a
^2*b)^(1/3))/(2*a^3*b - 2*a^2*b^2 + (2*a^3*b - 2*a^2*b^2 - (a^2*b)^(2/3)*(a
^2 - a*b) - (a^3 - a^2*b)*(a^2*b)^(1/3))*cos(f*x + e) - (a^2*b)^(2/3)*(a^2
- a*b) - (a^2*b - a*b^2)*(a^2*b)^(1/3))) - (-1/(a*b^2))^(1/3)*log(144*(2*a^
3*b - 2*a^2*b^2 + (2*a^3*b - 2*a^2*b^2 - (a^2*b)^(2/3)*(a^2 - a*b) - (a^3 -
 a^2*b)*(a^2*b)^(1/3))*cos(f*x + e) - (a^2*b)^(2/3)*(a^2 - a*b) - (a^2*b -
a*b^2)*(a^2*b)^(1/3))^2 + 144*(((a^2*b)^(2/3)*(sqrt(3)*a^2 - sqrt(3)*a*b) -
 (sqrt(3)*a^3 - sqrt(3)*a^2*b)*(a^2*b)^(1/3))*cos(f*x + e) + (a^2*b)^(2/3)*
(sqrt(3)*a^2 - sqrt(3)*a*b) - (sqrt(3)*a^2*b - sqrt(3)*a*b^2)*(a^2*b)^(1/3)
)^2) + 2*(-1/(a*b^2))^(1/3)*log(abs(24*a^3*b - 24*a^2*b^2 + 24*(a^3*b - a^2
*b^2 + (a^2*b)^(2/3)*(a^2 - a*b) + (a^3 - a^2*b)*(a^2*b)^(1/3))*cos(f*x + e
```

```
) + 24*(a^2*b)^(2/3)*(a^2 - a*b) + 24*(a^2*b - a*b^2)*(a^2*b)^(1/3))) - 6*l
og(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/a + 2*log(abs(a + b + 3*a*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) - 3*b*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) + 3*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*b*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - b
*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3))/a)/f
```

3.459 $\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)} dx$

Optimal. Leaf size=23

$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

```
[Out] -Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```

Rubi [A] time = 0.0307096, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4138, 260}

$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

Mathematica [A] time = 0.0186448, size = 23, normalized size = 1.

$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -Log[b + a*Cos[e + f*x]^3]/(3*a*f)
```

Maple [A] time = 0.026, size = 37, normalized size = 1.6

$$-\frac{\ln\left(a+b\left(\sec\left(fx+e\right)\right)^3\right)}{3fa}+\frac{\ln\left(\sec\left(fx+e\right)\right)}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/3/f/a*ln(a+b*sec(f*x+e)^3)+1/f/a*ln(sec(f*x+e))
```

Maxima [A] time = 0.998469, size = 28, normalized size = 1.22

$$-\frac{\log\left(a\cos\left(fx+e\right)^3+b\right)}{3af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] -1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

Fricas [A] time = 0.552398, size = 51, normalized size = 2.22

$$-\frac{\log\left(a\cos\left(fx+e\right)^{3}+b\right)}{3\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] -1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

Sympy [A] time = 82.6005, size = 170, normalized size = 7.39

$$\begin{cases} \frac{\tilde{\infty} x\tan(e)}{\sec^{3}(e)} & \text{for } a=0 \wedge b=0 \wedge \\ \frac{\log\left(\tan^{2}(e+fx)+1\right)}{2af} & \text{for } b=0 \\ \frac{x\tan(e)}{a+b\sec^{3}(e)} & \text{for } f=0 \\ -\frac{1}{3bf\sec^{3}(e+fx)} & \text{for } a=0 \\ -\frac{\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sec(e+fx)\right)}{3af}+\frac{\log\left(\tan^{2}(e+fx)+1\right)}{2af}-\frac{\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\sec(e+fx)+4\sec^{2}(e+fx)\right)}{3af} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Piecewise((zoo*x*tan(e)/sec(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (log(ta
n(e + f*x)**2 + 1)/(2*a*f), Eq(b, 0)), (x*tan(e)/(a + b*sec(e)**3), Eq(f, 0
)), (-1/(3*b*f*sec(e + f*x)**3), Eq(a, 0)), (-log(-(-1)**(1/3)*a**(1/3)*(1/
b)**(1/3) + sec(e + f*x))/(3*a*f) + log(tan(e + f*x)**2 + 1)/(2*a*f) - log(
4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3)*s
ec(e + f*x) + 4*sec(e + f*x)**2)/(3*a*f), True))
```

Giac [B] time = 1.22856, size = 258, normalized size = 11.22

$$\frac{\frac{3\,\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}+1\right)}{a}-\frac{\log\left(\left|a+b+\frac{3\,a\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}-\frac{3\,b\left(\cos\left(fx+e\right)-1\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{3\,b\left(\cos\left(fx+e\right)-1\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a\left(\cos\left(fx+e\right)-1\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{b\left(\cos\left(fx+e\right)-1\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right|\right)}{a}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] 1/3*(3*log(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)/a - log(abs(a + b +
3*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 3*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + 3*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*b*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)
^3 - b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3))/a)/f
```

3.460 $\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=295

$$-\frac{b^2\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)}+\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)}-\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\mathrm{lo}}{3\sqrt[3]{a}}$$

```
[Out] -((b^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/(S
qrt[3]*a^(1/3)*(a^(4/3) + a^(2/3)*b^(2/3) + b^(4/3))*f)) + Log[1 - Cos[e +
f*x]]/(2*(a + b)*f) + Log[1 + Cos[e + f*x]]/(2*(a - b)*f) - ((a^(2/3) + b^(
2/3))*b^(2/3)*Log[b^(1/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)*f
) + ((a^(2/3) + b^(2/3))*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x]
 + a^(2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)*f) - (b^2*Log[b + a*Cos[
e + f*x]^3])/(3*a*(a^2 - b^2)*f)
```

Rubi [A] time = 0.517365, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {4138, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^2\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)}+\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)}-\frac{b^{2/3}\left(a^{2/3}+b^{2/3}\right)\mathrm{lo}}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] -((b^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/(S
qrt[3]*a^(1/3)*(a^(4/3) + a^(2/3)*b^(2/3) + b^(4/3))*f)) + Log[1 - Cos[e +
f*x]]/(2*(a + b)*f) + Log[1 + Cos[e + f*x]]/(2*(a - b)*f) - ((a^(2/3) + b^(
2/3))*b^(2/3)*Log[b^(1/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)*f
) + ((a^(2/3) + b^(2/3))*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x]
 + a^(2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)*f) - (b^2*Log[b + a*Cos[
e + f*x]^3])/(3*a*(a^2 - b^2)*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
```

```
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)(-1+x)} - \frac{1}{2(a-b)(1+x)} - \frac{b(b-ax+bx^2)}{(-a^2+b^2)(b+ax^3)}\right) dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b\,\text{Subst}\left(\int \frac{b-ax+bx^2}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{(a^2-b^2)\,f}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b\,\text{Subst}\left(\int \frac{b-ax}{b+ax^3}\,dx, x, \cos(e+fx)\right)}{(a^2-b^2)\,f} - \frac{b^2\,\text{Subs}}{}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a\,(a^2-b^2)\,f} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{\sqrt[3]{b}}{}\right.}{}$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,(a^2-b^2)\,f} -$$

$$= \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\,(a^2-b^2)\,f} +$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)f} + \frac{\log(1-\cos(e+fx))}{2(a+b)f} + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right.}{}$$

Mathematica [C] time = 0.388984, size = 290, normalized size = 0.98

$$\frac{3\left(a(a-b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right) + a(a+b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right) + b^2\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)\right) - b\text{RootSum}\left[\#1^3a - 6\#1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] (3*(a*(a + b)*Log[Cos[(e + f*x)/2]] + b^2*Log[Sec[(e + f*x)/2]^2] + a*(a -
b)*Log[Sin[(e + f*x)/2]]) - b*RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3 -
```

```
b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 + Ta
n[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 2*a*b*Log[1
 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2
- b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^
2) & ])/(3*a*(a - b)*(a + b)*f)
```

Maple [A] time = 0.081, size = 393, normalized size = 1.3

$$-\frac{b^2}{3f(a-b)(a+b)a}\ln\left(\cos\left(fx+e\right)+\sqrt[3]{\frac{b}{a}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}+\frac{b^2}{6f(a-b)(a+b)a}\ln\left(\left(\cos\left(fx+e\right)\right)^2-\sqrt[3]{\frac{b}{a}}\cos\left(fx+e\right)+\left(\frac{b}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/3/f*b^2/(a-b)/(a+b)/a/(b/a)^(2/3)*ln(cos(f*x+e)+(b/a)^(1/3))+1/6/f*b^2/(
a-b)/(a+b)/a/(b/a)^(2/3)*ln(cos(f*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+(b/a)^(2/3)
)-1/3/f*b^2/(a-b)/(a+b)/a/(b/a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(
1/3)*cos(f*x+e)-1))-1/3/f*b/(a-b)/(a+b)/(b/a)^(1/3)*ln(cos(f*x+e)+(b/a)^(1/
3))+1/6/f*b/(a-b)/(a+b)/(b/a)^(1/3)*ln(cos(f*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+
(b/a)^(2/3))+1/3/f*b/(a-b)/(a+b)*3^(1/2)/(b/a)^(1/3)*arctan(1/3*3^(1/2)*(2/
(b/a)^(1/3)*cos(f*x+e)-1))-1/3/f*b^2/(a-b)/(a+b)/a*ln(b+a*cos(f*x+e)^3)+1/f
/(2*a-2*b)*ln(1+cos(f*x+e))+1/f/(2*a+2*b)*ln(-1+cos(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.76252, size = 12783, normalized size = 43.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")

[Out] -1/36*(2*(a^3 - a*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f ^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54* b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18* b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b ^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^ 3*f - a*b^2*f))*f*log(1/2*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^ 2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^ 2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/ 54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/ 18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^ 3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/ (a^3*f - a*b^2*f))*a*b^2*f - 1/36*(a^4 - a^2*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2* f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5 *f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/ (a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)* (I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 - a*b*cos(f*x + e) + 2*b^2) - ((a^3 - a*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2)) *(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2 *b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/ ((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/ ((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f ^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f + 3*sqrt(1/3)*(a^3 - a*b^2)*f*sqrt(-((a^6 - 2*a^4*b^2 + a^2*b ^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3* f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2* a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2 *b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/ ((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f ^2 - 144*a^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5* f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f)/((a^6 - 2*a^4*b^2 + a^2*b^4)*f ^2)) - 18*b^2)*log(1/2*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f

```
^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 -
 a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*
b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*
b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b
^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^
3*f - a*b^2*f))*a*b^2*f - 1/36*(a^4 - a^2*b^2)*((b^4/(a^3*f - a*b^2*f)^2 +
b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^
3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^
3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^
3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1
/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*
sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 + 1/12*sqrt(1/3)*(a^4 - a^2*b
^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) +
1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*
f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*
a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2
*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/
((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f^2
*sqrt(-((a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^
2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^
4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2
*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2
*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^
5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1
) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 - 144*a^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a
*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3)
+ 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^
3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^
2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a
^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^
2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f
)/((a^6 - 2*a^4*b^2 + a^2*b^4)*f^2)) + 2*a*b*cos(f*x + e) + 2*b^2) - ((a^3
- a*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(
3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*
(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^
2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2
- a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54
*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f)
)*f - 3*sqrt(1/3)*(a^3 - a*b^2)*f*sqrt(-((a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/
(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*
b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*
f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1
/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*
(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^
2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 - 144*a
```

```
^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4
*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18
*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*
b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*
b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/
(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3)
+ 1) + 6*b^2/(a^3*f - a*b^2*f))*f)/((a^6 - 2*a^4*b^2 + a^2*b^4)*f^2)) - 18*
b^2)*log(-1/2*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*
sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*
f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2
 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4
*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) +
 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b
^2*f))*a*b^2*f + 1/36*(a^4 - a^2*b^2)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*
f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*
b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b
^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b
^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(
a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) +
 1) + 6*b^2/(a^3*f - a*b^2*f))^2*f^2 + 1/12*sqrt(1/3)*(a^4 - a^2*b^2)*((b^4
/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/27
*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2
*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(
1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)
*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b
^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f^2*sqrt(-((
a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b
^2*f^2))*(-I*sqrt(3) + 1)/(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f
^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1
/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1
/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a
^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2
/(a^3*f - a*b^2*f))^2*f^2 - 144*a^2*b^2 + 36*b^4 - 12*(a^3*b^2 - a*b^4)*((b
^4/(a^3*f - a*b^2*f)^2 + b^2/(a^4*f^2 - a^2*b^2*f^2))*(-I*sqrt(3) + 1)/(-1/
27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^2)*(a^3*f - a*b
^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 - b^2)^2*a*f^3))
^(1/3) + 9*(-1/27*b^6/(a^3*f - a*b^2*f)^3 - 1/18*b^4/((a^4*f^2 - a^2*b^2*f^
2)*(a^3*f - a*b^2*f)) - 1/54*b^2/(a^5*f^3 - a^3*b^2*f^3) + 1/54*b^2/((a^2 -
 b^2)^2*a*f^3))^(1/3)*(I*sqrt(3) + 1) + 6*b^2/(a^3*f - a*b^2*f))*f)/((a^6 -
 2*a^4*b^2 + a^2*b^4)*f^2)) - 2*a*b*cos(f*x + e) - 2*b^2) - 18*(a^2 + a*b)*
log(1/2*cos(f*x + e) + 1/2) - 18*(a^2 - a*b)*log(-1/2*cos(f*x + e) + 1/2))/
((a^3 - a*b^2)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)}{b\sec\left(fx+e\right)^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(b*sec(f*x + e)^3 + a), x)
```

3.461 $\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

Optimal. Leaf size=393

$$-\frac{b^2\left(2a^2+b^2\right)\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)^2}+\frac{b^{4/3}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)^2}-$$

```
[Out] (b^(4/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[
e + f*x])/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*f) - 1/(4*(a +
 b)*f*(1 - Cos[e + f*x])) - 1/(4*(a - b)*f*(1 + Cos[e + f*x])) - ((2*a + 5*
b)*Log[1 - Cos[e + f*x]])/(4*(a + b)^2*f) - ((2*a - 5*b)*Log[1 + Cos[e + f*
x]])/(4*(a - b)^2*f) - (b^(4/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)^2*f) + (b^(4/3)*(a^2 + 3
*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2
/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*f) - (b^2*(2*a^2 + b^2)*Log[b
 + a*Cos[e + f*x]^3])/(3*a*(a^2 - b^2)^2*f)
```

Rubi [A] time = 0.631667, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4138, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^2\left(2a^2+b^2\right)\log\left(a\cos^3(e+fx)+b\right)}{3af\left(a^2-b^2\right)^2}+\frac{b^{4/3}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}f\left(a^2-b^2\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```


```
[Out] (b^(4/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[
e + f*x])/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*f) - 1/(4*(a +
 b)*f*(1 - Cos[e + f*x])) - 1/(4*(a - b)*f*(1 + Cos[e + f*x])) - ((2*a + 5*
b)*Log[1 - Cos[e + f*x]])/(4*(a + b)^2*f) - ((2*a - 5*b)*Log[1 + Cos[e + f*
x]])/(4*(a - b)^2*f) - (b^(4/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)^2*f) + (b^(4/3)*(a^2 + 3
*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2
/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*f) - (b^2*(2*a^2 + b^2)*Log[b
 + a*Cos[e + f*x]^3])/(3*a*(a^2 - b^2)^2*f)
```

Rule 4138

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(f
*ff^(m + n*p - 1))^(-1), Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x
)^n)^p)/x^(m + n*p), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n},
 x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(b+ax^3)}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{2a+5b}{4(a+b)^2(-1+x)} - \frac{1}{4(a-b)(1+x)^2} + \frac{2a-5b}{4(a-b)^2(1+x)} + \frac{b^2\left(a^2+2b^2-3abx+\left(2a^2+b^2\right)x^2\right)}{\left(a^2-b^2\right)^2\left(b+ax^3\right)}\right.\right.}{f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f}$$

$$= -\frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2f}$$

$$= \frac{b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 f} - \frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1}$$

Mathematica [C] time = 2.11565, size = 336, normalized size = 0.85

$$\frac{8b^2\left((b-a)\text{RootSum}\left[\#1^3a-6\#1^2a-\#1^3b+12\#1a-8a\&, \frac{2\#1^2a^2\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)+\#1^2b^2\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)+8a^2\log\left(-\#1+\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)-6\#1a^2\log\left(\right.}{\#1^2a-\#1^2b-4\#1a+4a}\right.\right.}{a\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

```
[Out] ((-3*Csc[(e + f*x)/2]^2)/(a + b) + (12*(-2*a + 5*b)*Log[Cos[(e + f*x)/2]])/
(a - b)^2 - (12*(2*a + 5*b)*Log[Sin[(e + f*x)/2]])/(a + b)^2 + (8*b^2*(3*(2
*a^2 + b^2)*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 - 6*a
*#1^2 + a*#1^3 - b*#1^3 & , (8*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 4*a*b
*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 6*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*
#1 + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2 + b^2*Log[1 - #1 + Tan[(e
+ f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(a*(a^2 - b^2)^2)
 - (3*Sec[(e + f*x)/2]^2)/(a - b))/(24*f)
```

Maple [B] time = 0.098, size = 676, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x)
```

```
[Out] -1/3/f*b^2/(a-b)^2/(a+b)^2*a/(b/a)^(2/3)*ln(cos(f*x+e)+(b/a)^(1/3))-2/3/f*b
^4/(a-b)^2/(a+b)^2/a/(b/a)^(2/3)*ln(cos(f*x+e)+(b/a)^(1/3))+1/6/f*b^2/(a-b)
^2/(a+b)^2*a/(b/a)^(2/3)*ln(cos(f*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+(b/a)^(2/3)
)+1/3/f*b^4/(a-b)^2/(a+b)^2/a/(b/a)^(2/3)*ln(cos(f*x+e)^2-(b/a)^(1/3)*cos(f
*x+e)+(b/a)^(2/3))-1/3/f*b^2/(a-b)^2/(a+b)^2*a/(b/a)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(b/a)^(1/3)*cos(f*x+e)-1))-2/3/f*b^4/(a-b)^2/(a+b)^2/a/(b/a)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*cos(f*x+e)-1))-1/f*b^3/(a-b
)^2/(a+b)^2/(b/a)^(1/3)*ln(cos(f*x+e)+(b/a)^(1/3))+1/2/f*b^3/(a-b)^2/(a+b)^
2/(b/a)^(1/3)*ln(cos(f*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+(b/a)^(2/3))+1/f*b^3/(
a-b)^2/(a+b)^2*3^(1/2)/(b/a)^(1/3)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*cos(f*
x+e)-1))-2/3/f*b^2/(a-b)^2/(a+b)^2*a*ln(b+a*cos(f*x+e)^3)-1/3/f*b^4/(a-b)^2
/(a+b)^2/a*ln(b+a*cos(f*x+e)^3)-1/f/(4*a-4*b)/(1+cos(f*x+e))-1/2/f/(a-b)^2*
ln(1+cos(f*x+e))*a+5/4/f/(a-b)^2*ln(1+cos(f*x+e))*b+1/f/(4*a+4*b)/(-1+cos(f
*x+e))-1/2/f/(a+b)^2*ln(-1+cos(f*x+e))*a-5/4/f/(a+b)^2*ln(-1+cos(f*x+e))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [C] time = 7.89577, size = 21913, normalized size = 55.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(18*a^4 - 18*a^2*b^2 + 2*((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2 -
 (a^5 - 2*a^3*b^2 + a*b^4)*f)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)
 - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/
(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)
*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*
f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^
2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5
*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*
f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4
)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)
^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f
 + a*b^4*f))*log(1/12*(a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^
2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/
18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f -
2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a
*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*
b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((
a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1
/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b
^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/
(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 + 2*a^2*b^2 + 7*b^4 + 1/6*(a^5 + 16*
a^3*b^2 + 10*a*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*
b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4
/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6
*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27
*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)
*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 +
 a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*
b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))
^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f
```

))*f - (a^3*b + 8*a*b^3)*cos(f*x + e)) - 18*(a^3*b - a*b^3)*cos(f*x + e) +
(36*a^2*b^2 + 18*b^4 - 18*(2*a^2*b^2 + b^4)*cos(f*x + e)^2 - ((a^5 - 2*a^3*
b^2 + a*b^4)*f*cos(f*x + e)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*f)*((b^4/(a^6*f^2
 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f
+ a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^
4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2
)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^
3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)
- 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 +
b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*
b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54
*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*
b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f)) + 3*sqrt(1/3)*((a^5 - 2*a^3*b^2
 + a*b^4)*f*cos(f*x + e)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^4
 + 720*a^2*b^6 - 36*b^8 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b
^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^
5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^
5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2
*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^
4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2
)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) +
1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f
- 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f +
 a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3
) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(2
*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)
 - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/
(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)
*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*
f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^
2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5
*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*
f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4
)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)
^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f
 + a*b^4*f))*f)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*f^2))
)*log(1/12*(a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2
*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt
(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b
^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f
 + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 +
 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^
3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2
*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*
b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a

^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*
a^3*b^2*f + a*b^4*f))^2*f^2 + 2*a^2*b^2 + 7*b^4 + 1/6*(a^5 + 16*a^3*b^2 + 1
0*a*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^
2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 -
 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^
4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2
 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2
- b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^
3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a
^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^
2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*s
qrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f + 1/4*
sqrt(1/3)*((a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2
*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt
(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b
^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f
 + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 +
 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^
3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2
*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*
b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a
^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*
a^3*b^2*f + a*b^4*f))*f^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^
4 + 720*a^2*b^6 - 36*b^8 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*
b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a
^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a
^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b
^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^
2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) +
 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f
+ a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(
3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(
2*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2
) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)
/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4
)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4
f)) - 1/27(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a
^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^
5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2
*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^
4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2
)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*
f + a*b^4*f))*f)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*f^2)

) + 2*(a^3*b + 8*a*b^3)*cos(f*x + e)) + (36*a^2*b^2 + 18*b^4 - 18*(2*a^2*b^
2 + b^4)*cos(f*x + e)^2 - ((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2 - (a^
5 - 2*a^3*b^2 + a*b^4)*f)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (
2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/
54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4
/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f))
- 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 +
8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2
*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2
+ a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/
(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a
f^3))^(1/3)(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a
*b^4*f)) - 3*sqrt(1/3)*((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2 - (a^5 -
 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^4 + 720*a^2*b^6 - 36*b^8 - (a^10 - 4
*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2
+ a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I
*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*
a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*
b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f
)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a
^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^
2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2
*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^
4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f
 - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(2*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b^4
/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a
^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3
 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^
2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5
*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3
))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a
^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b
^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)
^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) -
6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f)/((a^10 - 4*a^8*b^2
+ 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*f^2)))*log(-1/12*(a^6 - 2*a^4*b^2 + a^2*
b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a
^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a
^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^
2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b
^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^
2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) +
 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f
 - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f
+ a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(

3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 2*a^2*b^2 - 7*b^4 - 1/6*(a^5 + 16*a^3*b^2 + 10*a*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f + 1/4*sqrt(1/3)*((a^6 - 2*a^4*b^2 + a^2*b^4)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*f)*sqrt((288*a^4*b^4 + 720*a^2*b^6 - 36*b^8 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))^2*f^2 - 12*(2*a^7*b^2 - 3*a^5*b^4 + a*b^8)*((b^4/(a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2) - (2*a^2*b^2 + b^4)^2/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^2)*(-I*sqrt(3) + 1)/(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3) - 9*(-1/54*b^4/(a^7*f^3 - 2*a^5*b^2*f^3 + a^3*b^4*f^3) + 1/18*(2*a^2*b^2 + b^4)*b^4/((a^6*f^2 - 2*a^4*b^2*f^2 + a^2*b^4*f^2)*(a^5*f - 2*a^3*b^2*f + a*b^4*f)) - 1/27*(2*a^2*b^2 + b^4)^3/(a^5*f - 2*a^3*b^2*f + a*b^4*f)^3 + 1/54*(a^2 + 8*b^2)*b^4/((a^2 - b^2)^4*a*f^3))^(1/3)*(I*sqrt(3) + 1) - 6*(2*a^2*b^2 + b^4)/(a^5*f - 2*a^3*b^2*f + a*b^4*f))*f)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*f^2)) - 2*(a^3*b + 8*a*b^3)*cos(f*x +

```
e)) + 9*(2*a^4 - a^3*b - 8*a^2*b^2 - 5*a*b^3 - (2*a^4 - a^3*b - 8*a^2*b^2 -
 5*a*b^3)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 9*(2*a^4 + a^3*b -
8*a^2*b^2 + 5*a*b^3 - (2*a^4 + a^3*b - 8*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)
*log(-1/2*cos(f*x + e) + 1/2))/((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e)^2
- (a^5 - 2*a^3*b^2 + a*b^4)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^3}{b\sec\left(fx+e\right)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(b*sec(f*x + e)^3 + a), x)
```

3.462 $\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left((d\tan(e+fx))^m \left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

Rubi [A] time = 0.060413, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx=\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx$$

Mathematica [A] time = 3.25057, size = 0, normalized size = 0.

$$\int \left(a+b(c\sec(e+fx))^n\right)^p (d\tan(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

Maple [A] time = 2.434, size = 0, normalized size = 0.

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\tan\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*(d*tan(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

3.463 $\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^5(e+fx)\,dx$

Optimal. Leaf size=226

$$\frac{\sec^4(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{n},-p,\frac{n+4}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)}{4f}-\frac{\sec^2(e+f}{}$$

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p))) - (Hypergeometric2F1[2/n, -p,
 (2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f
*x])^n)^p)/(f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p) + (Hypergeometric2F1[4/n, -
p, (4 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^4*(a + b*(c*Sec[e +
 f*x])^n)^p)/(4*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rubi [A] time = 0.5245, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4139, 6742, 367, 12, 266, 65, 365, 364}

$$\frac{\sec^4(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{4}{n},-p;\frac{n+4}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{4f}-\frac{\sec^2(e+fx)\left(a+b(c\sec(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```


```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p))) - (Hypergeometric2F1[2/n, -p,
 (2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f
*x])^n)^p)/(f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p) + (Hypergeometric2F1[4/n, -
p, (4 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^4*(a + b*(c*Sec[e +
 f*x])^n)^p)/(4*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```


Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
```

```
[2*n, p])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^2(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+b(cx)^n)^p}{x} - 2x\,(a+b(cx)^n)^p + x^3\,(a+b(cx)^n)^p\right)dx, x, \sec(e+fx\right.}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x^3\,(a+b(cx)^n)^p\,dx, x, \text{s}\right.}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf} + \frac{\text{Subst}\left(\int \frac{x^3(a+bx^n)^p}{c^3}\,dx, x, c\sec(e\right.}{cf}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x^3\,(a+bx^n)^p\,dx, x, c\sec\right.}{c^4 f}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn} + \frac{\left(\left(a+b(c\sec(e+fx))^n\right)^p\left(1+\frac{b(}{}\right.\right.}{}$$
$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)\left(a+b(c\sec(e+fx))^n\right)^{1+p}}{afn(1+p)} - \frac{{}_2F_1\left(\frac{2}{n}\right.}{}$$

Mathematica [A] time = 6.89216, size = 221, normalized size = 0.98

$$\frac{\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{4\left(\frac{a\left(c\sqrt{\sec^2(e+fx)}\right)^{-n}}{b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{a\left(c\sqrt{\sec^2(e+fx)}\right)^{-n}}{b}\right)}{np} + \sec^2(e+fx)\left(\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a} + \right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

```
[Out] ((a + b*(c*Sec[e + f*x])^n)^p*((4*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*(
c*Sqrt[Sec[e + f*x]^2])^n))])/(n*p*(1 + a/(b*(c*Sqrt[Sec[e + f*x]^2])^n))^p
) + (Sec[e + f*x]^2*(-4*Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*Sqrt[
Sec[e + f*x]^2])^n)/a)] + Hypergeometric2F1[4/n, -p, (4 + n)/n, -((b*(c*Sqr
```

```
t[Sec[e + f*x]^2])^n)/a)]*Sec[e + f*x]^2))/(1 + (b*(c*Sqrt[Sec[e + f*x]^2])
^n)/a)^p))/(4*f)
```

Maple [F] time = 0.573, size = 0, normalized size = 0.

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\left(\tan\left(fx+e\right)\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

3.464 $\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^3(e+fx)\,dx$

Optimal. Leaf size=143

$$\frac{\sec^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)}{2f}+\frac{\left(a+b(c\right.}{}$$

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*(c
*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)) + (Hypergeometric2F1[2/n, -p, (2
 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f*x]
)^n)^p)/(2*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rubi [A] time = 0.293915, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4139, 6742, 367, 12, 266, 65, 365, 364}

$$\frac{\sec^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{2}{n},-p;\frac{n+2}{n};-\frac{b(c\sec(e+fx))^n}{a}\right)}{2f}+\frac{\left(a+b(c\sec(e+fx))^n\right)^{p+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*(c
*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)) + (Hypergeometric2F1[2/n, -p, (2
 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f*x]
)^n)^p)/(2*f*(1 + (b*(c*Sec[e + f*x])^n)/a)^p)
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b(cx)^n)^p}{x} + x\,(a+b(cx)^n)^p\right) dx, x, \sec(e+fx)\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x\,(a+b(cx)^n)^p\,dx, x\right.}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf} + \frac{\text{Subst}\left(\int \frac{x(a+bx^n)^p}{c}\,dx, x, c\sec(\right.}{cf}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x\,(a+bx^n)^p\,dx, x, c\,\text{s}\right.}{c^2 f}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn} + \frac{\left(\left(a+b(c\sec(e+fx))^n\right)^p \left(1+\right.\right.}{}$$

$$= \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)\left(a+b(c\sec(e+fx))^n\right)^{1+p}}{afn(1+p)} + \frac{{}_2F_1\left(\frac{2}{n}\right.}{}$$

Mathematica [A] time = 4.12472, size = 171, normalized size = 1.2

$$\frac{\left(a+b(c\sec(e+fx))^n\right)^p \left(\sec^2(e+fx)\left(\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right) - \frac{2\left(a\right.}{}\right.}{2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

```
[Out] ((a + b*(c*Sec[e + f*x])^n)^p*((-2*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b*
(c*Sqrt[Sec[e + f*x]^2])^n))])/(n*p*(1 + a/(b*(c*Sqrt[Sec[e + f*x]^2])^n))^
p) + (Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n
)/a)]*Sec[e + f*x]^2)/(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)^p))/(2*f)
```

Maple [F] time = 0.546, size = 0, normalized size = 0.

$$\int \left(a + b\left(c\sec\left(fx + e\right)\right)^{n}\right)^{p} \left(\tan\left(fx + e\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx + e\right)\right)^{n} b + a\right)^{p} \tan\left(fx + e\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c\sec\left(fx + e\right)\right)^{n} b + a\right)^{p} \tan\left(fx + e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n} b+a\right)^{p} \tan\left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

3.465 $\int \left(a+b(c\sec(e+fx))^n\right)^p \tan(e+fx)\,dx$

Optimal. Leaf size=59

$$-\frac{\left(a+b(c\sec(e+fx))^n\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Rubi [A] time = 0.0769761, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4139, 367, 12, 266, 65}

$$-\frac{\left(a+b(c\sec(e+fx))^n\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+b(c\sec(e+fx))^n\right)^p \tan(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b(cx)^n)^p}{x}\,dx, x, \sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{cf} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, c\sec(e+fx)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, (c\sec(e+fx))^n\right)}{fn} \\
&= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b(c\sec(e+fx))^n}{a}\right)\left(a+b(c\sec(e+fx))^n\right)^{1+p}}{afn(1+p)}
\end{aligned}$$

Mathematica [A] time = 0.0895108, size = 59, normalized size = 1.

$$-\frac{\left(a+b(c\sec(e+fx))^n\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

Maple [F] time = 0.553, size = 0, normalized size = 0.

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\tan\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\left(c\sec\left(e+fx\right)\right)^{n}\right)^{p}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e),x)
```

```
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

3.466 $\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0443204, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]

[Out] Defer[Int][Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 3.82342, size = 0, normalized size = 0.

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]

[Out] Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [A] time = 0.584, size = 0, normalized size = 0.

$$\int \cot\left(fx+e\right)\left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\cot\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

3.467 $\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0560734, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 2.34648, size = 0, normalized size = 0.

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [A] time = 0.537, size = 0, normalized size = 0.

$$\int \left(\cot \left(f x+e\right)\right)^{3} \left(a+b \left(c \sec \left(f x+e\right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \sec \left(f x+e\right)\right)^{n} b+a\right)^{p} \cot \left(f x+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c \sec \left(f x+e\right)\right)^{n} b+a\right)^{p} \cot \left(f x+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*(c*sec(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \sec \left(fx + e \right) \right)^n b + a \right)^p \cot \left(fx + e \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

3.468 $\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\tan^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Rubi [A] time = 0.055236, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
```

```
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx=\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx$$

Mathematica [A] time = 1.94417, size = 0, normalized size = 0.

$$\int \left(a+b(c\sec(e+fx))^n\right)^p \tan^2(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
```

```
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

Maple [A] time = 0.493, size = 0, normalized size = 0.

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p}\left(\tan\left(fx+e\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^{n}b+a\right)^{p}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\left(c\sec\left(e+fx\right)\right)^{n}\right)^{p}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**2,x)
```

```
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \sec \left(fx+e\right)\right)^{n} b + a\right)^{p} \tan \left(fx+e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

3.469 $\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0152094, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 1.90101, size = 0, normalized size = 0.

$$\int \left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Integrate[(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [A] time = 0.484, size = 0, normalized size = 0.

$$\int \left(a+b\left(c\sec\left(fx+e\right)\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*sec(f*x+e))^n)^p,x)
```

```
[Out] int((a+b*(c*sec(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n} b+a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\left(c\sec\left(fx+e\right)\right)^{n} b+a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\left(c\sec\left(e+fx\right)\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))**n)**p,x)
```

```
[Out] Integral((a + b*(c*sec(e + f*x))**n)**p, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c\sec\left(fx+e\right)\right)^{n} b + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

3.470 $\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p,x\right)$$

```
[Out] Unintegrable[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi [A] time = 0.0553758, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Defer[Int][Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx=\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Mathematica [A] time = 1.87544, size = 0, normalized size = 0.

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

```
[Out] Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

Maple [A] time = 0.541, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{2} \left(a+b \left(c \sec \left(fx+e\right)\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

```
[Out] int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \sec \left(fx+e\right)\right)^{n} b+a\right)^{p} \cot \left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(c \sec \left(fx+e\right)\right)^{n} b+a\right)^{p} \cot \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

```
[Out] integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*(c*sec(f*x+e))**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(c \sec \left(fx + e \right) \right)^n b + a \right)^p \cot \left(fx + e \right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

```
[Out] integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
    ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```